3-D Finite element method for analyzing magnetic fields in electrical machines excited from voltage sources

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3-D FINITE ELEMENT METHOD FOR ANALYZING MAGNETIC FIELDS IN ELECTRICAL MACHINES EXCITED FROM VOLTAGE SOURCES

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ABSTRACT

A new method for analyzing 3-D magnetic fields and currents in electrical machines excited from voltage sources using the $A-\phi$ method has been developed.

The basic idea and the finite element formulation of the method are described. The effectiveness of the method is shown by some examples of application.

1. INTRODUCTION

When magnetic circuits are analyzed using Poisson's equation, the magnetizing current densities must be given. As electrical machines are usually excited from voltage sources, the magnetizing current densities are unknown. Therefore, it is difficult to analyze magnetic fields in such machines using the conventional finite element method.

The 2-D finite element method for analyzing such machines has already been developed[1]. In this method, the loop equations obtained from Kirchhoff's second law are combined with Maxwell's equations for the magnetic field analysis, and both magnetic fields and currents can be directly calculated.

In this paper, the method is expanded into 3-D analysis using the $A-\phi$ method[2]. The technique for symmetrizing the coefficient matrix in the finite element formulation is also investigated[3] so that the ICCG method[4] can be introduced as a solver of linear equations. The validity of the method is examined by analyzing the magnetizing current in an infinite solenoid. The 2-D method is also applicable to this model. As an example of 3-D application, the currents in the primary and the secondary windings of a loaded transformer are analyzed.

2. METHOD OF ANALYSIS

2.1 Fundamental Equations

3-D magnetic fields with eddy currents are governed by the following partial differential equations[2]:

$$\mathrm{rot}(\nu\,\mathrm{rot}\mathbf{A}) = \mathbf{J}_o - \sigma\left(\frac{\partial \mathbf{A}}{\partial t} + \mathrm{grad}\,\phi\right) \tag{1}$$

$$\mathrm{div}\left\{-\sigma\left(\frac{\partial \mathbf{A}}{\partial t} + \mathrm{grad}\,\phi\right)\right\} = 0 \tag{2}$$

where $\mathbf{A}$ and ϕ are the magnetic vector potential and the electric scalar potential respectively. $\mathbf{J}_o$ is the magnetizing current density. ν and σ are the reluctivity and the conductivity respectively.

The following equations can be obtained by Galerkin's method from Eqs.(1) and (2)[2].

$$\begin{aligned} \mathbf{G}_{oi} = & -\iiint_{\Omega} \mathrm{grad} N_i \times (\nu\,\mathrm{rot}\mathbf{A})\,dV \\ & + \iiint_{\Omega e} N_i \sigma\left(\frac{\partial \mathbf{A}}{\partial t} + \mathrm{grad}\,\phi\right) dV \\ & - \iiint_{\Omega c} N_i \mathbf{J}_o\, dV = \mathbf{0} \end{aligned} \tag{3}$$

$$G_{di} = \iiint_{\Omega e} \mathrm{grad} N_i \cdot \sigma\left(\frac{\partial \mathbf{A}}{\partial t} + \mathrm{grad}\,\phi\right) dV = 0 \tag{4}$$

where N_i is the interpolation function[5]. Ω denotes the analyzed region. Ωe and Ωc are the region of the conductors with eddy currents and that of the windings respectively.

Although the magnetizing current density $\mathbf{J}_o$ has only one component in 2-D analysis, $\mathbf{J}_o$ has three components J_{ox}, J_{oy} and J_{oz} in 3-D analysis, and these components change with the position in the exciting winding. If these three components are treated as unknown variables, the number of unknown variables is increased above that of equations[1] which are obtained from loop equations derived from Kirchhoff's second law and Maxwell's equations. Because each element in the winding has different values of J_{ox}, J_{oy} and J_{oz}. This is the cause for the difficulty in 3-D analysis. In many cases, however, the magnetizing current in the winding flows uniformly. Therefore, if the magnetizing current densities, of which the amplitudes can be assumed to be all the same everywhere in a winding, are denoted by one unknown variable, the analysis becomes possible, because the number of unknown variables is equal to that of the equations.

