3-D foundation-soil-foundation interaction

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ABSTRACT

A direct time domain BEM formulation is used for the solution of a class of dynamic foundation-soil-foundation interaction problems. A set of parametric studies, in the form of time and frequency dependent plots, is presented illustrating the influence of a number of parameters upon the response of the soil-foundations system.

INTRODUCTION

Dynamic soil-structure interaction (SSI) analysis pertaining to the single foundation problem has been well established on the basis of numerous rigorous investigations during the last three decades. In contrast, relatively few works regarding the phenomenon of dynamic foundation-soil-foundation interaction (FSFI) have been reported since the early works of Whitman [1] and Warburton *et al.* [2,3] have identified the importance of such a dynamic interaction. Among the later works on the subject one could mention the analytical solutions of Triantafyllidis and Prange [4,5], the FEM solutions of Lee and Wesley [6], Lysmer *et al.* [7], Lin *et al.* [8], and Roesset and Gonzalez [9], and the Green's function approaches of Adams and Christiano [10], Savidis and Richter [11], Whittaker and Christiano [12], and Wong and Luco [13]. The BEM has also been used by Ottenstreuer and Schmid [14], Sato *et al.* [15], and Yoshida *et al.* [16] in determining the effect of through-the-soil coupling of two square foundations. With regard to multiple foundations, Karabalis and Mohammadi [17] and Mohammadi [18] investigated the dynamic response of a group of railroad ties connected by an overlaying track structure. Frequency domain formulations have been employed by all the above mentioned works and the importance of resonance phenomena, directly associated to the presence of more than one foundations, has been studied.

In this work, the dynamic FSFI of massive, square, rigid foundations resting on a homogeneous, isotropic, linear elastic half-space is discussed. Parametric studies are presented regarding the mass of foundations, the distance between foundations, as well as the number of foundations. Transient results are obtained directly via a time domain BEM formulation. However, some results are also displayed in the more familiar frequency dependent form. Additional results can be found in Huang [19].

FORMULATION

Due to limited space and the "applied point of view" adopted for this publication, only an overview of the time domain BEM formulation used in this work is presented in this section. The interested reader is refered to Ref. [19] for more details concerning the development of the formulation and in-depth discussion.

Assuming a set of massive, rigid, foundations resting on the surface of a half space, as those shown in Fig. 1, a substructure procedure can be used for the representation of the foundation-half space system. For the half space, standard BEM procedures can be

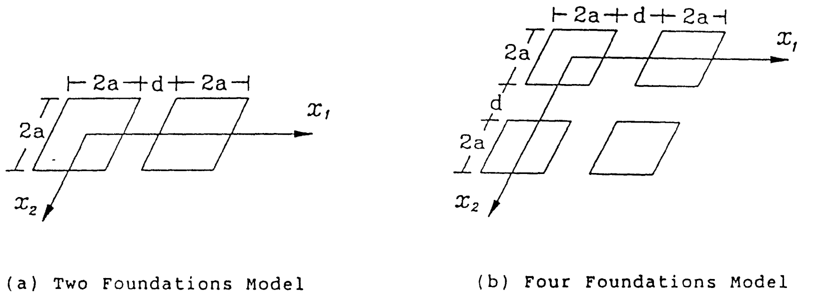
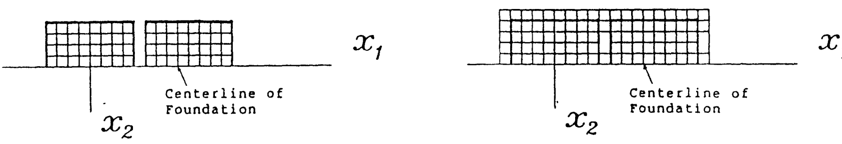


Figure 1. Geometry and nomenclature of a two- and four-foundation groups.

applied for the discretization of its infinite boundary. Thus, if full space fundamental solutions are chosen, as the Stokes pair [20] used in this work, a finite discretization of the contact surface between the foundation and the half space, if relaxed boundary conditions are used, or the contact surface and a portion of the surrounding free surface, if non-relaxed boundary conditions are in order, have proven sufficiently accurate [21] for the class of problems under investigation. Examples of such discretization patterns are shown in Fig.2. Among the various isoparametric quadrilateral elements available in



