3-D Target Localization in Wireless Sensor Networks Using RSS and AoA Measurements

Slavisa Tomic, Marko Beko, and Rui Dinis, *Senior Member, IEEE*

***Abstract*—This paper addresses target localization problems in both noncooperative and cooperative 3-D wireless sensor networks (WSNs), for both cases of known and unknown sensor transmit power, i.e., P_T. We employ a hybrid system that fuses distance and angle measurements, extracted from the received signal strength and angle-of-arrival information, respectively. Based on range and angle measurement models, we derive a novel nonconvex estimator based on the least squares criterion. The derived nonconvex estimator tightly approximates the maximum-likelihood estimator for small noise. We then show that the developed estimator can be transformed into a generalized trust region subproblem framework, by following the squared range approach, for noncooperative WSNs. For cooperative WSNs, we show that the estimator can be transformed into a convex problem by applying appropriate semidefinite programming relaxation techniques. Moreover, we show that the generalization of the proposed estimators for known P_T is straightforward to the case where P_T is not known. Our simulation results show that the new estimators have excellent performance and are robust to not knowing P_T. The new estimators for noncooperative localization significantly outperform the existing estimators, and our estimators for cooperative localization show exceptional performance in all considered settings.**

***Index Terms*—Angle-of-arrival (AoA), generalized trust region subproblem (GTRS), received signal strength (RSS), semidefinite programming (SDP), wireless localization, wireless sensor network (WSN).**

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S. Tomic is with the Laboratory of Robotics and Systems in Engineering and Science (LARSyS), Instituto Superior Técnico, Institute for Systems and Robotics (ISR/IST), 1049-001 Lisboa, Portugal (e-mail: stomic@isr.ist.utl.pt).

M. Beko is with Universidade Lusófona de Humanidades e Tecnologias, 1749-024 Lisboa, Portugal, and also with the Centre of Technology and Systems (CTS), Instituto Desenvolvimento De Novas Tecnologias (UNINOVA), 2829-516 Caparica, Portugal (e-mail: beko.marko@ulusofona.pt; mbeko@uninova.pt).

R. Dinis is with Instituto de Telecomunicações, 1049-001 Lisboa, Portugal, and also with the Departamento de Engenharia Electrotécnica, Faculdade de Ciências e Tecnologia (FCT), 2829-516 Caparica, Portugal (e-mail: rdinis@fct.unl.pt).

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I. Introduction

Wireless sensor networks (WSNs) generally refer to a wireless communication network that is composed of a number of devices, which are called sensors, allocated over a monitored region to measure some local quantity of interest [1]. Due to their autonomy in terms of human interaction and low device costs, WSNs find application in various areas such as event detection (fires, floods, and hailstorms) [2], monitoring (industrial, agricultural, health care, and environmental) [3], [4], energy-efficient routing [5], exploration (deep water, underground, and outer space) [6], and surveillance [7], to name a few. In many practical applications, data gathered by sensors are only relevant if they are associated with accurate sensors' locations; hence, the estimation of sensors' locations is a key requirement for a majority of practical applications [1].

Sensors are small, low-cost, and low-power nodes commonly deployed in a large number over a region of interest with limited to nonexisting control of their location in space, e.g., thrown out of an aeroplane for sensing in hostile environments [8]. Installing a global positioning system (GPS) receiver in each sensor would severely augment the network costs and restrict its applicability [9]. To maintain low implementation costs, only a small fraction of sensors are equipped with GPS receivers (called anchors), whereas the remaining sensors (called targets) determine their locations by using a kind of localization scheme that takes advantage of the known anchor locations [10]. Since the sensors have minimal processing capabilities, the key requirement is to develop localization algorithms that are fast, scalable, and abstemious in their computational and communication requirements.

Wireless localization schemes typically rely on range (distance) measurements [11], [12]. Depending on the available hardware, range measurements can be extracted from the different characteristics of radio signal, such as time of arrival (ToA) [13], time difference of arrival [14], round-trip time [15], angle of arrival (AoA) [16], or received signal strength (RSS) [17]–[21]. Recently, hybrid systems that fuse two measurements of the radio signal have been investigated [22]–[30]. Hybrid systems profit by exploiting the benefits of combined measurements, i.e., more available information. On the other hand, the price to pay for using such systems is the increased complexity of network devices, which increases the network implementation costs [1], [9].

