Eur. Phys. J. C (2018) 78:452
https://doi.org/10.1140/epjc/s10052-018-5933-7

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

# (3 + 1)-Dimensional topologically massive 2-form gauge theory: geometrical superfield approach

**R. Kumar**[1,a], **Debmalya Mukhopadhyay**[2,3,b]

[1] Department of Physics and Astrophysics, University of Delhi, New Delhi 110007, India
[2] Department of Theoretical Physics, Indian Association for the Cultivation of Science, 2A and B Raja S.C. Mullick Road, Jadavpur, Kolkata 700032, India
[3] Present address: Variable Energy Cyclotron Centre (VECC), 1/AF, Bidhannagar, Kolkata, West Bengal 700064, India

Received: 17 October 2017 / Accepted: 27 May 2018 / Published online: 5 June 2018


**Abstract** We derive the complete set of off-shell nilpotent and absolutely anticommuting Becchi–Rouet–Stora–Tyutin (BRST) and anti-BRST symmetry transformations corresponding to the combined "scalar" and "vector" gauge symmetry transformations for the $(3+1)$-dimensional (4D) topologically massive non-Abelian $(B \wedge F)$ theory with the help of geometrical superfield formalism. For this purpose, we use *three* horizontality conditions (HCs). The first HC produces the (anti-)BRST transformations for the 1-form gauge field and corresponding (anti-)ghost fields whereas the second HC yields the (anti-)BRST transformations for 2-form field and associated (anti-)ghost fields. The integrability of second HC produces third HC. The latter HC produces the (anti-)BRST symmetry transformations for the compensating auxiliary vector field and corresponding ghosts. We obtain *five* (anti-)BRST invariant Curci–Ferrari (CF)-type conditions which emerge very naturally as the off-shoots of superfield formalism. Out of five CF-type conditions, two are fermionic in nature. These CF-type conditions play a decisive role in providing the absolute anticommutativity of the (anti-)BRST transformations and also responsible for the derivation of coupled but equivalent (anti-)BRST invariant Lagrangian densities. Furthermore, we capture the (anti-)BRST invariance of the coupled Lagrangian densities in terms of the superfields and translation generators along the Grassmannian directions $\theta$ and $\bar{\theta}$.

## 1 Introduction

Every $p$-form ($p = 1, 2, 3, \ldots$) gauge theory remains invariant under a global symmetry known as BRST symmetry when we include the gauge-fixing term and Faddeev–Popov ghosts in the theory [1–5]. The physical significance of the BRST symmetry is to provide the unitarity in various interactions under consideration [6]. The BRST symmetry transformation of the fields is generated by the BRST charge $Q_b$ which is nilpotent $\left(Q_b^2 = 0\right)$. The nilpotency leads to the formation of BRST cohomology where the physical state $|phys\rangle$, defined by $Q_b|phys\rangle = 0$, is equivalent to another physical state $|phys\rangle'$ if $|phys\rangle' = |phys\rangle + Q_b|phys\rangle$. From this equivalence, we can identify the unphysical modes of a state (in the total quantum Hilbert space of states) whose contributions are mutually cancelled in a physical process [6]. Consequently, the unitarity is achieved in a given physical process.

The BRST formalism is one of the most elegant and mathematically rich methods to covariantly quantize any arbitrary $p$-form (non-)Abelian gauge theory. For a given *classical* gauge symmetry, we have two linearly independent global supersymmetric type *quantum* BRST and anti-BRST symmetries. The latter symmetries are nilpotent of order two (i.e. $s_b^2 = 0, \; s_{ab}^2 = 0$) and absolutely anticommuting (i.e. $s_b s_{ab} + s_{ab} s_b = 0$) in nature [7,8]. The absolute anticommutativity property of the (anti-)BRST transformations for the non-Abelian 1-form gauge theory and higher form ($p \geq 2$) (non-)Abelian gauge theories is satisfied due to the existence of Curci–Ferrari (CF)-type conditions [7,9–11]. Furthermore, the CF-type conditions also play an important role in the derivation of coupled (but equivalent) Lagrangian densities. These CF-type conditions emerge automatically within the framework of superfield formalism [11–13]. The emergence of CF-type of condition(s) is one of the characteristic features of a $p$-form (non-)Abelian gauge theory within the framework of superfield approach to BRST formalism.

Bonora–Tonin superfield approach to BRST formalism is a geometrical method to derive the proper off-shell nilpo-

[a] e-mail: raviphynuc@gmail.com
[b] e-mail: debphys.qft@gmail.com

tent and absolutely anticommuting (anti-)BRST symmetry transformations for a given gauge theory [12,14,15]. In this formalism, we generalize an ordinary D-dimensional Minkowskian space to the (D, 2)-dimensional superspace with the help of a pair of Grassmannian coordinates $(\theta, \bar{\theta})$ (with $\theta^2 = \bar{\theta}^2 = 0,\ \theta\bar{\theta} + \bar{\theta}\theta = 0$) in addition to the ordinary bosonic coordinates $x^\mu$ ($\mu = 0, 1, 2, 3, \ldots, D-1$). Further, we generalize the dynamical fields to their corresponding superfields onto the (D, 2)-dimensional supermanifold. By exploiting the power and strength of celebrated horizontality condition (HC) [12,14–23], we obtain the desired (anti-)BRST symmetry transformations. The HC implies that the components of super curvature along the Grassmannian directions are zero. Physically, the HC condition demands that the gauge-invariant quantities should be independent of the Grassmannian coordinates. In other words, the gauge-invariant quantities should not be affected by the presence of Grassmannian variables when they are generalized on the supermanifold.

The HC carries a very important physical significance in the gauge theories. Since, Faddeev–Popov–DeWitt ghost fields belongs to L(G), where $L(G)$ is a set of left-invariant one-forms being always isomorphic to tangent space at identity on group manifold $T_e(G)$ [24], then HC implies that the equivalent representation of gauge field is always connected to the identity i.e. there is no anomaly due to BRST transformation in gauge theory without the matter fields [24]. This conclusion can be similarly drawn from HC for Kalb–Ramond field. As a consequence, there is no anomaly of color current if the topologically massive model is applied in QCD without matter fields. Anomaly may be present when the model contains massless fermions with suitable action. In that case, Wess and Zumino found a consistent condition to be obeyed if quantum action of matter content gauge theory is not gauge invariant [1,24–26]. In that case, HC leads to provide Wess–Zumino consistency condition for anomaly or Stora–Zumino chain of descent equations [22].

In recent years, the "augmented" superfield formalism (which is an extended version of Bonora–Tonin superfield approach) has been extensively used for the interacting gauge theories such as 1-form gauge theory interacts with Dirac's fields and complex scalar fields [27], gauge-invariant Proca theory [28], gauge-invariant massive 2-form theory [29] and references therein. In this approach, in addition to the HC, the conserved currents and/or gauge-invariant restrictions play very important role in the derivation of the complete set of (anti-)BRST transformations.

During last few decades, the antisymmetric Kalb–Ramond field $B_{\mu\nu}(= -B_{\nu\mu})$ of rank two became quite popular because of its relevance in the context of (super-)string theories [30,31], (super-)gravity theories [32], dual description of a massless scalar field [33,34] and noncommutative theories [35]. It has been shown, within the framework of BRST formalism, that the 4D free Abelian 2-form gauge theory provides a tractable field-theoretic model for Hodge theory where de Rham cohomological operators of differential geometry and Hodge duality operation find their physical realizations in terms of the continuous and discrete symmetries, respectively [36]. Furthermore, it has also shown to be a quasi-topological field theory (q-TFT) which captures some features of Witten-type TFT and some aspects of Schwartz-type TFT [37].

The 2-form antisymmetric gauge field also plays an important role in the mass generation of the vector gauge bosons through a well-known topological $(B \wedge F)$ term [38–44]. In this model, the mass of gauge bosons and gauge-invariance co-exist together. The phenomenological aspects of this model have been discussed in [47] which shed light on the various kind of physical processes that are allowed by the standard model of particle physics. We have also studied the 4D (non-)Abelian topologically massive theory within the framework of BRST formalism [48,49]. In earlier work [10], the 4D non-Abelian topologically massive gauge theory has been studied in the context of superfield formalism where the "scalar" and "vector" gauge symmetries have been treated separately. As a consequence, the (anti-)BRST symmetry transformations corresponding to the above gauge symmetries are found to be off-shell nilpotent and absolutely anticommuting. We point out that when we combine the (anti-)BRST transformations corresponding to the scalar and vector gauge transformations, the resulting (anti-)BRST transformations are found to be off-shell nilpotent but they do not obey the absolute anticommutativity property. In our present investigation, we shall investigate this issue and derive the proper (anti-)BRST symmetries for the combined scalar and vector gauge transformations.

We know that pure Yang–Mills (YM) theory [50] obeys unitarity where the 1-form gauge field is taken to be massless. Due to having mass, the 1-form gauge field has a physical longitudinal mode. But the scattering among the longitudinal modes shows the violation of unitarity in tree level scattering processes [51,52]. We know this happens when we consider the tree level $2 \rightarrow 2$ scatterings of longitudinally polarized massive gauge bosons ($W^\pm$ and $Z^0$) in electroweak sector of the standard model excluding the process mediated by Higgs particle [53,54]. These are the Higgs mediated processes which save the unitarity of the scattering process among the longitudinally polarized electroweak bosons. In the Higgs mechanism [55,56], global symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken to the electromagnetic $U(1)$ group [57–60]. But we will consider the 4D dynamical $(B \wedge F)$ theory where mass of gauge boson is generated and keeping the global $SU(N)$ symmetry unbroken.

Using the geometric features of a gauge theory [16,61–63], we obtain *proper* (anti-)BRST transformations for all the fields. But it is not guaranteed whether the BRST charge keeps its nilpotency after quantum corrections. The assurance of unitarity at every order of quantum correction comes from the renormalizability of model. Pure YM theory containing massless 1-form gauge field is an example where the BRST symmetry and renormalizibility are maintained simultaneously. The mass generation of YM field (keeping global symmetry unbroken) shows unsatisfactory characteristics in quantum field theory. For example, non-Abelian Stückelberg model is found to be non-renormalizable [64–68] but it obeys unitarity. On the other hand, Curci–Ferrari model [69] containing Proca massive non-Abelian YM field shows renormalizibility but it fails unitary in $(3+1)$-dimensions [70,71]. There is a possibility of the mass generation by dynamical symmetry breaking in non-perturbative regime, but the mass tends to zero at the high energy limit of non-Abelian gauge theory [72]. The BRST symmetry plays an important part in the analysis of the various interactions according to a model under consideration. We should need the unitarity of the scattering matrix (S-matrix) in a renormalizable model.

Our present investigation is essential on the following grounds. First, to derive the proper off-shell nilpotent and absolutely anticommuting (anti-)BRST transformations for combined scalar and vector gauge transformations. Because in earlier work [10], the off-shell nilpotent (anti-)BRST transformations are found to be non-anticommuting. Second, to obtain the coupled and equivalent Lagrangian densities which respect both BRST and anti-BRST transformations. Third, to establish the CF-type of conditions because these conditions play an important role within the framework of BRST formalism.

The contents of our present endeavour are organized as follows. In Sect. 2, we briefly discuss about the mathematical aspects and geometrical significance of the BRST symmetries in the realm of differential geometry. In Sect. 3, we discuss about the 4D topologically massive (non)-Abelian $(B \wedge F)$ theories and associated local gauge symmetries. Section 4 deals with the derivation of the *proper* off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetry transformations of the Yang–Mills field, antisymmetric gauge field and compensating auxiliary vector field and their corresponding (anti-)ghost fields within the framework of geometrical superfield approach to BRST formalism. Section 5 is devoted to the derivation of the coupled (but equivalent) Lagrangian densities by using the basic tenets of BRST formalism. We capture, in Sect. 6, the (anti-)BRST invariance of the coupled Lagrangian densities, nilpotency and absolute anticommutativity properties of the (anti-)BRST symmetries within the framework of superfield formalism. Finally, in Sect. 7, we provide some concluding remarks.

In our Appendix A, we show the precise values of the various secondary field that are presented in the superfield expansions in terms of the dynamical and auxiliary fields of the (anti-)BRST invariant theory. Appendix B deals with the proof of the absolute anticommutativity of the (anti-)BRST transformations where the CF-type of conditions play decisive role. The (anti-)BRST invariance of the coupled Lagrangian densities is shown in Appendix C.