(a) Relaxed boundary conditions (b) Non-relaxed boundary conditions

Figure 2. An 8x8 element discretization of a two-foundation system.

the literature the four-node, linear element is chosen for the numerical study presented in this work. However, in order to better satisfy the causality condition of the fundamental solutions used in this time domain formulation and, at the same time, avoid the need for unnecessarily fine discretization patterns, a local discretization of each linear element into a number of subelements has been adopted resulting in a highly improved integration accuracy at low computational cost [22]. In addition to the above spatial discretization, a time domain formulation requires a temporal discretization as well. For the purpose of this work, the time axis has been discretized into equal intervals and all the field variables are assumed to be constant within each time step. On the basis of these approximations the time domain boundary integral equation of elastodynamics can be recast into a set of linear algebraic equations for the time step N as

$$0.5\{u\}^N = \sum_{n=1}^{N} \left\{[G]^n \{t\}^{N-n+1} - [T]^n \{u\}^{N-n+1}\right\} \tag{1}$$

where $\{u\}^N$ and $\{t\}^N$ are the displacement and traction vectors on the boundary, and $[G]^n$ and $[T]^n$ are the space- and time-discretized fundamental solutions [19].

For the entire set of massive, rigid foundations, the compatibility equations, for a total of M nodal points at the contact surface between the half space and each foundation $i=1,..I$, can be written as

$$\{u\} = [S]\{D\} \tag{2}$$

where

$$\{u\}^T = \left[\{u^1\}^T \quad \{u^2\}^T \quad . \quad . \quad \{u^I\}^T\right]$$

with $\{u^i\}$ being the $3Mx1$ displacement vector under the foundation i,

$$\{D\}^T = \left[\{D^1\}^T \quad \{D^2\}^T \quad . \quad . \quad \{D^I\}^T\right]$$

with $\{D^i\}$ being the $6x1$ rigid displacement vector for the foundation i, and

$$[S] = \begin{bmatrix} [S^1] & & & & \\ & [S^2] & & & \\ & & [\ ] & & \\ & & & [\ ] & \\ & & & & [S^I] \end{bmatrix}$$

with $[S^i]$ being the $3Mx6$ transformation matrix for foundation i as given, for example, in Ref. [19]. Similarly, the equations of equilibrium relating the tractions applied at the contact surface between the half space and the foundation to the externally applied forces can be written as

$$\{P\} = -[K]\{t\} \tag{3}$$

or in a more explicit form

$$\begin{Bmatrix} \{P^1\} \\ \{P^2\} \\ \{\ \} \\ \{\ \} \\ \{P^I\} \end{Bmatrix} = -\begin{bmatrix} [K^1] & & & & \\ & [K^2] & & & \\ & & [\ ] & & \\ & & & [\ ] & \\ & & & & [K^I] \end{bmatrix} \begin{Bmatrix} \{t^1\} \\ \{t^2\} \\ \{\ \} \\ \{\ \} \\ \{t^I\} \end{Bmatrix}$$

where $\{t^i\}$ is the 3Mx1 contact traction vector under the foundation i, $\{P^i\}$ is the 6x1 vector of external forces on foundation i, and the transfer matrix $[K^i]$ is an assemblange of M 6x3 submatrices the explicit form of which can be found in a number of sources, e.g. Ref. [19].

An appropriate combination of Eqs (1)-(3), taking also into consideration the inertia effect, yields for the time step N [19]

$$[M]\{\ddot{D}\}^N + [K_s]\{D\}^N = [K_s]\{\Delta D\}^N + \{P\}^N \tag{4}$$

where [M] is the 6Ix6I diagonal mass matrix of the foundations substructure, $\{\Delta D\}^N$ is the 6Ix1 rigid displacement vector at time step N of the equivalent massless foundations substructure due to the wave field on the surface of the half space caused by tractions applied at previous time steps, and $[K_S]$ is a constant of the foundation-half space system [19]. Thus, the right hand side of Eq. (4) is known at every time step N and the rigid body displacements $\{D\}^N$ can be obtained in a straightforward manner via a standard finite difference scheme.

