3D Pendulum Experimental Setup for Earth-based Testing of the Attitude Dynamics of an Orbiting Spacecraft

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***Abstract*— A 3D pendulum implemented using a triaxial air bearing system is proposed for Earth-based testing of orbiting spacecraft attitude dynamics and closed loop systems. This proposal is based on prior research on attitude dynamics and control of orbiting spacecraft, attitude dynamics and control of the 3D pendulum, and an experimental implementation of the 3D pendulum in our laboratory, referred to as the Triaxial Attitude Control Testbed (TACT). These research themes are integrated to assess the strengths and weaknesses of such an Earth-based testbed for spacecraft attitude dynamics and control hardware and software components. Several different cases, based on the importance of orbital effects and gravitational effects, are analyzed.**

I. Introduction

Spacecraft attitude dynamics and control has been a widely studied topic over the past several decades. Ever improving technology leads the way for future vehicles such as optical telescopes, space-based interferometry, and formation flight, capable of highly complex missions.

Design of such spacecraft attitude systems leaves engineers with a choice between Earth-based testing where gravity plays a dominant role and expensive space-based testing where problems could result in mission failure. This is especially important for small organizations that do not have the resources to carry out extensive analytical studies. Space vehicles can be assured to meet attitude mission requirements if suitable Earth-based testing is a part of the design process.

There are few test methods that simulate a space-like environment on Earth. Hardware-in-the-loop and software-in-the-loop methods allow testing of hardware and software components based on computer simulation of the spacecraft attitude dynamics and the space environment. These methods are valuable in testing the component input-output properties, but they are strongly dependent on the attitude models used in the simulation, and they typically do not expose the hardware or software components to the rotational dynamics environment experienced by an orbiting spacecraft.

The proposed methodology provides a physical Earth-based alternative to testing hardware and software components that are part of spacecraft attitude dynamics and control systems. This methodology is based on a triaxial attitude control testbed that is a physical implementation of a 3D pendulum using a spherical air bearing. Although air bearing rotational system have their limitations, they do allow the testing of spacecraft hardware and software in a low-torque environment similar to space. Air bearing systems offer a financially viable option for Earth-based testing of advanced spacecraft dynamics and control components.

Several air bearing testbeds are in use today, both in educational institutions and in government and industry laboratories. A historical survey of air bearing simulators is provided in [1], with a more in-depth look at various testbeds in [2], [3], [4], [5]. These systems represent only a fraction of the air bearing systems being used today. Each of the testbeds described in the literature shares a common attribute; their center of mass is intended to be coincident with their center of rotation. This attribute simulates a zero or low gravity spacecraft environment, but it may not be suitable when gravity gradient effects are significant, such as for spacecraft in low Earth orbit (LEO).

At the University of Michigan's Attitude Dynamics and Control Laboratory, a Triaxial Attitude Control Testbed (TACT) was developed in the late 1990's to explore various issues and concepts in spacecraft dynamics and control. Following the notation in [1], the TACT is classified as a dumbbell-type rotational air bearing system. The TACT consists of a rotational platform, supported by a triaxial air bearing, which allows nearly unrestricted three degrees of rotational motion. The TACT is limited only in pitch to $\pm 45\,\mathrm{deg}$, with both roll and yaw axes entirely free of motion constraints. Control actuators and instrumentation are mounted to the platform. Computer and communications

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systems are available for data acquisition, test operations, and post-test processing. This setup has been discussed in detail in [6] with mathematical models given in [7]. A picture of the TACT is provided in Figure 1.