The approaches in [17]–[21] and [31] consider the noncooperative and cooperative target localization problem, but the estimators are founded on RSS and distance measurements only.

The approaches in [22]–[24] are based on the fusion of RSS and ToA measurements. A hybrid system that merges range and angle measurements was investigated in [25]. Yu in [25] proposed two estimators for solving the noncooperative target localization problem in a 3-D scenario: linear least squares (LS) and optimization based. The LS estimator is a relatively simple and well-known estimator, whereas the optimization-based estimator was solved by the Davidson–Fletcher–Powell algorithm [32]. In [26], Wang *et al.* derived an LS and a maximum likelihood (ML) estimator for a hybrid scheme that combines RSS difference (RSSD) and AoA measurements. Nonlinear constrained optimization was used to estimate the target's location from multiple RSS and AoA measurements. Both LS and ML estimators in [26] are λ-dependent, where λ is a nonnegative weight assigned to regulate the contribution from RSS and AoA measurements. A selective weighted LS (WLS) estimator for the RSS/AoA localization problem was proposed in [28]. Gazzah *et al.* determined the target location by exploiting weighted ranges from the two *nearest* anchor measurements, which were combined with the serving base station AoA measurement. In [26]–[28], the authors investigated the noncooperative hybrid RSS/AoA localization problem for a 2-D scenario only. A WLS estimator for a 3-D RSSD/AoA noncooperative localization problem when the transmit power is unknown was presented in [29]. However, Chan *et al.* in [29] only investigated a small-scale WSN, with extremely low noise power. An estimator based on the semidefinite programming (SDP) relaxation technique for the cooperative target localization problem was proposed in [30]. Biswas *et al.* in [30] extended their previous SDP algorithm for pure range information into a hybrid one by adding angle information for triplets of points. However, due to the consideration of triplets of points, the computational complexity of the SDP approach increases rather substantially with the network size.

In this paper, we investigate the target localization problem in both noncooperative and cooperative 3-D WSNs. In the case of noncooperative WSNs, we assume that all targets exclusively communicate with anchors and that a single target is located at a time. In the case of cooperative WSNs, we assume that all targets communicate with any sensor within their communication range (whether it is an anchor or a target) and that all targets are simultaneously located. For both cases, a hybrid system that fuses distance and angle measurements, extracted from RSS and AoA information, respectively, is employed. By using the RSS propagation model and simple geometry, we derive a novel objective function based on the LS criterion. For the case of noncooperative WSNs, based on the squared range (SR) approach, we show that the derived nonconvex objective function can be transformed into a generalized trust region subproblem (GTRS) framework, which can be solved exactly by a bisection procedure [28]. For the case of cooperative localization, we show that the derived objective function can be transformed into a convex function by applying the SDP relaxation technique. Finally, we show that the generalization of the proposed estimators to the case where, alongside the targets' locations, the transmit power, i.e., P_T, is also unknown is straightforward for both noncooperative and cooperative localization.

Thus, the main contribution of our work is threefold. First, by using RSS and AoA measurement models, we derive a novel nonconvex objective function based on the LS criterion, which tightly approximates the ML criterion for small noise. In the case of noncooperative localization, we propose two novel estimators that significantly reduce the estimation error, compared with the state of the art. Finally, in the case of cooperative localization, we present the first hybrid RSS/AoA estimators for target localization in a 3-D cooperative WSN.

Throughout this paper, uppercase bold type, lowercase bold type, and regular type are used for matrices, vectors, and scalars, respectively. $\mathbb{R}^n$ denotes the n-dimensional real Euclidean space. The operators $\otimes$ and $(\bullet)^T$ denote the Kronecker product and transpose, respectively. The normal (Gaussian) distribution with mean μ and variance σ^2 is denoted by $\mathcal{N}(\mu, \sigma^2)$. $\mathrm{diag}(\boldsymbol{x})$ denotes a square diagonal matrix in which the elements of vector $\boldsymbol{x}$ form the main diagonal of the matrix, and the elements outside the main diagonal are zero. The N-dimensional identity matrix is denoted by $\boldsymbol{I}_N$ and the $M \times N$ matrix of all zeros by $\mathbf{0}_{M\times N}$ (if no ambiguity can occur, subscripts are omitted). $\|\boldsymbol{x}\|$ denotes the vector norm defined by $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$, where $\boldsymbol{x} \in \mathbb{R}^n$ is a column vector. For Hermitian matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, $\boldsymbol{A} \succeq \boldsymbol{B}$ means that $\boldsymbol{A} - \boldsymbol{B}$ is positive semidefinite.

