



50 years of OR in sport

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This paper reviews about 50 years of activity in OR as applied to sports. After some history and an overview, including discussions of what we mean by sport and what we mean by OR, four themes are presented: tactics and strategy, scheduling, forecasting and ‘other’. Within each theme many papers are discussed, showing the wide range of methods used and sports analysed. The issue is then raised of who our clients are and who they ought to be—it is suggested that not nearly enough is done for amateur sport. The paper ends with a conclusion and speculations about the next 50 years.

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1. Introduction

OR was conceived as an approach to aid the military, and was soon afterwards adopted for business problems. In some circles it became a branch of mathematics.

However, OR analysts are not concerned with war, business or mathematics 24 h a day, 7 days a week; in their private lives many of them are interested in sports of various kinds, either as participants or as enthusiastic fans (or both). It occurred to some of these people that the same types of work they were using for ‘serious’ problems could also be applied to activities they were really interested in. Given that every sports fan used to dream of representing their club or nation at a professional level in their chosen sport or sports, this suggested a way in which they could contribute even after their dreams of personal sporting success had been shattered.

The first time that this notion seems to have appeared in print was in 1954, in a letter to the then fledgling *Journal of the Operations Research Society of America* (Mottley, 1954). Mottley was a military man and his view was that ‘the resemblance of [American] football to ground combat is well known; baseball is analogous to a battle between two naval task forces; and basketball can be regarded as a simulation of aerial warfare’. While he was perhaps stretching the analogies rather further than they could tenably reach, his suggestion that ‘if coaches were to use scientific method to help them make decisions regarding the future course of the “operations” under their control, they might be able to make significant improvements in team performance’ has borne plenty of fruit.

For those OR people not interested in sports, this must have seemed a very frivolous idea, which is perhaps why it took some time for it to catch on. The next paper of note was not

for another 9 years, when Lindsey (1963) examined baseball strategies, and for many years after that the published output was quite small. However, the field was given a major boost in the 1970s by two collections of papers collecting together a wide range of studies (Machol and Ladany, 1976; Ladany and Machol, 1977) and eventually the subject developed its own momentum.

Now OR in Sport is a field of research and analysis undertaken by hundreds across the world. Every major conference has its ‘OR in Sport’ stream; there are conferences on mathematics in sport which feature a strong OR presence (eg the *Mathematics and Computers in Sport* series of conferences in Australasia since 1992 and the conference in Salford in 2007 on *Mathematics in Sport*); the Southern OR Group of the OR Society in the UK put on a 1-day workshop on OR in Sport in 2002; there have recently been special journal issues on the topic, for example, a 2003 issue of the *European Journal of Operational Research*, a 2005 issue of the *IMA Journal of Management Mathematics* and a 2006 issue of *Computers and Operations Research*; and we now have a journal devoted to the topic—the *Journal of Quantitative Analysis in Sports*.

This paper does not attempt to give a complete history of OR applied to sport, but instead it aims to give the reader a flavour of what is now a large and diverse field.

There is no universally accepted clear-cut distinction between a sport and a game, but a working definition of a sport is a competitive but essentially non-hostile activity that necessarily involves some kind of physical activity (even if only to a small extent) and cannot be undertaken at a distance, for example over the Internet. Thus, for example, darts and snooker are sports under this definition, whereas chess, poker and mathematics are not. The word ‘non-hostile’ is needed to exclude war and business; some sports may appear to be rather hostile in nature but underneath it all there is, or should be, no desire to cause great personal harm (though

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this can sometimes be difficult to believe when watching, for example, ice hockey or water polo).

2. What is OR and what characterizes OR as applied to sport?

It is often not easy to determine where OR ends and another discipline, for example, statistics or economics, begins. However, I think most OR people would agree that there needs to be a relation to decision-making, either directly (making recommendations for action) or indirectly (providing a framework within which better decisions can be made). In the case of sport, the decision-maker may be an individual sportsperson; a team captain, coach or manager; an administrator of a team, a league or a national association; possibly a sports journalist or a fan; or even a world governing body, or a local or national government.

At a basic level, OR is going on all around us, every time difficult decisions are made. This applies to virtually all sporting activities, whether at a professional level or whether played just for fun. Every time a volleyball team is picked, a golf club is chosen, a cricket field is set, a bet is placed on the Grand National or a five-a-side football tournament is organised, decisions are being made for which OR is applicable and, to some extent, whether consciously or unconsciously, applied.