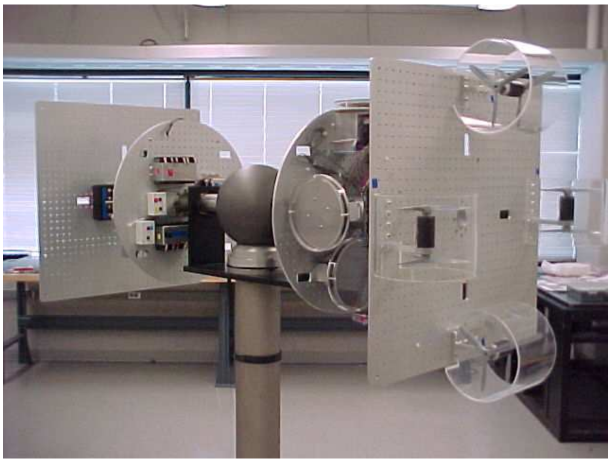

Fig. 1. Triaxial Attitude Control Testbed (TACT)

In recent publications, the TACT has been viewed as a physical implementation of a 3D pendulum [8], [9]. A 3D pendulum is a generalization of the classical planar pendulum and the spherical pendulum with the mass center not necessarily coincident with the pivot point. From the very onset of the TACT research, it has been proposed that the experimental testbed could be used for spacecraft dynamics and control experiments. This paper treats these issues and provides a summary of the conditions under which a 3D pendulum experimental setup can be used for Earth-based testing of hardware and software components of the attitude dynamics and control systems of an orbiting spacecraft.

The main contribution of this paper is the summary of an experimental Earth-based methodology that allows, within limits, the testing and evaluation of spacecraft attitude dynamics and control hardware devices and related software systems. One can test and evaluate different attitude control actuators and sensors as well as other control hardware and software instruments and components. For extended mission requirements, the operational life times of these components can be assessed. The operations of pointing, tracking, and attitude stabilization of the spacecraft can also be tested and evaluated in a controlled experimental environment. This is based on several different results for the rotational dynamics of orbiting spacecraft and for the 3D pendulum, as well as our prior experience with the TACT.

The presentation is divided into two parts based on the importance of orbital and gravity gradient effects on the attitude dynamics of a rigid spacecraft in a circular Earth orbit. Orbital effects are important since the attitude of the spacecraft in a circular orbit is conventionally defined with respect to a uniformly rotating local vertical local horizontal (LVLH) coordinate frame defined by the circular orbit. In Earth orbit, the gravitational force is not uniform and there is a gravity gradient moment about the spacecraft center of mass [10]. Gravity gradient refers to the attitude dependent potential arising from the differential gravitational force of the Earth on the spacecraft. Gravity gradient effects are the dominant environmental disturbance present in the altitude range $400 - 40,000$km. Below this altitude, aerodynamic drag is the most significant disturbance. These results are summarized in Figure 2, where the gravity gradient moment is shown for an attitude perturbation of $1\,\mathrm{deg}$.

