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3D gear measurement by CMM

W. Lotze[1] & F. Haertig[2]
[1]*Scientific Adviser, Carl Zeiss, Dresden, Germany*
[2]*Physikalisch–Technische Bundesanstalt, Braunschweig, Germany*

Abstract

Gear checking by means of co-ordinate measuring machines is becoming more universal and precise on the basis of a full surface gear model consisting of substitute helical involute flanks. The substitute helical involute flank is equivalent to the best fit evaluation model of any other features in co-ordinate measuring techniques. This model allows us to pick up measuring points anywhere on the flank and to calculate gear parameters beyond common gear standards. Thus there is no need to scan profile, lead and pitch exactly at the defined lines. A further advantage of the universal substitute gear model is the improved measuring and evaluation accuracy. It allows us, furthermore, to ensure the traceability of involute gear measurement.

1 Background

Modern co-ordinate measuring machines (CMM) and software allow the quality of gears to be checked at any place using models and algorithms similar to those used for any other workpieces such as gear boxes. The measurement of gears is not more difficult than that of common workpieces comprising various geometric elements such as planes, cylinders or sculptured surfaces, and even helical involute gear flanks.

The paper deals with the evaluation of gear measuring points by means of a 3D substitute gear model covering separate geometric elements of the gear flanks. The advantage of this 3D evaluation model and the respective software is that measuring points may be picked up anywhere on the gear flanks, without any restrictions on defined cross-sections or measuring planes. Thus the new mathematical model and the software allow gears to be measured as easily as any other geometric elements by means of any NC- or manually operated CMM equipped with a mechanical or optical probe.

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The basic principle of conventional gear measurement is mainly based on two characteristic lines on the flank, i.e. the involute in a transverse plane and the helix at the pitch cylinder. From the point of view of mathematical treatment both lines can be interpreted as lines in two dimensions. The conventional gear model may therefore be referred to as the *2D line model* of a gear. This is also the reason why the conventional gear measuring devices have been designed in such a way that the gear errors can be measured exactly at the lines corresponding to profile and flank line, and in a transverse plane when other errors are measured, e.g. errors of pitch, tooth thickness etc.. At the time when CMM application in gear testing was still in its infancy, the principle was to simulate the conventional gear measuring devices for measurement of profile and flank line as well as pitch and tooth thickness. Large measuring errors caused by gear misalignment and by probe stylus deformation in the case of large helix angles were unavoidable. Manually controlled CMM could not be used in this case either.

The conventional *line model* does not provide a sufficient basis for 3D gear measurement because CMM do not allow gear flanks to be measured alongside defined spatial lines on the gear flanks. The general principle of CMM is to pick up a sample of measuring points of the geometric elements of a workpiece. From these measuring points the mathematical expression of the geometrical element, referred to as substitute element, must be calculated (parameterised) by means of best fit algorithms. Consequently, any gear measurement by means of CMM must be modelled in the same way as described in the following:

1. Measurement of the spatial surface point co-ordinates {x, y, z} of the gear flanks by means of a spherical probe tip.
2. The probe head must be able to pick up measuring points independent of the spatial orientation of the gear flank (spur and helical gear or worm gear).
3. Evaluation of the measuring points by means of best fit algorithms according to the principles of orthogonal distance regression (ODR) for the gear flanks to be determined (helical involute surfaces).
4. Calculation of the searched sizes (length and angular) as defined in the drawing from the parameters of the substitute elements.

The complete and precise performance of a gear measurement by CMM thus requires a full 3D model and 3D measurement, independent of any defined cross-sections, lines or points. This new model is therefore referred to as *3D surface model* for gear testing by CMM. The paper gives a short overview of the new mathematical model, the measuring principle and the application of the INVOLUTE software package in spur and helical gear testing. Despite the new evaluation kernel the result output is oriented to the gear errors as defined in the existing standards [3, 4, 5].