## 2 Geometrical significance of BRST symmetries: mathematical aspects

In this section, we consider the geometrical significance of the BRST symmetry (see, e.g. [16,61–63] for details). We need to consider the principal $G$-bundle $(P, \pi, M)$ in pure YM theory where $F \equiv G$ is the fibre in the total space $P$ and $G$ is the structural Lie group over the base manifold which is spacetime (see Fig. 1). Here $\pi$ is the projection of $F$ on the $M$. We define a section $\sigma : M \to F$ such that

$$\pi(\sigma(y)) = id_M, \quad y \in G, \tag{1}$$

and Lie algebra valued connection 1-form $\omega$ on the bundle. The pull-back $\omega$ on $M$ i.e. $\sigma^*\omega$ represents YM field locally (or local trivialization). Here $id_M$ in the Eq. (1) represents an identity map of $M$. Let us consider the coordinates $y^i$ in the fibre and a point $x^\mu$ on $G$ which is lifted in $\sigma$ from $M$. The vector $\partial_{y^i}$ is tangent to the fibre and vertical whereas the vector $\partial_{x^\mu}$ is tangent to the section but neither horizontal nor vertical. The 1-forms $dy^i$ and $dx^\mu$ span the cotangent space $P^*$. Thus, 1-form $\omega$ can be decomposed as

$$\omega = \chi_i dy^i + \phi_\mu dx^\mu, \tag{2}$$

where $\chi = \chi_i dy^i$ is the Maurer–Cartan form, which is Faddeev–Popov ghost field on the bundle and $\phi_\mu$ is the 1-form gauge field. The ghost field $\chi$ is vertical and defined as

$$\chi_i(\partial_{x^\mu}) = 0, \tag{3}$$

whereas the gauge field is horizontal:

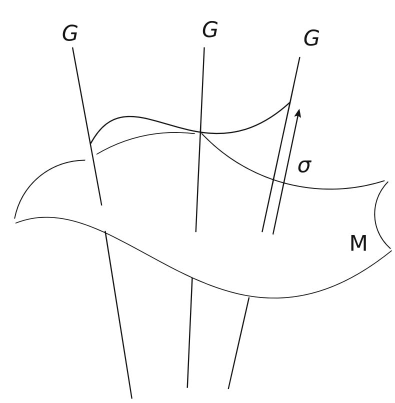


**Fig. 1** Fibres $G$ in the principal fibre bundle with base manifold $M$ and section $\sigma$

$$\phi_\mu(\partial_{y^i}) = 0. \quad (4)$$

We can also decompose the exterior derivative $d$ of a 0-form according to the Eq. (2) as

$$df = sf + bf, \quad (5)$$

where $s$ and $b$ are defined in the following fashion:

$$sf = \partial_{y^i} f dy^i, \qquad b = \partial_{x^\mu} f\, dx^\mu. \quad (6)$$

Using the cohomology with respect to exterior derivative, we obtain

$$s^2 = 0, \quad b^2 = 0, \quad sb + bs = 0, \quad (7)$$

In the above, $s$ defines the exterior differential normal to the sections and it is nilpotent of order two whereas $b$ is horizontal operator. We shall identify $s$ as the BRST operator.

Due to the construction of the fibre bundle, we can clearly see

$$\sigma^*(df) = \sigma^*(bf), \quad (8)$$

because

$$\sigma^*(sf) = 0. \quad (9)$$

Then the 2-form curvature with respect to the section $\sigma$ is given by

$$\Sigma = \Omega^1_{ij}(dy^i \wedge dy^j) + \Omega^2_{i\mu}(dy^i \wedge dx^\mu) + \Omega^3_{\mu\nu}(dx^\mu \wedge dx^\nu), \quad (10)$$

where $\Omega^1 = s\chi + \frac{1}{2}[\chi, \chi]$ and $\Omega^2 = s\phi + b\chi + [\phi, \chi]$. Here $[\ ,\ ]$ defines the Lie bracket. The Maurer–Cartan structural theorem states that the curvature $\Sigma$ is pure horizontal i.e.

$$\Omega^1 = 0, \quad \Omega^2 = 0, \quad (11)$$

which provide the BRST transformations of fields in the theory. In this paper we will use this horizontality conditions to get the BRST and anti-BRST transformations of fields in non-Abelian topologically massive $(B \wedge F)$ model.

## 3 4D topologically massive $(B \wedge F)$ theory

We first consider the topologically massive Abelian model in (3 + 1)-dimensions of spacetime [38–40] which contains a massive gauge field but keeping the gauge symmetry unbroken. In this model, the Abelian 1-form $A^{(1)} = dx^\mu A_\mu$ gauge field $A_\mu$ and antisymmetric 2-form $B^{(2)} = \frac{1}{2!}(dx^\mu \wedge dx^\nu)B_{\mu\nu}$ field are coupled, in a physically meaningful manner, through a well-known topological $B \wedge F = \frac{1}{4}\varepsilon^{\mu\nu\eta\kappa}B_{\mu\nu}F_{\eta\kappa}$ term. Here $B_{\mu\nu}$ is the Kalb–Ramond field and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor corresponding to the Abelian gauge field $A_\mu$. The mass of gauge field is put by hand in the model as a (constant) coupling parameter $m$ of the topological term. The topologically massive Abelian model has the Lagrangian density[1] [38–40]:

$$\mathcal{L}_0 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{12}H^{\mu\nu\kappa}H_{\mu\nu\kappa} + \frac{m}{4}\varepsilon^{\mu\nu\eta\kappa}F_{\mu\nu}B_{\eta\kappa}, \quad (12)$$

where $H_{\mu\nu\kappa} = \partial_\mu B_{\nu\kappa} + \partial_\nu B_{\kappa\mu} + \partial_\kappa B_{\mu\nu}$ is the field strength of the Kalb–Ramond field. The Abelian model is invariant under the following gauge transformations of the fields:

$$A_\mu \to A_\mu + \partial_\mu \Omega, \qquad B_{\mu\nu} \to B_{\mu\nu}, \quad (13)$$

and,

$$B_{\mu\nu} \to B_{\mu\nu} - (\partial_\mu \Omega_\nu - \partial_\nu \Omega_\mu), \qquad A_\mu \to A_\mu, \quad (14)$$

where $\Omega(x)$ and $\Omega_\mu(x)$ are the local gauge transformation parameters which vanish at infinity. The Euler–Lagrange equations of motion for $A_\mu$ and $B_{\mu\nu}$ fields are give by, respectively

$$\partial_\mu F^{\mu\nu} = -\frac{m}{6}\,\varepsilon^{\nu\mu\eta\kappa}H_{\mu\eta\kappa}, \qquad \partial_\mu H^{\mu\nu\eta} = \frac{m}{2}\,\varepsilon^{\nu\eta\kappa\rho}F_{\kappa\rho}. \quad (15)$$

After decoupling the above equations of motion for the fields, we get either

$$\left(\Box + m^2\right)F_{\mu\nu} = 0, \quad (16)$$

or,

$$\left(\Box + m^2\right)H_{\mu\nu\lambda} = 0, \quad (17)$$

which are clearly the gauge-invariant Klein–Gordon equations for massive $A_\mu$ and $B_{\mu\nu}$ fields. The counting of the degrees of freedom shows that massive $B_{\mu\nu}$ field has three degrees of freedom as same as massive vector field $A_\mu$ in physical (3 + 1)-dimensions of spacetime.

We now discuss the non-Abelian generalization of the above model. This theory is described by the following Lagrangian density[2] [41–44]

$$\mathcal{L} = -\frac{1}{4}\,F_{\mu\nu}\cdot F^{\mu\nu} + \frac{1}{12}\,H_{\mu\nu\eta}\cdot H^{\mu\nu\eta} + \frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}\,B_{\mu\nu}\cdot F_{\eta\kappa}, \quad (18)$$

[1] We adopt the conventions and notations such that the 4D flat Minkowski metric has mostly negative signatures: $\eta_{\mu\nu} = \eta^{\mu\nu} = \text{diga}\,(+1, -1, -1, -1)$. The Greek indices $\mu, \nu, \kappa, \ldots = 0, 1, 2, 3$ correspond to spacetime directions whereas the Latin indices $i, j, k, \ldots = 1, 2, 3$ stand for space directions only.

[2] The dot and cross products in the $SU(N)$ algebraic space between two non-null vectors $X$ and $Y$ are defined as: $X \cdot Y = X^a Y^a$, $X \times Y = f^{abc}X^aY^bT^c$. Here the structure constants $f^{abc}$ are chosen to be totally antisymmetric in their indices $a, b, c$ and $T^a$ are the generators of the gauge group $SU(N)$.

where $F^a_{\mu\nu}\, T^a \equiv F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g(A_\mu \times A_\nu)$ is the field strength tensor for the non-Abelian 1-form gauge field $A_\mu = A^a_\mu\, T^a$. The totally antisymmetric compensated curvature tensor $H_{\mu\nu\eta} \equiv H^a_{\mu\nu\eta} T^a$ for the non-Abelian gauge field $B_{\mu\nu} = B^a_{\mu\nu}\, T^a$ is defined as

$$H^a_{\mu\nu\eta}\, T^a \equiv H_{\mu\nu\eta} = D_\mu B_{\nu\eta} + D_\nu B_{\eta\mu} + D_\eta B_{\mu\nu} + g\,(F_{\mu\nu} \times K_\eta) + g\,(F_{\nu\eta} \times K_\mu) + g\,(F_{\eta\mu} \times K_\nu), \tag{19}$$

where 1-form $K^{(1)} = dx^\mu K_\mu \cdot T$ is the compensating auxiliary vector field $K_\mu = K^a_\mu T^a$ and $g$ is a dimensionless coupling constant. The gauge bosons $A_\mu$ acquire mass through the topological term $(B \wedge F)$ without taking any help of Higgs mechanism. The presence of topological term $\frac{m}{4}\, \varepsilon^{\mu\nu\rho\sigma} B_{\mu\nu} \cdot F_{\rho\sigma}$ also ensures us the CP-invariance of the model. It is because of the parity transformation of Kalb–Ramond field: $B_{0i} \rightarrow -B_{0i},\, B_{ij} \rightarrow B_{ij}$ [45]. The topological term does not break Lorentz invariance in (3+1)-dimensions unlike the topological term present in [46]. The compensating auxiliary vector field is required for the invariance of kinetic term for tensor field $B_{\mu\nu}$ under the non-Abelian vector gauge transformation: $B_{\mu\nu} \rightarrow B_{\mu\nu} - (D_\mu \Lambda_\nu - D_\nu \Lambda_\mu)$ (see below). The absence of propagator of the auxiliary vector field in Eq. (18) implies the absence of its role in the physical processes. We will see from the BRST transformation of $K_\mu$ that its all modes are unphysical. This model is shown to be renormalizable algebraically in [44] and unitary at tree level.

The non-Abelian generalization must keep all the symmetries that were present in the Abelian model. The above Lagrangian density respects two kinds of gauge symmetry transformations: (i) scalar gauge symmetry ($\delta_1$), and (ii) vector gauge symmetry ($\delta_2$). These symmetry transformations are listed as follows:

$$\begin{aligned}
&\delta_1 A_\mu = D_\mu \zeta \equiv \partial_\mu \zeta - g(A_\mu \times \zeta),\\
&\delta_1 B_{\mu\nu} = -g(B_{\mu\nu} \times \zeta), \quad \delta_1 K_\mu = -g(K_\mu \times \zeta),\\
&\delta_1 F_{\mu\nu} = -g(F_{\mu\nu} \times \zeta), \quad \delta_1 H_{\mu\nu\eta} = -g(H_{\mu\nu\eta} \times \zeta),\\
&\delta_2 B_{\mu\nu} = -(D_\mu \Lambda_\nu - D_\nu \Lambda_\mu), \qquad \delta_2 K_\mu = -\Lambda_\mu,\\
&\delta_2 A_\mu = 0, \quad \delta_2 F_{\mu\nu} = 0, \quad \delta_2 H_{\mu\nu\eta} = 0.
\end{aligned} \tag{20}$$

where $\zeta = \zeta \cdot T$ and $\Lambda_\mu = \Lambda_\mu \cdot T$ are the $SU(N)$-valued local "scalar" and "vector" gauge transformation parameters. Under these local gauge transformations, the Lagrangian density transforms as

$$\delta_1 \mathcal{L} = 0, \qquad \delta_2 \mathcal{L} = -\partial_\mu \Big[\frac{m}{2}\, \varepsilon^{\mu\nu\eta\kappa} \Lambda_\nu \cdot F_{\eta\kappa}\Big]. \tag{21}$$

Thus, the action integral ($S = \int d^4x \mathcal{L}$) remains invariant under the gauge transformations for the physically well-defined fields which vanish rapidly at infinity due to Gauss divergence theorem. Also, the combined gauge transformations $\delta = (\delta_1 + \delta_2)$ leaves the action integral invariant.

## 4 Off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetries: geometrical superfield formalism

In this section, we derive the complete set of off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetries with the help of Bonora–Tonin superfield approach to BRST formalism. For this purpose, we generalize our ordinary 4D spacetime to the (4, 2)D superspace. The latter is characterized by a pair of Grassmannian variables[3] $(\theta, \bar\theta)$ (with $\theta^2 = \bar\theta^2 = 0,\ \theta\bar\theta + \bar\theta\theta = 0$) in addition to the bosonic spacetime variables $x^\mu$ (with $\mu = 0, 1, 2, 3$) as [12,14,15]

$$x^\mu \rightarrow Z^M \equiv (x^\mu, \theta, \bar\theta), \quad \partial_\mu \rightarrow \partial_M \equiv (\partial_\mu, \partial_\theta, \partial_{\bar\theta}), \tag{22}$$

where the super-coordinates $Z^M$ parametrized the (4, 2)D supermanifold. The partial derivatives $\partial_\theta = \frac{\partial}{\partial\theta}$ and $\partial_{\bar\theta} = \frac{\partial}{\partial\bar\theta}$ (with $\partial_\theta^2 = \partial_{\bar\theta}^2 = 0,\ \partial_\theta\partial_{\bar\theta} + \partial_{\bar\theta}\partial_\theta = 0$) are the translational generators along the Grassmannian directions $\theta$ and $\bar\theta$, respectively. We shall see later on that these translational generators provide the geometrical meaning of the anti-BRST and BRST symmetry transformations, respectively.