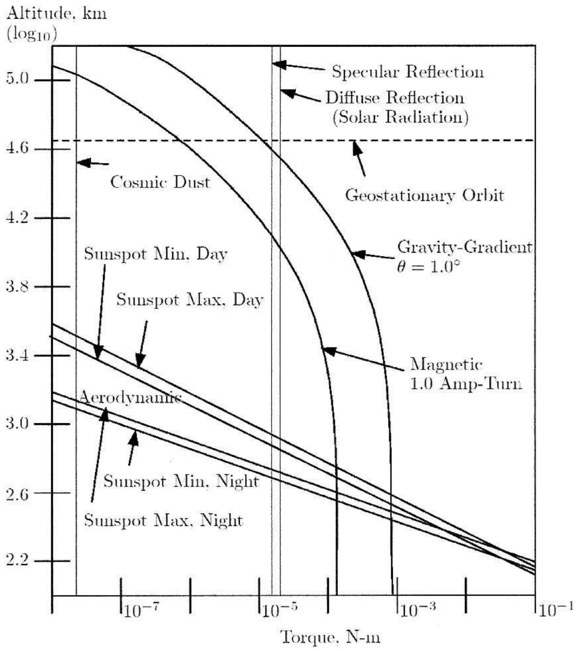

Fig. 2. Relative effects of various disturbance torques

II. Testbed for Spacecraft Attitude Dynamics without Orbital and Gravity Effects

In this section, we consider an orbiting spacecraft where the rotation of the LVLH frame and the gravity gradient moments are ignored; such an approximation is often justified in high Earth circular orbits (HEO). In this case, we show that a balanced 3D pendulum, and a TACT experiment, can be constructed whose attitude dynamics are identical with the spacecraft attitude dynamics. The balanced 3D pendulum requires that the center of mass be located at the pivot. We present equations of motion for the attitude dynamics of a spacecraft, ignoring orbital effects and gravity gradient moments. We also present equations of motion for the balanced 3D pendulum, or its physical implementation as a TACT. The equations for the attitude dynamics of the two systems are shown to be identical, thereby providing the basis for use of the TACT as a testbed for the attitude dynamics of an orbiting spacecraft.

A. On-Orbit Spacecraft Equations

Consider a rigid spacecraft in a high altitude circular orbit about the Earth. We assume the spacecraft is in a sufficiently high orbit that gravity gradient effects are negligible. First we introduce some useful notation. The cross product operator, denoted by $\hat{a}$, is represented by a 3×3 skew-symmetric matrix:

$$\hat{a} = \mathrm{S}(a) \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$

Euler's rotational equations of motion for an uncontrolled rigid spacecraft viewed as a free rigid body [10], [11] are given by

$$\begin{cases} J\dot{\omega} = J\omega \times \omega, \\ \dot{R} = R\hat{\omega}, \end{cases} \tag{1}$$

where $\omega \in \mathbb{R}^3$ represents the angular velocity of the spacecraft, expressed in spacecraft-fixed coordinates with respect to an inertial frame, $R \in \mathrm{SO(3)}$ represents the attitude of the spacecraft with respect to an inertial coordinate frame, and J is the inertia matrix of the spacecraft.

B. 3D Pendulum Equations

Now consider a 3D pendulum where the center of mass of the pendulum is exactly located at the pivot. In this case, there is no gravity moment on the pendulum and the equations of motion are given by

$$\begin{cases} J\dot{\omega} = J\omega \times \omega, \\ \dot{R} = R\hat{\omega}, \end{cases} \tag{2}$$

where $\omega \in \mathbb{R}^3$ represents the angular velocity of the pendulum, $R \in \mathrm{SO(3)}$ represents the attitude of the pendulum with respect to an inertial coordinate frame, and J is the inertia matrix of the pendulum.

C. Comparison

It is clear from the above formulations that (1) and (2) are identical systems of equations. If the moments of inertia of the pendulum are selected to be identical to, or uniformly scaled with, the moments of inertia of the spacecraft, then the global attitude dynamics of the 3D pendulum exactly represent the global attitude dynamics of the spacecraft. This implies that it is relatively straightforward to use an Earth-based triaxial attitude system to test flight software and hardware for suitable spacecraft missions.

This has been the underlying basis for most air bearing spacecraft attitude dynamics and control testbeds, past and present. In practice, however, it is not a trivial task to align the center of mass with the pivot. Much care must be taken to reduce the center of mass offset as much as possible to counteract any moments due to gravity.

III. Testbed for Spacecraft Attitude Dynamics including Orbital and Gravity Effects

In this section, we consider an orbiting spacecraft where the orbital effects and gravity gradient moments are assumed to be significant. The objective is to assess if there are 3D pendulum attitude dynamics that approximate the orbiting spacecraft attitude dynamics. This case is somewhat subtle.

We first present equations of motion for the attitude dynamics of an orbiting spacecraft, including effects due to rotation of the LVLH frame and including the gravity gradient moment. We then summarize the relative equilibria structure of the equations, and we give linear equations that describe small perturbations from a relative equilibrium. Next, we present equations of motion for the attitude dynamics of the 3D pendulum assuming that the center of mass is distinct from the pivot. The equilibria of the 3D pendulum are described and linear equations for small perturbations from an equilibrium are given.