Most published OR in Sport is not highly theoretical or mathematical, though there are a few exceptions. Some of it reports paid work for a client, though of course papers which do not go beyond pure consultancy do not tend to get published in the quality academic literature. Much of it is 'one-off' in nature, making the topic a little incoherent; thus it would be hard to conceive of a high-quality university module on OR in Sport in the same way as can be done for, say, OR in Supply Chain Management, OR in Forecasting, etc.

However, this does not mean that the OR world in general is not interested in reading papers relating to sport. Anecdotal evidence suggests that much of it is read far more widely than nearly all other OR papers. After all, sport is a topic of great interest and fascination to a very large number of people.

One interesting feature of OR in Sports is the major gulf between North America and the rest of the world. There are some exceptions, but by and large North American researchers mainly work on problems in American sports such as baseball, American Football and basketball, while the rest of the world tends to concentrate more on non-American sports such as football or cricket. This division becomes self-reinforcing: nobody would be likely to submit a paper on cricket to an American journal, and likewise nobody would be likely to submit a paper on American Football to a European journal. It is difficult to conceive of another branch of OR displaying this kind of dichotomy.

Since most of the early researchers into OR in Sports were Americans, the earliest published papers were predominantly about American sports, especially baseball. However, the rest

of the world has now caught up and a very wide range of sports has been studied. The reference list for this paper covers 25 different sports.

Of course, there is more to OR than the contents of academic papers. Many top professional sporting clubs and bodies are prosperous and employ large numbers of people. Among these are analysts of various kinds, many of whom doubtless engage in what we would regard as OR. For example, Lewis (2003) gives a detailed account of how a top baseball manager indulged in OR analysis to determine which players to draft, with a success rate that astonished the more traditionally minded baseball experts.

However, any advances that have been made beyond the published literature would be very difficult to access for reasons of confidentiality. Thus, this paper of necessity restricts itself to the public domain.

The main areas of study are in the analysis of tactics and strategy, scheduling and forecasting. There is also a wide variety of studies which are hard to categorize, which come under the heading 'Other'. This includes analyses of policy issues and the impact of sport upon society.

3. Tactics and strategy

Much of the early OR in sport followed Mottley's lead by considering questions of tactics. For example, Silverman and Schwartz (1973) considered under what circumstances American Football teams should lose on purpose; Freeze (1974) used simulation to find the best batting order for a baseball team; and Ladany (1975a) used probabilistic analysis to determine the best starting height for a pole vaulter (later updated in Hersh and Ladany (1989), when the rules changed). Other work published in the 1970s and 1980s on tactics involved weightlifting (Lilien, 1977)—what weights to choose; the long jump (Sphicas and Ladany, 1977)—how close to the takeoff line to aim for; squash rackets (Clarke and Norman, 1978; Wright, 1988)—whether to choose to play up to 9 or 10; and darts (Köhler, 1982)—which numbers to aim for.

This work continued and became more complex as computing power developed a rapid increase. Thus there are now many applications of dynamic programming (DP), which can be quite a time-consuming technique for problems of a serious size.

Many of these DP studies were undertaken by Steve Clarke and/or John Norman on tactical questions arising in cricket (Clarke and Norman, 1999, 2003; Norman and Clarke, 2007: when should a stronger batsman seek to protect a weaker one, or should a night watchman be used?), in road and fell running and orienteering (Davey *et al*, 1994; Hayes and Norman, 1984; Norman, 2004: which route should you choose, or when should you zigzag uphill?), in tennis (Norman, 1985: when should you go flat out for a very fast first serve?) and in Australian Rules football (Clarke and Norman, 1998: when should you deliberately give the opposition one point in order to reduce the danger of giving them six points?). Clarke's

paper on optimal scoring rates in cricket Clarke (1988), also based on a DP formulation, was awarded the OR Society President's Medal as one of the two best papers to be printed in *JORS* in that calendar year.

Washburn (1991) also applied DP to ice hockey to determine when to substitute an outfield player for a goalkeeper ('pulling the goalie'). Hirotsu and Wright (2002, 2003; also Wright and Hirotsu, 2003) used DP for studies on football and baseball—the latter implementation involving a decision matrix with over a million and a half rows and columns. Decisions analysed include: what use should be made of a pinch hitter in baseball, when should you make a substitution (in either football or baseball), or commit a professional foul in football?

