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1/f Frequency Noise Effects on Self-Heterodyne Linewidth Measurements for Coherent Communications

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LEXINGTON, MASSACHUSETTS

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FOR THE COMMANDER

Hugh L. Southall

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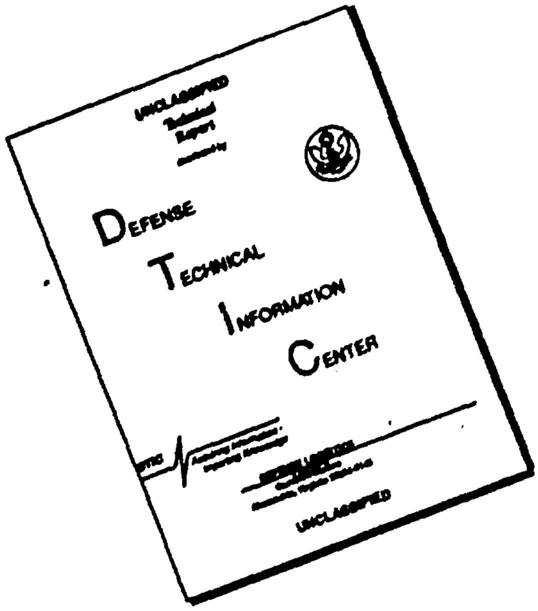

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1/f FREQUENCY NOISE EFFECTS ON SELF-HETERODYNE LINEWIDTH MEASUREMENTS FOR COHERENT COMMUNICATIONS

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Group 67

TECHNICAL REPORT 881

31 JULY 1990

LEXINGTON MASSACHUSETTS

ABSTRACT

The effects of 1/f frequency noise on self-heterodyne detection are described and the results are applied to the problem of laser-diode-linewidth measurement. Laser diode linewidths determined by self-heterodyne methods are not adequate predictors of coherent communications system performance because these measurements often include significant broadening due to 1/f frequency noise. In this report, the autocorrelation function and power spectrum of the detected self-heterodyne photocurrent are developed in terms of an arbitrary frequency noise spectrum and then evaluated for both the white and the 1/f components of the frequency noise. From numerical analysis, the power spectrum resulting from the 1/f frequency noise is shown to be approximately Gaussian and an empirical expression is given for its linewidth. These results are applied to the problem of self-heterodyne linewidth measurements for coherent optical communications, and the amount of broadening due to 1/f frequency noise is predicted. Two methods are then provided for estimating the portion of the measured self-heterodyne linewidth due to the white component of the frequency noise and the portion due to 1/f frequency noise.

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1. INTRODUCTION

Laser diode linewidths measured using delayed self-heterodyne detection often include significant broadening due to 1/f frequency noise. This complicates the evaluation of lasers for use in coherent communications systems because the linewidths determined by self-heterodyne methods will not be adequate predictors of performance. Coherent system performance is strongly dependent on the Lorentzian component of the laser lineshape resulting from white frequency noise, and is much less dependent on the component resulting from 1/f frequency noise. Most treatments of delayed self-heterodyne detection have assumed laser phase noise solely of quantum origin leading to a Lorentzian lineshape [1-4]. Recently, a few investigators have noted the broadening effect of 1/f frequency noise on the laser diode lineshape as measured by the self-heterodyne technique [5-8]. The performance of high-data-rate coherent communication systems is limited mostly by the white component of the frequency noise because the phase or frequency fluctuations resulting from 1/f frequency noise are very small over a symbol interval [8]. The 1/f frequency noise may impose a requirement for frequency tracking to keep the center frequency from drifting out of the receiver front-end bandwidth; however, this requirement is met fairly easily with automatic frequency tracking circuits [9].

This report describes the effects of 1/f frequency noise on self-heterodyne detection by developing the autocorrelation function and power spectrum of the photocurrent in terms of an arbitrary frequency noise spectrum, and then evaluating them for both the white and the 1/f components of the frequency noise. These results are then applied to the problem of self-heterodyne linewidth measurement for high-data-rate heterodyne communications. Long delays are required for these measurements to provide adequate resolution; however, these delays result in considerable broadening of the self-heterodyne lineshape due to 1/f. To overcome this, two methods are presented for estimating the white component of the frequency noise from the measured lineshape.

2. PHASE NOISE MODEL

The starting point of the analysis presented here is based on previous work which treated only white frequency noise, so some details are omitted [1]. The optical field from a single longitudinal mode semiconductor laser can be modeled as a quasi-monochromatic field with random phase fluctuations leading to broadening of the spectral line:

$$E(t) = E_o \exp j\left[\omega_o t + \phi(t)\right] \quad . \tag{1}$$

The analysis presented here assumes that the amplitude of the optical field is constant. The phase noise spectrum S_ϕ is the power spectrum of the phase fluctuations, and the frequency noise spectrum $S_{\dot{\phi}}$ is the power spectrum of the frequency fluctuations. The mean square phase fluctuation

$$\langle \Delta\varphi^2(\tau)\rangle = \langle[\phi(t+\tau) - \phi(t)]^2\rangle \tag{2}$$

is related to the frequency noise spectrum by [10]

$$\langle \Delta\phi^2(\tau)\rangle = \frac{2}{\pi}\int_{-\infty}^{\infty} \sin^2\left(\frac{\omega\tau}{2}\right) S_{\dot{\phi}}(\omega)\frac{d\omega}{\omega^2} \quad . \tag{3}$$

where < > denotes a time average. The two-sided frequency noise spectrum can be modeled at frequencies below the relaxation oscillation frequency, as power-dependent white noise and power-independent 1/f noise

$$S_{\dot{\phi}}(\omega) = S_o + \frac{k}{|\omega|} \quad . \tag{4}$$

Many treatments include only the white noise, even though a wide variety of semiconductor lasers have been shown to exhibit substantial levels of 1/f noise in addition to the white noise [11-17]. For all equations in this paper, S_o and k are expressed in units of $(rad/s)^2/Hz$ and $(rad/s)^3/Hz$, respectively. S_o and k are usually measured single-sided in units of Hz^2/Hz and Hz^3/Hz, so values given here are in these units. (To convert from measured values to the units used in the equations, the conversion factors are $(2\pi)^2/2$ and $(2\pi)^3/2$ for S_o and k, respectively.)

The actual frequency noise spectrum also has a resonance peak at the relaxation oscillation frequency and drops off above that frequency. The resonance frequency is typically well above 2 GHz, so from (3) it is clear that neglecting these features only affects $<\Delta\phi^2(\tau)>$ for τ in the vicinity of the reciprocal of the relaxation oscillation frequency and smaller. The effects of the relaxation oscillation on the frequency noise spectrum may be important for systems where the symbol rate approaches the relaxation oscillation frequency; however, these effects are neglected in this report.