The nonlinear equations for the attitude dynamics of the two systems are distinct with differing equilibria. We show that, in certain cases, the linear approximations of the two sets of equations are similar. In these cases, the linear equations for attitude perturbations from an equilibrium of the 3D pendulum can be viewed as an approximation of the linear equations for attitude perturbations from a relative equilibrium of an orbiting spacecraft. These observations provide a conceptual basis for use of a 3D pendulum experimental setup as a testbed for the attitude dynamics of an orbiting spacecraft.

A. On-Orbit Spacecraft Equations

Consider a rigid spacecraft in a two-body circular orbit about the Earth. We assume the spacecraft is in a sufficiently low orbit such that rotation of the LVLH frame and the gravity gradient moment are not negligible. Recall that a LVLH right hand coordinate frame is attached to the spacecraft center of mass with its first axis pointing in the direction of the spacecraft orbital velocity vector and third axis pointing towards the Earth's center. The second axis is mutually orthogonal to the first and third axes, normal to the orbital plane. This rotating LVLH coordinate frame is taken as the reference frame in defining the attitude of the orbiting spacecraft.

The rotational equations of motion for a rigid spacecraft, including gravity gradient moments [10], [11], are given by

$$\begin{cases} J\dot{\omega} = J\omega \times \omega - 3\omega_0^2 (JR^{\mathrm{T}} e_3 \times R^{\mathrm{T}} e_3), \\ \dot{R} = R\hat{\omega}. \end{cases} \tag{3}$$

where, $\omega \in \mathbb{R}^3$ represents the angular velocity of the spacecraft, expressed in spacecraft-fixed coordinates with respect to the LVLH frame, $R \in \mathrm{SO(3)}$ represents the attitude of the spacecraft with respect to the LVLH coordinate frame, J is the inertia matrix of the spacecraft, $e_2 \triangleq (0, 1, 0)^{\mathrm{T}}$

and $e_3 \triangleq (0,0,1)^{\mathrm{T}}$ are unit vectors, and ω_0 represents the constant rotation rate of the LVLH reference frame with respect to an inertial frame. This rotation rate is given by $\omega_0 \triangleq \sqrt{\frac{GM_e}{a^3}}$, where $G = 6.672 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is the universal gravitational constant, $M_e = 5.9742 \times 10^{24}$ kg is the mass of the Earth, and a represents the constant orbital radius.

The conditions for a relative equilibrium are given by a constant rotation matrix R_e such that the principal axes are aligned with the LVLH axes and the constant angular velocity vector of the spacecraft is given by $\omega = \omega_0 R_e^{\mathrm{T}} e_2$. Physically this means that a relative equilibrium corresponds to exact alignment of the spacecraft-fixed axes with the LVLH axes and a constant angular velocity about the axis that is normal to the orbital plane with magnitude given by the orbital angular rate.

It is convenient to assume that the spacecraft-fixed coordinate axes are the principal axes of the spacecraft. Thus the moment of inertia matrix is $J \triangleq \mathrm{diag}(J_1, J_2, J_3)$. The nonlinear spacecraft attitude equations of motion can be approximated, near a relative equilibrium, by linear equations

$$J\Delta\ddot{\Theta} + \mathcal{C}\Delta\dot{\Theta} + (\mathcal{K}_1 + \mathcal{K}_2)\Delta\Theta = 0, \tag{4}$$

where

$$\mathcal{C} \triangleq \begin{bmatrix} 0 & 0 & \omega_0(J_1 - J_2 + J_3) \\ 0 & 0 & 0 \\ -\omega_0(J_1 - J_2 + J_3) & 0 & 0 \end{bmatrix},$$

$$\mathcal{K}_1 \triangleq \begin{bmatrix} 3\omega_0^2(J_2 - J_3) & 0 & 0 \\ 0 & 3\omega_0^2(J_1 - J_3) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{K}_2 \triangleq \begin{bmatrix} \omega_0^2(J_2 - J_3) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \omega_0^2(J_2 - J_1) \end{bmatrix},$$

