

# 3D Finite Element Analysis of Substandard RC Columns Strengthened by Fiber Reinforced Polymer Sheets

Athanasios I. Karabinis<sup>1</sup>

Theodoros C. Rousakis<sup>2</sup>

Georgia E. Manolitsi<sup>3</sup>

**Abstract:** Numerical analyses are performed to predict the stress-strain behavior of square reinforced concrete columns strengthened by fiber reinforced polymer (FRP) sheet confinement. The research focuses on the contribution of FRP sheets to the prevention of elastic buckling of longitudinal steel bars under compression, in cases of inadequate stirrup spacing. A new Drucker-Prager type plasticity model is proposed for confined concrete and is used in constructed finite element model. Suitable plasticity and elasticity models are used for steel reinforcing bars and fiber reinforced polymers correspondingly. The finite element analyses results are compared against published experimental results of columns subjected to axial compression, to validate the proposed finite element model. Stress concentrations in concrete core and on FRP jacket are investigated considering circular or square sectioned, plain or reinforced concrete columns. Geometry of section as well as presence of steel bars and stirrups affects remarkably the variation and magnitude of stress on FRP as percentage of its tensile strength.

**CE Database subject headings:** finite elements, concrete columns, fiber reinforced polymers, confinement, rehabilitation, plasticity.

<sup>1</sup>Professor, Laboratory of Reinforced Concrete, Dept of Civil Engineering, Democritus University of Thrace (DUTH), 67100 Xanthi, Greece. E-mail: [karabin@civil.duth.gr](mailto:karabin@civil.duth.gr)

<sup>2</sup>Dr. Civil Engineer, Laboratory of Reinforced Concrete, Dept of Civil Engineering, Democritus University of Thrace (DUTH), Vas. Sofias 12, 67100 Xanthi, Greece. E-mail: [trousak@civil.duth.gr](mailto:trousak@civil.duth.gr), tel.: 2541079645, fax: 2541079638.

<sup>3</sup>Civil Engineer, PhD Candidate, Laboratory of Reinforced Concrete, Dept of Civil Engineering, Democritus University of Thrace (DUTH), 67100 Xanthi, Greece. E-mail: [gmanolit@civil.duth.gr](mailto:gmanolit@civil.duth.gr)

## **Introduction**

In old type reinforced concrete columns designed according to early code provisions no special care was taken of the effect of strong seismic excitations and corresponding ductility demands or detailing of reinforcement. Earthquake damaged columns often present non-ductile behavior with shear cracking of concrete within critical regions or buckling of longitudinal steel bars under compression, both related with inadequate transverse steel reinforcement (stirrups etc.). Structural upgrading of such elements involves, among others, application of additional external confinement on concrete. Such an intervention not only increases strength and deformability of concrete but upgrades its shear strength as well, withstands buckling of longitudinal bars and increases bond stress between concrete and longitudinal reinforcement to prevent local slip.

Externally bonded fiber reinforced polymer (FRP) carbon or glass sheets is a common choice by designers in repair or strengthening of earthquake damaged columns when additional confinement is required. Research focuses on the effectiveness of FRP sheets as confining reinforcement of columns having circular or rectangular sections (Rochette and Labossière 2000; Matthys 2000, Fam and Rizkalla 2001; Karabinis and Rousakis 2002; Karabinis and Rousakis 2003; Campione and Miraglia 2003; Teng and Lam 2004; Rousakis 2005; Silva and Rodriguez 2006, Lam et al 2006, Tastani et al 2006, Al-Salloum 2007 among others). Significant semi-empirical or constitutive models have been proposed (Samaan et al 1998; Spoelstra and Monti 1999; Saafi et al 1999; Fam and Rizkalla 2001; Wang and Restrepo 2001; Karabinis and Rousakis 2002; De Lorenzis and Tepfers 2003; Fujikake et al 2004; Teng and Lam 2004; Rousakis 2005; Braga et al 2006, Vintzileou and Panagiotidou 2007 among others) to describe mechanical behavior of concrete under uniform FRP confinement (circular sections) or non-

uniform FRP confinement (rectangular sections) with or without conventional steel reinforcement.

Behavior of FRP confined concrete in micro-structural level has been investigated through finite element (FE) analysis (Rochette and Labossière 1996; Mirmiran et al 2000; Parvin and Wang 2001; Montoya et al 2004, Karam and Tabbara 2005). More recently, study by Bardaro et al (2006) was focused on analysis of solid or hollow core FRP confined concrete cylinders. Objectives of above studies are the reliable prediction of stress-strain behavior of confined columns as well as the variation of stress and strain in transverse sections or on the FRP jacket.

In the present paper, numerical analyses of concrete columns with conventional longitudinal and transverse steel reinforcement and external confinement by FRP sheets is performed. The study focuses on the behavior of old type columns with extremely low concrete strength and stirrup spacing leading to bars' elastic buckling failure when compressed. The investigation includes also viability of the use of external FRP confinement in delaying or preventing elastic buckling of bars under compression. A new Drucker – Prager type model with an advanced approach in estimating plasticity parameters is inserted in FE code. Results from numerical analyses are compared against performed experiments. Stress concentrations in concrete core and on FRP jacket are also investigated for analyses cases that have been carried out.

### **Finite Element Analyses and Advanced Plasticity Model**

FE analysis enables the construction of models taking into account geometrical and material non-linearity or non-linear interaction between materials in their contact points or surfaces. Also, the effect of several parameters on mechanical behavior of materials or structural elements can be investigated in macro-structural or micro-structural level choosing suitable discretization.

Reliable FE analysis requires consideration of suitable models for materials and their interactions.

The models for concrete, steel and FRP inserted in Abaqus (ABAQUS / PRE, Users' manual 1997) finite element software are presented as follows.