The component of the phase jitter $\Delta\phi(\tau)$ due to the white frequency noise is commonly assumed to be a zero mean, stationary, Gaussian random process. The component of the phase jitter due to the 1/f frequency noise is also stationary and exhibits Gaussian statistics even though the phase and frequency noise $\phi(\tau)$ and $\dot{\phi}(\tau)$ due to the 1/f frequency noise are not stationary [17]. Under these conditions

$$\langle \exp[\pm j\Delta\phi(t,\tau)]\rangle = \exp\left[-\frac{1}{2}\langle\Delta\phi^2(\tau)\rangle\right] \quad . \tag{5}$$

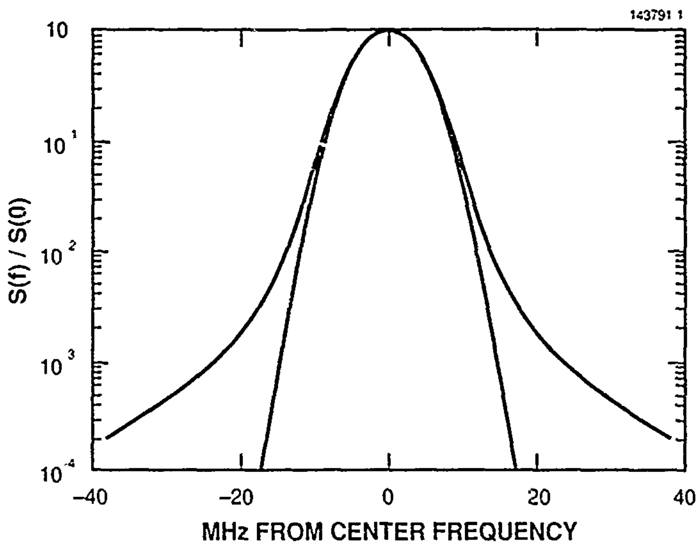

Figure 1. Field spectrum due to 1/f frequency noise only (upper curve) compared with Gaussian approximation (lower curve). $k = 2 \times 10^{12}$ Hz^2, and the observation time is 1 ms.

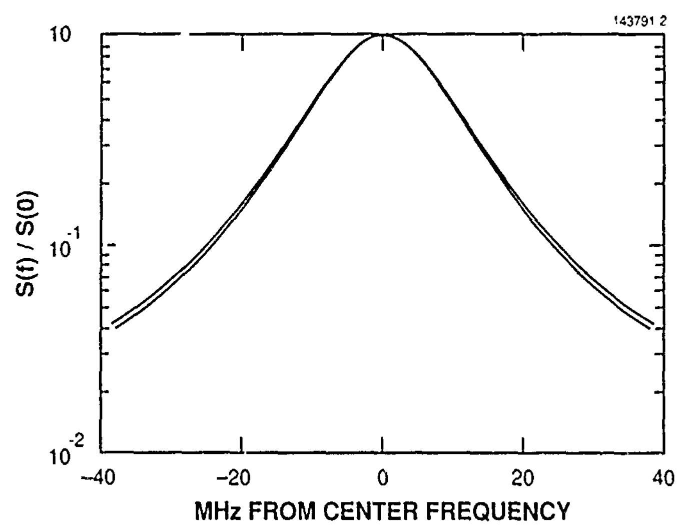

Figure 2. Field spectrum for a laser with a 7-MHz natural linewidth and $k = 2 \times 10^{12}$ Hz^2. The lower curve is the result obtained using a Gaussian approximation for the field spectrum due to 1/f frequency noise.

By Fourier transforming the field correlation function $<E^*(t)\ E(t+\tau)>$, where * indicates the complex conjugate, the laser field spectrum is found. The resulting spectrum is the convolution of the Lorentzian spectrum associated with the white frequency noise and the approximately Gaussian spectrum arising from the 1/f noise. The result of convolving a Lorentzian spectrum and a Gaussian spectrum is known as a Voigt spectrum. The Voigt profile is used extensively in investigations of radiative transfer in the upper atmosphere. In this area of study, the Voigt profile results from independent Lorentzian and Doppler broadening (Doppler broadening leads to a Gaussian lineshape). The Voigt function is not available in an analytic form; however, many useful approximations are available. The accuracy of using a Voigt profile to model the laser diode lineshape is limited by the approximations used in modeling the spectrum due to 1/f frequency noise with a Gaussian.

The linewidth of the Lorentzian part of the lineshape taken by itself is $S_o/2\pi$ Hz FWHM (full width half maximum), and the linewidth of the approximately Gaussian part is roughly $\{4\ \ln(2)\ (k/2\pi^3)\ [1 + \ln(T_{obs}\sqrt{k/2\pi^3})]\}^{1/2}$ Hz FWHM when observed for T_{obs} seconds and $T_{obs}\sqrt{k/2\pi^3} >> 1$ [17] [k is the level of the 1/f noise as in (4)]. Figure 1 shows the lineshape due to the 1/f part of the frequency noise compared with the Gaussian approximation which is only good near line center; however, about 90 percent of the power in the lineshape is contained within the region where the error in the Gaussian approximation is less than 10 percent. Hence, the lineshape which results from convolving the Gaussian approximation with a Lorentzian differs very little from the lineshape calculated without the approximation. The lower curve in Figure 2 is the lineshape due to combined white and 1/f frequency noise components and using the Gaussian approximation, and the upper curve is the same field spectrum computed without the approximation. As can be seen, the approximation is a valid approach to computing the complete lineshape except in unusual cases where the 1/f noise dominates a large part of the frequency noise spectrum. Thus, the complete laser diode lineshape is well approximated by a Voigt profile for common levels of 1/f noise.

3. DETECTED FIELD MODEL AND PHOTOCURRENT AUTOCORRELATION FUNCTION

Following the development of [1], the detected field for the delayed self-heterodyne linewidth measurement setup as shown in Figure 3 is the sum of a laser field and a time-delayed and frequency-shifted image of itself,

$$E_T = E(t) + \alpha E(t + \tau_o)\exp(j\Omega t) \tag{6}$$

and α is a real factor which accounts for the amplitude ratio between the two fields. The time delay between the two fields is τ_o, and Ω is the mean frequency difference between the two fields. Assuming stationary fields, the autocorrelation of the photocurrent depends only on the intensity correlation function of the detected total field $G_{E_T}^{(2)}(\tau)$

$$R_I(\tau) = e\eta G_{E_T}^{(2)}(0)\delta(t) + \eta^2 G_{E_T}^{(2)}(\tau) \tag{7}$$

where e is the electronic charge, η is the detector sensitivity, $\delta(t)$ is the Dirac delta function, and the optical intensity correlation function is

$$G_{E_T}^{(2)}(\tau) = \langle E_T(t)E_T^*(t)E_T(t+\tau)E_T^*(t+\tau)\rangle \quad . \tag{8}$$

The first term in (7) is the shot noise associated with the dc component of the photocurrent.

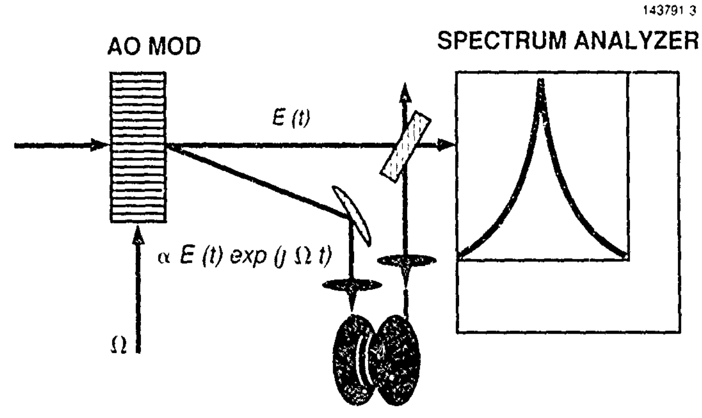

Figure 3 Delayed self-heterodyne linewidth measurement setup.