For easy understanding, using a simple model of the winding shown in Fig.1, the method for reducing the number of unknown variables of $\mathbf{J}_o$ is explained in more detail. It is assumed that the magnetizing current flows in the x-y plane, and the z-component of the current can be neglected. If the sign of the current flowing anti-clockwise around the z-axis is positive, the magnetizing current density $\mathbf{J}_o$ is represented as follows:

$$\mathbf{J}_o = J_o(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) \tag{5}$$

where θ is the angle from the x-axis. $\mathbf{i}$ and $\mathbf{j}$ are the x- and y-directional unit vectors respectively. The amplitude I_o of the magnetizing current is represented as follows:

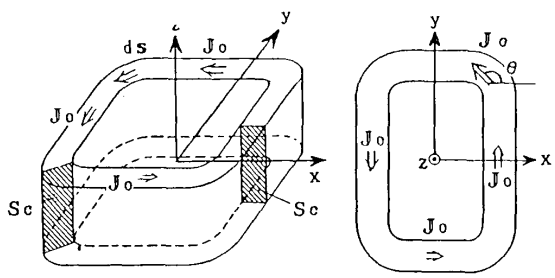
$$I_o = \frac{S_c J_o}{n_c} \tag{6}$$

where S_c and n_c are the cross-sectional area and the number of turns of the winding shown in Fig.1 respectively.

From Eqs.(5) and (6), the third term $\mathbf{G}_{vi}$ of the mid side of Eq.(3) becomes as follows:

$$\mathbf{G}_{vi} = -\frac{n_c}{S_c}\iiint_{\Omega c} N_i I_o(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta)\,dV \tag{7}$$

In Eq.(7), the unknown variable is only the amplitude I_o of the magnetizing current.



(a) bird's-eye view (b) plane view
Fig.1 Winding.

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2.2 Relationship among Magnetic Vector Potentials, Magnetizing Currents and Applied Voltages

Figure 2 shows an equivalent circuit of a machine excited from a voltage source. The finite element region which is enclosed by the broken line in Fig.2 corresponds to the winding shown in Fig.1. R_c is the dc resistance of the winding. R_o and L_o are the resistance and the inductance which cannot be included in the finite element region. The following equation can be obtained from Kirchhoff's second law:

$$\eta = V_o - \frac{d\Psi}{dt} - (R_o + R_c) I_o - L_o \frac{d I_o}{dt} = 0 \qquad (8)$$

where Ψ is the interlinkage flux of the winding. By using the magnetic vector potential $\mathbf{A}$, Ψ can be rewritten as follows:

$$\Psi = \frac{n_c}{S_c} \iint_S \left(\int \mathbf{A} \cdot d\mathbf{s} \right) dS \qquad (9)$$

where $\mathbf{s}$ is an unit tangent vector along the winding shown in Fig.1. S means the cross section of the winding.

Ψ can be determined from the x- and y-components A_x and A_y of $\mathbf{A}$ in the winding as follows:

$$\Psi = \frac{n_c}{S_c} \iiint_{\Omega c} (A_x \cos\theta + A_y \sin\theta) dV \qquad (10)$$

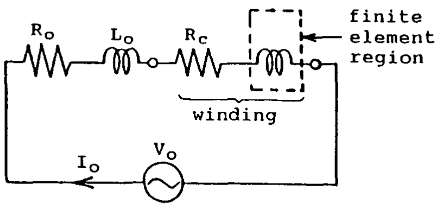


Fig.2 Equivalent circuit.

2.3 Discretization

In the nonlinear analysis using the Newton-Raphson iteration technique, the increments of the unknown variables δA_{xj}, δA_{yj}, δA_{zj}, $\delta \phi_j$ and δI_o at the instant t are obtained from the following equation[1]:

$$\begin{bmatrix} \left[\dfrac{\partial G_i}{\partial u_j}\right] & \left\{\dfrac{\partial G_i}{\partial I_o}\right\} \\ \left\{\dfrac{\partial \eta}{\partial u_j}\right\} & \dfrac{\partial \eta}{\partial I_o} \end{bmatrix} \begin{bmatrix} \{\delta u_j\} \\ \delta I_o \end{bmatrix} = \begin{bmatrix} -\{G_i\} \\ -\eta \end{bmatrix}$$