Concrete is modeled as a Drucker-Prager type material exploiting findings by Rousakis (2005) derived from experimental investigation of plastic parameters on concrete under tri-axial compression provided by uniform elastic confinement by carbon FRP jacket. According to previous study the initial plasticity model by Karabinis and Kioussis (1996) was further developed for application to FRP confined concrete following previous research (Karabinis and Rousakis 2002, Rousakis et al 2007). That is, while in Karabinis and Kioussis (1996) a set of several material parameters was estimated empirically for different concrete strengths to describe steel confinement, in developed model concrete parameters required by plasticity theory are calibrated utilizing the method of indirect estimation of plastic behavior from monotonic compressive loading experiments of FRP confined concrete (Rousakis 2005). According to the above mentioned approach the developed Drucker-Prager loading function includes a hardening parameter function capable of describing hardening and softening behavior of concrete. Also, a non associated flow rule is used with a different Drucker-Prager type potential function. Friction parameter  $\theta$  and plastic dilatation parameter  $\alpha$  are calculated through closed form expressions related to the plain concrete strength (parameter  $\alpha$  is related to the confinement modulus  $E_l$  as well). All closed form expressions that enable for easy estimation of elastic and plastic parameters are based on indirect estimation method from monotonic FRP confined concrete behavior. Above method was validated against properly performed repeated increasing load-unload-reload cycles to assess real plastic behavior of FRP confined concrete for a wide range of concrete strengths (from 25 MPa to 82 MPa). Similarly the plastic behavior of plain concrete is indirectly estimated considering hardening and softening behavior. Thus, no optimization methods are necessary to estimate material parameters. All parameters are estimated given the

plain concrete strength and confinement modulus. The same two unknowns have to be identified in the simpler model adopted in this paper according to indirect estimation method. Concrete is modeled using a solid eight-node element (C3D8R) with linear reduced Gauss integration points (six points per element) and enhanced hourglass control. A Drucker – Prager failure criterion was adopted in the form of:

$$F = \sqrt{J_{2D}} f(K) + \theta J_1 - \kappa = 0 \quad (1)$$

where  $J_{2D}$  is the second invariant of stress deviator,  $J_1$  is the first invariant of stress and  $\theta$  is the friction parameter. Function  $f(K)$  is an indirect expression of Lode's angle combining second and third invariant of deviatoric stress (symbolized as  $J_{3D}$ ) taking herein the form of:

$$f(K) = \sqrt{3/2} [1 + 1/K - (1 - 1/K) (3\sqrt{3/2}) J_{3D} / (J_{2D}^{3/2})] \quad (2)$$

It accounts for the variation of shear strength of concrete for different load paths and a given hydrostatic pressure and determines the shape of failure function in deviatoric plane. The shape of deviatoric plane changes from a circle to a curved triangle for different values of material parameter  $K$  (fig. 1).

Hardening-softening parameter  $\kappa$  is derived by the relation:

$$\kappa = (1/\sqrt{3} - \theta) \sigma_c \quad (3)$$

where  $\sigma_c$  is the corresponding stress carried by concrete when considered unconfined under uni-axial compression. Herein, unfavorably to all forth-mentioned models, a simpler  $\kappa$  parameter is adopted, that does not incorporate neither the softening function (exponentially related to the lateral confining stress provided to concrete and to a material parameter related to concrete strength), nor a hardening function related to plastic dilatation  $a$ . However, the stress-plastic strain behavior of plain concrete is incorporated in analysis based on the same indirect estimation approach (Rousakis 2005) so as to take into account hardening-softening behavior of confined

concrete, while friction parameter  $\theta$  depends on plain concrete strength  $f_{co}$ . The initial standard stress-strain response of plain concrete that provides the corresponding plastic behavior, the modulus of elasticity and the poisson ratio are estimated according to CEB-FIP Model Code 90.

A Drucker-Prager type plastic potential function  $G$  is used:

$$G = \sqrt{J_{2D}} f(K) + f(\alpha) J_1 \quad (4)$$

where  $f(\alpha)$  is an expression of the parameter of plastic dilatation  $\alpha$  of concrete that affects the direction of plastic strain vector and was found to be strongly depended on the plain concrete strength  $f_{co}$  and the modulus of confinement  $E_l$  with closed form relation (Rousakis 2005). A non-associated flow rule is considered, meaning that the direction of plastic strain vector is normal to a limit surface (plastic potential surface  $G$ ) that differs from the failure surface  $F$ .

Steel reinforcement is modeled using the same solid eight-node element that was used for concrete, inserting the post-yield stress-strain behaviour of a material that in general yields and hardens according to Yalcin and Saatcioglu (2000) relations (fig. 8). In case of premature buckling (elastic or plastic) of steel rebars under compression, the stress – strain softening response according to Yalcin and Saatcioglu (2000) relations proposed for steel bar segments in between stirrups, is also investigated as a lower bound case. That is, for ratio of stirrup spacing ( $s$ ) to bar diameter ( $\varnothing_L$ ) greater than 8 ( $s/\varnothing_L > 8$ ) elastic buckling occurs. When  $4.5 \leq s/\varnothing_L \leq 8$  the inelastic behavior of steel varies from elastic perfectly-plastic to similar to that of steel in tension. For  $s/\varnothing_L < 4.5$  the behavior of steel is the same either in tension or in compression (fig. 8).

FRP jacket is modeled as quadrilateral lamina element with membrane properties (M3D4R), linear reduced Gauss integration points (one point per element) and enhanced hourglass control. Material response is considered orthotropic linearly elastic up to failure. Displacement

compatibility is considered between concrete and composite material in the axial or lateral direction.

FE analysis includes the following steps: (i) the geometric characteristics of the members of the model are determined, (ii) the properties of the materials are defined, (iii) the connections and contacts among members are assigned, (iv) the type of discretization of the model as well as the density of discretization is determined depending on the number of seeds in each surface and the suitable meshing technique, always depending on the case of study. The main target of meshing is the simplicity and compatibility of nodes' positions among elements in contact so as to avoid convergence problems during analysis, (v) the mode of imposed load or displacement is defined. Monotonic axial load is imposed concentrically on a rigid steel plate that in turn loads uniformly exclusively concrete surface or steel rebars (through perfect contact) and not FRP jacket, (vi) the boundary conditions of the model are determined. In the case of reinforced concrete columns, where one quarter of the section is modeled (because of symmetry in loading and geometry of the FE column model), the FE column model has to be constrained suitably in symmetry planes (displacement in axial direction and normal to that is allowed for all boundary nodes, except for the bottom and top nodes). Newton's solution method is used as a basis, while parameters required in convergence and integration accuracy algorithms are given default values to optimize solutions.

### **Finite Element Analyses Results and Discussion**

An experimental investigation of reinforced concrete columns externally confined by FRP sheets subjected to axial compressive load is currently performed in laboratory of Reinforced Concrete of Democritus University of Thrace (Karabinis and Rousakis 2006). The test program includes concrete specimens with conventional longitudinal and transverse steel reinforcement in two

spacings. Also, concrete specimens with dual confinement provided by steel stirrups (in two spacings) and additional external unidirectional carbon or glass FRP sheets in three different confining levels are included (the direction of fibers is perpendicular to columns axis). Moreover, plain concrete specimens were confined by carbon or glass FRP sheets for comparison. Present study is concentrated on the analytical modeling of the mechanical behavior of reinforced concrete columns with extremely low concrete strength, inadequate stirrup spacing and elastic buckling failure of compressed longitudinal steel bars. The main target is to investigate the efficiency of external carbon FRP confinement in delaying or preventing buckling of bars.