Other techniques applied include various forms of statistical and optimization techniques, with at times several methods being used together. One of the most extreme examples of this is the work of Swartz *et al* (2006), who used loglinear models, Markov chains, metaheuristic search and Monte Carlo simulation to determine the best batting order in cricket.

Most of the above examples concern decisions to be taken in real time during a match. However, there are many tactical and strategic decisions to be made in the cold light of day between matches or even between seasons.

Boon and Sierksma (2003) (see also Sierksma, 2006) produced a decision support system used in practice to help football coaches and managers to assess the potential or actual contributions of particular players to their teams, hence helping not only in team selection but also in scouting and purchasing new players. Cooper *et al* (2008) used Data Envelopment Analysis for similar purposes in basketball.

Many researchers (Audas *et al*, 2002; Bruinshoofd and ter Weel, 2003; Hope, 2003; Tena and Forrest, 2007) have used statistical models to determine when a football manager should be sacked. Between them they conclude that Dutch clubs sack their managers too readily, English clubs used to do so and Spanish clubs get it about right. More recently English clubs may have been giving their managers slightly too long a 'honeymoon' period. Hope (2003) provided a useful decision aid to clubs considering this action.

Other sports to be analysed include: American Football (Brimberg *et al*, 1999)—where to place 'punt returners'; basketball (Annis, 2006)—when to foul; volleyball (Lee and Chin, 2004)—whether to serve when there is a choice; cycling (Scarf and Grehan, 2005)—which route to choose; yachting (Golding, 2002)—route choice again; pentathlon (Ladany, 1975b)—optimizing training schedules; swimming (Nowak *et al*, 2006)—team selection; curling (Kostuk *et al*, 2001)—when to be cautious, when aggressive; and skating (Kuper and Sterken, 2004)—choice of clothing.

As can be seen, there are plenty of tactical and strategic decisions to be analysed and there is no sport which cannot be helped by OR!

4. Scheduling

4.1. Fixture scheduling—practical case studies

All sports leagues have to schedule their fixtures. Often this is a simple process, using readily available patterns; however, in practice there may be more objectives and/or constraints than the standard patterns can readily deal with, and thus specialized software is often required, using an OR algorithm of some kind.

Constraints can derive from issues such as ground availability and the requirements of TV companies or sponsors. Objectives can include minimizing distance travelled, producing good home/away patterns, minimizing dates when two particular teams are both at home, etc. Sometimes changes may need to be made after the season has started, in which case another objective may be to minimize the amount of change required. On top of that there may be several different types of preference expressed by the teams involved, for which relative weights have to be assigned.

In some cases it may be practical to use an exact optimization approach such as integer programming, goal programming, etc. Such approaches have been used for ice hockey (Fleurent and Ferland, 1993); basketball (Nemhauser and Trick, 1998; van Voorhis, 2002); football (della Croce and Oliveri, 2006); and American Football (Urban and Russell, 2003).

Other authors have devised specific heuristics, for example, della Croce *et al* (1999) for tennis; Russell and Leung (1994) for baseball; Bartsch *et al* (2006) for football; and Armstrong and Willis (1993) for cricket. In yet other cases, typically when the problem formulation was very complex, a metaheuristic approach such as simulated annealing or tabu search was used, for cricket (Willis and Terrill, 1994; Wright, 1994, 2005), football (Biajoli *et al*, 2004; Kendall, 2008) and for basketball (Wright, 2006).

However, not all sports fixture scheduling problems need such a sophisticated approach. Thompson (1999) initially used tabu search to schedule the Rugby Union World Cup, but eventually realized that he could produce a solution manually which was just as good—and indeed the organizers of the World Cup preferred the manual solution!

4.2. Fixture timetabling—theoretical studies

Led by Mike Trick at Carnegie Mellon University, there are now several people researching into a set of theoretical problems relating to sports fixture scheduling. These studies often concentrate upon one important objective and ignore any other objectives, which are usually plentiful in real applications but are often different every time, making them unsusceptible to the more traditional forms of OR analysis.