4. AUTOCORRELATION FUNCTION AND SPECTRUM FOR WHITE FREQUENCY NOISE

The autocorrelation function is found by substituting (6) into (8). This results in 16 terms, 10 of which average out to zero. By using (5), the result as given in [1] is

$$G_{E_T}^{(2)}(\tau) = E_o^4\left[\left(1+\alpha^2\right)^2 + 2\alpha^2 \cos \Omega\tau \exp(A)\right]$$

where

$$A = -\langle \Delta\phi^2(\tau_o)\rangle - \langle \Delta\phi^2(\tau)\rangle + \frac{1}{2}\langle \Delta\phi^2(\tau-\tau_o)\rangle + \frac{1}{2}\langle \Delta\phi^2(\tau+\tau_o)\rangle \quad . \tag{9}$$

By substituting (3) into (9) and combining the integrals, the intensity correlation function can be written in terms of the frequency noise spectrum

$$G_{E_T}^{(2)}(\tau) = E_o^4\left\{\left(1+\alpha^2\right)^2 + 2\alpha^2 \cos\Omega\tau \exp\left[-\frac{4}{\pi}\int_{-\infty}^{\infty} \sin^2\left(\frac{\omega\tau}{2}\right)\sin^2\left(\frac{\omega\tau_o}{2}\right)\frac{S_\phi(\omega)}{\omega^2}d\omega\right]\right\} \quad . \tag{10}$$

This agrees with Kikuchi [7]. At this point it is useful to observe that the second term in this autocorrelation is similar to the field correlation for a single laser with the addition of a sine-squared term which acts as a filter on the frequency noise spectrum. For small delay times, this sine-squared term effectively filters out most of the 1/f component of the frequency noise spectrum.

For the white part of the frequency noise spectrum, the integral in (10) is easily evaluated and yields

$$G_{E_T}^{(2)}(\tau) = E_o^4\left[\left(1+\alpha^2\right)^2 + 2\alpha^2 \cos\Omega\tau \exp\begin{pmatrix} -S_o|\tau| & \text{for } |\tau| < \tau_o \\ -S_o\,\tau_o & \text{for } |\tau| > \tau_o \end{pmatrix}\right] \quad \text{for } S_\phi = S_o \quad . \tag{11}$$

By using the Wiener-Khintchine theorem, the power spectral density resulting from the white component is the Fourier transform of (11) [1,2]

$$S(\omega) = E_o^4\left[\left(1+\alpha^2\right)^2 \delta(\omega) + 2\alpha^2 \exp(-S_o\tau_o)\delta(\omega-\Omega) + 2\alpha^2 \times \frac{2S_o}{(S_o)^2+(\omega-\Omega)^2}\right.$$
$$\left.\times\left\{1-\exp(-S_o\tau_o)\left[\cos(\omega-\Omega)\tau_o + \frac{S_o}{\omega-\Omega}\sin(\omega-\Omega)\tau_o\right]\right\}\right] \quad . \tag{12}$$

In the limit of large delay times the component of the spectrum due to the white frequency noise becomes exactly Lorentzian, with width equal to twice the individual laser linewidth. For delay times which are comparable to or shorter than the coherence time, the quasi-Lorentzian part is broadened and scalloped and power is shifted into the delta function at the modulation frequency. If the delay is much shorter than the coherence time, the spectrum consists solely of delta functions at the modulation frequency and at dc.

5. AUTOCORRELATION FUNCTION AND SPECTRUM FOR 1/f FREQUENCY NOISE

For the 1/f part of the frequency noise spectrum evaluation of the integral in (10) requires use of the identity

$$\frac{\sin \pi a x}{\pi x} = \int_{-(a/2)}^{a/2} \exp(\pm j 2\pi x t) dt \tag{13}$$

yielding [18]

$$G_{E_T}^{(2)}(\tau) = E_o^4 \left\{ \left(1+\alpha^2\right)^2 + 2\alpha^2 \cos\Omega\tau \left[\left(\tau+\tau_o\right)^{-k(\tau+\tau_o)^2/2\pi} \times \left(\left|\tau-\tau_o\right|\right)^{-k(\tau-\tau_o)^2/2\pi} \tau^{k\tau^2/\pi} \tau_o^{k\tau_o^2/\pi} \right] \right\} \quad \text{for } S_{\dot{\phi}} = \frac{k}{|\omega|} \quad . \tag{14}$$

Figure 4 is a plot of the bracketed expression in (14) for $k = 3 \times 10^{12}$ Hz^2, showing the dependence of the autocorrelation, and thus the frequency spectrum, on the delay time. For times much greater than the delay time, the slope of the log of the autocorrelation as plotted in Figure 4 is $-k\tau_o^2/\pi$.

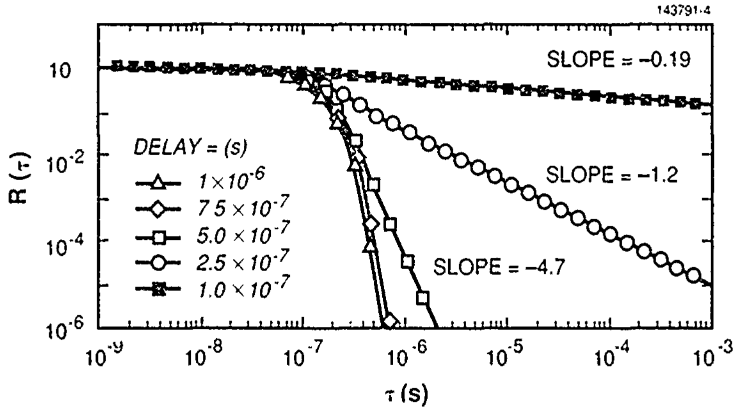

Figure 4. Autocorrelation function for 1/f noise only; $k = 3 \times 10^{12}$ Hz^2.

The power spectrum for the 1/f component has been obtained by fast Fourier transforming the autocorrelation function. The result is shown in Figure 5 for several delay times and $k = 2 \times 10^{12}$ Hz^2. (Note that this result neglects the dc component and the shot noise.)

The linewidth of the self-heterodyne spectrum due to 1/f noise alone is shown in Figure 6 as a function of the delay for several reasonable values of k.