Initially, FE analysis is executed on FRP confined plain concrete columns of circular section, to validate proposed approach and especially consistency of the estimation of required elastic and plastic material parameters that provide the expansive behavior of confined concrete (case of isotropic biaxial elastic confinement of concrete). Then, the same approach is applied on reinforced concrete columns with carbon FRP confinement or unstable steel bars under compression.

### ***Modeling of Circular Concrete Columns Confined by FRP Jacket***

FE analysis includes circular columns of 25.2 MPa concrete strength confined by 1, 2 or 3 layers (labeled herein L1, L2 and L3) of high modulus carbon sheet with elastic modulus  $E_j=377$  GPa and nominal layer thickness  $t_j=0.17$ mm from the experimental investigation of Rousakis and Tepfers (2004). For the above confinement application with unidirectional FRP sheet, the elastic modulus normal to the direction of the fibers (along the column axis)  $E_k=13$ GPa and Poisson ratio  $\nu_{jk}=0.25$  were also required and taken from the study of Sichelschmidt (2000). In fig. 2, the discretization of concrete and of FRP jacket is presented. The required parameters according to proposed model were estimated as follows:  $\theta=0.2874$  and  $\alpha=0.083$  or  $-0.9281$  or  $-0.554$  (for 1, 2



or 3 layers of carbon sheet correspondingly). Also the circular columns reported by Matthys (2000) are analyzed having 34.8 MPa concrete strength confined by 1 layer of unidirectional carbon sheet with 198 GPa elastic modulus and thickness of 0.117 mm (column C240b) and by 1 layer of carbon sheet with 471 GPa elastic modulus and thickness of 0.235 mm. The parameters for the analysis were:  $\theta=0.2897$  and  $\alpha=1.262$  for 198 GPa modulus carbon sheet or  $\alpha=-0.152$  for 477 GPa modulus carbon sheet. Eid et al (2006) had tested columns with concrete strengths 32 MPa (series N) and 48 MPa (series N). Herein the specimens N1, N2, N3 are used, confined with 1, 2 or 3 layers of unidirectional carbon sheet with 78 GPa elastic modulus and layer thickness of 0.381 mm, as well as specimen M1. For specimens N1, N2, N3 friction parameter was  $\theta=0.289$  and plastic dilatation was  $\alpha=1.003$  or 0.376 or 0.009 (for 1, 2 or 3 layers of carbon sheet correspondingly) while for specimen M1,  $\theta=0.293$  and  $\alpha=1.362$ . In cases that material properties in directions normal to the fibers or Poisson ratios were missing from the referenced studies, and given that in examined applications these values are not expected to affect the overall predicted stress-strain behaviour of the columns, the values were chosen with respect to the study of Sichel Schmidt (2000).

In fig. 3, 4 and 5 FE analysis results are compared against abovementioned experimental data. The agreement between experimental results and numerical analysis prediction of stress-strain behavior of concrete is remarkable for the proposed model for both low and high FRP confinement or for variation of concrete strength. An important feature in predictive performance of the proposed model is the accuracy in stress – lateral strain behaviour that provides the load at failure, for determined effective ultimate lateral strain. Stress-axial strain curves and values at failure are also accurate for corresponding experimental lateral strain levels (see Table 1).

### ***Modeling of Reinforced Concrete Columns Confined by FRP Jacket***

Based on proposed approach validated by numerical analysis results on uniformly confined concrete, the investigation of the mechanical behavior of reinforced concrete columns is presented performing FE analysis. Experimental results from the first part of a promising program of reinforced concrete columns are used. Columns have a square section of 200mm with 30mm corner radius and 320mm height. Their concrete strength was extremely low, 13.4 MPa. The specimens were reinforced with four conventional longitudinal bars of 14 mm diameter with nominal yield strength of 500 MPa. Transverse steel stirrups with 6 mm diameter and 220 MPa nominal yield strength were placed in two different spacings of 200 mm or 95 mm. Twelve steel reinforced specimens were confined by unidirectional carbon FRP sheet (in 1, 3 and 5 layers) and unidirectional glass FRP sheet (in 3, 6 and 9 layers). Carbon FRP sheet had a modulus of elasticity of 240 GPa and nominal layer thickness 0.117 mm (Scherer 1999). Corresponding properties for glass FRP sheet was 65 GPa and 0.1538 mm. In columns with 200 mm stirrup spacing without external FRP confinement elastic buckling of bars under compression is expected ( $s/\phi_L=14.3$ ). More details about experiments performed by the authors can be found in Karabinis and Rousakis (2006), as herein only the specimens that are the most critical to premature buckling of the compressed bars are further discussed.

The analysis is focused on the investigation of the efficiency of external FRP confinement in delaying or preventing buckling of bars in specimens with 200 mm stirrup spacing and use of carbon FRP sheet confinement. In fig. 6, the discretization of the parts of the FE model is presented for stirrup spacing of 200 mm for the concrete, the steel reinforcement and the FRP jacket.

As the concrete strength was extremely low and out of the limits of the proposed relations for indirect estimation of plastic behavior of concrete (lower than 25 MPa, Rousakis 2005), recalibration of the friction parameter was required (value of  $\theta=0.121$ ).

The experimental results are presented in fig. 7 for plain concrete specimens without or with one layer of FRP jacket (named as PC or PCFRP) as well as for steel reinforced concrete specimens (with stirrup spacing of 200 mm) without or with FRP jacket (named as RC or RCFRP). Buckling of steel bars occurs in reinforced concrete columns without FRP confinement as in this case the contribution of compressed steel rebars in bearing the imposed axial load is marginal. Strength increases from 13.35 MPa for plain concrete column to 14.32 MPa for steel reinforced concrete column while the calculated contribution of longitudinal bars for full utilization of their axial load carrying capacity (assumption of no premature buckling and full hardening inelastic response) was estimated around 8.7 MPa. Comparing the behavior of plain concrete columns confined by FRP sheet to the behavior of steel reinforced FRP confined columns, it could be marked that the contribution of bars is significant, revealing an increase in strength while deformability levels are quite similar. The strength of plain specimen confined by one layer of FRP sheet is 18.33 MPa while in reinforced concrete column the strength rises to 24.1 MPa (increase of 5.77 MPa). It seems that even if full utilization of bars was not succeeded - as the strength increase is lower than 8.7 MPa - initial elastic buckling of bars and premature failure was restricted.