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2 The 3D model of the helical gear

It is the state of the art in the design, manufacture and measurement of workpieces to split them into individual simple geometric elements and to use suitable mathematical representations in the various software packages (CAD, CAM, CAQ). For measuring purposes, a distinction must be made between the real geometric elements of a machined workpiece and the substitute geometric elements represented by a set of geometrical equations for computer representation. The real geometric elements of a workpiece in fact errors of position, size and form, as well surface roughness. The substitute geometric elements are more or less mathematical approximations to the real workpiece, which are to make possible the calculations of the actual workpiece errors of size, position and form. Such 3D models must even be used for gear flanks and complete gears in order that the same scope of workpiece information as for any other workpieces is obtained [6].
A common gear consists of two sets of geometric elements, such as:

- The reference elements for the gear axis, e.g. the bore or shaft of a gear (except the axis of splines). These elements determine the gear axis and the co-ordinate system for gear evaluation.
- The substitute gear flanks of spur and helical gears as a set of involute flanks in space as shown in fig. 1.

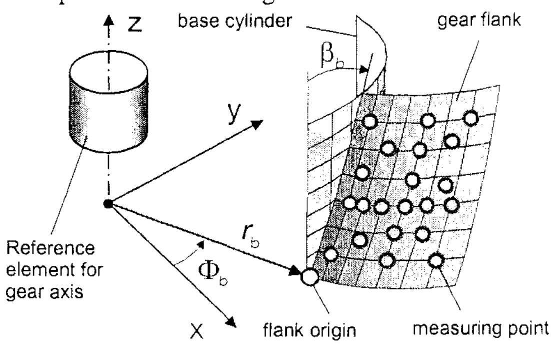

Figure 1: Substitute gear flank and measuring points

The substitute gear flank [1, 2] is determined by the flank origin as the point of intersection of base helix and *xy*-plane. The mathematical parametric equation for the helical involute flank contains only three coefficients as given below:

- base radius r_b of the flank origin;
- polar angle φ_b of the origin;
- base helix angle β_b.

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$$\mathrm{F}(\alpha,\zeta)=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}r_b\ \sec\alpha\ \cos[\varphi_{b0}+C\,\zeta-h\ Ev\alpha]\\r_b\ \sec\alpha\ \sin[\varphi_{b0}+C\,\zeta-h\ Ev\alpha]\\\zeta\end{pmatrix} \quad (1)$$

with

	$C = \tan(\beta_b)/r_b$	helix coefficient
	$\mathrm{Ev}(\alpha) = \tan(\alpha)-\alpha$	involute function
	r_b	base radius
	α, ζ	Gaussian surface parameters (pressure angle, axis co-ordinate)
	h	flank direction ($h=\pm 1$ for left- and right-hand flank)

Further coefficients may be used for describing flank modifications (e.g. crowning). When this single flank model is used, the whole substitute gear with z teeth consists of 2 z substitute left- and right-hand flanks (see fig. 2) defined by a set of 6^*z coefficients (z = number of teeth).

Which are the advantages of the full 3D model of the real gear flanks with individual coefficients to be calculated from measuring points selected on a sample or even on each flank:

- use of complete computerized gear model of the whole gear is possible for any further calculations;
- calculation of any cross-section with transverse plane, pitch cylinder, straight lines or circles for the determination of profiles and further geometric elements as is usual in evaluations in co-ordinate measuring technique;
- calculation of any lengths and angles for determination of pitch, tooth thickness, base length, radial and diametral size over balls, run-out, etc.;
- determination of any form deviation in defined cross-sections for profile and lead errors.

One of the main advantages of the measuring point evaluation by means of the 3D surface model is that for profile and lead measurement the measuring points must not be picked up exactly in the transverse plane for profile testing or on the pitch cylinder for lead testing.