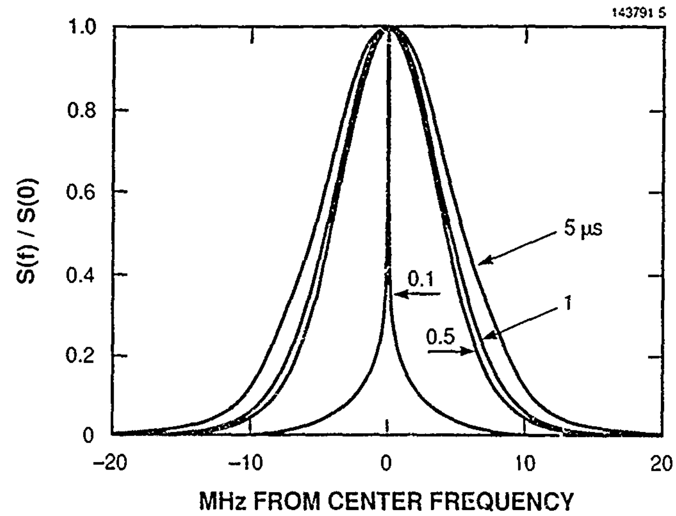

Figure 5. Normalized self-heterodyne lineshape due to 1/f alone for $k = 2 \times 10^{12}$ Hz^2 *and delay times = 0.1, 0.5, 1, and 5 μs.*

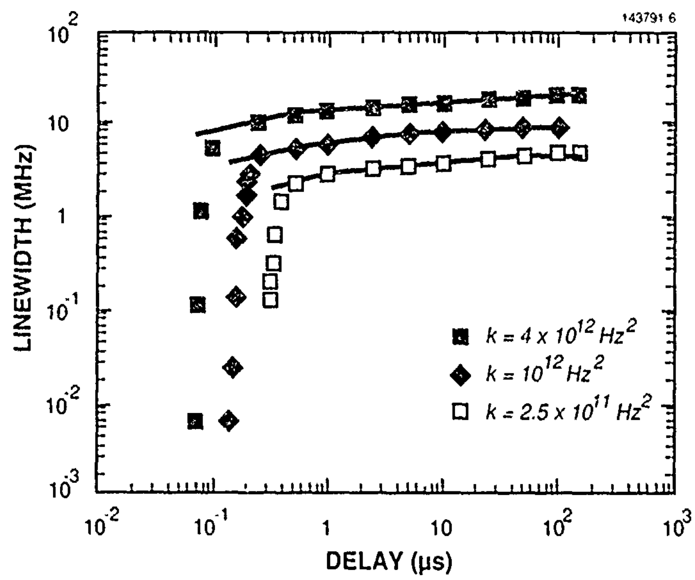

Figure 6. Self-heterodyne linewidth due to 1/f noise only.

For the extreme case of short delays such that $k\tau_o^2/\pi << 1$, the autocorrelation function is composed of two parts: a narrow spike and a dc component. Figure 7 shows the autocorrelation for cases such that $k\tau_o^2/\pi$ is equal to 10^{-1} and 10^{-2}. The appearance of the autocorrelation is markedly different from Figure 4 because of the difference in the delay and the linear scaling. The narrow spike produces a broad base in the power spectrum. The dc component corresponds to a narrow spike in the power spectrum at the modulation frequency and, as $k\tau_o^2/\pi$ becomes much less than 1, the power is shifted completely into the spike (which becomes a delta function in the limit of small $k\tau_o^2/\pi$).

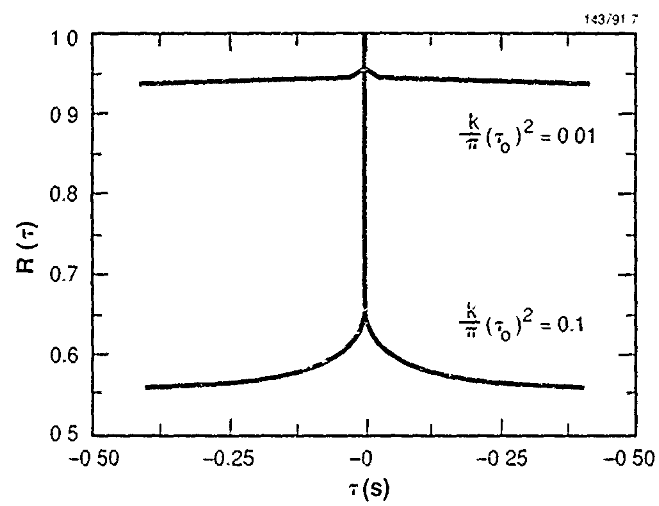

Figure 7. Autocorrelation function for 1/f frequency noise only ($k = 2 \times 10^{12}$ Hz^2 for both curves, $\tau_o = 6.4$ ns for the upper curve and 20 ns for the lower curve).

The contribution of the 1/f laser frequency noise to the laser field spectrum is approximately Gaussian, so it was expected that the contribution to the self-heterodyne spectrum would also be approximately Gaussian. Figure 8 shows the 1/f lineshape and, for comparison, a Gaussian and Lorentzian with the same width at half maximum. The 1/f lineshape is very close to the Gaussian near the center of the line, but is much stronger than the Gaussian farther away from the line center. This lineshape does not drop off as fast as a Gaussian, but it has much less power in the wings than a Lorentzian. The Gaussian component of the lineshape has nearly the same lineshape and $\sqrt{2}$ times the width of the field spectrum of the individual laser for cases where $k\tau_o^2/\pi >> 1$.

A Gaussian approximation for the autocorrelation was derived empirically using (14). For $k\tau_o^2/\pi >> 1$,

$$G_{E_T}^{(2)}(\tau) \approx E_o^4 \left\{ \left(1+\alpha^2\right)^2 + 2\alpha^2 \cos\Omega\tau \exp\left[-\tau^2 \frac{k}{\pi} \left(4.3 + 2.1 \ln \frac{4.3\, k\tau_o^{2.1}}{\pi} \right) \right] \right\} \quad \text{for } S_\phi = \frac{k}{|\omega|} \quad . \ (15)$$

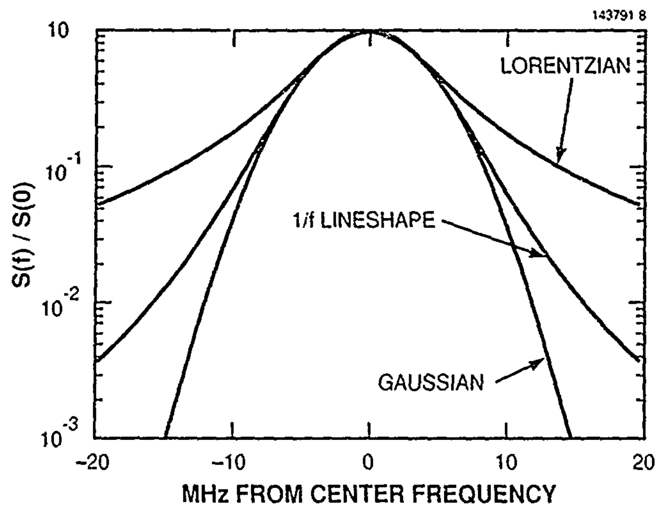

Figure 8. Normalized self-heterodyne lineshape due to 1/f alone compared with Lorentzian and Gaussian curves with the same 3-dB width (shown for comparison) ($k = 2 \times 10^{12}$ Hz^{2}, delay = 1 μs)

The power spectral density resulting from this approximation is also Gaussian. The linewidth predicted from this approximation is

$$G_{lw} = \frac{1}{\pi}\sqrt{\frac{2k \ln 2}{\pi}\left(4.3 + \ln\frac{4.3\, k\tau_o^{2.1}}{\pi}\right)} \text{ Hz FWHM} \quad . \tag{16}$$

This approximation is plotted in Figure 6 as a solid line. Good accuracy is obtained for large delays. For fixed delay τ_o, the Gaussian linewidth is roughly proportional to $\sqrt{k}$. This $\sqrt{k}$ dependence leads to the $\sqrt{2}$ relationship between the Gaussian component of the field spectrum and the Gaussian component of the self-heterodyne lineshape.