Some of these problems belong to the family of 'break minimization problems', where a break is defined as an

occurrence of two consecutive home matches or two consecutive away matches for a team. Papers on this topic include Elf *et al* (2003), who used a branch-and-cut approach; Miyashiro and Matsui (2006), who used semidefinite programming; Rasmussen and Trick (2007), who used integer programming and constraint programming; and Briskorn and Drex1 (2007), who used a branch-and-price method.

Other theoretical problems considered include ‘the travelling tournament problem’, where distance travelled is an important factor but home/away patterns may not be. Ribeiro and Urrutia (2007) and Lim *et al* (2006) used metaheuristic search to solve problems of this type. Easton *et al* (2001) gave a good description of this problem and its variants.

Such studies do indeed have the potential to be of value to real-life studies, but there is the danger that they may become not much more than just another branch of pure mathematics, though of course there is nothing at all wrong with pure mathematics. In the words of Miss Jean Brodie: ‘for those who like that sort of thing, that is the sort of thing they like’ (Spark, 1961).

4.3. Scheduling of sports officials

Sports fixtures are of course not the only things that need to be scheduled to be sure that competitions can run smoothly. All sports have their officials, whether known as referees as in football or rugby, umpires as in cricket and hockey, judges in equestrian events, etc. Scheduling these officials so as to ensure fairness, high quality, suitable experience and low cost is of great importance to any kind of sporting competition or body.

Thus it is surprising that the issue has not featured more widely in the academic literature. I have been able to find just two papers for baseball—Evans *et al* (1984) and Evans (1988); three for football—Zakarya *et al* (1989), Gil Lafuente (2004), and Yavuz *et al* (2008); two for cricket—Wright (1991) and Wright (2007); plus one (Duarte *et al*, 2007) which may be applicable to several sports. Heuristic or metaheuristic methods have been used, except in the case of Gil Lafuente who uses an exact method. Wright’s (1991) paper generated sufficient interest in the wider OR world to be awarded the OR Society President’s Medal, and it was also one of 12 papers to be reprinted in *JORS* in 2000 to mark the Society’s 50th anniversary.

Objectives here can relate to issues such as travel, frequency of officiating the same team, or being with a specific other official, number of consecutive days on or off, acceding to requests for specific time off, etc. Again there may be constraints laid down by the tournament organizers, sponsors and others, perhaps concerning the different levels of official and what they are eligible to do, which officials should be chosen for the most important fixtures, etc. As with fixture timetabling, problem formulation can often be the most challenging part of the solution procedure.

5. Forecasting

Gambling has always been a major feature of most sports. Because of the amounts of money involved, it is likely that all sorts of OR modelling is taking place within bookmakers’ organizations which is being regarded as highly confidential. However, there are some studies in the public domain of potential use for gamblers and/or bookmakers.

For many years Clarke (1993) used to predict the results of Australian Rules football matches, using the Hooke and Jeeves method, under the alias ‘Tinhead the Tipster’. These forecasts were published in the Melbourne press and performed at least as well as the experts’ predictions. More recently, Flitman (2006) used neural nets and LP for the same purpose.

Dixon and Robinson (1998) reported on a statistical model used to predict the results of football matches and to update those predictions during the course of a match. Fitt *et al* (2006) considered the same issue within the specific context of spread betting. Klaassen and Magnus (2003) and Barnett and Clarke (2005) used statistical analysis to predict the winner of a tennis match. Other work includes Philpott *et al* (2004), who used simulation to forecast the winners of yachting races, and Lo *et al* (1995), who used ranking probability models to help those who gamble on horse races.

Away from gambling, two papers described methods for predicting success at the Olympic Games: Condon *et al* (1999), who used regression and neural nets to forecast national success, and Heazlewood (2006), who reported on non-linear models used to predict winning times and distances for athletics and swimming.

6. Other OR work

As with almost any field of study, complete categorization is impossible, and thus we have an ‘other’ category. Pride of place here must go to two academics whose names are known to literally hundreds of millions of people across the world, probably several orders of magnitude more than the names of any other members of the OR community. They are of course Frank Duckworth and Tony Lewis.