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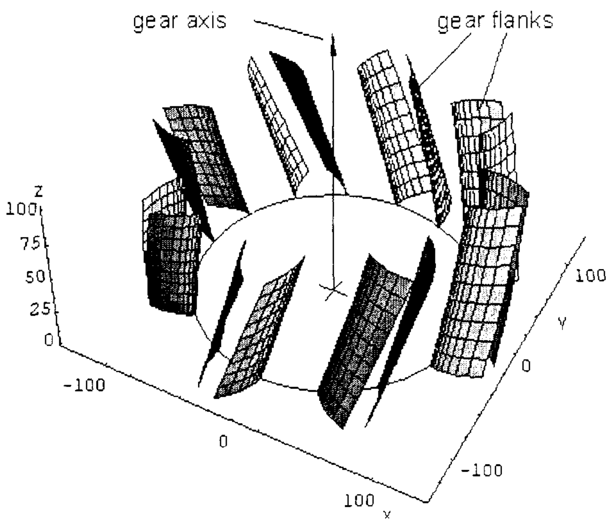

Figure 2: Substitute model of the whole gear consisting of 2 z substitute gear flanks

All misalignment errors of the gear are exactly compensated by means of the 3D flank evaluation. There is also no need to clamp the probe in a predefined measuring plane. A touch trigger probe or a 3D analogue probe with three degrees of freedom allows measuring points on the gear flank to be picked up without any error even in the case of very large helix angles (e.g. worm gears). The only condition is that the gear axis must be measured at first and the measuring points must be transferred into the workpiece coordinate system with the gear axis serving as the z-axis.

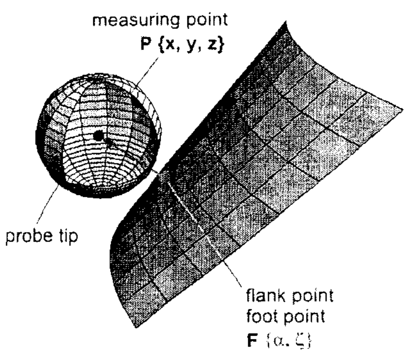

Figure 3: Measuring point and flank point

The coefficients of each flank must be calculated from the measuring points by best fit routines based on orthogonal distance regression (ODR) [6]. Thus the distance f of the measuring points P$\{x, y, z\}$ from the best fit helical involute flank must be calculated in direction normal to the corresponding flank points F$\{\alpha, \zeta\}$ as shown in fig 3. The corresponding flank points are also referred to as foot points. Therefore the surface parameters $\{\alpha, \zeta\}$ of the foot points must be determined as well. The ODR best fit evaluation in the normal direction, taking the probe tip radius into consideration, is the only way of correct evaluation although it seems in contradiction to the conventional way of determining profile deviations in the transverse plane.

3 Flank parameterisation by means of best fit calculation

To evaluate the gear measuring points (picked up using probe balls of different diameter anywhere on the flank) and depending on the parameters searched, different types of best fit routines with different constraints and objective functions must be used together with probe radius compensation. Furthermore, the distinction between gears and splines must be taken into account. As with geometric elements the following objective functions may be used:

- least squares method (Gaussian method), so far preferred in gear measurement
  $$Q_1 = \sum f_i^2 \Rightarrow Min \qquad (2)$$
- minimum zone method (Tsebyshev method), widely used to determine of the form deviation according to ISO 1101 of any geometric elements, available but not yet used in gear measurement
  $$Q_2 = Max\ (|f_i|) \Rightarrow Min \qquad (3)$$
- L1 approximation as a robust best fit method with low sensitivity to outliers, also not commonly used today
  $$Q_3 = \sum |f_i| \Rightarrow Min \qquad (4)$$
- full adjacent gear profile optimisation in the case of splines based on tooth thickness and axis optimisation; also not yet used today
  $$Q_4 = Max\ (s_i) \Rightarrow Min \qquad (5)$$

The deviation f_i of measuring points must be calculated as distance orthogonal to the flank surface, and the best fit evaluation must be carried out as the so-called orthogonal distance regression (ODR). This distance can be calculated as the distance between the measuring point itself and the respective foot point on the flank. The calculation of the Gaussian parameters $\{\alpha, \zeta\}$ of the foot points F therefore is always part of the best fit evaluation of measuring points P(x, y, z).