In our upcoming subsections, we shall exploit the horizontality conditions and integrability condition for the derivation of proper (anti-)BRST transformations.

### 4.1 Derivation of the (anti-)BRST transformations of YM field and corresponding ghost fields

For the derivation of (anti-)BRST symmetry transformations of the YM gauge field, we generalize the exterior derivative $d = dx^\mu \partial_\mu$ (with $d^2 = 0$) and 1-form connection $A^{(1)} = dx^\mu A^a_\mu T^a$ to the super-exterior derivative $\tilde d$ (with $\tilde d^2 = 0$) and super 1-form $\tilde{\mathcal{A}}^{(1)} = dx^\mu \tilde{\mathcal{A}}^a_\mu T^a$ on the (4, 2)D supermanifold in the following fashion:

$$\begin{aligned}
\tilde d &= dZ^M \partial_M \equiv dx^\mu\, \partial_\mu + d\theta\, \partial_\theta + d\bar\theta\, \partial_{\bar\theta},\\
\tilde{\mathcal{A}}^{(1)} &= dZ^M A_M \equiv dx^\mu\, \tilde{\mathcal{A}}_\mu(x, \theta, \bar\theta) + d\theta\, \bar{\tilde{\mathcal{F}}}(x, \theta, \bar\theta) + d\bar\theta\, \tilde{\mathcal{F}}(x, \theta, \bar\theta),
\end{aligned} \tag{23}$$

where the superfields $\tilde{\mathcal{A}}_\mu(x, \theta, \bar\theta), \tilde{\mathcal{F}}(x, \theta, \bar\theta)$ and $\bar{\tilde{\mathcal{F}}}(x, \theta, \bar\theta)$, as the super-multiplets of super 1-form, are the generalization of 1-form gauge field $A_\mu(x)$, ghost field $C(x)$ and anti-ghost field $\bar C(x)$, respectively, on the (4, 2)D supermanifold. One can expand these superfields along the Grassmannian directions $(\theta, \bar\theta)$ as

$$\begin{aligned}
\tilde{\mathcal{A}}_\mu(x, \theta, \bar\theta) &= A_\mu(x) + \theta \bar R_\mu(x) + \bar\theta R_\mu(x) + \theta\bar\theta S_\mu(x),\\
\tilde{\mathcal{F}}(x, \theta, \bar\theta) &= C(x) + \theta \bar B_1(x) + \bar\theta B_1(x) + \theta\bar\theta s(x),\\
\bar{\tilde{\mathcal{F}}}(x, \theta, \bar\theta) &= \bar C(x) + \theta \bar B_2(x) + \bar\theta B_2(x) + \theta\bar\theta \bar s(x),
\end{aligned} \tag{24}$$

[3] The Grassmannian variables obey the following Hermiticity properties: $\theta^\dagger = \theta$ and $\bar\theta^\dagger = -\bar\theta$.

where the secondary fields $\bar{R}_\mu, R_\mu, s, \bar{s}$ are fermionic and the remaining secondary fields $S_\mu, B_1, \bar{B}_1, B_2, \bar{B}_2$ are bosonic in nature.

To determine the values of these secondary fields, we invoke the following HC

$$\tilde{d}\tilde{\mathcal{A}}^{(1)} + \frac{i}{2}\, g\left[\tilde{\mathcal{A}}^{(1)},\, \tilde{\mathcal{A}}^{(1)}\right] = dA^{(1)} + \frac{i}{2}\, g\left[A^{(1)},\, A^{(1)}\right] \Longrightarrow \tilde{\mathcal{F}}^{(2)} = F^{(2)}, \tag{25}$$

where the super 2-form $\tilde{\mathcal{F}}^{(2)} = \frac{1}{2!}(dZ^M \wedge dZ^N)\, \tilde{\mathcal{F}}_{MN}$ is the generalization of $F^{(2)} = \frac{1}{2!}(dx^\mu \wedge dx^\nu)F_{\mu\nu}$ on the supermanifold. The HC in the literature is also known as *soul-flatness* condition which states that the r.h.s. is independent of the Grassmannian variables when it is generalized onto (4, 2)D supermanifold. To be more precise, the HC demands that all the Grassmannian components of the super 2-form curvature $\tilde{\mathcal{F}}_{MN}$ are equal to zero (i.e., $\tilde{\mathcal{F}}_{\mu\theta} = \tilde{\mathcal{F}}_{\mu\bar{\theta}} = \tilde{\mathcal{F}}_{\theta\theta} = \tilde{\mathcal{F}}_{\bar{\theta}\bar{\theta}} = \tilde{\mathcal{F}}_{\theta\bar{\theta}} = 0$). As we already know that the kinetic term ($-\frac{1}{4}\, F_{\mu\nu}\cdot F^{\mu\nu}$) for the gauge field $A_\mu$ remains invariant under the combined gauge transformations ($\delta$). Thus, the kinetic term would also remain invariant under (anti-)BRST transformations. Physically, the HC implies that the gauge-invariant quantity must be independent of the Grassmannian variables $(\theta, \bar{\theta})$ (i.e. $-\frac{1}{4}\, \tilde{\mathcal{F}}_{MN}\cdot\tilde{\mathcal{F}}^{MN} = -\frac{1}{4}\, F_{\mu\nu}\cdot F^{\mu\nu}$) when it is generalized on the (4, 2) supermanifold. It is worthwhile to point out that the Grassmannian variables are just a mathematical artifact and they cannot be physically realized in our physical 4D spacetime. In fact, they are used to construct the (4, 2)-dimensional superspace.

By exploiting the above HC, we obtain the values of the secondary fields [cf. (A.1)]. The substitution of the values of secondary fields in the super-expansions of the superfields, we obtain[4]

$$\begin{aligned}
\tilde{\mathcal{A}}^{(h)}_\mu(x,\theta,\bar{\theta}) &= A_\mu + \theta D_\mu\bar{C} + \bar{\theta} D_\mu C \\
&\quad + \theta\bar{\theta}\left(D_\mu B - g(D_\mu C\times\bar{C})\right), \\
\tilde{\mathcal{F}}^{(h)}(x,\theta,\bar{\theta}) &= C + \theta\bar{B} + \bar{\theta}\,\frac{g}{2}\big(C\times C\big) - \theta\bar{\theta} g\big(\bar{B}\times C\big), \\
\tilde{\bar{\mathcal{F}}}^{(h)}(x,\theta,\bar{\theta}) &= \bar{C} + \theta\,\frac{g}{2}\big(\bar{C}\times\bar{C}\big) + \bar{\theta}\, B \\
&\quad + \theta\bar{\theta} g\big(B\times\bar{C}\big),
\end{aligned} \tag{26}$$

where the superscript $(h)$ on the superfields denotes the super-expansions obtained after the application of HC (25). We have made the identifications: $\bar{B}_1 = \bar{B}$ and $B_2 = B$ for the Nakanishi–Lautrup (NL) fields $B$ and $\bar{B}$. These fields are required for the off-shell nilpotency of the (anti-)BRST transformations. Similarly, the super-curvature $\tilde{\mathcal{F}}^{(h)}_{\mu\nu}$ corresponding to the superfield $\mathcal{A}^{(h)}_\mu$ can be written as

$$\begin{aligned}
\tilde{\mathcal{F}}^{(h)}_{\mu\nu}(x,\theta,\bar{\theta}) &= F_{\mu\nu} - \theta g\big(F_{\mu\nu}\times\bar{C}\big) - \bar{\theta} g\big(F_{\mu\nu}\times C\big) \\
&\quad + \theta\,\bar{\theta}\big(g^2(F_{\mu\nu}\times C)\times\bar{C} \\
&\quad - g(F_{\mu\nu}\times B)\big).
\end{aligned} \tag{27}$$

From the above super-expansions, one can easily read-off all the (anti-)BRST transformations for the YM field and corresponding (anti-)ghost fields. These are listed as follows

$$\begin{aligned}
& s_b A_\mu = D_\mu C, \quad s_b C = \frac{g}{2}(C\times C), \quad s_b\bar{C} = B, \\
& s_b B = 0, \quad s_b\bar{B} = -g(\bar{B}\times C), \\
& s_b F_{\mu\nu} = -g(F_{\mu\nu}\times C), \\
& s_{ab} A_\mu = D_\mu\bar{C}, \quad s_{ab}\bar{C} = \frac{g}{2}(\bar{C}\times\bar{C}), \quad s_{ab} C = \bar{B}, \\
& s_{ab}\bar{B} = 0, \quad s_{ab} B = -g(B\times\bar{C}), \\
& s_{ab} F_{\mu\nu} = -g(F_{\mu\nu}\times\bar{C}).
\end{aligned} \tag{28}$$

Geometrically, the BRST transformation ($s_b$) for any generic field $\Sigma(x)$ is equivalent to the translational of corresponding superfield $\tilde{\Sigma}^{(h)}(x,\theta,\bar{\theta})$ along $\bar{\theta}$-direction while keeping $\theta$-direction fixed. In a similar fashion, the anti-BRST transformation ($s_{ab}$) can be obtained by taking the translational of the superfield along $\theta$-direction while $\bar{\theta}$-direction remains intact. As a consequence, the following mappings are valid between the Grassmannian translational generators $(\partial_\theta, \partial_{\bar{\theta}})$ and the (anti-)BRST symmetry transformations, namely;

$$\begin{aligned}
& \frac{\partial}{\partial\bar{\theta}}\,\tilde{\Sigma}^{(h)}(x,\theta,\bar{\theta})\Big|_{\theta=0} = s_b\,\Sigma(x), \\
& \frac{\partial}{\partial\theta}\,\tilde{\Sigma}^{(h)}(x,\theta,\bar{\theta})\Big|_{\bar{\theta}=0} = s_{ab}\,\Sigma(x), \\
& \frac{\partial}{\partial\bar{\theta}}\,\frac{\partial}{\partial\theta}\,\tilde{\Sigma}^{(h)}(x,\theta,\bar{\theta}) = s_b s_{ab}\,\Sigma(x).
\end{aligned} \tag{29}$$

The (anti-)BRST transformations of the NL auxiliary fields $B$ and $\bar{B}$ have been derived from the requirements of the nilpotency and absolute anticommutativity of the (anti-)BRST transformations.

We point out that the absolute anticommutativity property of the BRST and anti-BRST transformations is satisfied due to the validity of the following CF condition [7] [cf. (A.1)]:

$$B + \bar{B} - g\big(C\times\bar{C}\big) = 0. \tag{30}$$

It is a physical condition on the theory in the sense that it is BRST as well as anti-BRST invariant quantity (i.e. $s_{(a)b}[B + \bar{B} - g\big(C\times\bar{C}\big)] = 0$). This is an original CF condition which was emerged automatically first time for the non-Abelian 1-form gauge theory within the framework of superfield approach to BRST formalism [12]. For the sake of brevity, the restriction $\tilde{\mathcal{F}}_{\theta\bar{\theta}} = 0$ leads to the above CF condition.

[4] The Nakanishi–Lautrup fields $B, \bar{B}$ are real and the anticommuting ghost fields satisfy the following Hermiticity properties: $C^\dagger = C$, and $\bar{C}^\dagger = -\bar{C}$.