Note that the linewidth due to the 1/f component is not independent of the delay even for large delays. This is due to the very low frequencies present in the 1/f noise. The analysis presented here is general in that it is valid for any delay. Results of any analysis which assumes mutual incoherence of the two optical fields (such as [19]) should be used with caution because, of course, the strong low-frequency components of the 1/f noise are correlated even for very long delays.

6. AUTOCORRELATION FUNCTION AND SPECTRUM FOR COMBINED WHITE AND 1/f FREQUENCY NOISE

For the combined white and 1/f frequency noise actually present in most semiconductor lasers, the autocorrelation function is simply a combination of the results already given.

$$G_{E_T}^{(2)}(\tau) = E_o^4\left[\left(1+\alpha^2\right)^2 + 2\alpha\cos(\Omega\tau)LG\right]$$

where

$$L = \exp\begin{Bmatrix} -S_o|\tau| & \text{for } |\tau| \le \tau_o \\ -S_o\,\tau_o & \text{for } |\tau| \ge \tau_o \end{Bmatrix}$$

and

$$G = \left(\tau+\tau_o\right)^{-k(\tau-\tau_o)^2/2\pi}\left(\left|\tau-\tau_o\right|\right)^{-k(\tau-\tau_o)^2/2\pi}\ \tau^{k\tau_o^2/\pi}\ \tau_o{}^{k\tau_o^2/\pi} \quad \text{for } S_\phi = S_o + \frac{k}{|\omega|} \quad . \tag{17}$$

The power spectral density for the combined white and 1/f frequency noise can be evaluated by finding the FFT of (17). Figures 9 and 10 are plots of the linewidth which would be observed using self-heterodyne detection for lasers with natural linewidths of 10 and 2 MHz, respectively. The component of the linewidth due to the 1/f frequency noise by itself was shown in Figure 6 for the same delays and k values used here.

The linewidth due to combined white and 1/f frequency noise can be approximated for long delay times using the Gaussian approximation given earlier. The relationship between the complete Voigt lineshape and the Gaussian and Lorentzian lineshapes is approximated by the expression

$$\alpha_V = \frac{1}{2}\left(1.0692\,\alpha_L + \sqrt{0.86639\,\alpha_L^2 + 4\,\alpha_G^2}\right) \tag{18}$$

where α_V, α_G, and α_L are the Voigt, Gaussian, and Lorentzian linewidths, respectively. If the 1/f lineshape were exactly Gaussian, the error in (18) would be less than about 0.01 percent [20]. The linewidths predicted using these approximations are plotted as solid lines in Figures 9 and 10. The error seen in this approximation is due to the slightly non-Gaussian lineshape resulting from the 1/f component of the frequency noise.

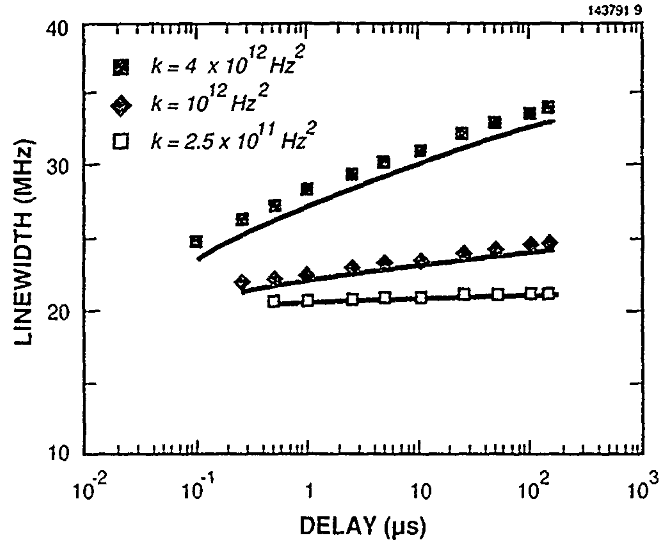

Figure 9. Self-heterodyne linewidth vs delay for $So/2\pi$ = 10 MHz.

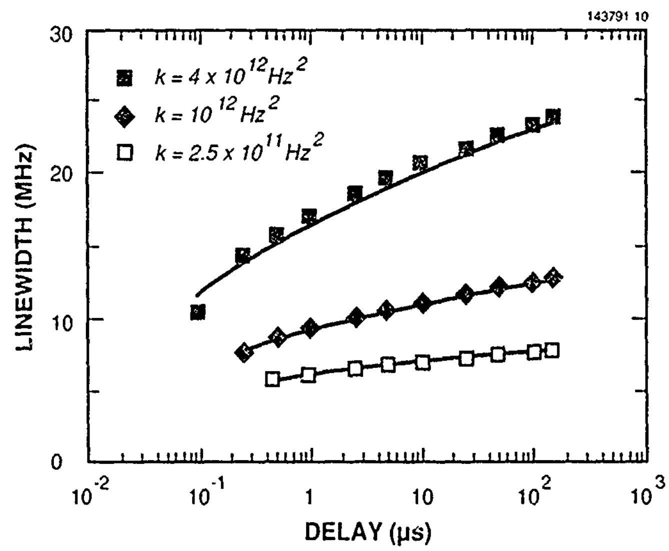

Figure 10 Self-heterodyne linewidth vs delay for $So/2\pi$ = 2 MHz.

7. EFFECTS OF 1/f ON LINEWIDTH MEASUREMENTS

The broadening of the delayed self-heterodyne lineshape resulting from 1/f frequency noise has particular significance in the context of linewidth measurements for high-data-rate coherent communications systems. Since the performance of these systems is largely unaffected by 1/f noise, it is desirable to know the natural linewidth which would result from the white component of the frequency noise alone. In general, the linewidths estimated from self-heterodyne measurements are simply one-half of the FWHM of the self-heterodyne lineshape. From Figures 9 and 10 it is obvious that this linewidth is often substantially larger than the Lorentzian or natural component of the linewidth.

As first observed by Kikuchi and Okoshi, the broadening of the self-heterodyne linewidth by 1/f frequency noise can lead to a residual linewidth at high powers [5-8]. The natural linewidth, as predicted by the Schawlow-Townes formula with the broadening factors introduced by Henry [21], varies inversely with laser output power in the main longitudinal mode. The 1/f frequency noise is normally constant with power [17]. Thus, 1/f noise produces a residual linewidth at the high power limit and also causes a finite intercept when self-heterodyne linewidths are plotted against inverse power. For Figure 11 the linewidth which would be measured by the self-heterodyne technique with a 5-μs delay is plotted for a constant level of 1/f noise ($k = 10^{12}$ Hz^2) and a varying natural linewidth. This simulates a plot of linewidth vs inverse power. For small natural linewidths a residual is observed and a line fitted to all of the data is seen to intercept the dependent axis at 3.0 MHz. The residual linewidth in the limit of high power is simply the 1/f self-heterodyne linewidth divided by 2. For the case in Figure 11, the residual linewidth would be 3.9 MHz.