NUMERICAL EXAMPLES

The dynamic response of two and four 5x5 ft square, massive, rigid, surface foundations on a homogeneous elastic half space is studied. The elastic constants of the half space are: Poisson's ratio $\nu=1/3$, mass density $\rho=10.368$ lb·sec^2/ft^4, and modulus of elasticity $E=2.5898\times10^9$ lb/ft^2. An 8x8 element mesh, with 9x9 subelements per element, is used over the contact surface of each foundation with the half space. The transient forcing function is specified as

$$P_1(t) = P_3(t) = M_2(t) = M_3(t) = \begin{cases} 100, & \text{first time step} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

in consistent units. Frequency dependent solutions are due to harmonic forcing functions of unit magnitude and are plotted versus a dimensionless frequency defined as

$$\alpha_0 = \omega a / c_2 \tag{6}$$

where ω is the circular frequency of the harmonic forcing function, a is the half-width of the square foundation, and c_2 is the shear wave velocity in the half space. The frequency dependent dimensionless compliance C^i_{jk} is defined as follows:

Translation $\quad : C^i_{jk} = \mu a \dfrac{\Delta_j}{P_k}$

Rotation $\quad : C^i_{jk} = \mu a^3 \dfrac{\varphi_j}{M_k}$

where in translation j,k=1,2 for the loaded foundation and j,k=5,6 for the unloaded foundation, while in rotation j,k=3,4 for the loaded foundation and j,k=7,8 for the unloaded foundation, the superscript i indicates the number of foundations involved in the calculations, μ is the shear modulus of the half space, and P_k and M_k are, respectively, the externally applied force and moment. In addition the mass ratio M is defined as

$$M = m / \rho R^3 \tag{7}$$

where m is the mass of the foundation, and R is the equivalent radius of a circular foundation, i.e.,

$$R = \sqrt{\frac{A}{\pi}}$$

with A being the contact area under the foundation. A comparison study of relaxed versus non-relaxed boundary conditions yielded almost identical results and, thus, in this section only results produced under the assumption of relaxed boundary conditions are presented.

First the effect of the distance d between two foundations is studied. In Fig. 3 the response versus time of the excited foundation is shown for the vertical, horizontal, rocking, and torsional degrees of freedom. In this case, a single mass ratio of M=10, for a relatively heavy foundation, is used. The distance ratio d/a is varied from very small to infinite, the later case representing, of course, a single foundation. The same results, convoluted with a harmonic function are plotted in Fig. 4 versus frequency. As it was expected, using as reference the single foundation results, the distance ratio has its greatest effect when it is relatively small. For values of $d/a \geq 1$ the effect is almost negligible. It is also obvious that the vertical and torsional modes are almost insensitive to the presence of the adjacent unloaded foundation. In Figs 5 and 6 the response of the