The remainder of this work is organized as follows. In Section II, the RSS and AoA measurement models are introduced, and the target localization problem is formulated. Section III presents the development of the proposed estimators in the case of noncooperative localization for both known and unknown P_T values. In Section IV, we describe the derivation of the proposed estimators in the case of cooperative localization for both known and unknown P_T values. In Sections V and VI, complexity and performance analyses are presented, respectively, together with the relevant results to compare the performance of the newly presented estimators to the state of the art. Finally, Section VII summarizes the main conclusions.

II. Problem Formulation

We consider a WSN with N anchors and M targets, where the known locations of anchors are denoted, respectively, by $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_N$, and the unknown locations of targets are denoted by $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_M$ ($\boldsymbol{x}_i, \boldsymbol{a}_j \in \mathbb{R}^3$, $i = 1, \ldots, M$ and $j = 1, \ldots, N$). For ease of expression, let us define a vector $\boldsymbol{x} = [\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \ldots, \boldsymbol{x}_M^T]^T$ ($\boldsymbol{x} \in \mathbb{R}^{3M\times 1}$) as the vector of all unknown target locations, such that $\boldsymbol{x}_i = \boldsymbol{E}_i^T \boldsymbol{x}$, where $\boldsymbol{E}_i = \boldsymbol{e}_i \otimes \boldsymbol{I}_3$, and $\boldsymbol{e}_i$ is the ith column of the identity matrix I_M. We determine these locations by using a hybrid system that fuses range and angle measurements. Combining two measurements of the radio signal provides more information to the user, and it is likely to enhance the estimation accuracy, as shown in Fig. 1.

Fig. 1 shows how (a) a range-based, (b) an angle-based, and (c) a hybrid (range and angle) system operates for the case where $M = 1$ and $N = 4$. In range-based localization, each range measurement, i.e., $\hat{d}_i$, defines a circle as a possible location of the unknown target. Thus, a set of range measurements, i.e., $\{\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_N\}$, defines multiple circles, and the area determined by their intersection accommodates the target [see Fig. 1(a)]. Similarly, with angle-based localization,

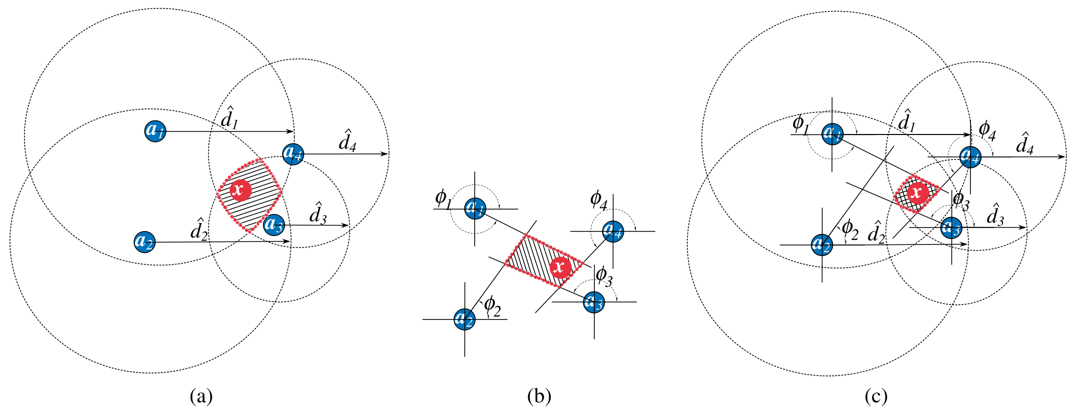

Fig. 1. Illustration of different localization systems in 2-D space. (a) Range-based localization. (b) Angle-based localization. (c) Hybrid localization.

each angle measurement, i.e., ϕ_i, defines a line as the set of possible locations of the unknown target [see Fig. 1(b)]. In Fig. 1(c), one can see that when the two measurements of the radio signal are integrated, the set of all possible solutions (the area determined by the intersection) is significantly reduced; hence, hybrid systems are more likely to improve the estimation accuracy.