Before analysis of reinforced concrete columns, proposed approach is applied to the square plain concrete columns (without steel reinforcement) with confinement by one layer of carbon FRP sheet (PCFRP, fig. 7). In that case, where a varying and non-uniform confinement of concrete exists at the section level and along the height of column, the estimation of  $K$  parameter (taken equal to 0.778) that accounts for load-path dependence of shear strength is necessary. The required plastic dilatation parameter  $\alpha$ , according to proposed model for plain or reinforced columns, is estimated equal to 1.5. The accuracy of the prediction of the proposed model against presented experimental data is satisfactory, reproducing the general stress-strain behavior as well

as strength and axial strain at failure (for experimental lateral strain levels, Table 1).

In numerical analysis of reinforced concrete column the yield, strain hardening and ultimate stress of steel bars were  $f_y = 555$  MPa,  $f_{sh} = 555$  MPa (at 0.01 strain) and  $f_u = 654$  MPa (at 0.08 strain). Corresponding values for ties were  $f_y = 220$  MPa,  $f_{sh} = 220$  MPa (at 0.01 strain) and  $f_u = 237$  MPa (at 0.1 strain). The Young modulus of steel was 200 GPa. A Yalcin and Saatcioglou (2000) hardening function was considered for steel. In fig. 7, the analytical predictions are presented for reinforced concrete column with FRP confinement by 1 layer of carbon sheet in comparison with the experimental curves. It is observed that when analysis is performed assuming full utilization of bars and prevention of buckling, the contribution to axial load carrying capacity of column is higher for the initial load stages - that is in the region around the unconfined compressive strength - compared to that obtained from experiments. Above difference in strength is explained by the initial hysteresis in activation of longitudinal bars in experiments, as the load is applied on them indirectly and their curved anchorage detail on the bottom and on the top of the concrete specimens causes in some extend bending of the bars that reduces their contribution in the axial direction for the initial load stages (before full activation of the FRP jacket).

On the other hand, in the FE model the bars are directly loaded as they are considered directly fixed on the loading plate. Above stress state of bars in initial load stages of experiments requires even better restrictive reaction provided by FRP jacket against premature bars buckling, already occurred in specimens with 200 mm stirrup spacing without FRP confinement. FE analysis performed for assumption of premature buckling prevention, provides strength and ductility values at failure (for experimental lateral strain levels) that compare well with the experimental ones (FEA RCFRP, fig. 7). Therefore, it could be concluded that the gain of only 5.77 MPa bearing stress when FRP confinement is used (instead of 8.7 MPa, if full utilization of bars in

axial direction is assumed) in experiments, is owed to the interaction among concrete core, steel bar and FRP jacket and not to partial elastic buckling. Concrete dilates under compression; bars are deformed by the dilation of concrete and tend to buckle in lower axial load levels. However, bars' buckling is efficiently restricted as FRP jacket resists both lateral expansion of concrete and lateral deformation of bar.

Restriction of longitudinal bars' buckling even by low FRP confinement comes out by comparison with FE analysis assuming buckling of bars utilizing approach by Yalcin and Saatcioglou, 2000 (FEA RCFRP-BUCKLE, fig. 7). In the above analysis a linear degrading behavior of steel under compression is assumed after yielding (fig. 8). FE analysis results reveal a premature failure of FRP confined column at far lower levels of ductility (fig. 7) compared to experimental ones. In that case, premature failure of FRP jacket is concluded also if numerical analysis results of strain at failure are compared to experimental behavior of concrete columns confined by FRP without steel reinforcement presence (fig. 7). It seems that bars' buckling – if not restricted - causes a reduction of column ductility to almost half.

Finally, the square reinforced concrete column series FSC4 from the study of Tastani et al (2006) is analyzed using the proposed model. The column had side dimension of 200 mm with 25 mm corner radius, 21.2 MPa concrete strength and included four longitudinal bars of 12 mm diameter with 562 MPa yield strength. The transverse steel reinforcement had 6 mm diameter and 220 MPa yield strength. In absence of detailed data for steel, the strain hardening stress, ultimate stress, hardening functions and Young modulus were considered for longitudinal bars (full hardening response was assumed) and ties as in analysis of RCFRP column. The ties were spaced at 140 mm corresponding to value of  $s/\varnothing_L=11.67$ . The analyzed column was strengthened externally by four layers of carbon sheet with layer thickness 0.13 mm and modulus of elasticity

235 GPa. The following concrete plasticity parameters were estimated for column FSC4 according to proposed model:  $\theta=0.2865$  and  $\alpha=0.654$ .

The FSC4 column series were chosen as they had presented a significant scatter of the measured failure load and strain, owed to the randomness of the bars buckling effect. Column FSC4-14 presented premature failure with 1461 KN strength at a strain of 0.0113 just before fracture of FRP jacket. Then a sudden irrecoverable drop of the load took place. The full contribution considering unbuckled compressed bars was calculated around 254 KN. In absence of tested FRP confined concrete columns without steel reinforcement, the strength of the column subjected to axial load and dually confined by ties and FRP jacket was estimated analytically around 1809 KN using the empirical relation proposed in Karabinis & Rousakis (2006) and assuming no bars buckling. The premature failure of the column prevented the full strength development that corresponds to composite confining action of ties and FRP jacket as well as of the compressed bars. The FE analysis results using the proposed model compare well with the experimental curve (fig. 9, Table 1).

From both numerical analyses of reinforced concrete columns it is concluded that the discrete 3d modeling of steel bar assuming full hardening response for steel, accounts well for the reduction of load carrying capacity of bend bar under compression.

### ***Comparative study of stress concentrations on concrete and FRP jacket affecting failure criterion in confinement***

In the following section, column RCFRP is further investigated. It should be noticed that no modeling of sheet overlap and additional layers of FRP material was considered. Also jacket was considered bonded to concrete. In fig. 10 the strain variation and deformed shape is presented for steel reinforcement as well as the stress variation for FRP jacket just before failure. Variation of

stress concentrations in concrete core and on FRP sheet are further discussed for reinforced concrete column.