It follows for the scalar and signed distance f of a measuring point $P \equiv \mathbf{x} = \{x, y, z\}^T$ picked up by a probe with tip radius r_T and foot point parameters $\{\alpha, \zeta\}$ that

$$f = \mathbf{n}^T (\mathbf{x} - \mathrm{F}(\alpha, \zeta)) - r_T \qquad (6)$$

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and the normal vector **n**

$$\mathbf{n} = \frac{\dfrac{\partial \mathbf{F}}{\partial \alpha} \times \dfrac{\partial \mathbf{F}}{\partial \zeta}}{\left| \dfrac{\partial \mathbf{F}}{\partial \alpha} \times \dfrac{\partial \mathbf{F}}{\partial \zeta} \right|} \qquad (7)$$

In the case of single flank evaluation by means of any of the best fit methods mentioned (except spline evaluation), the following constraints must be distinguished:

- full flank best fit with three degrees of freedom for the determination of the coefficients r_b, φ_b, and β_b;
- constrained flank best fit with locked helix angle β_b (for the evaluation of the profile error in relation to the nominal profile);
- constrained flank best fit with locked base radius r_b (for the evaluation of the lead error in relation to the nominal profile);
- constrained flank best fit with locked base radius r_b and helix angle β_b (for the evaluation of the pitch and tooth thickness error).

In the case of splines, a full best fit can be carried out simultaneously all teeth to determine the spline axis and the effective tooth thickness (similar to a full go-gauge). All the best fit routines mentioned are implemented as a software kernel for gear evaluation by means of the powerful INVOLUTE software package.

The overview of the various best fit methods shows many possibilities and ways of gear measurement evaluation, beyond the evaluation strategies referred in the conventional gear standards. The new methods based on the full 3D feature model of gears must be taker into account in future in order to avoid contradictions and misunderstandings as regards the results of gear testing. On the other hand, the new parameters and results provide more information for machining and quality control in gear production.

It should be mentioned that the ODR best fit evaluation converts the $\{x, y, z\}$ co-ordinates of the measuring points into involute co-ordinates $\{\alpha, \zeta, f_n\}$, f_n being the deviation in the normal direction of the flank. In order to determine the common gear errors in the transverse plane, this value must be converted using the following equation

$$f_y = f_n \frac{1}{\cos \beta_b} \qquad (8)$$

It is yet another advantage that the same evaluation procedure can be used for worm gears as well. The best fit evaluation of measuring points works even in the case of helix angles close to 90°.

Besides the individual point deviations f_n a set of 6 z (six times the number of teeth) coefficients will be available for further evaluation. These coefficients do not directly describe the gear parameters and gear deviations as used for the gear tolerances stated in various standards. But these normal parameters can be calculated by the substitute gear model.

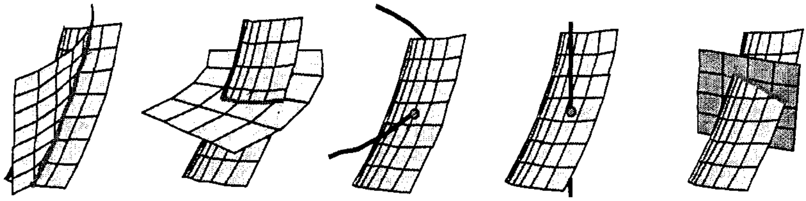

Figure 4: Intersection of involute flanks with other geometric elements

Fig. 4 shows a few examples of the calculation of intersections of a flank with other geometric features for the determination of profile and lead errors, pitch errors, etc.. As an example the calculation of the radial single-ball size is demonstrated by the geometric model shown in fig. 5. Two adjacent flanks may be reconstructed from the actual coefficients. To determine the size over a single ball, the ball is moved into the gap between two adjacent flanks. The position of the measuring ball can be calculated by iterative calculation as the point of intersection of a transverse plane and the two equidistant helical involute flanks.