$$(i, j = 1, 2, \cdots, nu) \qquad (11)$$

where nu is the number of unknown nodes at which the potentials are unknown. $\{G_i\}$ and $\{u_j\}$ are denoted as follows:

$$\{G_i\} = \{G_{oxi},\ G_{oyi},\ G_{ozi},\ G_{di}\}^T \qquad (12)$$

$$\{u_j\} = \{A_{xj},\ A_{yj},\ A_{zj},\ \phi_j\}^T \qquad (13)$$

As $[\partial G_i / \partial u_j]$ in Eq.(11) is the same as that of the conventional finite element method[2] in which the magnetizing current is given, let us calculate other coefficients.

When the first-order tetrahedral elements are used, G_{vi} is discretized from Eq.(7) as follows:

$$G_{vi} = -\frac{n_c I_o}{4 S_c} \sum_{e=1}^{N_c} V^{(e)} (\mathbf{i} \cos\theta^{(e)} + \mathbf{j} \sin\theta^{(e)}) \delta_i^{(e)} \qquad (14)$$

where N_c is the number of elements in the winding. $V^{(e)}$ and $\theta^{(e)}$ are the volume and the angle from the x-axis at the centre of gravity of the element e respectively. $\delta_i^{(e)}$ is unity when the node i is included in the element e and zero when the node i is outside the element e.

Ψ is discretized from Eq.(10) as follows:

$$\Psi = \frac{n_c}{4 S_c} \sum_{e=1}^{N_c} V^{(e)} \sum_{k=1}^{4} (A_{xke} \cos\theta^{(e)} + A_{yke} \sin\theta^{(e)}) \qquad (15)$$

When the backward substitution method is used as the time difference method, the coefficients $\{\partial G_i / \partial I_o\}$, $\{\partial \eta / \partial u_j\}$ and $\partial \eta / \partial I_o$ are obtained from Eqs.(3), (4) and (12) to (15) as follows:

$$\frac{\partial G_{oxi}}{\partial I_o} = -\frac{n_c}{4 S_c} \sum_{e=1}^{N_c} V^{(e)} \cos\theta^{(e)} \delta_i^{(e)} \qquad (16)$$

$$\frac{\partial G_{oyi}}{\partial I_o} = -\frac{n_c}{4 S_c} \sum_{e=1}^{N_c} V^{(e)} \sin\theta^{(e)} \delta_i^{(e)} \qquad (17)$$

$$\frac{\partial G_{zoi}}{\partial I_o} = \frac{\partial G_{di}}{\partial I_o} = 0 \qquad (18)$$

$$\frac{\partial \eta}{\partial A_{xj}} = -\frac{n_c}{4 \Delta t S_c} \sum_{e=1}^{N_c} V^{(e)} \cos\theta^{(e)} \delta_i^{(e)} \qquad (19)$$

$$\frac{\partial \eta}{\partial A_{yj}} = -\frac{n_c}{4 \Delta t S_c} \sum_{e=1}^{N_c} V^{(e)} \sin\theta^{(e)} \delta_i^{(e)} \qquad (20)$$

$$\frac{\partial \eta}{\partial A_{zi}} = \frac{\partial \eta}{\partial \phi_j} = 0 \qquad (21)$$

$$\frac{\partial \eta}{\partial I_o} = -(R_o + R_c) - \frac{L_o}{\Delta t} \qquad (22)$$

where Δt is the time interval.

The coefficient matrix in Eq.(11) is not symmetric, because Eq.(16) is different from Eq.(19), and Eq.(17) is also different from Eq.(20). If the coefficients in the row related to η in Eq.(11) are multiplied by Δt, the coefficient matrix in Eq.(11) becomes symmetric. Therefore, the ICCG method can be applied to solve Eq.(11).

3. EXAMPLES OF APPLICATION

3.1 Infinite Solenoid

In order to check the program of the newly developed method, the magnetizing current in an infinite solenoid shown in Fig.3 is analyzed. The 2-D method[1] is also applicable to this model.