From figs. 11(a,c), it seems that there is a high compressive stress field in concrete in the diagonal from the corner of the section to the center. Higher values are marked in the darker regions between deformed steel reinforcement (only bar in 11(a) or bar and stirrup in 11(c)) and FRP jacket resisting expansion. However, there is a lower compressive stress field (brighter region) where vector's direction tends almost in parallel to direction of outer side at the middle-height of the FE column model (fig. 11b). In fig. 11(d) the presence of stirrup results in a compressive stress field in parallel to stirrup direction, i.e. in parallel to outer side of column. The stress in those regions is somewhat lower compared to the one developed in the core's diagonal. This stress variation could justify the identification of a zone where uniaxial confinement could be considered for concrete rather than no confinement at all (Karabinis and Kiousis 1996) even for absence of stirrups or steel bars.

Tensile stress of FRP at failure at the middle-height of the FE column model (fig. 12(a)) varies from around 75% of its strength in direct tension (tensile strength is 3720 MPa according to the manufacturer), measured at the middle of the side, to around 50% at the corner of the section. Compressive stress of FRP at failure is higher around the corner region of the section (fig. 12(b)), however the magnitude is extremely low and its effect could be ignored.

In figure 13, the comparative variation of tensile stress on FRP jacket of circular or square concrete sections is presented for paths at the middle-height of the FE column models or at the height of the stirrups. The stress on the FRP confinement is divided by the tensile strength of FRP material subjected to direct tension, for comparison among different unidirectional carbon sheets. In all cases, the analytical stress at failure is lower than the tensile strength of the jacket. All numerical analyses of columns without steel reinforcement were terminated to the corresponding

experimental lateral strain at failure. In both steel reinforced columns, analysis was terminated because of the failure of the concrete core as no specific failure criterion was adopted for FRP sheet. Convergence problems did not appear during all performed analyses. Failure of experimentally tested columns used in comparison of fig. 13 was observed after the fracture of the external FRP jacket reinforcement. However for reinforced concrete columns no information was available to suggest that local concrete core fracture around the corner was not the cause of actual and prior failure of the column. Concerning the circular concrete sections confined by high modulus of elasticity carbon FRP, the stress on the jacket is constant and around 85% of its potential strength. When square carbon FRP confined concrete section is analyzed then the stress varies from nearly 90% of tensile strength to around 70% at the corner of the section. Around the corner of the section, there is a significant drop of stress despite the 30 mm curvature radius. Similar variation is observed in the numerical study of plain concrete FRP confined columns by Karam & Tabbara (2005) due to shape effect. “Average” developed stress at failure is lower than that of circular section. In the case of reinforced concrete square sections stress on the jacket is dramatically decreased to values lower than 60% of the tensile strength. Variation of the stress on FRP jacket between stirrups is in general similar to that of a column without steel bars. However, the values are extremely lower and they decrease with a higher rate along the flat part of the perimeter and up to the curved edge due to the presence of the bar. Just before the corner of the section there is a local increase of stress. The stress at the stirrup level is even lower (less than 50%), although the variation is different. Higher stresses are developed at the corner while stresses are lower at the middle of the sections’ side due to stirrup’s presence. The sharpest variation of stress is marked in reinforced concrete columns between stirrups and outside corner region. Sharp variation of stress along section’s perimeter or along height of column seems to lead to lower developed stresses and finally lower “average” FRP stress at failure.



Above variations of the FRP jackets should be viewed considering that no attempt was made to model “microscopically” the behaviour of FRP jacket (i.e. discrete resin layers and their properties, composite sheets and bond in between and to concrete, sheet overlap region etc). Therefore no premature failures of FRP jacket could be predicted by the FE analysis associated with overlap failure or local stresses due to overlap configuration, or due to concrete to jacket bond degradation in less confined (and therefore heavily cracked concrete) regions, or due to further intensification of above stress concentrations due to shape effect. However the inserted properties of cured FRP sheet, modeled as lamina, account “macroscopically” for the biaxial effect on the jacket caused by axial compressive strain transferred from the concrete. As mentioned above, compressive stress developed on the jacket is considered too low to affect failure stress of the jacket with unidirectional fibers in investigated columns.

## **Conclusions**

Finite element analysis is used to study the mechanical behavior of reinforced concrete columns with bars suffering from premature buckling under compression as well as the effect of external confinement by FRP sheet jacket. An advanced Drucker – Prager type loading-failure function and a non-associated flow rule are used to model concrete behavior. Plasticity theory’s material parameters are estimated with closed form relations. The model provides accurate prediction for concrete columns under uniform elastic confinement by FRP materials. The agreement of analytical predictions with experimental behavior of steel reinforced columns and external FRP confinement is also satisfactory. It seems that the discrete 3d modeling of steel bar assuming full hardening response for steel, accounts well for the reduction of load carrying capacity of bend bar under compression. The contribution of FRP confinement in prevention of longitudinal bars’ buckling is investigated. FRP sheet confinement even in low volumetric ratios

can enhance remarkably the mechanical behavior of reinforced columns of low concrete strength by providing lateral restriction required to prevent bars' buckling. According to lower bound numerical analysis, if bars' buckling was not restricted, a reduction of column ductility to almost half would happen.

The variation of stress on FRP sheet for different paths is investigated. The geometry of section as well as presence of steel bars and stirrups affects remarkably the variation and stress on FRP as percentage of its tensile strength. Sharp variation of stress along section's perimeter or along height of column owed to section shape or presence of steel reinforcement, seems to lead to lower developed stresses and finally lower "average" FRP stress at failure. Stress developed on FRP sheet at failure, ranges from less than 90% of its strength for columns without reinforcement, to less than 60% for steel reinforced columns where the variation of stress along the perimeter is even sharper.

After primary validation of proposed approach, additional FE analyses are required to address the effect of main variables of such types of columns to the failure stress of FRP sheet and the effect of steel buckling to their mechanical behavior through a systematic parametric study. Main variables of further research should include varying properties of steel bars (to cover all existing columns), tie spacing to bar diameter ratio, tie properties and diameter, concrete properties, shape of confined cross-section, existing detailing of bars, thickness and stiffness of external FRP confinement, detailing of FRP confinement and improvement of strengthening effectiveness through cross-section ovalization schemes etc.

## **Notation**

*The following symbols are used in this paper*

$F$  = loading/failure function of plasticity theory;

$J_1$	=	first invariant of stress;
$J_{2D}$	=	second invariant of deviatoric stress;
$J_{3D}$	=	third invariant of deviatoric stress;
$K$	=	material parameter defining the shape of deviatoric plane;
$G$	=	plastic potential function of plasticity theory;
$E_l$	=	modulus of confinement $E_l=2E_jnt_j/d$ , where $E_j$ =FRP fiber tensile modulus of elasticity, $t_j$ = nominal thickness of FRP jacket per layer, $n$ =number of FRP layers, $d$ =diameter of concrete column
$f_{co}$	=	plain concrete average cylindrical compressive strength;
$s$	=	stirrup spacing;
$\varnothing_L$	=	longitudinal steel bar diameter;
$\theta$	=	friction parameter of concrete, controlling pressure sensitivity;
$\kappa$	=	hardening –softening parameter of loading/failure function;
$\alpha$	=	plastic dilatation parameter of concrete.