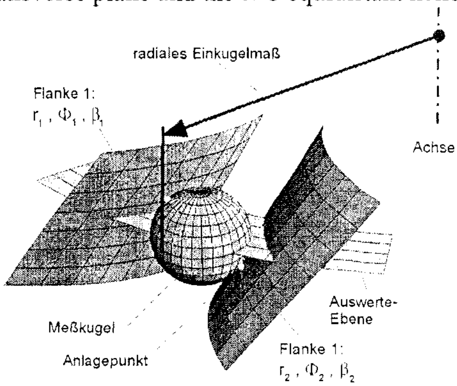

Figure 5: Calculation of the radial single-ball size

The radial size over the ball is calculated as the distance of the midpoint from the gear axis and the measuring ball radius. From this size, the run-out as well as the size over two balls can be determined for each gap. Further gear parameters can be determined by similar geometric models and calculations. After determination of the whole substitute gear model (including crowning etc., if necessary) all known gear parameters and deviations can be calculated.

The above mentioned 3D surface model and software kernel, which is strictly based on the rules of CMM measurement and modelling, offers yet another advantage. The full mathematical 3D gear model allows the measuring uncertainty of the individual gear parameters to be calculated or estimated as for any other geometric elements such as cylinders etc.. Furthermore, if the CMM has taken fully calibrated, the traceability of gear measurements by means of

CMM and certified software can be guaranteed. The only gap is the certification of the software by means of certified test data. A group of experts (preferably from national metrology institutes) should work out a number of xyz datasets for the testing and certification of gear evaluation software as it was done for other geometric elements a few years ago [7].

4 INVOLUTE 97 software package

Several years ago, the INVOLUTE software package was developed for gear measurements by means of manually or CNC-controlled co-ordinate measuring machines. It is meanwhile used by a number of CMM manufacturers as standard package for gear measurement. It has been redesigned now for the Windows NT/2000 operating system, and it uses the INVOLUTE kernel for data evaluation based on the full 3D surface model described above. It can also be used as a two-dimensional (2D) package for the evaluation of optical gear measurements by means of image processing systems. The measuring parameters of the program are shown in table 1.

Table 1: Parameter limits of gears to be tested by the INVOLUTE program

Parameter	Symbol	min	max
number of teeth	z	3	500
normal module	m_n	0.1	-
pressure angle	α	10°	40°
helix angle	β_b	0°	85°
number of teeth in segment	z_s	1	z

The standard version of the program has been compiled for the English and German language. It can be run in the stand-alone mode as well as linked to the CMM software (via COM). Touch trigger probes as well as analogue 3D probes or optical probes can be used. The user may define the individual evaluation tasks such as profile testing, lead testing, pitch testing or flank testing, the number of measuring points per flank and even the number of fully tested teeth.

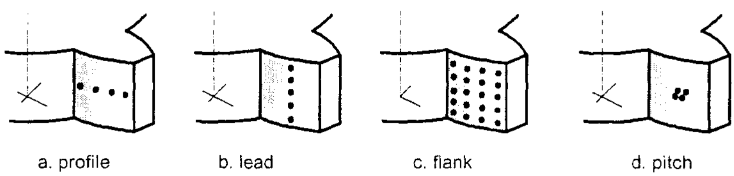

Figure 6: Measuring strategy for the testing of profile, lead, topography and pitches

A single involute profile can be tested as well as a full gear with all profiles, leads and pitches. The measuring points may be picked up point by point with a touch trigger probe or by scanning with analog probes. In this case, low-pass filters such as the Gaussian filter can also be activated for profile and lead measurement with different cut-off.