### 4.2 (Anti-)BRST symmetries of antisymmetric gauge field and associated ghost fields

In this subsection, we focus on the derivation of the BRST and anti-BRST transformations for $B_{\mu\nu}$ and corresponding (anti-)ghost fields. For this purpose, we use another HC as given below

$$H^{(3)} = \tilde{\mathcal{H}}^{(3)}, \tag{31}$$

which again implies that the kinetic term for the 2-form field $B_{\mu\nu}$ is a gauge-invariant quantity. Here $\tilde{\mathcal{H}}^{(3)} = \frac{1}{3!}\,(dZ^L \wedge dZ^M \wedge dZ^N)\mathcal{H}_{LMN}$ defines the 3-form super-curvature on the (4, 2)D supermanifold corresponding to the 3-form curvature $H^{(3)} = \frac{1}{3!}\,(dx^\mu \wedge dx^\nu \wedge dx^\eta) H_{\mu\nu\eta}$. These (super-)curvature are defined in the following fashion:

$$\tilde{\mathcal{H}}^{(3)} = \tilde{d}\tilde{\mathcal{B}}^{(2)} + ig\left[\tilde{\mathcal{A}}^{(1)}_{(h)},\, \tilde{\mathcal{B}}^{(2)}\right] + ig\left[\tilde{\mathcal{K}}^{(1)},\, \tilde{\mathcal{F}}^{(2)}_{(h)}\right],$$
$$H^{(3)} = dB^{(2)} + ig\left[A^{(1)},\, B^{(2)}\right] + ig\left[K^{(1)},\, F^{(2)}\right], \tag{32}$$

where $\tilde{\mathcal{A}}^{(1)}_{(h)}$ is the super 1-form obtained after the application of first HC (25) and $\tilde{\mathcal{F}}^{(2)}_{(h)}$ defines the corresponding super-curvature. It is straightforward to check that $H^{(3)}$ produces the curvature tensor (19). The super 1-form $\tilde{\mathcal{K}}^{(1)}$ and super 2-form $\tilde{\mathcal{B}}^{(2)}$ can be written as follows:

$$\tilde{\mathcal{K}}^{(1)} = dx^\mu\, \tilde{\mathcal{K}}_\mu(x,\theta,\bar\theta) + d\theta\, \tilde{\bar{\xi}}(x,\theta,\bar\theta) + d\bar\theta\, \tilde{\xi}(x,\theta,\bar\theta),$$
$$\begin{aligned}\tilde{\mathcal{B}}^{(2)} &= \frac{1}{2!}\,(dZ^M \wedge dZ^N)\, \tilde{\mathcal{B}}_{MN}(x,\theta,\bar\theta)\\ &\equiv \frac{1}{2!}\,(dx^\mu \wedge dx^\nu)\, \tilde{\mathcal{B}}_{\mu\nu}(x,\theta,\bar\theta)\\ &\quad + (dx^\mu \wedge d\theta)\, \tilde{\bar{\mathcal{F}}}_\mu(x,\theta,\bar\theta) + (dx^\mu \wedge d\bar\theta)\, \tilde{\mathcal{F}}_\mu(x,\theta,\bar\theta)\\ &\quad + (d\theta \wedge d\bar\theta)\, \tilde{\Phi}(x,\theta,\bar\theta) + (d\theta \wedge d\theta)\, \tilde{\bar{\beta}}(x,\theta,\bar\theta)\\ &\quad + (d\bar\theta \wedge d\bar\theta)\, \tilde{\beta}(x,\theta,\bar\theta),\end{aligned} \tag{33}$$

where $\tilde{\mathcal{K}}^{(1)}$ and $\tilde{\mathcal{B}}^{(2)}$ are the generalizations of $K^{(1)}$ and $B^{(2)}$, respectively on the supermanifold. Again, the super-multiples, as the components of the above super 1-form and super 2-form, can be expanded along the directions of Grassmannian variables $(\theta, \bar\theta)$ as

$$\begin{aligned}\tilde{\mathcal{B}}_{\mu\nu}(x,\theta,\bar\theta) &= B_{\mu\nu}(x) + \theta \bar{R}_{\mu\nu}(x) + \bar\theta R_{\mu\nu}(x) + \theta\bar\theta S_{\mu\nu}(x),\\ \tilde{\mathcal{K}}_\mu(x,\theta,\bar\theta) &= K_\mu(x) + \theta\, \bar{P}_\mu(x) + \bar\theta\, P_\mu(x) + \theta\bar\theta\, Q_\mu(x),\\ \tilde{\mathcal{F}}_\mu(x,\theta,\bar\theta) &= C_\mu(x) + \theta\, \bar{b}^{(1)}_\mu(x) + \bar\theta\, b^{(1)}_\mu(x) + \theta\bar\theta\, q_\mu(x),\\ \tilde{\bar{\mathcal{F}}}_\mu(x,\theta,\bar\theta) &= \bar{C}_\mu(x) + \theta\, \bar{b}^{(2)}_\mu(x) + \bar\theta\, b^{(2)}_\mu(x) + \theta\bar\theta\, \bar{q}_\mu(x),\\ \tilde{\Phi}(x,\theta,\bar\theta) &= \phi(x) + \theta\, \bar{f}_1(x) + \bar\theta\, f_1(x) + \theta\bar\theta\, b_1(x),\\ \tilde{\beta}(x,\theta,\bar\theta) &= \beta(x) + \theta\, \bar{f}_2(x) + \bar\theta\, f_2(x) + \theta\bar\theta\, b_2(x),\\ \tilde{\bar{\beta}}(x,\theta,\bar\theta) &= \bar{\beta}(x) + \theta\, \bar{f}_3(x) + \bar\theta\, f_3(x) + \theta\bar\theta\, b_3(x),\\ \tilde{\xi}(x,\theta,\bar\theta) &= \xi(x) + \theta\, \bar{R}_1(x) + \bar\theta\, R_1(x) + \theta\bar\theta\, S_1(x),\\ \tilde{\bar{\xi}}(x,\theta,\bar\theta) &= \bar{\xi}(x) + \theta\, \bar{R}_2(x) + \bar\theta\, R_2(x) + \theta\bar\theta\, S_2(x),\end{aligned} \tag{34}$$

where the secondary fields $R_{\mu\nu}$, $\bar{R}_{\mu\nu}$, $P_\mu$, $\bar{P}_\mu$, $q_\mu$, $\bar{q}_\mu$, $f_1$, $\bar{f}_1$, $f_2$, $\bar{f}_2$, $f_3$, $\bar{f}_3$, $S_1$, $S_2$ are fermionic in nature and $S_{\mu\nu}$, $Q_\mu$, $b^{(1)}_\mu, \bar{b}^{(1)}_\mu, b^{(2)}_\mu, \bar{b}^{(2)}_\mu, b_1, b_2, b_3, R_1, \bar{R}_1, R_2, \bar{R}_2$ are the bosonic secondary fields.

By using the second HC (31) together with (26) and (27), we obtain the values of the above secondary fields except $P_\mu$, $\bar{P}_\mu$ and $Q_\mu$ [cf. (A.2)]. As a result, we get the desired super-expressions of the above superfields (34):

$$\begin{aligned}\tilde{\mathcal{B}}^{(h)}_{\mu\nu}(x,\theta,\bar\theta) &= B_{\mu\nu} + \theta\big[-(D_\mu \bar{C}_\nu - D_\nu \bar{C}_\mu)\\ &\quad + g(\bar{C} \times B_{\mu\nu}) + g(\bar\xi \times F_{\mu\nu})\big]\\ &\quad + \bar\theta\big[-(D_\mu C_\nu - D_\nu C_\mu) + g(C \times B_{\mu\nu})\\ &\quad + g(\xi \times F_{\mu\nu})\big]\\ &\quad + \theta\bar\theta\big[-(D_\mu B_\nu - D_\nu B_\mu) + g(D_\mu C \times \bar{C}_\nu)\\ &\quad - g(D_\nu C \times \bar{C}_\mu) + g(B \times B_{\mu\nu})\\ &\quad + g^2\big(\bar\xi \times (F_{\mu\nu} \times C)\big) - g^2\big(\bar{C} \times (C \times B_{\mu\nu})\big)\\ &\quad + g\big(\bar{C} \times (D_\mu C_\nu - D_\nu C_\mu)\big)\\ &\quad + g(R \times F_{\mu\nu}) - g^2\big(\bar{C} \times (\xi \times F_{\mu\nu})\big)\big],\\ \tilde{\mathcal{F}}^{(h)}_\mu(x,\theta,\bar\theta) &= C_\mu + \theta\, \bar{B}_\mu + \bar\theta\big[-D_\mu\beta + g(C \times C_\mu)\big]\\ &\quad + \theta\bar\theta\big[D_\mu\lambda - g(D_\mu \bar{C} \times \beta)\\ &\quad - g(\bar{B} \times C_\mu) - g(\bar{B}_\mu \times C)\big],\\ \tilde{\bar{\mathcal{F}}}^{(h)}_\mu(x,\theta,\bar\theta) &= \bar{C}_\mu + \theta\big[-D_\mu\bar\beta + (\bar{C} \times \bar{C}_\mu)\big] + \bar\theta\, B_\mu\\ &\quad + \theta\bar\theta\big[-D_\mu\bar\lambda + g(D_\mu C \times \bar\beta)\\ &\quad + g(B \times \bar{C}_\mu) + g(B_\mu \times \bar{C})\big],\\ \tilde{\beta}^{(h)}(x,\theta,\bar\theta) &= \beta + \theta\,\lambda + \bar\theta\, g\big(C \times \beta\big)\\ &\quad + \theta\bar\theta\big[g(C \times \lambda) - g(\bar{B} \times \beta)\big],\\ \tilde{\bar\beta}^{(h)}(x,\theta,\bar\theta) &= \bar\beta + \theta\, g\big(\bar{C} \times \bar\beta\big) + \bar\theta\,\bar\lambda\\ &\quad + \theta\bar\theta\big[-g(\bar{C} \times \bar\lambda) + g(B \times \bar\beta)\big],\\ \tilde{\Phi}^{(h)}(x,\theta,\bar\theta) &= \phi + \theta\,\bar\rho + \bar\theta\,\rho\\ &\quad + \theta\bar\theta\big[g(B \times \phi) - g(\bar{C} \times \bar\rho)\\ &\quad - g(C \times \bar\lambda) + g^2\big(C \times (C \times \bar\beta)\big)\big],\\ \tilde\xi^{(h)}(x,\theta,\bar\theta) &= \xi + \theta\,\bar{R} + \bar\theta\big[-\beta + (C \times \xi)\big]\\ &\quad + \theta\bar\theta\big[\lambda - g(\bar{R} \times C) - g(\bar{B} \times \xi)\big],\\ \tilde{\bar\xi}^{(h)}(x,\theta,\bar\theta) &= \bar\xi + \theta\big[-\bar\beta + (\bar{C} \times \bar\xi)\big] + \bar\theta\, R\\ &\quad + \theta\bar\theta\big[-\bar\lambda - g(R \times \bar{C}) - g(B \times \bar\xi)\big].\end{aligned} \tag{35}$$

In the above, we have chosen $b^{(2)}_\mu = B_\mu, \bar{b}^{(1)}_\mu = \bar{B}_\mu, R_2 = R$, $\bar{R}_1 = \bar{R}$ for the bosonic NL-type auxiliary fields $B_\mu$, $\bar{B}_\mu$, $R$, $\bar{R}$ and $f_1 = \rho$, $\bar{f}_1 = \bar\rho$, $\bar{f}_2 = \lambda$, $f_3 = \bar\lambda$ for the additional fermionic NL-type fields $\rho, \bar\rho, \lambda, \bar\lambda$. Again, these (bosonic) fermionic auxiliary fields are required for the off-shell nilpotency of the (anti-)BRST transformations. One can also express the 3-form super-curvature in terms of the Grassmannian variables as

$$\begin{aligned}\tilde{\mathcal{H}}^{(h)}_{\mu\nu\eta}(x,\theta,\bar\theta) &= H_{\mu\nu\eta} - \theta g(H_{\mu\nu\eta}\times\bar C)\\ &\quad -\bar\theta g(H_{\mu\nu\eta}\times C) + \theta\bar\theta\big[-g(H_{\mu\nu\eta}\times B)\\ &\quad + g^2(H_{\mu\nu\eta}\times C)\times\bar C\big]. \qquad (36)\end{aligned}$$

As a consequence of the above super-expansions, we obtain the following BRST and anti-BRST symmetry transformations, namely;

$$\begin{aligned}
s_b B_{\mu\nu} &= -(D_\mu C_\nu - D_\nu C_\mu) + g(C\times B_{\mu\nu})\\
&\quad + g(\xi\times F_{\mu\nu}),\\
s_b C_\mu &= -D_\mu\beta + g(C\times C_\mu), \quad s_b\bar C_\mu = B_\mu,\\
s_b\beta &= g(C\times\beta), \quad s_b\bar\beta = \bar\lambda, \quad s_b\phi = \rho,\\
s_b\xi &= -\beta + g(C\times\xi), \quad s_b\bar\xi = R,\\
s_b\bar R &= \lambda - g(\bar R\times C) - g(\bar B\times\xi),\\
s_b\lambda &= g(\lambda\times C) - g(\bar B\times\beta),\\
s_b H_{\mu\nu\eta} &= -g(H_{\mu\nu\eta}\times C), \quad s_b[B_\mu, R, \rho, \bar\lambda] = 0,\\
s_b\bar\rho &= g(B\times\phi) + g(\bar\rho\times C) - g(\rho\times\bar C)\\
&\quad - g^2\big(C\times(\bar C\times\phi)\big),\\
s_b\bar B_\mu &= -D_\mu\big(\rho - g(C\times\phi)\big) - g^2\big((C\times\bar C)\times C_\mu\big)\\
&\quad + g(B\times C_\mu) - g(\bar B_\mu\times C) + g(\bar C\times D_\mu\beta),\\
s_{ab} B_{\mu\nu} &= -(D_\mu\bar C_\nu - D_\nu\bar C_\mu) + g(\bar C\times B_{\mu\nu})\\
&\quad + g(\bar\xi\times F_{\mu\nu}),\\
s_{ab}\bar C_\mu &= -D_\mu\bar\beta + g(\bar C\times\bar C_\mu), \quad s_{ab}C_\mu = \bar B_\mu,\\
s_{ab}\bar\beta &= g(\bar C\times\bar\beta), \quad s_{ab}\beta = \lambda,\\
s_{ab}\phi &= \bar\rho, \quad s_{ab}\bar\xi = -\bar\beta + g(\bar C\times\bar\xi),\\
s_{ab}\xi &= \bar R, \quad s_{ab}R = \bar\lambda - g(R\times\bar C) - g(B\times\xi),\\
s_{ab}\bar\lambda &= g(\bar\lambda\times\bar C) - g(B\times\bar\beta),\\
s_{ab} H_{\mu\nu\eta} &= -g(H_{\mu\nu\eta}\times\bar C), \quad s_{ab}[\bar B_\mu, \bar R, \bar\rho, \lambda] = 0,\\
s_{ab}\rho &= g(\bar B\times\phi) - g(\bar\rho\times C) + g(\rho\times\bar C)\\
&\quad - g^2\big(\bar C\times(C\times\phi)\big),\\
s_{ab}B_\mu &= -D_\mu\big(\bar\rho - g(\bar C\times\phi)\big) - g^2\big((C\times\bar C)\times\bar C_\mu\big)\\
&\quad + g(\bar B\times\bar C_\mu) - g(B_\mu\times\bar C) + g(C\times D_\mu\bar\beta). \qquad (37)
\end{aligned}$$

These transformations are also off-shell nilpotent and absolutely anticommuting in nature.