Throughout this work, it is assumed that the range measurements are obtained from the RSS information exclusively, since ranging based on RSS requires the lowest implementation costs [1]. However, the RSS measurement model can be replaced with the path-loss model by using the relationship $L_{ij} = 10\log_{10}(P_T/P_{ij})$ (dB), where L_{ij} and P_{ij} are, respectively, the path loss and received power between two sensors i and j, which are within the communication range of each other (from the transmitting sensor), and P_T is the transmission power of a sensor [35], [36]. Thus

$$L_{ij}^{\mathcal{A}} = L_0 + 10\gamma\log_{10}\frac{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}{d_0} + n_{ij}, \text{ for } (i,j)\in\mathcal{A} \quad (1a)$$

$$L_{ik}^{\mathcal{B}} = L_0 + 10\gamma\log_{10}\frac{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}{d_0} + n_{ik}, \text{ for } (i,k)\in\mathcal{B} \quad (1b)$$

where L_0 denotes the path-loss value at a short reference distance d_0 ($\|\boldsymbol{x}_i - \boldsymbol{a}_j\| \geq d_0$, $\|\boldsymbol{x}_i - \boldsymbol{x}_k\| \geq d_0$); γ is the path-loss exponent (PLE) between two sensors, which indicates the rate at which the path loss increases with distance; and n_{ij} and n_{ik} are the lognormal shadowing terms modeled as $n_{ij} \sim \mathcal{N}(0, \sigma^2_{n_{ik}})$ and $n_{ik} \sim \mathcal{N}(0, \sigma^2_{n_{ik}})$. Furthermore, the sets $\mathcal{A} = \{(i,j) : \|\boldsymbol{x}_i - \boldsymbol{a}_j\| \leq R, \text{for } i = 1,\ldots,M,\ j = 1,\ldots,N\}$ and $\mathcal{B} = \{(i,k) : \|\boldsymbol{x}_i - \boldsymbol{x}_k\| \leq R, \text{for } i,k = 1,\ldots,M, i \neq k\}$, where R is the communication range of a sensor, denote the existence of target/anchor and target/target connections, respectively.

To obtain the AoA measurements (both azimuth and elevation angles), we assume that either multiple antennas or a directional antenna is implemented at anchors [25], [34]. To make use of the AoA measurements from different sensors, the orientation information is required, which can be obtained by implementing a digital compass at each sensor [25], [34]. However, a digital compass introduces an error in the AoA measurements due to its static accuracy. For the sake of simplicity and without loss of generality, we model the angle measurement error and the orientation error as one random variable in the rest of this paper.

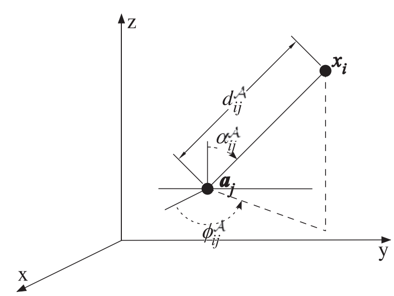

Fig. 2. Illustration of a target and anchor locations in 3-D space.