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**Table 1.** Predictive performance of proposed plasticity model for FE analysis

Model	Experiment		Experimental results		FE analysis results		Prediction / Experiment	
			Strength (MPa)	Ultimate axial strain (mm/m)	Strength (MPa)	Ultimate axial strain (mm/m)	Strength ratio	Axial strain ratio
Modified Rousakis (2005)	Rousakis and Tepfers (2004)	L1 (2)	40.2	13.2	43.0	14.9	1.09	1.13
		L2 (2)	58.0	19.9	56.8	19.5	0.98	0.98
	Matthys (2000)	C240b	46.1	10.2	45.9	11.8	1.00	1.16
		C640b	45.8	6.0	45.6	4.1	1.00	0.68
	Eid et al (2006)	N1 (3)	40.3	10.7	42.3	10.5	1.05 <sup>a</sup>	-
		N2 (3)	59.0	20.1	59.0	20.1	1.00 <sup>a</sup>	-
		N3 (3)	74.3	22.9	70.9	22.8	0.95 <sup>a</sup>	-
	Karabinis and Rousakis (2006)	M1	56.8	6.0	54.0	6.0	0.95 <sup>a</sup>	-
		PCFRP (2)	18.3	13.6	17.4	12.1	0.95	0.91
	Tastani et al (2006)	RCFRP (2)	24.1	11.4	24.4	9.6	1.01	0.86
		FSC4-14	1461 <sup>b</sup> (KN)	11.3	1509 <sup>b</sup> (KN)	11.5	1.03	1.02

<sup>a</sup> FE analysis is terminated at experimental strain, as no lateral strains are available

<sup>b</sup> values in KN



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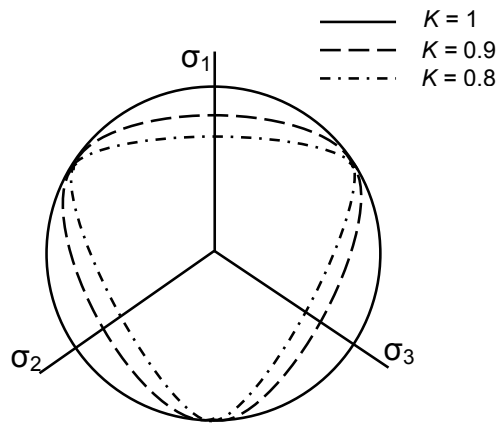
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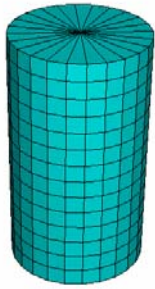
**Fig. 12.** Variation of principal tensile stress at failure, of the FRP jacket at the middle height of the column RCFRP (a). Corresponding principal compressive stress (b).

**Fig. 13.** Variation of principal tensile stress of FRP sheet at failure, for different paths of nodes for columns L1, PCFRP and RCFRP.

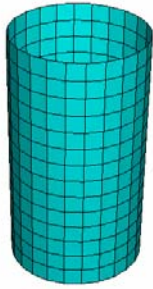


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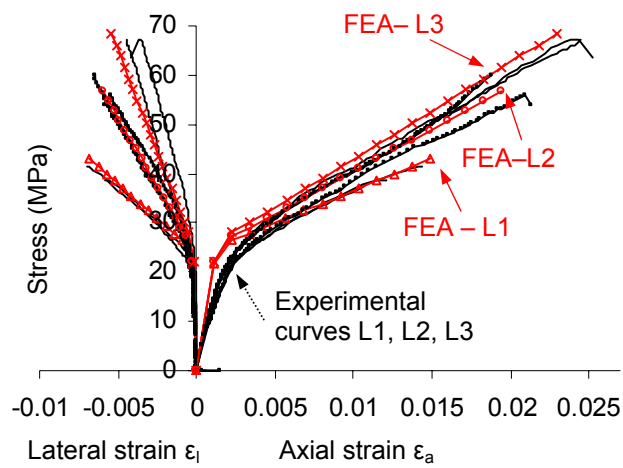
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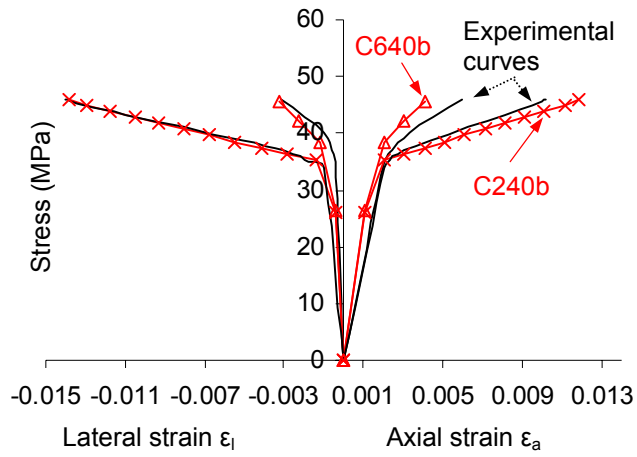
(b)