### 4.3 (Anti-)BRST transformations of $K_\mu$ and associated ghosts

We have, so far, determined the BRST and anti-BRST transformations for the YM, Kalb–Ramond and their associated (anti-)ghost fields. But the proper (anti-)BRST transformations of the compensating auxiliary vector field are still unknown. This is because of the fact that the second HC is incapable to determine the precise value of the secondary fields $P_\mu$, $\bar P_\mu$ and $Q_\mu$.

It is to be noted that the field strength tensors transform covariantly (i.e. $\delta F_{\mu\nu} = -g(F_{\mu\nu}\times\zeta)$ and $\delta H_{\mu\nu\eta} = -g(H_{\mu\nu\eta}\times\zeta)$) under the combined gauge transformations $\delta$. In a similar manner, it is interesting to point out that the following quantity

$$\begin{aligned}&\delta[(D_\mu K_\nu - D_\nu K_\mu) - B_{\mu\nu}]\\ &\quad = -[(D_\mu K_\nu - D_\nu K_\mu) - B_{\mu\nu}]\times\zeta, \qquad (38)\end{aligned}$$

transforms covariantly under the combined gauge transformations, too.

In the language of differential forms, one can write

$$\begin{aligned}&dK^{(1)} + ig\Big[A^{(1)},\ K^{(1)})\Big] - B^{(2)}\\ &\quad = \frac{1}{2!}(dx^\mu\wedge dx^\nu)[(D_\mu K_\nu - D_\nu K_\mu) - B_{\mu\nu}], \qquad (39)\end{aligned}$$

which is clearly a 2-form quantity. Generalizing this 2-form quantity on the (4, 2)D superspace which in turn produces the third HC

$$\begin{aligned}&\tilde d\tilde{\mathcal{K}}^{(1)} + ig\Big[\tilde{\mathcal{A}}^{(1)}_{(h)},\ \tilde{\mathcal{K}}^{(1)}\Big] - \tilde{\mathcal{B}}^{(2)}_{(h)}\\ &\quad = dK^{(1)} + ig\Big[A^{(1)},\ K^{(1)}\Big] - B^{(2)}. \qquad (40)\end{aligned}$$

It is worthwhile to mention that the above HC can also be obtained from the integrability of the second HC (31) [41]. Exploiting the above HC and setting all the Grassmannian differential equal to zero, we obtain the precise values of the renaming secondary fields [cf. (A.3)] and we have the following super-expansion of $\tilde{\mathcal{K}}_\mu$ as given below

$$\begin{aligned}\tilde{\mathcal{K}}^{(h)}_\mu(x,\theta,\bar\theta) &= K_\mu + \theta\big[D_\mu\bar\xi - \bar C_\mu - g(K_\mu\times\bar C)\big]\\ &\quad + \bar\theta\big[D_\mu\xi - C_\mu - g(K_\mu\times C)\big]\\ &\quad + \theta\bar\theta\big[D_\mu R - B_\mu\\ &\quad - g(D_\mu C\times\bar\xi) - g(K_\mu\times B)\\ &\quad - g\big(D_\mu\xi - C_\mu - g(K_\mu\times C)\big)\times\bar C\big]. \qquad (41)\end{aligned}$$

Thus, we obtain the following BRST and anti-BRST transformations for the compensating auxiliary field:

$$\begin{aligned}s_b K_\mu &= D_\mu\xi - C_\mu - g(K_\mu\times C),\\ s_{ab} K_\mu &= D_\mu\bar\xi - \bar C_\mu - g(K_\mu\times\bar C). \qquad (42)\end{aligned}$$

The above transformations as listed in (37) and (42) are off-shell nilpotent and absolutely anticommuting. However, the absolute anticommutativity property is satisfied on the constrained hypersurface defined by the CF-type condition (30) and the following additional CF-type conditions [cf. (B.5)]:

$$\begin{aligned}&\bar B_\mu + B_\mu + D_\mu\phi - g(\bar C\times C_\mu) - g(C\times\bar C_\mu) = 0,\\ &\bar R + R + \phi - g(\bar C\times\xi) - g(C\times\bar\xi) = 0,\\ &\rho + \lambda - g(C\times\phi) - g(\bar C\times\beta) = 0,\\ &\bar\rho + \bar\lambda - g(\bar C\times\phi) - g(C\times\bar\beta) = 0. \qquad (43)\end{aligned}$$

These CF-type conditions emerge from the second and third HCs [cf. (A.2) and (A.3)]. Furthermore, it is to be noted that the first two CF-type conditions are bosonic whereas last two are fermionic in nature.

## 5 Coupled but equivalent Lagrangian densities

Using the basic principles and ingredients of BRST formalism, the most appropriate (anti-)BRST invariant Lagrangian densities which incorporate the gauge-fixing and Faddeev–Popov ghosts terms can be written as

$$\mathcal{L}_{(B)} = \mathcal{L} + s_b s_{ab}\Big[\frac{1}{2}\,A_\mu\cdot A^\mu + \bar{C}\cdot C + \frac{1}{2}\,\phi\cdot\phi + 2\,\bar{\beta}\cdot\beta + \bar{C}_\mu\cdot C^\mu - \frac{1}{4}\,B^{\mu\nu}\cdot B_{\mu\nu}\Big], \quad (44)$$

$$\mathcal{L}_{(\bar{B})} = \mathcal{L} - s_{ab} s_b\Big[\frac{1}{2}\,A_\mu\cdot A^\mu + \bar{C}\cdot C + \frac{1}{2}\,\phi\cdot\phi + 2\,\bar{\beta}\cdot\beta + \bar{C}_\mu\cdot C^\mu - \frac{1}{4}\,B^{\mu\nu}\cdot B_{\mu\nu}\Big]. \quad (45)$$

It is worthwhile to mention that all terms in the square brackets are Lorentz scalar and they are chosen in such a way that each term carries zero ghost number and mass dimension equal to two (in natural units: $\hbar = c = 1$) for the 4D theory. Furthermore, the (anti-)BRST symmetry transformations (decrease) increase the ghost number by one unit when they operate on any generic field. Also, the operation of nilpotent transformations raises mass dimension by one when they act on any field. One can see these observations directly from the expressions of the (anti-)BRST symmetry transformations given in (28), (37) and (42). The Lagrangian densities in its full blaze of glory (in the Feynman-t' Hooft gauge) can written as

$$\begin{aligned}\mathcal{L}_{(B)} = &-\frac{1}{4}\,F^{\mu\nu}\cdot F_{\mu\nu} + \frac{1}{12}\,H^{\mu\nu\eta}\cdot H_{\mu\nu\eta}\\ &+\frac{m}{4}\,\varepsilon_{\mu\nu\eta\kappa}B^{\mu\nu}\cdot F^{\eta\kappa} + \frac{1}{2}\big[B\cdot B + \bar{B}\cdot\bar{B}\big]\\ &- B\cdot(\partial_\mu A^\mu) + \big[B^\mu - g(C\times\bar{C}^\mu)\big]\cdot\big[B_\mu + D_\mu\phi\\ &- g(C\times\bar{C}_\mu) + D^\nu B_{\mu\nu}\big]\\ &+\frac{1}{2}\big[(D^\mu\bar{C}^\nu - D^\nu\bar{C}^\mu) - g(\bar{\xi}\times F^{\mu\nu})\big]\cdot\big[(D_\mu C_\nu\\ &- D_\nu C_\mu) - g(\xi\times F_{\mu\nu})\big]\\ &- \partial^\mu\bar{C}\cdot D_\mu C + D^\mu\bar{\beta}\cdot D_\mu\beta\\ &+\frac{g}{2}\big[R - g(C\times\bar{\xi})\big]\cdot(B^{\mu\nu}\times F_{\mu\nu})\\ &-\big[\bar{\lambda} - g(C\times\bar{\beta})\big]\cdot\big[\rho - g(C\times\phi) - D_\mu C^\mu\big]\\ &-\big[\rho - g(C\times\phi)\big]\cdot D_\mu\bar{C}^\mu,\end{aligned} \quad (46)$$

$$\begin{aligned}\mathcal{L}_{(\bar{B})} = &-\frac{1}{4}\,F^{\mu\nu}\cdot F_{\mu\nu} + \frac{1}{12}\,H^{\mu\nu\eta}\cdot H_{\mu\nu\eta}\\ &+\frac{m}{4}\,\varepsilon_{\mu\nu\eta\kappa}B^{\mu\nu}\cdot F^{\eta\kappa} + \frac{1}{2}\big[B\cdot B + \bar{B}\cdot\bar{B}\big]\\ &+ \bar{B}\cdot(\partial_\mu A^\mu) + \big[\bar{B}^\mu - g(\bar{C}\times C^\mu)\big]\cdot\big[\bar{B}_\mu + D_\mu\phi\\ &- g(\bar{C}\times C_\mu) - D^\nu B_{\mu\nu}\big]\\ &+\frac{1}{2}\big[(D^\mu\bar{C}^\nu - D^\nu\bar{C}^\mu) - g(\bar{\xi}\times F^{\mu\nu})\big]\cdot\big[(D_\mu C_\nu\\ &- D_\nu C_\mu) - g(\xi\times F_{\mu\nu})\big]\\ &- D^\mu\bar{C}\cdot\partial_\mu C + D^\mu\bar{\beta}\cdot D_\mu\beta\\ &-\frac{g}{2}\big[\bar{R} - g(\bar{C}\times\xi)\big]\cdot(B^{\mu\nu}\times F_{\mu\nu})\\ &-\big[\bar{\rho} - g(\bar{C}\times\phi)\big]\cdot\big[\lambda - g(\bar{C}\times\beta) + D_\mu C^\mu\big]\\ &+\big[\lambda - g(\bar{C}\times\beta)\big]\cdot D_\mu\bar{C}^\mu.\end{aligned} \quad (47)$$

These are the coupled Lagrangian densities because the pairs of the NL-type auxiliary fields $(B, \bar{B})$, $(B_\mu, \bar{B}_\mu)$, $(R, \bar{R})$, $(\lambda, \rho)$, $(\bar{\lambda}, \bar{\rho})$ are related to each other through CF-type conditions (cf. (30) and (43)). Further, the couple Lagrangian densities are equivalent because they respect (anti-)BRST symmetry transformations on constrained surface defined by CF-type conditions (see Appendix C below).

## 6 (Anti-)BRST invariance of the Lagrangian densities, nilpotency and absolute anticommutativity of the (anti-)BRST symmetries: superfield approach

It is evident from the expressions of the Lagrangian densities (44) and (45) that the (anti-)BRST invariance can now be proven in a rather simpler way. This is because of the fact that under the operation of (anti-)BRST transformations, $\mathcal{L}$ transforms to a total spacetime derivative and rest part in (44) and (45) turns out to be zero due to the nilpotency and anticommutativity properties of the (anti-)BRST transformations.