Fig. 2 illustrates a target and anchor locations in 3-D space. As shown in Fig. 2, $\boldsymbol{x}_i = [x_{i1}, x_{i2}, x_{i3}]^T$ and $\boldsymbol{a}_j = [a_{j1}, a_{j2}, a_{j3}]^T$ are, respectively, the unknown coordinates of the ith target and the known coordinates of the jth anchor, whereas $d_{ij}^{\mathcal{A}}$, $\phi_{ij}^{\mathcal{A}}$, and $\alpha_{ij}^{\mathcal{A}}$ represent the distance, azimuth angle, and elevation angle between the ith target and the jth anchor, respectively. The ML estimate of the distance between two sensors can be obtained from the RSS measurement model (1) as follows [1]:

$$\hat{d}_{ij}^{\mathcal{A}} = d_0 10^{\frac{L_{ij}^{\mathcal{A}} - L_0}{10\gamma}}, \text{ for } (i,j)\in\mathcal{A} \quad (2a)$$

$$\hat{d}_{ik}^{\mathcal{B}} = d_0 10^{\frac{L_{ik}^{\mathcal{B}} - L_0}{10\gamma}}, \text{ for } (i,k)\in\mathcal{B}. \quad (2b)$$

Applying simple geometry, azimuth and elevation angle measurements can be modeled as [25]

$$\phi_{ij}^{\mathcal{A}} = \arctan\left(\frac{x_{i2} - a_{j2}}{x_{i1} - a_{j1}}\right) + m_{ij}, \text{ for } (i,j)\in\mathcal{A} \quad (3a)$$

$$\phi_{ik}^{\mathcal{B}} = \arctan\left(\frac{x_{i2} - x_{k2}}{x_{i1} - x_{k1}}\right) + m_{ik}, \text{ for } (i,k)\in\mathcal{B} \quad (3b)$$

$$\alpha_{ij}^{\mathcal{A}} = \arccos\left(\frac{x_{i3} - a_{j3}}{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}\right) + v_{ij}, \text{ for } (i,j)\in\mathcal{A} \quad (4a)$$

$$\alpha_{ik}^{\mathcal{B}} = \arccos\left(\frac{x_{i3} - x_{k3}}{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}\right) + v_{ik}, \text{ for } (i,k)\in\mathcal{B} \quad (4b)$$

respectively, where m_{ij}, m_{ik} and v_{ij}, v_{ik} are, respectively, the measurement errors of azimuth and elevation angles, which are modeled as $m_{ij} \sim \mathcal{N}(0, \sigma^2_{m_{ij}})$, $m_{ik} \sim \mathcal{N}(0, \sigma^2_{m_{ik}})$ and $v_{ij} \sim \mathcal{N}(0, \sigma^2_{v_{ij}})$, $v_{ik} \sim \mathcal{N}(0, \sigma^2_{v_{ik}})$.

Given the observation vector $\boldsymbol{\theta} = [\boldsymbol{L}^T, \boldsymbol{\phi}^T, \boldsymbol{\alpha}^T]^T$ ($\boldsymbol{\theta} \in \mathbb{R}^{3(|\mathcal{A}|+|\mathcal{B}|)}$), where $\boldsymbol{L} = [L^{\mathcal{A}}_{ij}, L^{\mathcal{B}}_{ik}]^T$, $\boldsymbol{\phi} = [\phi^{\mathcal{A}}_{ij}, \phi^{\mathcal{B}}_{ik}]^T$, $\boldsymbol{\alpha} = [\alpha^{\mathcal{A}}_{ij}, \alpha^{\mathcal{B}}_{ik}]^T$, and $|\bullet|$ denotes the cardinality of a set (the number of elements in a set), the conditional probability density function (pdf) is given as

$$p(\boldsymbol{\theta}|\boldsymbol{x}) = \prod_{i=1}^{3(|\mathcal{A}|+|\mathcal{B}|)} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left\{-\frac{(\theta_i - f_i(\boldsymbol{x}))^2}{2\sigma_i^2}\right\} \quad (5)$$

where

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \vdots \\ L_0 + 10\gamma \log_{10}\frac{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}{d_0} \\ \vdots \\ L_0 + 10\gamma \log_{10}\frac{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}{d_0} \\ \vdots \\ \arctan\left(\frac{x_{i2}-a_{j2}}{x_{i1}-a_{j1}}\right) \\ \vdots \\ \arctan\left(\frac{x_{i2}-x_{k2}}{x_{i1}-x_{k1}}\right) \\ \vdots \\ \arccos\left(\frac{x_{i3}-a_{j3}}{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}\right) \\ \vdots \\ \arccos\left(\frac{x_{i3}-x_{k3}}{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}\right) \\ \vdots \end{bmatrix}, \quad \boldsymbol{\sigma} = \begin{bmatrix} \vdots \\ \sigma_{n_{ij}} \\ \vdots \\ \sigma_{n_{ik}} \\ \vdots \\ \sigma_{m_{ij}} \\ \vdots \\ \sigma_{m_{ik}} \\ \vdots \\ \sigma_{v_{ij}} \\ \vdots \\ \sigma_{v_{ik}} \\ \vdots \end{bmatrix}.$$