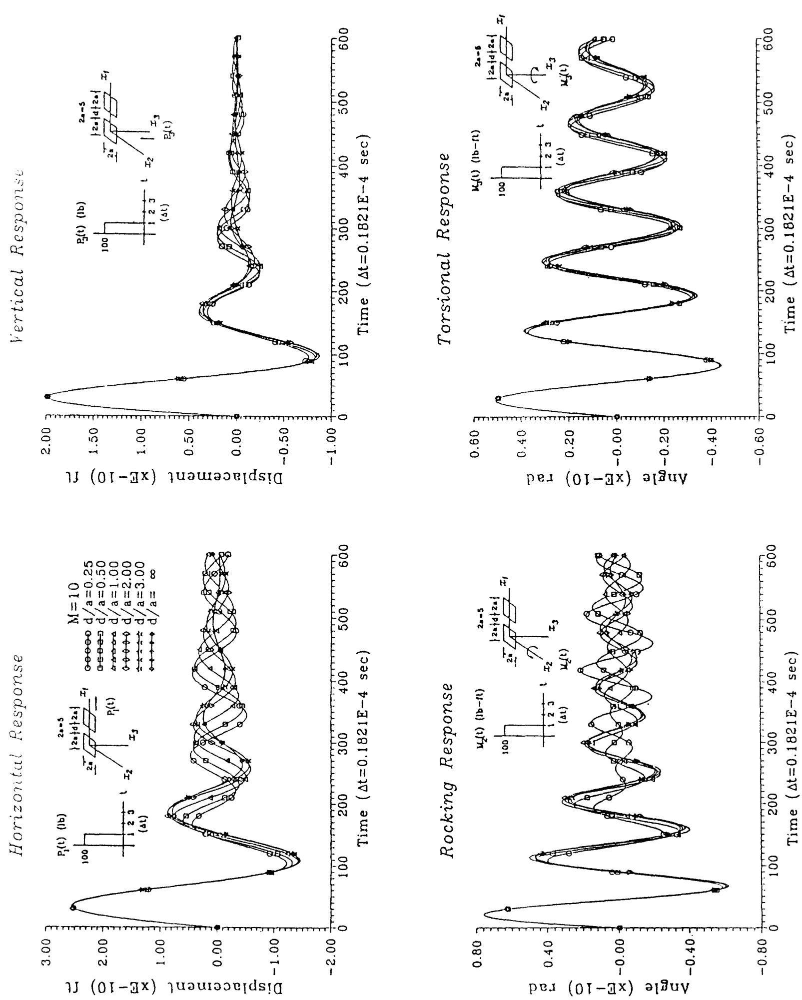


Figure 3. Horizontal, vertical, rocking, and torsional response versus time of the loaded foundation, in a group of two foundations, to a rectangular impulse force.

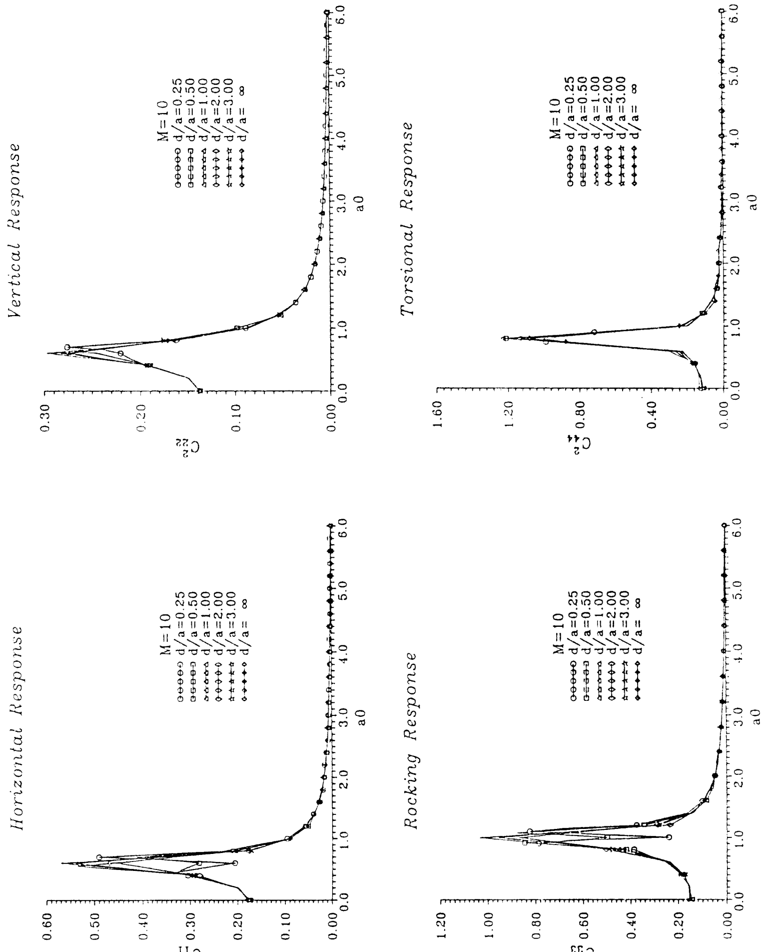


Figure 4. Horizontal, vertical, rocking, and torsional response versus frequency of the loaded foundation, in a group of two foundations.