This procedure was repeated for a range of 1/f noise levels and several different delay times, and the intercepts which resulted are plotted in Figure 12. The line fits were produced using a range of five Lorentzian linewidths from 2 to 25 MHz. The intercepts are dependent on the range of points fitted and so should be seen as approximate values only.

If the 1/f noise level has been measured independently or if an estimate of the level is available, the Gaussian portion of the self-heterodyne lineshape can be estimated using (16). An estimate of the Lorentzian component is then easily found using (18) or the other approximations found in [20].

In some circumstances, direct measurement of the natural linewidth may be possible by tailoring the length of the delay used for the self-heterodyne measurement. For instance, if it were known from independent measurements that $k < 2.5 \times 10^{11}$ Hz^2, then choosing a delay of 0.3 μs would make the 1/f contribution to the measured linewidth negligible, as can be seen from Figure 6. In this case, the smallest Lorentzian linewidth which could be measured directly would be about 5 MHz [2]. This approach of tailoring the delay to reduce effects of 1/f noise has limited value because many semiconductor lasers have $k > 10^{12}$ Hz^2 leading to selection of a delay shorter than 0.15 μs. This delay would only allow direct measurement of Lorentzian linewidths greater than about 10 MHz.

The broadening effect of the 1/f frequency noise is most pronounced near the center of the self-heterodyne lineshape. If signal and noise levels permit, a more accurate estimate of the Lorentzian part of the linewidth of the laser diode under test can be obtained from the width 10 or 20 dB down from the maximum. Figures 13 and 14 show that the linewidths determined from the width 20 dB down are much

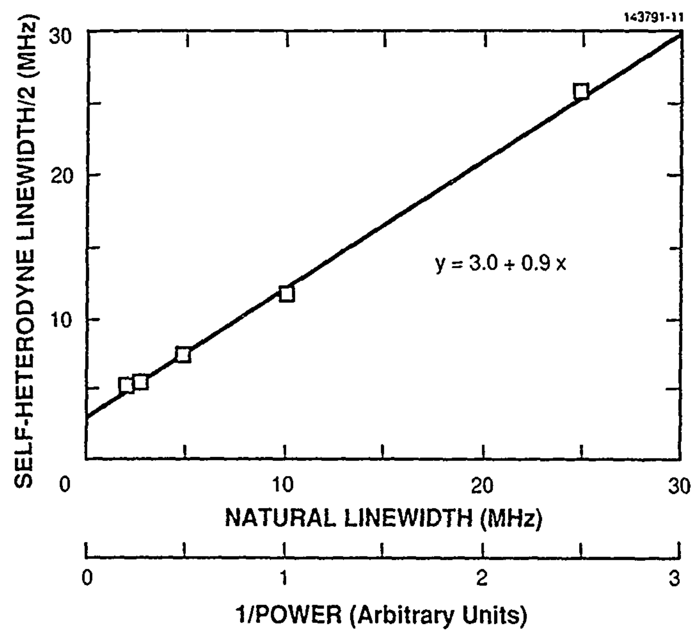

Figure 11. Self-heterodyne linewidth/2 vs natural linewidth showing finite intercept due to 1/f noise ($k = 10^{12}$ Hz^2, delay = 5 μs).

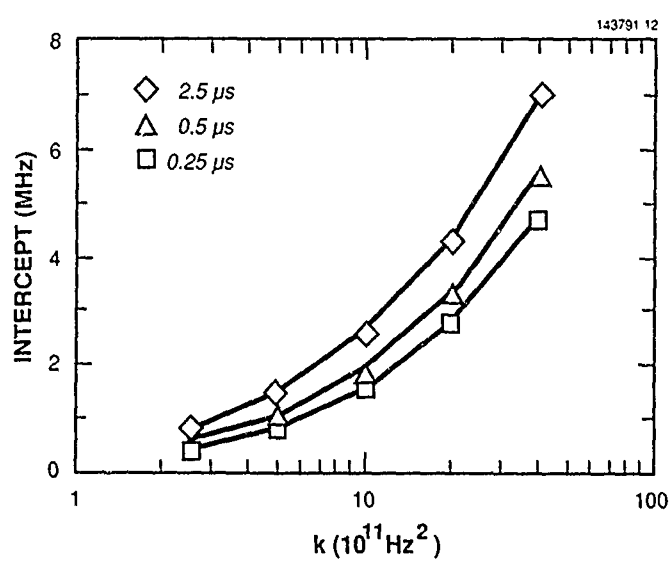

Figure 12. Infinite power intercept due to 1/f frequency noise.

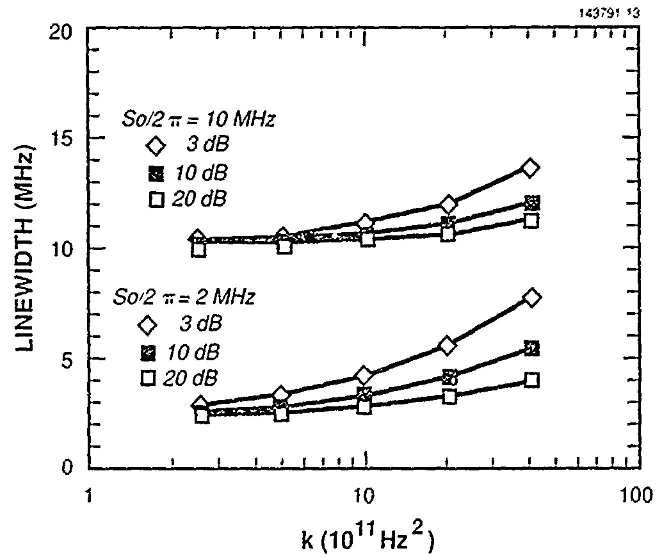

Figure 13. Linewidth vs k determined from 3-, 10-, and 20-dB widths of the self-heterodyne lineshape for 2- and 10-MHz Lorentzian linewidths. Delay = 0 5 μs.

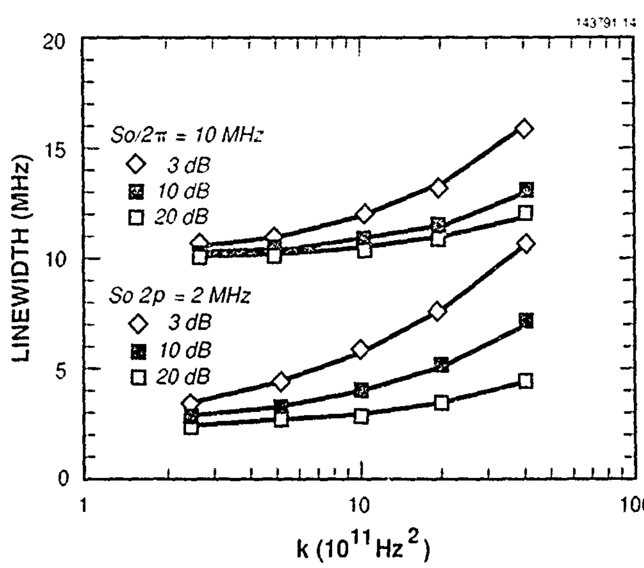

Figure 14. Linewidth vs k determined from 3-, 10-, and 20-dB widths of the self-heterodyne lineshape for 2- and 10-MHz Lorentzian linewidths. Delay = 50 μs.