The most common estimator used in practice is the ML estimator, since it has the property of being asymptotically efficient (for large enough data records) [37], [38]. The ML estimator forms its estimate as the vector $\hat{\boldsymbol{x}}$, which maximizes the conditional pdf in (5); hence, the ML estimator is obtained as

$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \sum_{i=1}^{3(|\mathcal{A}|+|\mathcal{B}|)} \frac{1}{\sigma_i^2} [\theta_i - f_i(\boldsymbol{x})]^2 . \quad (6)$$

Although the ML estimator is approximately the minimum-variance unbiased estimator [37], the LS problem in (6) is nonconvex and has no closed-form solution. In the remainder of this work, we will show that the LS problem in (6) can be efficiently solved by applying certain approximations. More precisely, for noncooperative WSNs, we propose a suboptimal estimator based on the GTRS framework leading to an SR-WLS estimator, which can be solved exactly by a bisection procedure [33]. For the case of cooperative WSNs, we propose a convex relaxation technique leading to an SDP estimator that can be efficiently solved by interior-point algorithms [39]. Not only that the new approaches efficiently solve the traditional RSS/AoA localization problem, but they can also be used to solve the localization problem when P_T is not known, with straightforward generalization.

A. Assumptions

We outline here some assumptions for the WSN (made for the sake of simplicity and without loss of generality).

1) The network is connected, and it does not change during the computation time.
2) Measurement errors for the RSS and AoA models are independent, and $\sigma_{n_{ij}} = \sigma_n$, $\sigma_{m_{ij}} = \sigma_m$, and $\sigma_{v_{ij}} = \sigma_v \ \forall \ (i,j) \in \mathcal{A} \cup \mathcal{B}$.
3) The range measurements are extracted from the RSS information exclusively, and all target/target measurements are symmetric.
4) All sensors have identical P_T values.
5) All sensors are equipped with either multiple antennas or a directional antenna, and they can measure the AoA information.

In assumption (1), we assume that the sensors are static and that there is no node/link failure during the computation period, and all sensors can convey their measurements to a central processor. Assumptions (2) and (4) are made for the sake of simplicity. Assumption (3) is made without loss of generality; it is readily seen that if $L^{\mathcal{B}}_{ik} \neq L^{\mathcal{B}}_{ki}$, then it serves to replace $L^{\mathcal{B}}_{ik} \leftarrow (L^{\mathcal{B}}_{ik} + L^{\mathcal{B}}_{ki})/2$ and $L^{\mathcal{B}}_{ki} \leftarrow (L^{\mathcal{B}}_{ik} + L^{\mathcal{B}}_{ki})/2$ when solving the localization problem. Assumption (4) implies that L_0 and R are identical for all sensors. Finally, assumption (5) is made for the case of cooperative localization, where only some targets are able to directly connect to anchors; thus, they are forced to cooperate with other targets within their communication range.

III. Noncooperative Localization

By noncooperative WSN, we imply a network comprising a number of targets and anchors, where each target is allowed to communicate with anchors exclusively, and a single target is localized at a time. For such a setting, we can assume that the targets are passive nodes that only emit radio signals and that all radio measurements are collected by anchors.

In the remainder of this section, we develop a suboptimal estimator to solve the noncooperative localization problem in (6), whose *exact* solution can be obtained by a bisection procedure. We then show that its generalization for the case where P_T is not known is straightforward.