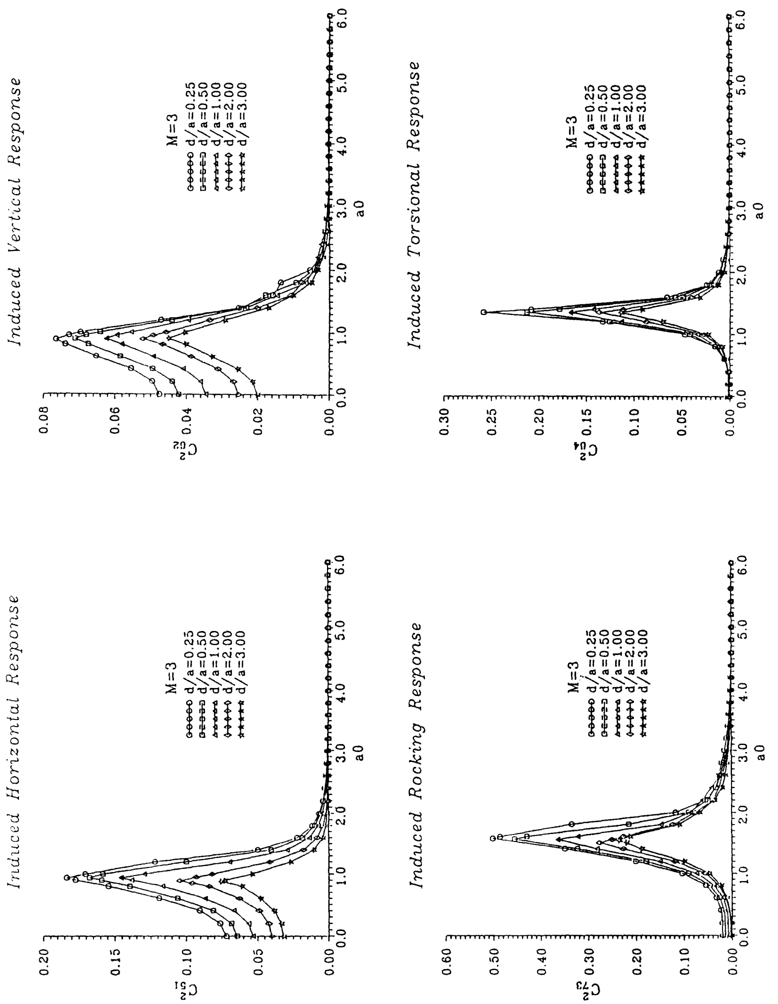


Figure 5. Horizontal, vertical, rocking, and torsional response versus frequency of the unloaded foundation, in a group of two foundations with M=3.