**Fig. 2.** Discretization of concrete (a) and composite material jacket (b).

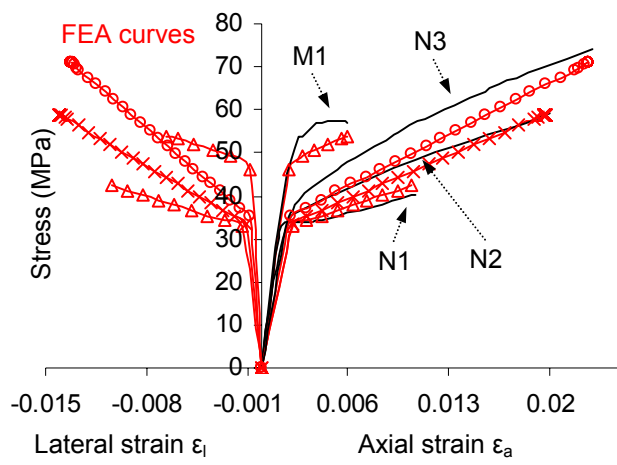
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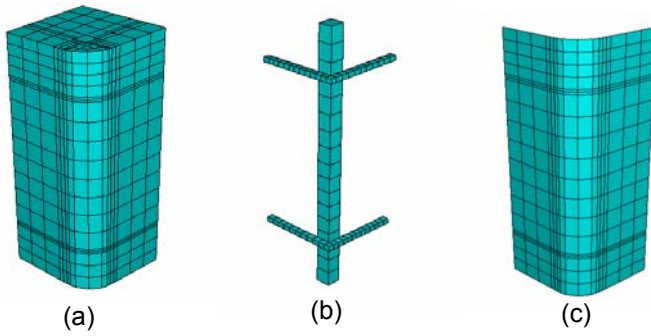
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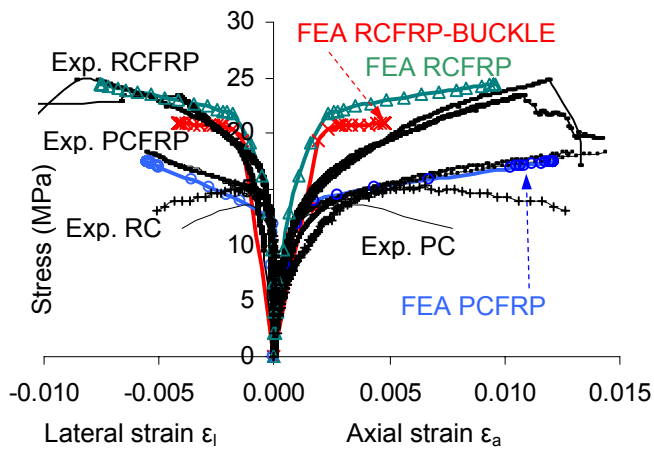
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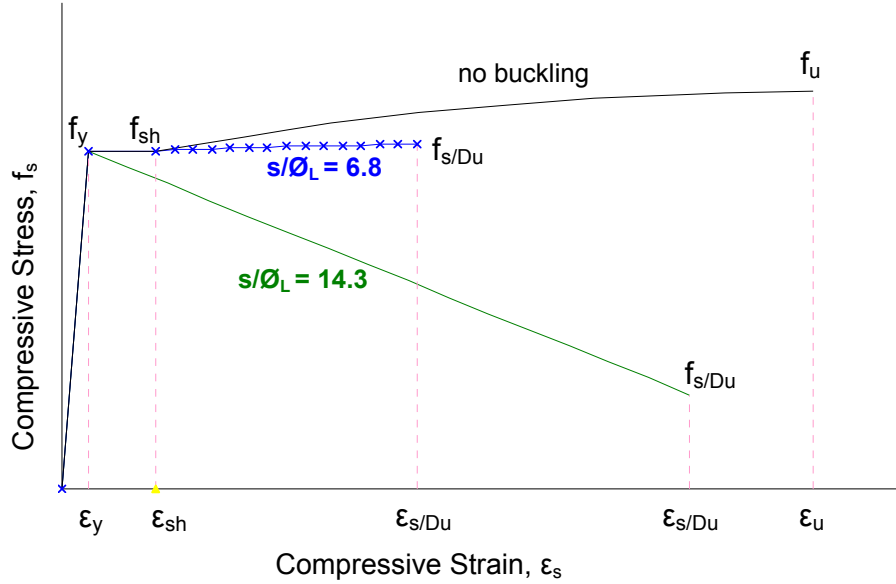




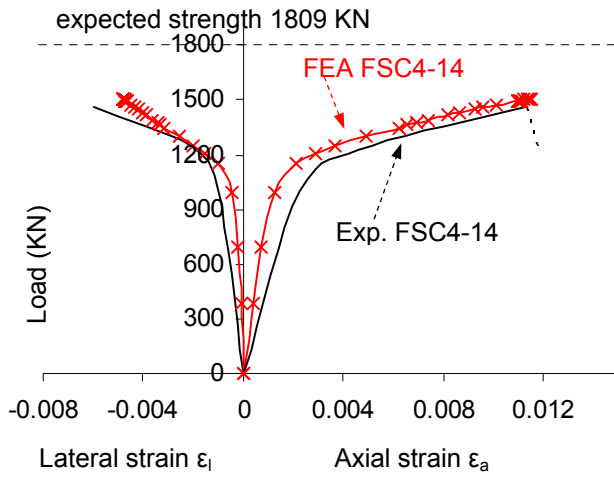
**Fig. 7.** Stress – axial and lateral strain diagrams for plain and reinforced concrete columns confined by 1 layer of carbon FRP sheet from Karabinis and Rousakis (2006).

for  $s/\varnothing_L \leq 8$   $f_s = E_s \varepsilon_s$  for  $\varepsilon_s \leq \varepsilon_y$   
 $f_s = f_y + (\varepsilon_s - \varepsilon_y) \left( \frac{f_{sh} - f_y}{\varepsilon_{sh} - \varepsilon_y} \right)$  for  $\varepsilon_y < \varepsilon_s \leq \varepsilon_{sh}$   
 $f_s = f_y + (f_{S/Du} - f_{sh}) \left[ 2 \frac{\varepsilon_s - \varepsilon_{sh}}{\varepsilon_{S/Du} - \varepsilon_{sh}} - \left( \frac{\varepsilon_s - \varepsilon_{sh}}{\varepsilon_{S/Du} - \varepsilon_{sh}} \right)^2 \right]$  for  $\varepsilon_s > \varepsilon_{sh}$   
 $f_{S/Du} = f_{sh} + (f_u - f_{sh}) [48e^{-0.9(s/\varnothing_L)}]$  for  $s/\varnothing_L < 4.5$ :  $f_{S/Du} = f_u$ ,  $\varepsilon_{S/Du} = \varepsilon_u$ ,  
 $\varepsilon_{S/Du} = \varepsilon_{sh} + (\varepsilon_u - \varepsilon_{sh}) [6e^{-0.4(s/\varnothing_L)}]$  similar to relation in tension