and $\Delta\Theta \triangleq [\theta_1 \quad \theta_2 \quad \theta_3]^{\mathrm{T}}$ are 3-2-1 Euler angles that describe small angle perturbations from the relative equilibrium. The terms in the symmetric matrix $\mathcal{K}_1$ arise from the gravity gradient moment. The terms in the skew-symmetric matrix $\mathcal{C}$ and the symmetric matrix $\mathcal{K}_2$ arise as a consequence of the uniform rotation of the LVLH frame at the orbital angular rate about the axis normal to the orbital plane.

Conditions for local stability of a relative equilibrium depend on the values of the three moments of inertia. These stability conditions are summarized in [10], [11]. Some of the 24 distinct relative equilibrium solutions are stable in the sense of Lyapunov according to the above linear equations; other relative equilibrium solutions are unstable.

B. 3D Pendulum Equations

Now consider a 3D pendulum where the center of mass is not located at the pivot. The dynamics and kinematics equations of motion for the 3D pendulum, introduced in [7] and [8], are

$$\begin{cases} J\dot{\omega} = J\omega \times \omega + mg\rho \times R^{\mathrm{T}} e_3, \\ \dot{R} = R\hat{\omega}, \end{cases} \tag{5}$$

where $\omega \in \mathbb{R}^3$ represents the angular velocity of the pendulum, expressed in body-fixed coordinates, and $R \in \mathrm{SO(3)}$ represents the attitude of the pendulum with respect to an inertial coordinate frame. Here, J is the inertia matrix of the rigid 3D pendulum, m is its total mass, ρ is the constant vector from the pivot to the center of mass in the pendulum-fixed coordinate frame, and $g = 9.81$ m s^{-2} is the constant acceleration of gravity at Earth's surface.

Equilibrium solutions for a 3D pendulum satisfy $\omega = 0$, and $0 = mg\rho \times R^{\mathrm{T}} e_3$. Therefore, it is seen that an equilibrium solution occurs when the center of mass vector ρ is co-linear with the gravity vector $R^{\mathrm{T}} e_3$. If the center of mass vector is in the same direction as the gravity vector, we obtain a 1D hanging equilibrium manifold, with the center of mass located below the pivot. If the center of mass vector is in the opposite direction to the gravity vector, we obtain a 1D inverted equilibrium manifold, with the center of mass located above the pivot.

It is convenient to assume that the pendulum-fixed coordinate axes are the principal axes of the rigid pendulum. Thus the moment of inertia matrix is $J \triangleq \mathrm{diag}(J_1, J_2, J_3)$. We further assume that the center of mass lies along the third principal axis. Using 3-2-1 Euler angles to describe small angle perturbations from an equilibrium, the nonlinear 3D pendulum attitude equations of motion can be approximated by linear equations

$$J\Delta\ddot{\Theta} + \mathcal{K}\Delta\Theta = 0. \tag{6}$$

For perturbations from a hanging equilibrium

$$\mathcal{K} \triangleq \begin{bmatrix} mg\|\rho\| & 0 & 0 \\ 0 & mg\|\rho\| & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

and for perturbations from an inverted equilibrium

$$\mathcal{K} \triangleq \begin{bmatrix} -mg\|\rho\| & 0 & 0 \\ 0 & -mg\|\rho\| & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

C. Comparison

It is clear from the above formulations that (3) and (5) describe different sets of nonlinear equations. In fact, the attitude dynamics of the orbiting spacecraft and the attitude dynamics of the 3D pendulum have fundamentally different global properties, including different equilibria structures. The spacecraft attitude dynamics has 24 distinct relative equilibrium solutions, while the angular momentum of the 3D pendulum about the vertical axis is conserved and the 3D pendulum has a 1D manifold of hanging equilibria and a 1D manifold of inverted equilibria. This suggests that the 3D pendulum can not be used to represent the exact attitude dynamics of an orbiting spacecraft, at least when orbital rotation effects and gravity gradient moments are included. However, if we examine the linear equations, we see that the linear attitude dynamics of the orbiting spacecraft and the linear attitude dynamics of the 3D pendulum are similar in some respects.