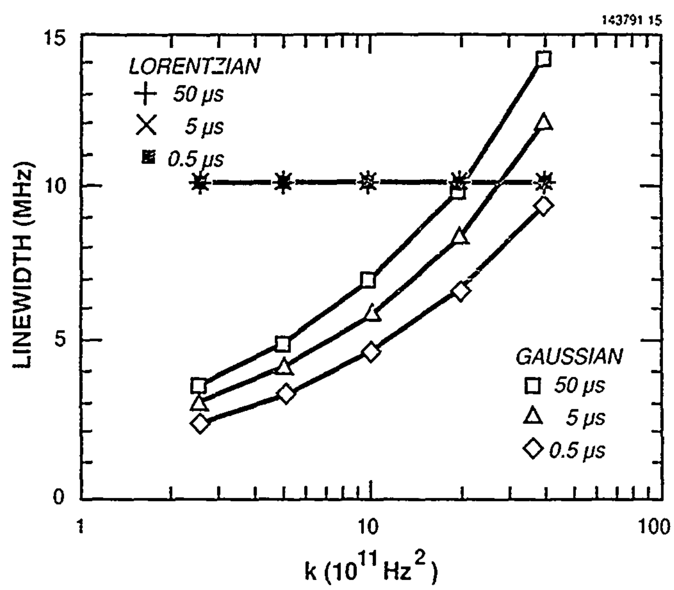

Figure 15 Lorentzian and Gaussian components estimated from Voigt fitting of self-heterodyne lineshape for So/2π = 10 MHz and delays of 0.5, 5, and 50 μs.

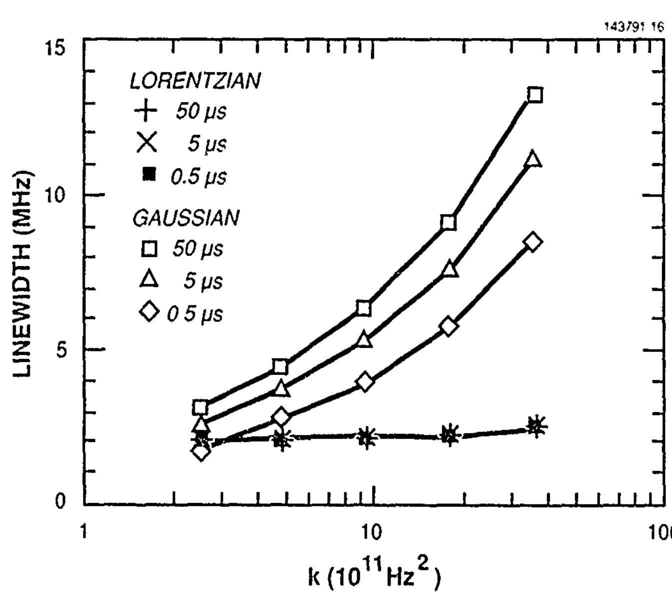

Figure 16. Lorentzian and Gaussian components estimated from Voigt fitting of self-heterodyne lineshape for So/2π = 2 MHz and delays of 0.5, 5, and 50 μs.

closer to the Lorentzian linewidth. The linewidths were determined from the 3-, 10-, and 20-dB widths of numerically generated lineshapes and simply assuming a Lorentzian lineshape. (The 3-dB linewidth is the self-heterodyne linewidth divided by 2; the 10-dB linewidth is the self-heterodyne linewidth 10 dB down divided by 6; the 20-dB linewidth is the self-heterodyne linewidth 20 dB down divided by $2\sqrt{99}$.)

Good estimates of the Lorentzian and Gaussian components can be obtained by fitting a Voigt profile to the measured self-heterodyne lineshape. Efficient and accurate algorithms are available for estimating the Voigt lineshape for any combination of Lorentzian and Gaussian contributions [22-26]. Some of the approximations for the Voigt profile also provide derivatives with respect to all of the parameters allowing application of nonlinear least-squares fitting procedures. These procedures use the Voigt function and derivatives for each point in the lineshape and iterate until a satisfactory level of accuracy is obtained.

An alternative method of using the Voigt approximations to estimate the Gaussian and Lorentzian parts of a measured self-heterodyne lineshape is given here. This approach requires fewer computations than the nonlinear least-squares fitting procedures. The Lorentzian component is initially estimated from the 20-dB linewidth by simply assuming a Lorentzian lineshape. Then, the Gaussian component is estimated by using the 3-dB linewidth and (18). Use of this estimate of the Gaussian component allows a new estimate of the Lorentzian component to be found by using a Voigt approximation and iterating to find the Lorentzian value required to produce a 20-dB width equal to the measured width. The new Lorentzian value is then used to refine the estimate of the Gaussian component, and these steps are repeated until the estimates converge. Convergence is fairly rapid because the 20-dB linewidth is dominated more by the Lorentzian contribution, while the 3-dB linewidth is strongly affected by the Gaussian component.

To test its accuracy over a wide range of inputs, this Voigt fitting procedure was applied to self-heterodyne lineshapes computed using the results presented here (see Figures 15 and 16). The estimates of the Lorentzian component are very close to the correct value, with a small error appearing for the 2-MHz case where the Gaussian component begins to be much larger than the Lorentzian component. The Voigt approximation used here was the one found in [25].

This Voigt fitting procedure was also applied to actual self-heterodyne lineshapes. The setup used a 500-m fiber delay line providing 2.5 μs of delay. Figure 17 shows a measured lineshape and the Voigt lineshape resulting from the fit. The laser for this test was a 30-mW Hitachi HL8314E.

Figures 18 and 19 show the results of applying the Voigt fitting procedure to a typical set of linewidth vs power measurements. A Spectra Diode SDL-5410-C 100-mW laser diode was used for these tests. The 3- and 20-dB width data used to produce the fits were derived from the lineshape assuming a Lorentzian shape as discussed above. The linewidth components plotted in Figure 19 are for the individual laser. The Lorentzian linewidth is one-half of the Lorentzian component of the self-heterodyne lineshape, and the Gaussian component is $\sqrt{2}/2$ times the Gaussian component of the full

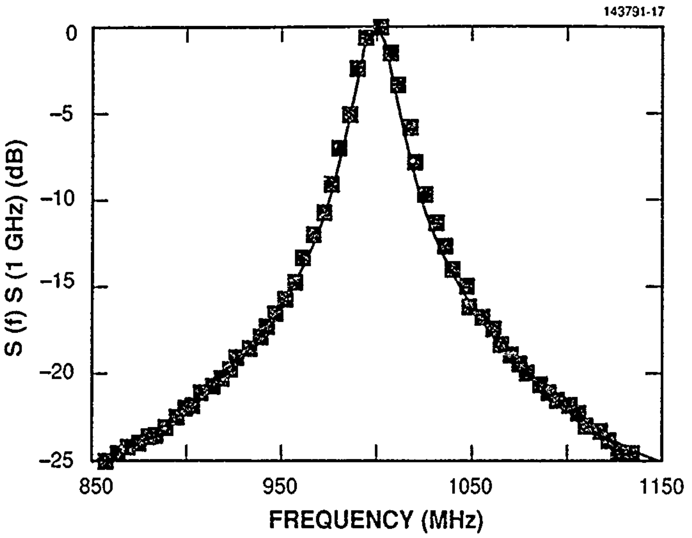

Figure 17. Measured self-heterodyne lineshape and Voigt fit; 3-dB linewidth = 10.2 MHz, 20-dB linewidth = 8.0 MHz, Voigt linewidth = 12.1 MHz, Lorentzian linewidth = 7.2 MHz, Gaussian linewidth = 7.6 MHz.