A. Noncooperative Localization With Known P_T

Note that the targets communicate with anchors exclusively in a noncooperative network; hence, the set $\mathcal{B}$ in path-loss model (1) is empty. Therefore, when the noise power is sufficiently small, from (1a), we have

$$\lambda^{\mathcal{A}}_{ij}\|\boldsymbol{x}_i - \boldsymbol{a}_j\| \approx d_0 \ \text{ for } (i,j) \in \mathcal{A} \quad (7)$$

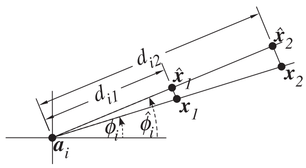

Fig. 3. Illustration of azimuth angle measurements: short-range versus long-range.

where $\lambda_{ij}^{\mathcal{A}} = 10^{(L_0 - L_{ij}^{\mathcal{A}})/10\gamma}$. Similarly, from (3a) and (4a), respectively, we get

$$\boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) \approx 0 \quad (8)$$

$$\boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) \approx \|\boldsymbol{x}_i - \boldsymbol{a}_j\| \cos\left(\alpha_{ij}^{\mathcal{A}}\right) \quad (9)$$

where $\boldsymbol{c}_{ij} = [-\sin(\phi_{ij}^{\mathcal{A}}), \cos(\phi_{ij}^{\mathcal{A}}), 0]^T$, and $\boldsymbol{k}_{ij} = [0, 0, 1]^T$.

Next, we can rewrite (7) as

$$\lambda_{ij}^{\mathcal{A}\,2} \|\boldsymbol{x}_i - \boldsymbol{a}_j\|^2 \approx d_0^2. \quad (10)$$

Introduce weights, $\boldsymbol{w} = [\sqrt{w_{ij}}]$, where each w_{ij} is defined as

$$w_{ij} = 1 - \frac{\hat{d}_{ij}^{\mathcal{A}}}{\sum_{(i,j)\in\mathcal{A}} \hat{d}_{ij}^{\mathcal{A}}}$$

such that more importance is given to nearby links. The reason for defining the weights in this manner is because both RSS and AoA short-range measurements are trusted more than long-range measurements. The RSS measurements have relatively constant standard deviation with distance [1]. This implies that multiplicative factors of RSS measurements are constant with range. For example, for a multiplicative factor of 1.5, at a range of 1 m, the measured range would be 1.5 m, and at an actual range of 10 m, the measured range would be 15 m, which is a factor ten times greater [1]. In the case of AoA measurements, the reason is more intuitive, and we call the reader's attention to Fig. 3.

In Fig. 3, an azimuth angle measurement made between an anchor and two targets located along the same line but with different distances from the anchor is illustrated. The true and measured azimuth angles between the anchor and the targets are denoted by ϕ_i and $\hat{\phi}_i$, respectively. Our goal is to determine the locations of the two targets. Based on the available information, the location estimates of the two targets are at points $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$. However, in Fig. 3, it is shown that the estimated location of the target physically closer to the anchor ($\hat{\boldsymbol{x}}_1$) is much closer to its true location than that further away. In other words, for a given angle, the more two sensors are physically further apart, the greater the set of all possible solutions will be (more likely to impair the localization accuracy).

Replace $\|\boldsymbol{x}_i - \boldsymbol{a}_j\|$ in (9) with $\hat{d}_{ij}^{\mathcal{A}}$ described in (2a), to obtain the following WLS problem according to (8)–(10):

$$\begin{aligned}\hat{\boldsymbol{x}}_i = \underset{\boldsymbol{x}_i}{\operatorname{argmin}} & \sum_{(i,j):(i,j)\in\mathcal{A}} w_{ij}\left(\lambda_{ij}^{\mathcal{A}\,2}\|\boldsymbol{x}_i - \boldsymbol{a}_j\|^2 - d_0^2\right)^2 \\ & + \sum_{(i,j):(i,j)\in\mathcal{A}} w_{ij}\left(\boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j)\right)^2 \\ & + \sum_{(i,j):(i,j)\in\mathcal{A}} w_{ij}\left(\boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - \hat{d}_{ij}^{\mathcal{A}}\cos(\alpha_{ij}^{\mathcal{A}})\right)^2. \quad (11)\end{aligned}$$