The above BRST and anti-BRST invariances of the coupled Lagrangian densities can also be discussed in the context of superfield formalism. Thus, for the sake of brevity, we generalize the Lagrangian densities on the (4, 2)D supermanifold as

$$\begin{aligned}\tilde{\mathcal{L}}_{(B)} = \tilde{\mathcal{L}} + \frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\theta}\Big[&\frac{1}{2}\,\tilde{\mathcal{A}}^{(h)}_\mu\cdot\tilde{\mathcal{A}}^{\mu(h)} + \tilde{\bar{\mathcal{F}}}^{(h)}\cdot\tilde{\mathcal{F}}^{(h)}\\ &+\frac{1}{2}\,\tilde{\Phi}^{(h)}\cdot\tilde{\Phi}^{(h)} + 2\,\tilde{\bar{\beta}}^{(h)}\cdot\tilde{\beta}^{(h)}\\ &+\tilde{\bar{\mathcal{F}}}^{(h)}_\mu\cdot\tilde{\mathcal{F}}^{\mu(h)} - \frac{1}{4}\,\tilde{\mathcal{B}}^{\mu\nu(h)}\cdot\tilde{\mathcal{B}}^{(h)}_{\mu\nu}\Big],\end{aligned} \quad (48)$$

$$\begin{aligned}\tilde{\mathcal{L}}_{(B)} = \tilde{\mathcal{L}} - \frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar{\theta}}\Big[&\frac{1}{2}\,\tilde{\mathcal{A}}^{(h)}_\mu\cdot\tilde{\mathcal{A}}^{\mu(h)} + \tilde{\bar{\mathcal{F}}}^{(h)}\cdot\tilde{\mathcal{F}}^{(h)}\\ &+\frac{1}{2}\,\tilde{\Phi}^{(h)}\cdot\tilde{\Phi}^{(h)} + 2\,\tilde{\bar{\beta}}^{(h)}\cdot\tilde{\beta}^{(h)}\\ &+\tilde{\bar{\mathcal{F}}}^{(h)}_\mu\cdot\tilde{\mathcal{F}}^{\mu(h)} - \frac{1}{4}\,\tilde{\mathcal{B}}^{\mu\nu(h)}\cdot\tilde{\mathcal{B}}^{(h)}_{\mu\nu}\Big],\end{aligned} \quad (49)$$

where the super-Lagrangian density $\tilde{\mathcal{L}}$ is given by

$$\tilde{\mathcal{L}} = -\frac{1}{4}\,\tilde{\mathcal{F}}^{(h)}_{\mu\nu}\cdot\tilde{\mathcal{F}}^{\mu\nu(h)} + \frac{1}{12}\,\tilde{\mathcal{H}}^{(h)}_{\mu\nu\eta}\cdot\tilde{\mathcal{H}}^{\mu\nu\eta(h)} + \frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}\,\tilde{\mathcal{B}}^{(h)}_{\mu\nu}\cdot\tilde{\mathcal{F}}^{(h)}_{\eta\kappa}. \quad (50)$$

By virtue of the HCs [cf. (25) and (31)], the first two terms in the super-Lagrangian density $(\tilde{\mathcal{L}})$ are independent of the

Grassmannian variables $(\theta, \bar{\theta})$. The key reason behind this is that these terms are gauge-invariant (and obviously (anti-)BRST invariant). The super-topological term in (50) can be expressed, in terms of Grassmannian variables, as

$$\begin{aligned}\frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}\mathcal{B}^{(h)}_{\mu\nu}\cdot\mathcal{F}^{(h)}_{\eta\kappa} = \frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}B_{\mu\nu}\cdot F_{\eta\kappa}\\ -\theta\,\partial_\mu\Big[\frac{m}{2}\varepsilon^{\mu\nu\eta\kappa}F_{\mu\nu}\cdot\bar{C}_\kappa\Big]\\ -\bar{\theta}\,\partial_\mu\Big[\frac{m}{2}\varepsilon^{\mu\nu\eta\kappa}F_{\mu\nu}\cdot C_\kappa\Big]\\ +\theta\bar{\theta}\,\partial_\mu\Big[\frac{m}{2}\varepsilon^{\mu\nu\eta\kappa}F_{\mu\nu}\cdot\bar{B}_\kappa\Big].\end{aligned} \quad (51)$$

The (anti-)BRST invariance of the super-topological term can be captured in the context of superfield formalism as

$$\begin{aligned}\frac{\partial}{\partial\bar{\theta}}\Big[\frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}\mathcal{B}^{(h)}_{\mu\nu}\cdot\mathcal{F}^{(h)}_{\eta\kappa}\Big]\Big|_{\theta=0} &= -\,\partial_\mu\Big[\frac{m}{2}\varepsilon^{\mu\nu\eta\kappa}F_{\mu\nu}\cdot C_\kappa\Big]\\ &= s_b\Big[\frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}B_{\mu\nu}\cdot F_{\eta\kappa}\Big],\\ \frac{\partial}{\partial\theta}\Big[\frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}\mathcal{B}^{(h)}_{\mu\nu}\cdot\mathcal{F}^{(h)}_{\eta\kappa}\Big]\Big|_{\bar{\theta}=0} &= -\,\partial_\mu\Big[\frac{m}{2}\varepsilon^{\mu\nu\eta\kappa}F_{\mu\nu}\cdot\bar{C}_\kappa\Big]\\ &= s_{ab}\Big[\frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}B_{\mu\nu}\cdot F_{\eta\kappa}\Big],\\ \frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\theta}\Big[\frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}\mathcal{B}^{(h)}_{\mu\nu}\cdot\mathcal{F}^{(h)}_{\eta\kappa}\Big] &= +\,\partial_\mu\Big[\frac{m}{2}\varepsilon^{\mu\nu\eta\kappa}F_{\mu\nu}\cdot\bar{B}_\kappa\Big]\\ &= s_b s_{ab}\Big[\frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}B_{\mu\nu}\cdot F_{\eta\kappa}\Big].\end{aligned} \quad (52)$$

Thus, under the operation of Grassmannian translational generators $\partial_{\bar{\theta}}$, $\partial_\theta$, the super-topological term remains quasi-invariant (i.e. transforms to a total spacetime derivative). This implies that the topological term remains invariant modulo a total spacetime derivative term under the operations of BRST and/or anti-BRST transformations. Consequently, the super-Lagrangian densities (48) and (49) remain invariant (up to a total spacetime derivative) under the action of Grassmannian derivatives due to the nilpotency (i.e. $\partial_{\bar{\theta}}^2 = 0$, $\partial_\theta^2 = 0$) and anticommutativity (i.e. $\partial_{\bar{\theta}}\partial_\theta + \partial_\theta\partial_{\bar{\theta}} = 0$) of the Grassmannian translation generators. This implies the (anti-)BRST invariance of the coupled Lagrangian densities within the framework of superfield formalism.

We can also capture the nilpotency and absolute anticommutativity properties of the (anti-)BRST symmetry transformations in the language of Grassmannian translational generators. Mathematically, to corroborate this statement, the following relations are true, namely;

$$\begin{aligned}\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\,\tilde{\Sigma}^{(h)}(x,\theta,\bar{\theta}) = 0 \Longleftrightarrow s_b^2\,\Sigma(x) = 0,\\ \frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta}\,\tilde{\Sigma}^{(h)}(x,\theta,\bar{\theta}) = 0 \Longleftrightarrow s_{ab}^2\,\Sigma(x) = 0,\end{aligned} \quad (53)$$

$$\begin{aligned}\Big(\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar{\theta}}\Big)\tilde{\Sigma}^{(h)}(x,\theta,\bar{\theta}) = 0\\ \Longleftrightarrow \big(s_b\,s_{ab} + s_{ab}\,s_b\big)\Sigma(x) = 0,\end{aligned} \quad (54)$$

where $\Sigma(x)$ is any generic field present in the 4D (anti-)BRST invariant theory and $\tilde{\Sigma}^{(h)}(x,\theta,\bar{\theta})$ is the corresponding superfield defined on the (4, 2)D supermanifold.

## 7 Conclusions

In our present investigation, we have exploited the superfield formalism to derive the off-shell nilpotent and absolutely anticommuting BRST as well as anti-BRST symmetry transformations corresponding to the combined "scalar" and "vector" gauge transformations for the 4D topologically massive non-Abelian gauge theory. In this approach, we have invoked the power and strength of *three* horizontality conditions in order to derive the complete set of the (anti-)BRST transformations. By using the basic tenets of BRST formalism, we have obtained the most general BRST and anti-BRST invariant Lagrangian densities (in the Feynman gauge) for the topologically massive model (cf. (46) and (47)), respectively, where the ghost number and mass dimension of the dynamical fields are taken into account.

The BRST and anti-BRST invariant Lagrangian densities are coupled but equivalent due to the very existence of *five* constrained field equations defined by CF-type conditions (cf. (30) and (43)). *Two* of them are fermionic in nature (cf. (43)). These CF conditions provide us the relations between the pairs of NL-type auxiliary fields. All CF-type conditions play very important role:

1. in the proof of anticommutativity (i.e. linear independence) of the BRST and anti-BRST transformations (cf. (A.4) and (B.5)), and
2. in the derivation of coupled (but equivalent) Lagrangian densities.

These CF conditions are (anti-)BRST invariant and, thus, they are physical restrictions on the (anti-)BRST invariant theory.

We have provided the geometrical origin of the BRST and anti-BRST symmetry transformations in the language of Grassmannian translational generators $\partial_{\bar{\theta}}$ and $\partial_\theta$, respectively. The properties of the (anti-)BRST transformations are also captured in terms of the Grassmannian translational generators. Further, by exploiting the key properties of Grassmannian translation generators, we have also captured the (anti-)BRST invariance of the coupled Lagrangian densities within the framework of superfield formalism in a simple and straightforward manner.

We have observed that the vector gauge symmetry of the Kalb–Ramond field $B^a_{\mu\nu}$ in the non-Abelian generalization of the topologically model exists due to the introduction of an auxiliary vector field $K^a_\mu$ in the Lagrangian density (18) with the expression of the field strength given in Eq. (19). From the (anti-)BRST transformations of $K^a_\mu$ as given in Eq. (42):

$$s_b K^a_\mu = (D_\mu \xi)^a - C^a_\mu - g(K_\mu \times C)^a,$$
$$s_{ab} K^a_\mu = (D_\mu \bar{\xi})^a - \bar{C}^a_\mu - g(K_\mu \times \bar{C})^a,$$

we observe that all the modes of the auxiliary field are unphysical.

We have not included matter fields in this gauge theory. Fermions can be introduced in the model via the coupling $\bar{\psi}\sigma^{\mu\nu}\psi B_{\mu\nu}$ where $\sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$. This coupling is invariant under $CP$ transformation and remains invariant under the gauge transformations

$$A_\mu \to U A_\mu U^\dagger + \frac{i}{g}(\partial^\mu U)U^\dagger,$$
$$B_{\mu\nu} \to U B_{\mu\nu} U^\dagger, \qquad \psi \to U\psi, \qquad \bar{\psi} \to \bar{\psi} U^\dagger, \qquad (55)$$

but the interaction term does not obey the vector gauge symmetry of $B_{\mu\nu}$ field. It will be interesting to see how the interaction $\bar{\psi}\sigma^{\mu\nu}\psi B_{\mu\nu}$ contribute to the chromomagnetic moment and mass renormalization of quarks in QCD. We can also think of modification of the interaction as $\bar{\psi}\sigma^{\mu\nu}\psi\big[B_{\mu\nu} - (D_\mu K_\nu - D_\nu K_\mu)\big]$ to get an interaction term remained invariant under the vector gauge symmetry of $B^{\mu\nu}$ field. In the both cases, we should see how those interactions contribute to the beta function in the non-Abelian gauge theory because the interaction terms are new with respect to existing literature . We do not know from our present knowledge how the chiral symmetry of fermion field can be broken in this model. It should also be part in the investigation how mass of gluon is renormalized in this topologically massive model.

**Acknowledgements** RK would like to thank UGC, Government of India, New Delhi, for financial support under the PDFSS scheme.



## Appendix A: Determination of various secondary fields

Exploiting the first HC (25) for the superfields (24), we obtain the values of secondary fields in terms of the dynamical and auxiliary fields of the 4D (anti-)BRST invariant theory. These are listed as follows:

$$\begin{aligned}
&R_\mu = D_\mu C, \quad \bar{R}_\mu = D_\mu \bar{C}, \quad B_1 = \frac{g}{2}(C \times C),\\
&s = -g(\bar{B}_1 \times C), \quad \bar{s} = g(B_2 \times \bar{C}),\\
&\bar{B}_2 = \frac{g}{2}(\bar{C} \times \bar{C}), \quad \bar{B}_1 + B_2 - g(C \times \bar{C}) = 0,\\
&S_\mu = D_\mu B_2 - g(D_\mu C \times \bar{C})\\
&\quad \equiv -D_\mu \bar{B}_1 + g(C \times D_\mu \bar{C}).
\end{aligned} \qquad (A.1)$$

The second relation in the third line of the above equation is nothing but the well-known CF condition. It is the *hallmark* of non-Abelian 1-form gauge theory and emerged very naturally within the framework of superfield approach to BRST formalism.