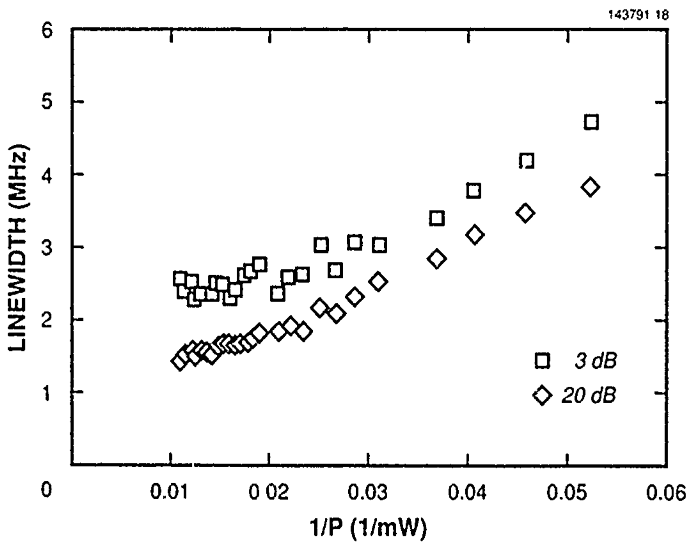

Figure 18. Measured 3- and 20-dB linewidths vs 1/power.

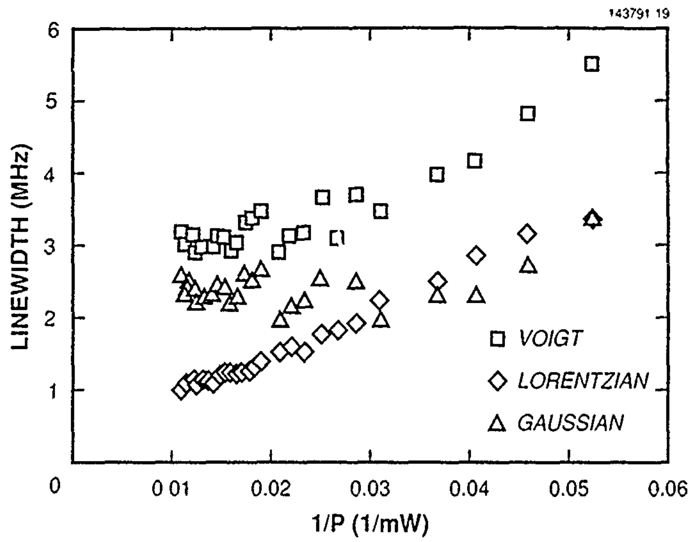

Figure 19. Voigt, Lorentzian, and Gaussian linewidths vs 1/power, determined using the Voigt fitting procedure.

self-heterodyne lineshape. The true width of the individual laser line is the Voigt linewidth calculated from the components using (18). Notice that the 3-dB linewidth, which is the commonly used result from this measurement, underestimates the full linewidth of the laser and overestimates the Lorentzian component which is the most important to coherent communications performance. The Voigt linewidth is larger because the Gaussian components add in quadrature, as noted earlier. It is important to note again that the Gaussian linewidth measured with the self-heterodyne setup is dependent on the length of the delay; so, to be meaningful, any Gaussian linewidths quoted for this measurement must be accompanied by the value of the delay.

8. CONCLUSIONS

The autocorrelation function and power spectrum resulting from delayed self-heterodyne detection were developed in terms of an arbitrary frequency noise spectrum. These expressions were then evaluated for both the white and 1/f components of the laser frequency noise spectrum. The autocorrelation function for the 1/f frequency noise was given and numerical results were presented for the associated power spectrum. This power spectrum, due to 1/f frequency noise alone, was approximately Gaussian and an empirically derived expression was given for its width. Numerical results were also provided for the power spectrum resulting from the combined effect of 1/f and white frequency noise.

High-data-rate heterodyne communications system performance is limited by the white part of the frequency noise without much effect from the 1/f noise. For this reason, the contribution of the 1/f noise to the self-heterodyne linewidth measurement is undesirable. The 1/f frequency noise leads to a residual linewidth at high power and a finite intercept when linewidth measurements are plotted vs inverse power. This intercept could range from 0.5 to 7 MHz for reasonable conditions. If an estimate of the level of 1/f noise level is available, the approximations provided herein can be used to estimate the portion of the self-heterodyne linewidth due to the white portion of the frequency noise. For certain limited circumstances, it was shown that a careful choice of the self-heterodyne delay allows measurement of the natural Lorentzian lineshape without broadening due to 1/f noise.

For more general circumstances, two methods of estimating the Lorentzian component from the self-heterodyne lineshape were presented. The first was simply to deduce the laser linewidth from the width of the lineshape 10 or 20 dB down from the peak where the effects of the 1/f noise are not as great. This method performs well when the Gaussian contribution is much less than the Lorentzian contribution. The second method used approximations for the Voigt profile and searched for the Lorentzian and Gaussian components needed to match the 3- and 20-dB widths of the Voigt profile to the data. This method extracts the Lorentzian component well even for cases where the Gaussian component is large. More accurate extraction of the Voigt components may be possible by nonlinear least-squares fitting procedures, and this possibility deserves further investigation. Any Voigt fitting procedures will ultimately be limited by the slightly non-Gaussian character of the 1/f contribution to the self-heterodyne lineshape. The lineshape fitting techniques are dependent on the assumption that the frequency noise spectrum is composed of only 1/f and white components.

The results presented show that the linewidth measured for a particular laser can depend substantially upon the delay used for the measurement and the level of the 1/f noise. It is suggested that all linewidths reported in the literature from self-heterodyne measurements should be accompanied by information on the length of the delay used and, if possible, an estimate of the 1/f frequency noise level of the laser.

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The effects of 1/f frequency noise on self-heterodyne detection are described and the results are applied to the problem of laser-diode-linewidth measurement. Laser diode linewidths determined by self-heterodyne methods are not adequate predictors of coherent communications system performance because these measurements often include significant broadening due to 1/f frequency noise. In this report, the autocorrelation function and power spectrum of the detected self-heterodyne photocurrent are developed in terms of an arbitrary frequency noise spectrum and then evaluated for both the white and the 1/f components of the frequency noise. From numerical analysis, the power spectrum resulting from the 1/f frequency noise is shown to be approximately Gaussian and an empirical expression is given for its linewidth. These results are applied to the problem of self-heterodyne linewidth measurements for coherent optical communications, and the amount of broadening due to 1/f frequency noise is predicted. Two methods are then provided for estimating the portion of the measured self-heterodyne linewidth due to the white component of the frequency noise and the portion due to 1/f frequency noise.

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