For additional insight, assume that the spacecraft is an axially symmetric rigid body. The linear equations for the spacecraft attitude perturbation dynamics near a relative equilibrium for which the axis of symmetry is along the local vertical LVLH axis are considered. In this case, $J_1 = J_2$, which we denote by the transverse moment of inertia J_t. We denote the axial moment of inertia by $J_3 = J_a$. In this case the linear perturbation equations for the spacecraft are

$$J\Delta\ddot{\Theta} + \mathcal{C}\Delta\dot{\Theta} + (\mathcal{K}_1 + \mathcal{K}_2)\Delta\Theta = 0, \tag{7}$$

where

$$\mathcal{C} \triangleq \begin{bmatrix} 0 & 0 & \omega_0 J_a \\ 0 & 0 & 0 \\ -\omega_0 J_a & 0 & 0 \end{bmatrix},$$

$$\mathcal{K}_1 \triangleq \begin{bmatrix} 3\omega_0^2(J_t - J_a) & 0 & 0 \\ 0 & 3\omega_0^2(J_t - J_a) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{K}_2 \triangleq \begin{bmatrix} \omega_0^2(J_t - J_a) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

As shown previously, the terms in the symmetric matrix $\mathcal{K}_1$ arise from the gravity gradient moment while the terms in the skew-symmetric matrix $\mathcal{C}$ and the symmetric matrix $\mathcal{K}_2$ arise as a consequence of the uniform rotation of the LVLH frame at the orbital angular rate about the axis normal to the orbital plane.

Even if the orbital rotation effects are not ignored, these linear equations for the orbiting spacecraft attitude dynamics have two zero eigenvalues, which is consistent with the fact that the angular momentum about the vertical axis of the 3D pendulum is conserved. In this respect, the local attitude dynamics of the orbiting axially symmetric spacecraft are captured by the local attitude dynamics of the 3D pendulum.

Further, if the inequality $J_t > J_a$, then the relative equilibrium of the attitude dynamics of the orbiting spacecraft has local properties that are reflected by any hanging equilibrium of the 3D pendulum. This is a physically meaningful case in which the TACT can provide a testbed for the attitude dynamics of an orbiting spacecraft. If the inequality $J_t < J_a$, then the relative equilibrium of the attitude dynamics of the orbiting spacecraft has unstable local properties that are reflected by any inverted equilibrium of the 3D pendulum. Test operations in such an unstable situation are not usually important.

Although the linearized attitude dynamics of an axisymmetric orbiting spacecraft and the linearized attitude dynamics of the 3D pendulum are not identical, they differ only in the terms that arise due to the rotation of the LVLH frame. In different words, the attitude dynamics of the 3D pendulum are identical to the attitude dynamics of the axisymmetric orbiting spacecraft if the rotation of the LVLH frame is ignored. This makes clear in what sense the 3D pendulum attitude dynamics and the orbiting spacecraft attitude dynamics are related. This property is perhaps not surprising since the attitude dynamics of the spacecraft are expressed in a rotating frame, while the attitude dynamics of the 3D pendulum are expressed in a non-rotating, inertial frame.

IV. Testbed for Spacecraft Closed Loop Attitude Dynamics

The prior discussion has focused on use of the TACT, or any physical implementation of the 3D pendulum, to test hardware or software components of a rigid spacecraft in a circular orbit operating in open loop. The objective is that the 3D pendulum attitude dynamics represent the attitude dynamics of the spacecraft in a circular orbit. As described in Sections II and III, this test objective can be realized, at least with some qualifications.