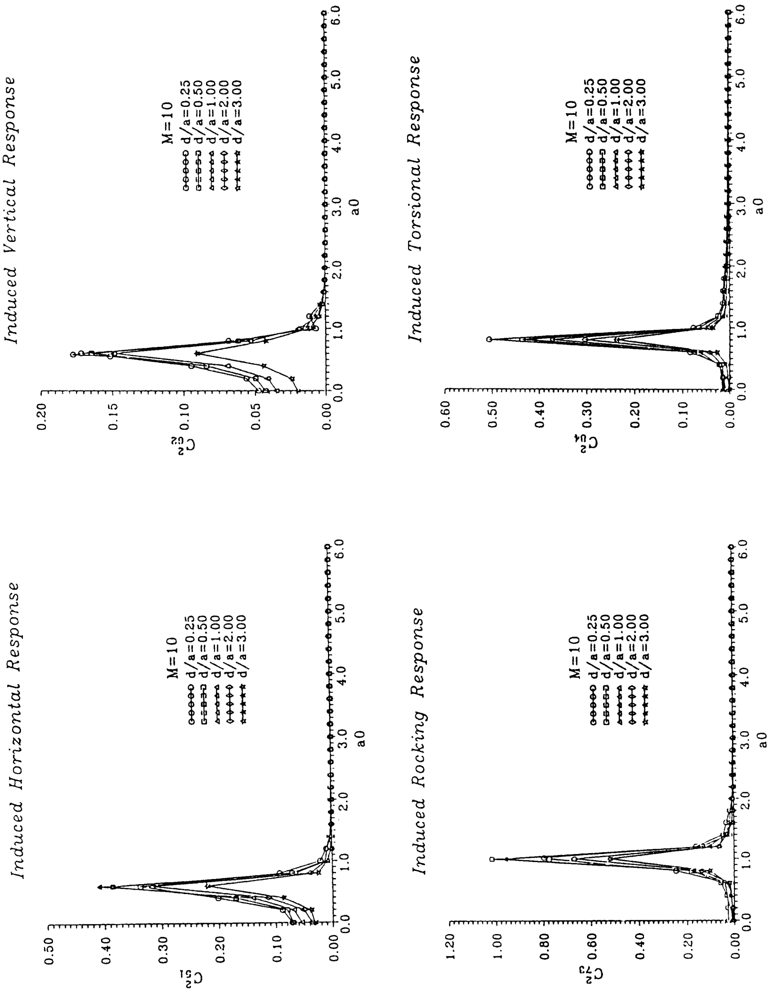


Figure 6. Horizontal, vertical, rocking, and torsional response versus frequency of the unloaded foundation, in a group of two foundations with M=10.

unloaded foundation (induced motion) is plotted versus frequency for the mass ratios M=3 and 5. It is apparent that the effect of the distance ratio has a much more pronounced effect in both cases. As it was also expected the "resonance peak" of the heavier mass is more noticeable, as compared to the lighter mass, and over a narrower band of frequencies.

The effect of mass ratio on the response of the loaded foundation is depicted in Fig. 7 where the response versus frequency for all uncoupled modes of vibration is plotted. In all cases the distance ratio is kept constant at d/a=0.25 and these results are compared to those of an identical single foundation. The basic trends of the results for the group of two foundations are similar to those for a single foundation, but substantial differences exist, in the sense of shifting and "bending", at the neighborhood of the "resonance peak" particularly for the horizontal and rocking modes.

The response versus time of a group of four foundations is plotted in Fig. 8. Only the response of the loaded foundation is shown using as reference the response of an identical single foundation or a group of two identical foundations at the same distance ratio. It can be observed that, for the geometry and material constants used in this example, the response of the four foundation group is close to the response of the two foundation group, while once again the horizontal and rocking modes present the most substantial differences.

CONCLUSIONS

The successful application of a time domain BEM formulation to a class of dynamic foundation-soil-foundation interaction problems has been demonstrated. It is shown that the proposed method can achieve particularly good results using only a minimum of computer resources and, thus, lends itself to applications involving extensive parametric studies and even more complicated geometries and boundary conditions. The interested reader will find in Huang [19] the solution of a number of problems involving a variety of geometries, loading conditions, including seismic wave excitations, and detailed parametric studies.