The given WLS estimator is nonconvex and has no closed-form solution. However, we can express (11) as a quadratic programming problem whose *global* solution can be efficiently computed [33]. Using the substitution $\boldsymbol{y}_i = [\boldsymbol{x}_i^T, \|\boldsymbol{x}_i\|^2]^T$, the problem in (11) can be rewritten as

$$\underset{\boldsymbol{y}_i}{\text{minimize}} \; \|\boldsymbol{W}(\boldsymbol{A}\boldsymbol{y}_i - \boldsymbol{b})\|^2$$

subject to

$$\boldsymbol{y}_i^T \boldsymbol{D} \boldsymbol{y}_i + 2\boldsymbol{l}^T \boldsymbol{y}_i = 0 \quad (12)$$

where $\boldsymbol{W} = \boldsymbol{I}_3 \otimes \operatorname{diag}(\boldsymbol{w})$

$$\boldsymbol{A} = \begin{bmatrix} \vdots & \vdots \\ -2\lambda_{ij}^{\mathcal{A}\,2}\boldsymbol{a}_j^T & \lambda_{ij}^{\mathcal{A}\,2} \\ \vdots & \vdots \\ \boldsymbol{c}_{ij}^T & 0 \\ \vdots & \vdots \\ \boldsymbol{k}_{ij}^T & 0 \\ \vdots & \vdots \end{bmatrix}, \; \boldsymbol{b} = \begin{bmatrix} \vdots \\ d_0^2 - \lambda_{ij}^{\mathcal{A}\,2}\|\boldsymbol{a}_j\|^2 \\ \vdots \\ \boldsymbol{c}_{ij}^T\boldsymbol{a}_j \\ \vdots \\ \boldsymbol{k}_{ij}^T\boldsymbol{a}_j + \hat{d}_{ij}^{\mathcal{A}}\cos(\alpha_{ij}^{\mathcal{A}}) \\ \vdots \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{I}_3 & \boldsymbol{0}_{3\times1} \\ \boldsymbol{0}_{1\times3} & 0 \end{bmatrix}, \; \boldsymbol{l} = \begin{bmatrix} \boldsymbol{0}_{3\times1} \\ -1/2 \end{bmatrix}$$

i.e., $\boldsymbol{A} \in \mathbb{R}^{3|\mathcal{A}|\times4}$, $\boldsymbol{b} \in \mathbb{R}^{3|\mathcal{A}|\times1}$, and $\boldsymbol{W} \in \mathbb{R}^{3|\mathcal{A}|\times3|\mathcal{A}|}$.

The objective function and the constraint in (12) are both quadratic. This type of problem is known as GTRS [33], [40], and it can be solved exactly by a bisection procedure [33]. We denote (12) as "SR-WLS1" in the remaining text.

B. Noncooperative Localization With Unknown P_T

To maintain low implementation costs, testing and calibration are not the priority in practice. Thus, sensors' transmit power values are often not calibrated, i.e., not known. Not knowing P_T in the RSS measurement model corresponds to not knowing L_0 in the path-loss model (1) (see [9], [12], and the references therein).

The generalization of the proposed estimators for known L_0 is straightforward for the case where L_0 is not known. Notice that (7) can be rewritten as

$$\beta_{ij}^{\mathcal{A}}\|\boldsymbol{x}_i - \boldsymbol{a}_j\| \approx \eta d_0, \text{ for } (i,j)\in\mathcal{A} \quad (13)$$

where $\beta_{ij}^{\mathcal{A}} = 10^{-(L_{ij}^{\mathcal{A}}/10\gamma)}$, and $\eta = 10^{-(L_0/10\gamma)}$ is an unknown parameter that needs to be estimated.

Substitute $\|\boldsymbol{x}_i - \boldsymbol{a}_j\|$ with $\hat{d}_{ij}^{\mathcal{A}}$ in (9). Then, we can rewrite (9) as

$$\beta_{ij}^{\mathcal{A}}\boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) \approx \eta d_0 \cos(\alpha_{ij}^{\mathcal{A}}). \quad (14)$$

To assign more importance to nearby links, introduce weights $\widetilde{\boldsymbol{w}} = [\sqrt{\widetilde{w}_{ij}}]$, where

$$\widetilde{w}_{ij} = 1 - \frac{L_{ij}^{\mathcal{A}}}{\sum_{(i,j)\in\mathcal{A}} L_{ij}^{\mathcal{A}}}.$$