The effective voltage and the frequency of the power source are 100(V) and 50(Hz) respectively. The number of turns and the dc resistance of the winding are 100 and 1(Ω) respectively. The material of the core hatched in Fig.3 is assumed to be air or steel of which the relative permeability and the conductivity are 5000 and 0.25×10^8(S/m) respectively.

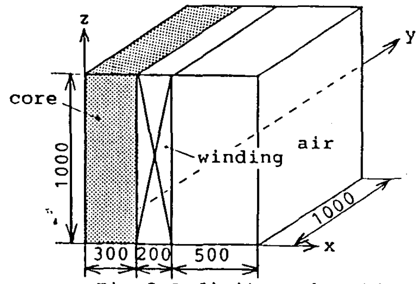


Fig.3 Infinite solenoid.

Table 1 shows the magnetizing currents obtained by 3-D and 2-D calculations at the instants of $\omega t=0$ and $\pi/2$(rad). Zero time is taken to be the instant when the applied voltage is equal to zero.

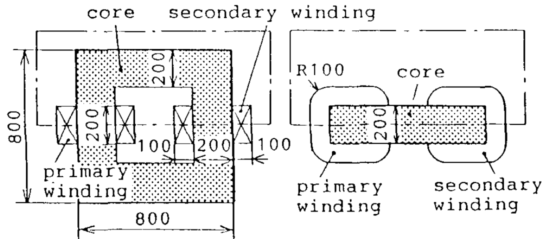
Table 1 Comparison between 3-D and 2-D calculations

No.	material of core	eddy current	magnetizing current (A)			
			3-D		2-D	
			$\omega t=0$	$\omega t=\frac{\pi}{2}$	$\omega t=0$	$\omega t=\frac{\pi}{2}$
1	air	no	69.0	86.1	69.0	86.3
2	steel	no	52.9	23.8	52.9	23.8
3	steel	yes	25.7	13.5	25.5	13.5

3.2 Loaded Transformer

The currents in the primary and the secondary windings of a loaded transformer shown in Fig.4 are analyzed. The chained lines denote the analyzed region.

The effective voltage and the frequency of the power source are 100(V) and 50(Hz) respectively. The numbers of turns of the primary and the secondary windings are both equal to 30. The dc resistances Rc of the primary and the secondary windings are both equal to 1(Ω). The load is pure resistance, and its value Ro is equal to 0 or 9(Ω). The magnetic characteristic of the core is assumed to be linear, and its relative permeability is 5000. Eddy current in the core is not taken into account.



(a) front view (b) plane view
Fig.4 Transformer model.

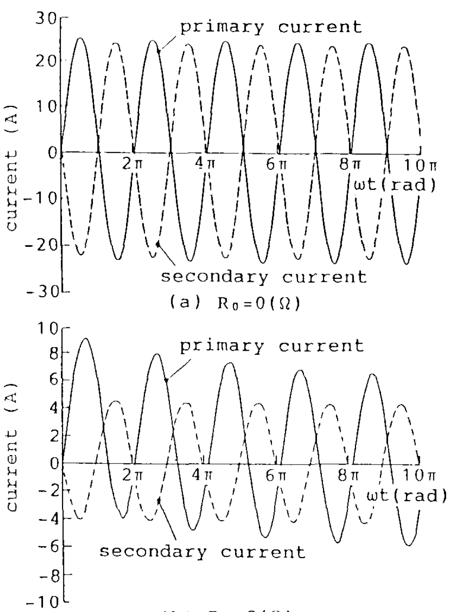


Fig.5 Current waveforms.

Figure 5 shows the current waveforms obtained by transient analysis using the step-by-step method[6] of which the time interval Δt is 1(msec). The solid and the broken lines show the currents in the primary and the secondary windings respectively. Figure 5 suggests that if the resistance Ro is decreased, the currents reaches steady state rapidly.

4. CONCLUSIONS

The method for analyzing 3-D magnetic fields and currents in electrical machines excited from voltage sources has been established. As the currents can be treated as unknown variables, magnetic fields in electrical machines under actual operating conditions can be analyzed using this method.

If the method proposed here is expanded into the time-periodic finite element method[7] which can calculate steady state phenomena directly, the method becomes more effective. This expansion will be reported later.

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