As noted in Section II, the balanced model of the TACT has the same global dynamics as the orbiting spacecraft model, if one excludes gravity-gradient and orbital effects. Thus, a balanced TACT can be used as an experimental testbed to study attitude dynamics and tracking control problems. Such studies have been carried out, and adaptive controllers for angular velocity tracking and identification have been implemented on the TACT for the orbiting spacecraft model excluding gravity-gradient and orbital effects [12].

For the spacecraft model given in Section III, we observe that one can test hardware and software components using a physical implementation of the 3D pendulum such as the TACT, operating in closed loop based on feedback control of the spacecraft attitude dynamics. It should be clear that this is achieved if the feedback controller proposed for the spacecraft attitude dynamics is suitably realized in hardware and software on the testbed. Most spacecraft control actuators and sensors are rigidly mounted on the spacecraft; in terms of the TACT, these actuators and sensors, and the associated flight control hardware and software, should be rigidly mounted on the TACT in a consistent way. In this fashion, the closed loop attitude dynamics of the spacecraft in orbit are represented by the closed loop attitude dynamics of the 3D pendulum experimental setup.

Closed-loop attitude control experimental results for the TACT are presented in [9], and an attitude control strategy for an orbiting satellite including orbital effects and gravity gradient effects is given in [13].

V. Additional Comments

Even if the spacecraft orbit rotation effects are ignored, it may not be possible to construct a 3D pendulum testbed that exactly reproduces the resulting attitude dynamics of the orbiting spacecraft. In particular, important scaling issues arise in selecting the mass, moment of inertia, and the vector ρ from the pivot to the center of mass of the 3D pendulum. If the mass and moment of inertia of the 3D pendulum are selected to be identical to the those of the rigid spacecraft, then the vector ρ is necessarily small. In practice, it is not possible to modify the mass distribution of the TACT to achieve a precisely specified vector ρ with small magnitude in a specified direction. Small mass elements can be added to the TACT to achieve the desired result, but this is a very delicate and difficult calibration problem.

Such careful TACT design and calibration may not be necessary for most spacecraft testing purposes. The testing objective is likely to require a dynamic attitude environment that, only approximately, reproduces the attitude dynamics of the orbiting spacecraft near a stable relative equilibrium. In this sense, any small imbalance corresponding to a small magnitude for the vector ρ implies that the attitude dynamics of the TACT near a hanging equilibrium approximates the attitude dynamics of an orbiting spacecraft near a stable relative equilibrium for some unknown orbit. In this sense, testing for robustness simply requires that a set of tests be performed for a variety of TACT imbalances.

A more involved testing methodology could combine the physical testing approach described here with the more traditional hardware-in-the-loop approach. This could be accomplished using additional feedback loops whose purpose is to modify the TACT attitude dynamics so that they more accurately reflect the orbiting spacecraft attitude dynamics. This approach could be used to incorporate the rotation effects of the LVLH frame, that may otherwise not be captured in a TACT test, and it could be used to provide a global representation of the gravity gradient moments. Of course, the addition of these feedback loops would add extra complexity and cost to the experimental set up.

VI. Concluding Remarks

Air bearing systems can provide the basis for Earth-based testing and evaluation of technology for spacecraft attitude dynamics and control systems. A triaxial air bearing testbed offers flexibility in carrying out experimental testing of pointing, tracking, and attitude stabilization systems for orbiting spacecraft.

Attitude dynamics and control hardware, such as actuators and sensors, and the control software intended for operation in the space environment, can be assessed in an Earth-based experimental test. This methodology provides a means for studying issues such as control power-on, control power-off, normal control operation, robustness to variations in spacecraft mass, moment of inertia, location of center of mass, orbital parameters, non-rigid body effects due to spacecraft flexibility or fuel slosh, control failure analysis, and reliability of control actuators, control sensors, and control software.

In some cases, exact spacecraft attitude dynamics can be reproduced by a 3D pendulum experimental setup. In other cases it may not be possible to reproduce the exact attitude dynamics of the spacecraft, but the testbed may provide a suitable approximation to the spacecraft attitude dynamics for testing purposes.

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