Using the second HC (31) together with (26) and (27), we obtain the values for the secondary fields for the expansions of superfields (34), namely;

$$\begin{aligned}
&\bar{f}_3 = g(\bar{C} \times \bar{\beta}), \quad R_1 = -\beta - g(C \times \xi),\\
&\bar{R}_2 = -\bar{\beta} + g(\bar{C} \times \bar{\xi}), \quad f_2 = g(C \times \beta),\\
&\bar{f}_1 + f_3 - g(\bar{C} \times \phi) - g(C \times \bar{\beta}) = 0,\\
&f_1 + \bar{f}_2 - g(C \times \phi) - g(\bar{C} \times \beta) = 0,\\
&S_1 = g(C \times \bar{R}_1) - g(\bar{B} \times \xi) + \bar{f}_2,\\
&S_2 = g(\bar{C} \times R_2) - g(B \times \bar{\xi}) - f_3,\\
&\bar{b}^{(1)}_\mu + b^{(2)}_\mu + D_\mu \phi - g(\bar{C} \times C_\mu) - g(C \times \bar{C}_\mu) = 0,\\
&b_1 = g(C \times \bar{f}_1) - g(\bar{B} \times \phi) + g(\bar{C} \times \bar{f}_2)\\
&\quad - g^2\big(\bar{C} \times (\bar{C} \times \beta)\big)\\
&\quad \equiv -g(\bar{C} \times f_1) + g(B \times \phi) - g(C \times f_3)\\
&\quad + g^2\big(C \times (C \times \bar{\beta})\big),\\
&b_2 = g(C \times \bar{f}_2) - g(\bar{B} \times \beta)\\
&\quad \equiv g(B \times \beta) - g(C \times f_1) - g(\bar{C} \times f_2)\\
&\quad + g^2\big(C \times (C \times \phi)\big),\\
&b_3 = g(B \times \bar{\beta}) - g(\bar{C} \times f_3)\\
&\quad \equiv g(\bar{C} \times \bar{f}_1) + g(C \times \bar{f}_3) - g(\bar{B} \times \bar{\beta})\\
&\quad - g^2\big(\bar{C} \times (\bar{C} \times \phi)\big),\\
&b^{(1)}_\mu = -D_\mu \beta + g(C \times C_\mu),\\
&\bar{b}^{(2)}_\mu = -D_\mu \bar{\beta} + g(\bar{C} \times \bar{C}_\mu),\\
&q_\mu = D_\mu \bar{f}_2 - g(D_\mu \bar{C} \times \beta) + g(C \times \bar{b}^{(1)}_\mu) - g(\bar{B} \times C_\mu),\\
&\bar{q}_\mu = -D_\mu f_3 + g(D_\mu C \times \bar{\beta}) - g(\bar{C} \times b^{(2)}_\mu) + g(B \times \bar{C}_\mu),\\
&R_{\mu\nu} = -(D_\mu C_\nu - D_\nu C_\mu) + g(C \times B_{\mu\nu}) + g(\xi \times F_{\mu\nu}),\\
&\bar{R}_{\mu\nu} = -(D_\mu \bar{C}_\nu - D_\nu \bar{C}_\mu) + g(\bar{C} \times B_{\mu\nu}) + g(\bar{\xi} \times F_{\mu\nu}),\\
&S_{\mu\nu} = g(B \times B_{\mu\nu}) - (D_\mu B_\nu - D_\nu B_\mu) + g(R_2 \times F_{\mu\nu})\\
&\quad + g(D_\mu C \times \bar{C}_\nu) - g(D_\nu C \times \bar{C}_\mu)\\
&\quad + g^2\big(\bar{\xi} \times (F_{\mu\nu} \times C)\big) + g\bar{C} \times \big((D_\mu C_\nu - D_\nu C_\mu)\\
&\quad - g(C \times B_{\mu\nu}) - g(\xi \times F_{\mu\nu})\big)\\
&\quad \equiv (D_\mu \bar{B}_\nu - D_\nu \bar{B}_\mu) - g(\bar{B} \times B_{\mu\nu}) - g(\bar{R}_1 \times F_{\mu\nu})\\
&\quad + g(D_\nu \bar{C} \times C_\mu) - g(D_\mu \bar{C} \times C_\nu)\\
&\quad - g^2\big(\xi \times (F_{\mu\nu} \times \bar{C})\big) - gC \times \big((D_\mu C_\nu - D_\nu C_\mu)
\end{aligned}$$

$$- g(C \times B_{\mu\nu}) - g(\xi \times F_{\mu\nu})\big). \quad \text{(A.2)}$$

It is to be noted that the third and seventh equations in the above are the CF-type conditions. These constrained field equations emerge naturally when we set the coefficients of the wedge products $(d\theta \wedge d\bar{\theta} \wedge d\bar{\theta})$, $(d\theta \wedge d\theta \wedge d\bar{\theta})$ and $(dx^\mu \wedge d\theta \wedge d\theta)$ equal to zero due to the HC (31).

Similarly, the third HC (40) produces the precise values of the remaining secondary fields as

$$\begin{aligned}
&P_\mu = D_\mu \xi - C_\mu - g(K_\mu \times C),\\
&\bar{P}_\mu = D_\mu \bar{\xi} - \bar{C}_\mu - g(K_\mu \times \bar{C}),\\
&\bar{R} + R + \phi - g(\bar{C} \times \xi) - g(C \times \bar{\xi}) = 0,\\
&Q_\mu = D_\mu R - g(D_\mu C \times \bar{\xi}) + g(B \times K_\mu) - B_\mu\\
&\quad -g\big(\bar{C} \times (D_\mu \xi - C_\mu - g(K_\mu \times C))\big)\\
&\quad \equiv -D_\mu \bar{R} + g(D_\mu \bar{C} \times \xi) - g(\bar{B} \times K_\mu) + \bar{B}_\mu\\
&\quad +g\big(C \times (D_\mu \bar{\xi} - \bar{C}_\mu - g(K_\mu \times \bar{C}))\big).
\end{aligned} \quad \text{(A.3)}$$

The field equation in the third line of the above equation is also the CF-type condition and it emerges naturally from the coefficient of Grassmannian differentials $(d\theta \wedge d\bar{\theta})$ in Eq. (40).

## Appendix B: Absolute anticommutativity property of (anti-)BRST transformations

It is well-known that BRST and anti-BRST transformations by construction are off-shell nilpotent and absolutely anticommuting. The latter property is satisfied due to the existence of five CF-type conditions. The anticommutator of the BRST and anti-BRST transformations for the gauge field $A_\mu$ can be written as

$$\{s_b,\ s_{ab}\} A_\mu = D_\mu \big[B + \bar{B} - g\big(C \times \bar{C}\big)\big]. \quad \text{(A.4)}$$

Thus, it is clear that $\{s_b,\ s_{ab}\} A_\mu = 0$ on the constraint hypersurface defined by CF condition: $B + \bar{B} - g\big(C \times \bar{C}\big) = 0$. Similarly, for the sake of completeness, we note that the followings are true:

$$\begin{aligned}
\{s_b,\ s_{ab}\} B_{\mu\nu} &= g[B + \bar{B} - g(C \times \bar{C})] \times B_{\mu\nu}\\
&\quad + g[R + \bar{R} - g(C \times \bar{\xi}) - g(\bar{C} \times \xi)] \times F_{\mu\nu}\\
&\quad - D_\mu[B_\nu + \bar{B}_\nu - g(\bar{C} \times C_\nu) - g(\bar{C} \times \bar{C}_\nu)]\\
&\quad + D_\nu[B_\mu + \bar{B}_\mu - g(\bar{C} \times C_\mu) - g(\bar{C} \times \bar{C}_\mu)],\\
\{s_b,\ s_{ab}\} C_\mu &= g[B + \bar{B} - g(C \times \bar{C})] \times C_\mu\\
&\quad - D_\mu[\rho + \lambda - g(C \times \phi) - g(\bar{C} \times \beta)],\\
\{s_b,\ s_{ab}\} \bar{C}_\mu &= g[B + \bar{B} - g(C \times \bar{C})] \times \bar{C}_\mu\\
&\quad - D_\mu[\bar{\rho} + \bar{\lambda} - g(\bar{C} \times \phi) - g(C \times \bar{\beta})],\\
\{s_b,\ s_{ab}\} \rho &= g\big[B + \bar{B} - g(C \times \bar{C})\big] \times \big(\rho - g(C \times \phi)\big)\\
&\quad - g^2\big(\big[B + \bar{B} - g(C \times \bar{C})\big] \times \phi\big) \times C,\\
\{s_b,\ s_{ab}\} \bar{\rho} &= g\big[B + \bar{B} - g(C \times \bar{C})\big] \times \big(\bar{\rho} - g(\bar{C} \times \phi)\big)\\
&\quad - g^2\big(\big[B + \bar{B} - g(C \times \bar{C})\big] \times \phi\big) \times \bar{C},\\
\{s_b,\ s_{ab}\} \phi &= g\big[B + \bar{B} + i(C \times \bar{C})\big] \times \phi,\\
\{s_b,\ s_{ab}\} B_\mu &= g\big[B + \bar{B} - g(C \times \bar{C})\big] \times B_\mu\\
&\quad - g D_\mu\big[\bar{\rho} + \bar{\lambda} - g(C \times \bar{\beta}) - g(\bar{C} \times \phi)\big] \times C\\
&\quad + g^2\big(\big[B + \bar{B} - g(C \times \bar{C})\big] \times C\big) \times \bar{C}_\mu,\\
\{s_b,\ s_{ab}\} \bar{B}_\mu &= g\big[B + \bar{B} - g(C \times \bar{C})\big] \times \bar{B}_\mu\\
&\quad - g D_\mu\big[\rho + \lambda - g(\bar{C} \times \beta) - g(C \times \phi)\big] \times \bar{C}\\
&\quad + g^2\big(\big[B + \bar{B} - g(C \times \bar{C})\big] \times \bar{C}\big) \times C_\mu,\\
\{s_b,\ s_{ab}\} K_\mu &= g\big[B + \bar{B} - g(C \times \bar{C})\big] \times K_\mu\\
&\quad + D_\mu\big[R + \bar{R} + \phi - g(C \times \bar{\xi}) - g(\bar{C} \times \xi)\big]\\
&\quad - \big[B_\mu + \bar{B}_\mu + D_\mu \phi\\
&\quad - g(C \times \bar{C}_\mu) - g(\bar{C} \times C_\mu)\big].
\end{aligned} \quad \text{(B.5)}$$

Thus, the anticommutativity property of the BRST and anti-BRST transformations for the fields $A_\mu$, $B_{\mu\nu}$, $C_\mu$, $\bar{C}_\mu$, $\rho$, $\bar{\rho}$, $\phi$, $B_\mu$, $\bar{B}_\mu$, $K_\mu$ is satisfied only on the constrained hypersurface defined by the CF-type conditions. For remaining fields (i.e. $\beta$, $\bar{\beta}$, $\xi$, $\bar{\xi}$, $\lambda$, $\bar{\lambda}$, $R$, $\bar{R}$) this property is trivially satisfied. We again emphasize that all five CF-type conditions play an important role in providing the anticommutativity of the (anti-)BRST transformations and also responsible for the coupled (but equivalent) Lagrangian densities.

## Appendix C: (Anti-)BRST invariance of coupled Lagrangian densities

The Lagrangian densities $\mathcal{L}_{(B)}$ respects the BRST symmetry transformations, as one check that it remains quasi-invariant. To be more precise, $\mathcal{L}_{(B)}$ transforms to a total spacetime derivative under the BRST transformations as follows

$$\begin{aligned}
s_b \mathcal{L}_{(B)} = -\partial_\mu \Big[ & B \cdot D^\mu C + \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} F_{\nu\eta} \cdot C_\kappa\\
& - \big(B^\mu - g(C \times \bar{C}^\mu)\big) \cdot \big(\rho - g(C \times \phi)\big)\\
& - \big(B_\nu - g(C \times \bar{C}_\nu)\big) \cdot \big(D^\mu C^\nu - D^\nu C^\mu\\
& - g(\xi \times F^{\mu\nu})\big) - \big(\bar{\lambda} - g(C \times \bar{\beta})\big) \cdot D^\mu \beta \Big].
\end{aligned} \quad \text{(C.6)}$$

As a consequence, the action integral remains invariant (i.e. $s_b \int d^4x \mathcal{L}_{(B)} = 0$) due to Gauss divergence theorem. It is interesting to note that under the anti-BRST symmetry transformations $\mathcal{L}_{(B)}$ transforms to a total spacetime derivative plus some additional terms

$$
\begin{aligned}
s_{ab}\mathcal{L}_B = -\partial_\mu\Big[& B\cdot\partial^\mu\bar{C} + \frac{m}{2}\,\varepsilon^{\mu\nu\eta\kappa}\,F_{\nu\eta}\cdot\bar{C}_\kappa \\
&- \big(B_\nu - g(C\times\bar{C}_\nu)\big)\cdot\big(D^\mu\bar{C}^\nu - D^\nu\bar{C}^\mu \\
&- g(\bar{\xi}\times F^{\mu\nu})\big) \\
&+ \big(\bar{\rho} - g(\bar{C}\times\phi)\big)\cdot\Big(B^\mu + D^\mu\phi - g(C\times\bar{C}^\mu) \\
&+ D_\nu B^{\mu\nu}\Big) - \big(\lambda - g(\bar{C}\times\beta)\big)\cdot D^\mu\bar{\beta}\Big] \\
&+ D_\mu\big[\big(B + \bar{B} - g(C\times\bar{C})\big)\cdot\partial^\mu\bar{C}\big] \\
&- \frac{g^2}{2}\big[\big(B + \bar{B} - g(C\times\bar{C})\big)\times\bar{\xi}\big]\cdot(B^{\mu\nu}\times F^{\mu\nu}) \\
&+ g\big[\big(B + \bar{B} - g(C\times\bar{C})\big)\times\bar{\beta}\big]\cdot\big[\rho - g(C\times\phi) \\
&- D_\mu C^\mu\big] \\
&- \big[\bar{\lambda} - g(C\times\bar{\beta})\big]\cdot D_\mu\big[B^\mu + \bar{B}^\mu + D^\mu\phi \\
&- g(C\times\bar{C}^\mu) - g(\bar{C}\times C^\mu)\big] \\
&- D_\mu\big[B_\nu + \bar{B}_\nu + D_\nu\phi - g(C\times\bar{C}_\nu) \\
&- g(\bar{C}\times C_\nu)\big]\cdot\big[D^\mu\bar{C}^\nu - D^\nu\bar{C}^\mu - g(\bar{\xi}\times F^{\mu\nu})\big] \\
&- \frac{g}{2}\big[R + \bar{R} + \phi - g(C\times\bar{\xi}) - g(\bar{C}\times\xi)\big] \\
&\cdot\big[\big(D^\mu\bar{C}^\nu - D^\nu\bar{C}^\mu - g(\bar{\xi}\times F^{\mu\nu})\big)\times F_{\mu\nu}\big] \\
&- \big[\lambda + \rho - g(C\times\phi) - g(\bar{C}\times\beta)\big]\cdot D_\mu(D^\mu\bar{\beta}) \\
&+ \frac{g}{2}\big[\bar{\rho} + \bar{\lambda} - g(\bar{C}\times\phi) \\
&- g(C\times\bar{\beta})\big]\cdot(B^{\mu\nu}\times F^{\mu\nu}). \qquad \text{(C.7)}
\end{aligned}
$$

Due to the validity of CF conditions, all the extra terms, except total derivative term, vanish. Thus, $\mathcal{L}_{(B)}$ also respects the anti-BRST transformations on the constrained hypersurfaces defined by CF conditions (30) and (43).