Evaluation of the measuring points requires that the nominal gear parameters be available as a nominal data file. The parameters, tolerances, designations and additional information can be defined by a menu for nominal parameters. A universal system for K-charts has been implemented to make checking of profile modification possible. Options are available for the testing of special modifications such as crowning and relief.

A very flexible and language-independent system has been implemented for result output. It allows special output forms to be designed in any language for text and graphics output. The profile, flank and pitch error charts may be printed in any orientation, size and number. The flank topography can also be drawn when a regular or random grid of flank measuring points is measured (about 100 up to more than 1000 points per flank). The program is also equipped with powerful error-detecting routines for the nominal data and the measurement data.

5 Reducing measuring uncertainty

Taking an approach which treats the flank of gears as a surface, the German national metrological institute, the Physikalisch-Technische Bundesanstalt is developing new measuring methods to ensure the traceability of involute gears. This is a must in order that the ever more exacting accuracy requirements for the measurement of standards can be met. To date, gears have been traced back via one standard each for profile, helix slope and pitch, a special standard calibration facility being available for each of these standards. These facilities allow the national primary standards to be traced back very accurately. The substitution method is applied to pass the measure on within the traceability chain, from industrial standards and master gears to the product [8, 9]. The main drawback of this method, apart from the time and effort to be spent, is the accuracy loss at each stage of the traceability chain, due in particular to the fact that the dimensions of standards and product may differ considerably. An essential aim therefore is to clearly shorten the traceability chain. This aim can be reached by making it possible to calibrate standards, which closely approximate the work piece (product-like standards), already at the level of the national institutes (Fig. 7).

In future, the gears will be traced back using a coordinate measuring machine whose axes are monitored interferometrically, and a highly precise rotary table. As is common practice in the link-up of prismatic work pieces such as circle, cone or sphere, the flanks of a gear will in future be treated as areas. This approach offers two essential advantages:

- The evaluation in the classical way by profile, helix slope and pitch remains possible.

- The flank is treated as a surface. It is represented on the basis of its characteristic parameters, such as base circle, lead and position.

Traceability to measurement results along the classical lines is an important prerequisite which makes a comparison with the evaluation guidelines possible which have been followed for decades. The description of the involute as a surface bridges the gap to modern 3D measuring techniques which need not put up with the technical limitations of the past.

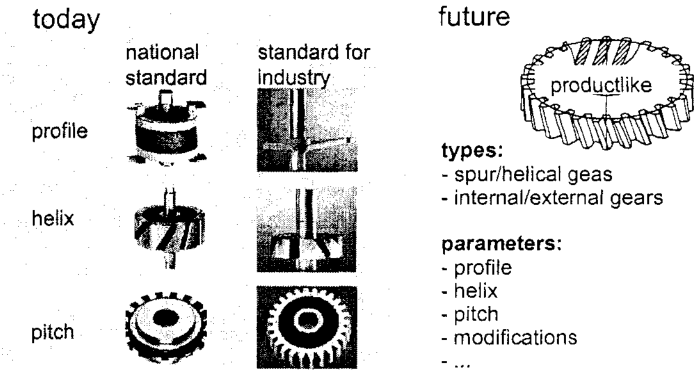

Figure 7: Gear artefacts today and in future

6 Final remarks

The traditional quality control of gears has been increasingly turned into a field of application of the co-ordinate measuring technique. Because of the very conventional gear standards and testing recommendations the evaluation software commonly used simulates only the conventional and more or less two-dimensional testing procedures. When the more general 3D analytic modelling of geometric elements is applied, gear evaluation is based on substitute helical involute flanks and finally on a full 3D substitute gear model. This gives much more flexibility for measurement and evaluation of gears by CMM. There is no need anymore to measure profile, lead or pitch exactly at predefined flank lines or flank points. The new INVOLUTE 97 software package for the Windows NT/2000 operating system is a very powerful tool for the testing of helical and spur gears by means of manually and CNC-controlled CMM based on the full 3D substitute gear model.

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