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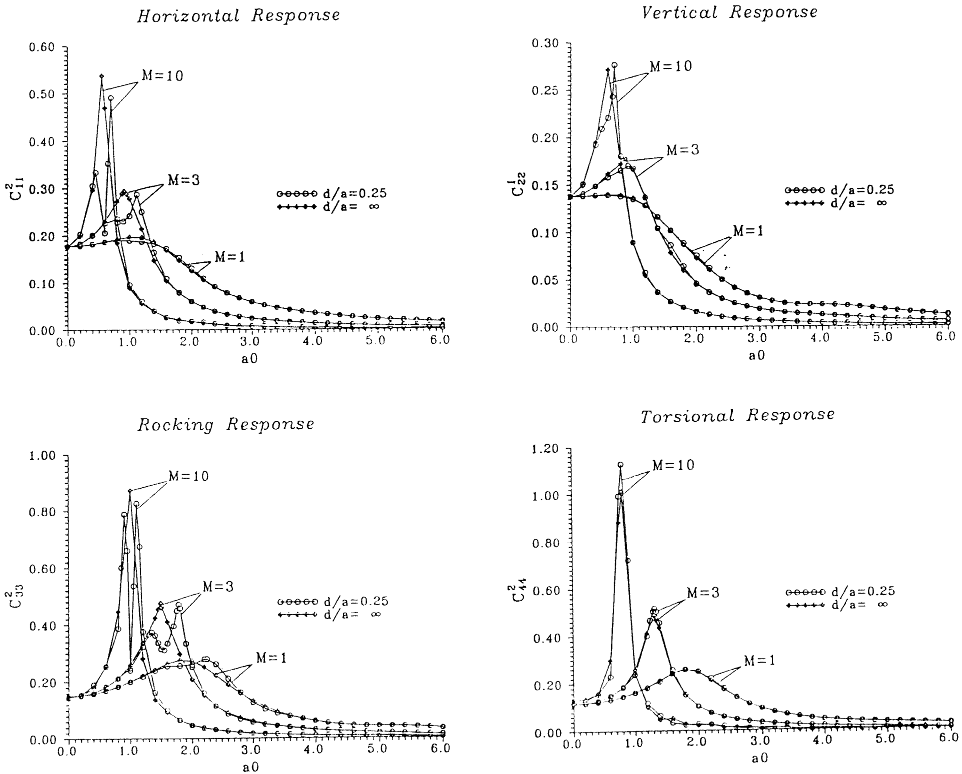

Figure 7. Horizontal, vertical, rocking, and torsional response versus frequency of the loaded foundation, in a group of two foundations with M=1,3, or 10.

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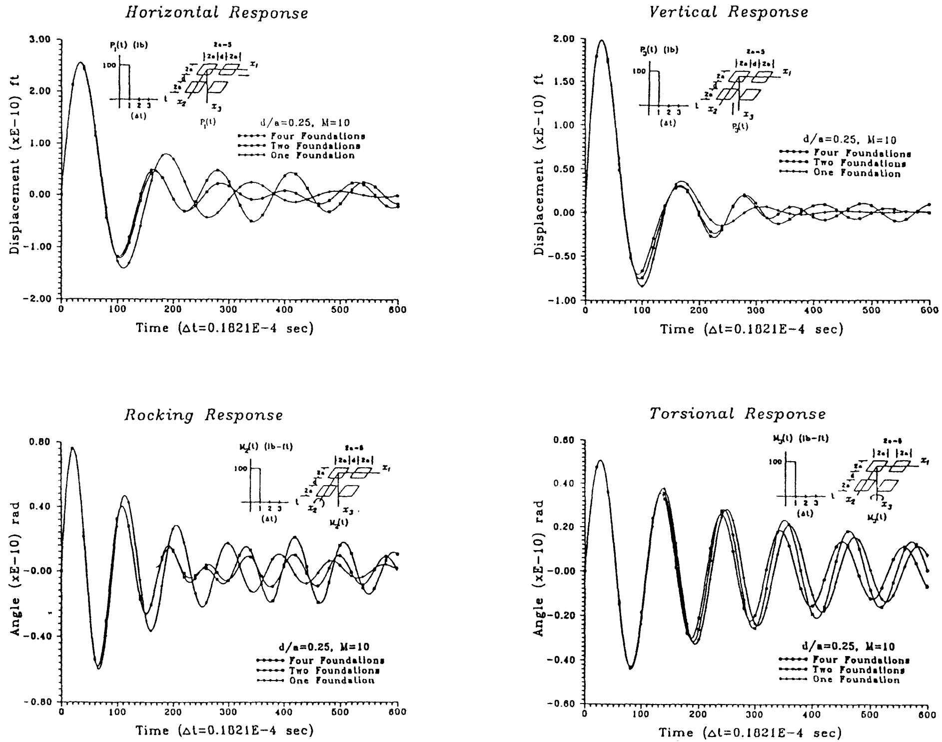

Figure 8. Horizontal, vertical, rocking, and torsional response versus time of the loaded foundation, in a group of four foundations, to a rectangular impulse force.

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