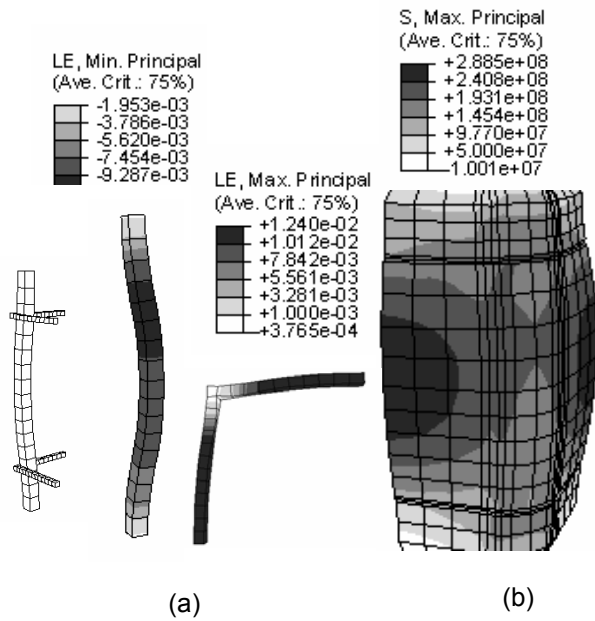
for  $s/\varnothing_L \geq 8$   
 $f_s = f_y - (\varepsilon_s - \varepsilon_y) \left[ -23000 + 11000 \ln \left( \frac{s}{\varnothing_L} \right) \right]$  for  $\varepsilon_y < \varepsilon_s \leq \varepsilon_{S/Du}$   
 $f_{S/Du} = 28 \left( \frac{s}{\varnothing_L} \right)^{-1.7} f_y$   $\varepsilon_{S/Du} = \left[ 40 - 6 \ln \left( \frac{s}{\varnothing_L} \right) \right] \varepsilon_y$



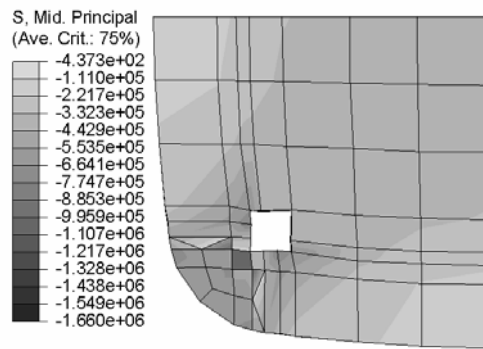
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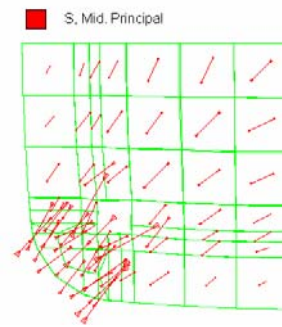
**Fig. 9.** Stress – axial and lateral strain diagrams for reinforced concrete column confined by 4 layers of carbon FRP sheet from Tastani et al (2006).



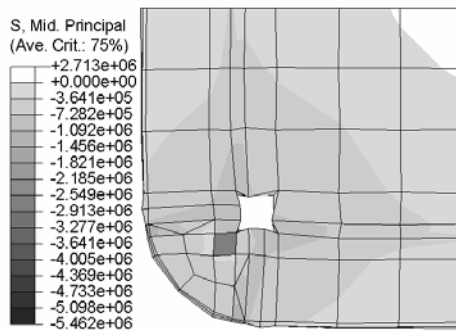
**Fig. 10.** Variation of strain on steel reinforcement (a) and of stress on FRP jacket (b) just before failure in RCFRP column.



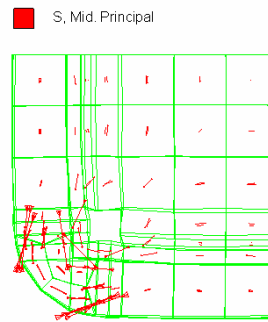
(a)



(b)

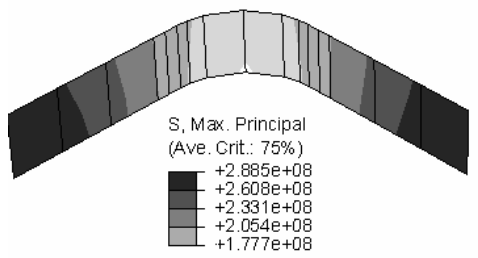


(c)

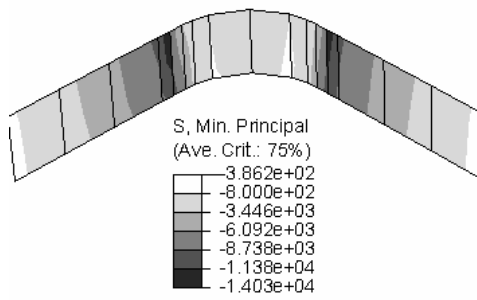


(d)

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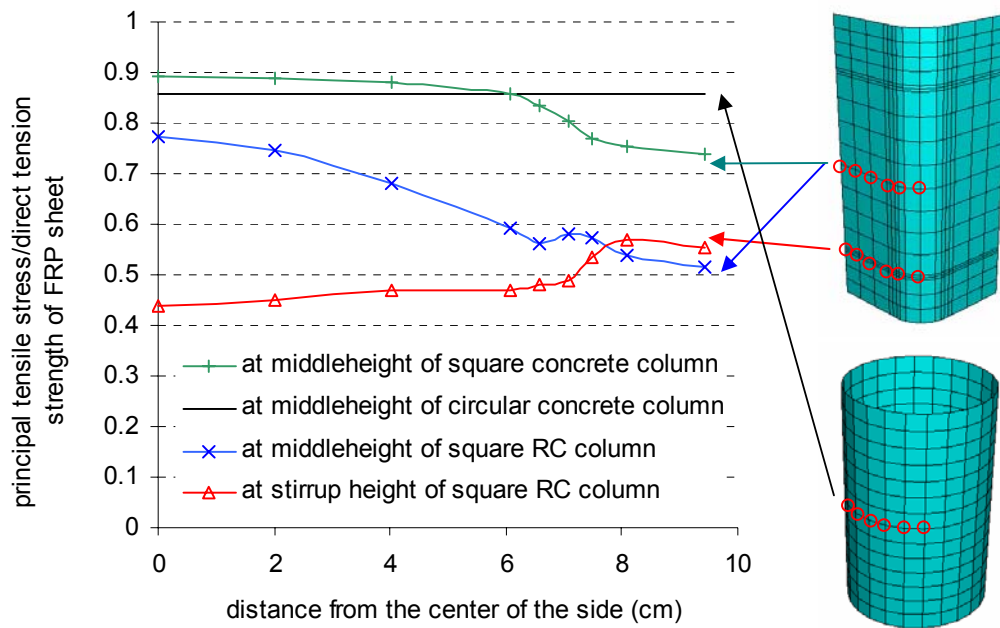
(a)



(b)

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**Fig. 13.** Variation of principal tensile stress of FRP sheet at failure, for different paths of nodes for columns L1, PCFRP and RCFRP.