In a similar fashion, the anti-BRST transformations leave $\mathcal{L}_{(\bar{B})}$ to a total spacetime derivative

$$
\begin{aligned}
s_{ab}\mathcal{L}_{(\bar{B})} = \partial_\mu\Big[& \bar{B}\cdot D^\mu\bar{C} - \frac{m}{2}\,\varepsilon^{\mu\nu\eta\kappa}\,F_{\nu\eta}\cdot\bar{C}_\kappa \\
&- \big(\bar{B}^\mu - g(\bar{C}\times C^\mu)\big)\cdot\big(\bar{\rho} - g(\bar{C}\times\phi)\big) \\
&- \big(\bar{B}_\nu - g(\bar{C}\times C_\nu)\big)\cdot\big(D^\mu\bar{C}^\nu - D^\nu\bar{C}^\mu \\
&- g(\bar{\xi}\times F^{\mu\nu})\big) + \big(\lambda - g(\bar{C}\times\beta)\big)\cdot D^\mu\bar{\beta}\Big]. \qquad \text{(C.8)}
\end{aligned}
$$

Thus, $\mathcal{L}_{(\bar{B})}$ respects off-shell nilpotent anti-BRST symmetry transformations. It is to be noted that under the BRST transformations $\mathcal{L}_{\bar{B}}$ transforms in the following fashion:

$$
\begin{aligned}
s_b\mathcal{L}_{\bar{B}} = -\partial_\mu\Big[& -\bar{B}\cdot\partial^\mu C + \frac{m}{2}\,\varepsilon^{\mu\nu\eta\kappa}\,F_{\nu\eta}\cdot C_\kappa \\
&+ \big(\bar{B}_\nu - g(\bar{C}\times C_\nu)\big)\cdot\big(D^\mu C^\nu - D^\nu C^\mu \\
&- g(\xi\times F^{\mu\nu})\big) \\
&+ \big(\rho - g(C\times\phi)\big)\cdot\Big(\bar{B}^\mu + D^\mu\phi - g(C\times\bar{C}^\mu) \\
&- D_\nu B^{\mu\nu}\Big) - \big(\bar{\lambda} - g(C\times\bar{\beta})\big)\cdot D^\mu\beta\Big] \\
&- D_\mu\big[\big(B + \bar{B} - g(C\times\bar{C})\big)\cdot\partial^\mu C\big] \\
&+ \frac{g^2}{2}\big[\big(B + \bar{B} - g(C\times\bar{C})\big)\times\xi\big]\cdot(B^{\mu\nu}\times F^{\mu\nu}) \\
&- g\big[\big(B + \bar{B} - g(C\times\bar{C})\big)\times\beta\big]\cdot\big[\bar{\rho} - g(\bar{C}\times\phi) \\
&+ D_\mu\bar{C}^\mu\big] \\
&- \big[\lambda - g(\bar{C}\times\beta)\big]\cdot D_\mu\big[B^\mu + \bar{B}^\mu + D^\mu\phi \\
&- g(C\times\bar{C}^\mu) - g(\bar{C}\times C^\mu)\big] + D_\mu\big[B_\nu + \bar{B}_\nu \\
&+ D_\nu\phi - g(C\times\bar{C}_\nu) \\
&- g(\bar{C}\times C_\nu)\big]\cdot\big[D^\mu C^\nu - D^\nu C^\mu - g(\xi\times F^{\mu\nu})\big] \\
&+ \frac{g}{2}\big[R + \bar{R} + \phi - g(C\times\bar{\xi}) - g(\bar{C}\times\xi)\big]\cdot\big[\big(D^\mu C^\nu \\
&- D^\nu C^\mu - g(\xi\times F^{\mu\nu})\big)\times F_{\mu\nu}\big] \\
&- \big[\bar{\lambda} + \bar{\rho} - g(\bar{C}\times\phi) - g(C\times\bar{\beta})\big]\cdot D_\mu(D^\mu\beta) \\
&- \frac{g}{2}\big[\rho + \lambda - g(C\times\phi) \\
&- g(\bar{C}\times\beta)\big]\cdot(B^{\mu\nu}\times F^{\mu\nu}). \qquad \text{(C.9)}
\end{aligned}
$$

It is clear that Lagrangian density $\mathcal{L}_{(\bar{B})}$ also respects the BRST symmetry transformations due to the validity of CF-type conditions. As a consequence, the coupled Lagrangian densities respect BRST and anti-BRST symmetries on the constrained hypersurface defined by the CF-type conditions. This shows that the coupled Lagrangian densities are equivalent on the constrained hypersurface.

## References


1. S. Weinberg, *The Quantum Theory of Fields*, vol. 2 (Cambridge University Press, New York, 1996)
2. C. Becchi, A. Rouet, R. Stora, Phys. Lett. B **52**, 344 (1974)
3. C. Becchi, A. Rouet, R. Stora, Commun. Math. Phys. **42**, 127 (1975)
4. C. Becchi, A. Rouet, R. Stora, Ann. Phys. (N. Y.) **98**, 287 (1976)
5. I.V. Tyutin, Lebedev Institute Preprint (Report No.: FIAN-39, 1975). arXiv:0812.0580 [hep-th]
6. T. Kugo, I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979)
7. G. Curci, R. Ferrari, Phys. Lett. B **63**, 91 (1976)
8. I. Ojima, Prog. Theor. Phys. **64**, 625 (1980)
9. R.P. Malik, Eur. Phys. J. C **55**, 687 (2008)
10. S. Krishna, A. Shukla, R.P. Malik, Int. J. Mod. Phys. A **26**, 4419 (2011)
11. R. Kumar, S. Krishna, A. Shukla, R.P. Malik, Eur. Phys. J. C **72**, 1980 (2012)
12. L. Bonora, M. Tonin, Phys. Lett. B **98**, 48 (1981)
13. R.P. Malik, Eur. Phys. J. C **60**, 457 (2009)
14. L. Bonora, P. Pasti, M. Tonin, Nuovo Cim. A **63**, 353 (1981)
15. R. Delbourgo, P.D. Jarvis, J. Phys. A Math. Gen. **15**, 611 (1981)
16. J. Thierry-Mieg, J. Math. Phys. **21**, 2834 (1980)
17. L. Baulieu, J. Thierry-Mieg, Nucl. Phys. B **197**, 477 (1982)
18. J. Hoyos, M. Quiros, J. Ramirez Mittelbrunn, F.J. de Urries, J. Math. Phys. **23**, 1504 (1982)
19. J. Hoyos, M. Quiros, J. Ramirez Mittelbrunn, F.J. de Urries, Nucl. Phys. B **218**, 159 (1983)
20. P. Cotta Ramusinio, C. Reina, J. Geom. Phys. **1**, 121 (1984)
21. L. Baulieu, Phys. Rep. **129**, 1 (1985)
22. J. Manes, R. Stora, B. Zumino, Commun. Math. Phys. **102**, 157 (1985)
23. L. Bonora, P. Cotta Ramusino, M. Rinaldi, J. Stasheff, Commun. Math. Phys. **112**, 237 (1987)

24. R.A. Bertlmann, *Anomalies in Quantum Field theory* (Oxford University Press, Oxford, 1996)
25. J. Wess, B. Zumino, Phys. Lett. B. **37**, 95 (1971)
26. A. Bilal, arXiv:0802.0634 [hep-th]
27. R.P. Malik, J. Phys. A Math. Gen. **37**, 5261 (2004)
28. A. Shukla, S. Krishna, R.P. Malik, Adv. High Energy Phys. **2015**, 258536 (2015)
29. R. Kumar, S. Krishna, Eur. Phys. J. C **77**, 387 (2017)
30. M.B. Green, J.H. Schwarz, E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, 1987)
31. J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, 1998)
32. A. Salam, E. Sezgin, *Supergravities in Diverse Dimensions* (World Scientific, Singapore, 1989)
33. S. Deser, Phys. Rev. **187**, 1931 (1969)
34. A. Aurilia, Y. Takahashi, Prog. Theor. Phys. **66**, 693 (1981)
35. N. Seiberg, E. Witten, JHEP **9909**, 032 (1999)
36. S. Gupta, R.P. Malik, Eur. Phys. J. C **58**, 517 (2008)
37. R.P. Malik, J. Phys. A Math. Gen. **36**, 5095 (2003)
38. E. Cremmer, J. Sherk, Nucl. Phys. B **72**, 117 (1974)
39. T.J. Allen, M.J. Bowick, A. Lahiri, Phys. Lett. B **237**, 47 (1990)
40. T.J. Allen, M.J. Bowick, A. Lahiri, Mod. Phys. Lett. A **6**, 559 (1991)
41. J. Thierry-Mieg, L. Baulieu, Nucl. Phys. B **228**, 259 (1981)
42. I. Oda, S. Yahikozawa, Phys. Lett. B **234**, 69 (1990)
43. E. Harikumar, A. Lahiri, M. Sivakumar, Phys. Rev. D **63**, 105002 (2001)
44. A. Lahiri, Phys. Rev. D **63**, 105020 (2001)
45. M. Kalb, P. Ramond, Phys. Rev. D **9**, 2273 (1973)
46. S.M. Carroll, G.B. Field, R. Jackiw, Phys. Rev. D **41**, 1231 (1989)
47. D. Mukhopadhyay, A. Lahiri, Phys. Rev. D **90**, 025015 (2014)
48. S. Gupta, R. Kumar, R.P. Malik, Eur. Phys. J. C **70**, 491 (2010)
49. R. Kumar, R.P. Malik, Eur. Phys. Lett. **94**, 11001 (2011)
50. C.N. Yang, R.L. Mills, Phys. Rev. **95**, 631 (1954)
51. J.S. Bell, Nucl. Phys. B **60**, 427 (1973)
52. C.H. Llewellyn Smith, Phys. Lett. B **46**, 233 (1973)
53. S. Joglekar, Ann. Phys. **83**, 427 (1973)
54. B. Lee, C. Quigg, H.B. Thacker, Phys. Rev. D **16**, 1519 (1977)
55. P.W. Higgs, Phys. Rev. Lett. **13**, 508 (1964)
56. F. Englert, R. Brout, Phys. Rev. Lett. **13**, 321 (1964)
57. S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967)
58. A. Salam, Weak and electromagnetic interactions, in *Elementary Particle Physics: Relativistic Groups and Analyticity*, ed. by N. Svartholm (Almqvist and Wiksell, Sweden, 1968)
59. S. Glashow, Nucl. Phys. **22**, 579 (1961)
60. S. Weinberg, *The Quantum Theory of Fields*, vol. 1 (Cambridge University Press, New York, 1995)
61. M. Daniel, C.M. Viallet, Rev. Mod. Phys. **52**, 1 (1980)
62. D. Bleecker, *Gauge Theory and Variational Principles* (Dover, New York, 1981)
63. M. Spivak, *A Comprehensive Introduction to Differential Geometry*, vol. 2 (Publish or Perish, Houston, 1999)
64. H. Umezawa, S. Kamefuchi, Nucl. Phys. **23**, 399 (1961)
65. M. Veltman, Nucl. Phys. B **7**, 637 (1968)
66. M. Veltman, Nucl. Phys. B **21**, 288 (1970)
67. Ken-ichi Sizuya, Nucl. Phys. B **121**, 125 (1977)
68. H. Ruegg, M. Ruiz-Altaba, Int. J. Mod. Phys. A **19**, 3265 (2004)
69. G. Curci, R. Ferrari, Nuovo Cim. A **32**, 151 (1975)
70. I. Ojima, Z. Phys. C **13**, 173 (1982)
71. J. de Boer, K. Skenderis, P. van Nieuwenhuizen, A. Waldron, Phys. Lett. B **367**, 175 (1996)
72. J.M. Cornwall, Phys. Rev. D **26**, 1453 (1982)