Cricket had for many years struggled to find a sensible way of determining the winners of limited-overs matches curtailed by rain, bad light or other factor. A number of oversimplistic methods were tried but all were found wanting. One spectacular example of this was the 1992 World Cup semi-final between England and South Africa when a sudden flurry of rain near the end changed South Africa’s target from 22 runs in 13 balls—challenging but certainly possible—to 22 runs in 1 ball—quite impossible!

Our two heroes came up with a new method, based on a curve-fitting approach, that was complicated enough to be fair but not too complicated to be used. After a few trials it was indeed seen to be both fair and practical and it became accepted by all involved in professional cricket across the world. Their method is outlined in Duckworth and Lewis

(1998), and although others have come up with allegedly superior models (eg Carter and Guthrie, 2004), the Duckworth/Lewis model is now firmly entrenched in the game of cricket. The phrase ‘duckworth/lewis’ will soon become lower-case like sellotape, biro or hoover—proper names that are now regarded by many as a part of the English language.

Other authors have looked at the effects of actual or potential changes in rules or playing conditions. For example, Albright and Winston (1978) considered what would happen in basketball if the scoring team were to retain possession after scoring, and found that the teams’ probabilities of winning would be almost unchanged; Eiselt and Laporte (1991) considered ways of optimizing the design of dartboards with the objective to maximize risk; and Percy (2007b) analysed the effects of rule changes in badminton designed to make the game more entertaining.

Yet others have examined ways of maximizing fairness: for example, Pollock (1974) investigated fair handicapping methods for golf; Wright (1992) devised fair sets of fixtures for an unbalanced cricket tournament, using local search techniques; Saltzman and Bradford (1996) considered both fairness and cost when examining alternative structures for an American Football league; and Percy (2007a) considered fair handicapping systems for disabled sports. McGarry (1998) examined the fairness or otherwise of various tournament designs, and Glickman (2008) used simulation to derive a method for ensuring that the best player in a knock-out tournament has the highest probability of winning the tournament.

Sports-related social issues are considered by Bennett *et al* (1980), who used hypergames to model football hooliganism, and by Taylor and Keown (1978), who used goal programming to help with the planning of urban sports facilities. Finally, issues of interest to TV broadcasters were studied by Buraimo *et al* (2008), who considered how football match attendances are affected by the presence of a live match on TV, and Scarf and Xin (2008), who discussed measures of the importance of a football match.

7. Who are we working for?

For the most part ‘OR in Sport’ means ‘OR in professional sport’. Nearly all published OR papers concern themselves with professional sports; even those which apply to a sport in general (such as those involving an analysis of tactics) are usually calibrated using professional data, so that their results and recommendations may not apply at a lower level.

However, professional sporting activities form only a tiny fraction of the total. For example, consider cricket in the UK. During the summer there are typically about 15 professional matches played every week, while my local weekly newspaper alone includes results of about 80 amateur matches. Given that many matches (eg in schools, colleges, villages and various friendly matches) do not even reach the local papers, and given also that my local paper covers an area representing well under 1% of the national population, it would appear

that the professional game must make up no more than 0.1% of the total.

Why then should we restrict our studies to this tiny fraction? Can OR not be applied to amateur sporting activities? Yes indeed: there are at least three published papers which consider problems faced at the amateur level and it is interesting that some of the most important issues faced are rather different from those at professional level.

Vasko (2003) used mathematical programming on a problem concerning the selection of teams and team positions so as to ensure that every member of a children’s baseball team would be involved to a roughly equal extent—not the kind of objective that a professional team would consider important!

Schönberger *et al* (2004) reported on a system using memetic algorithms to schedule fixtures for a table tennis league. Here factors such as ‘limited access to sport facilities and temporary non-availability of sportsmen’ were key—everyday concerns in amateur sport but not usually important for professional sports.

Wright (2007) reported on a system based on metaheuristic search to allocate umpires for an amateur cricket league in Devon. Money was so tight for this league that an important criterion to be used was the possibility of one umpire being able to give a lift to the other umpire when travelling to and from a match at the other end of the county. Again it is difficult to conceive of this being an important objective in a professional league.

There may be other unpublished work that has been carried out for amateur clients—for example Johns (2001) reported at the OR Society Conference on a fixture scheduling system he created for the Kent Cricket League. However, it seems that there is plenty of scope for a major expansion of OR in this area.

Amateur sport affects literally millions of people in the UK alone, so it may also represent an opportunity for OR to become much more widely known, and in a very favourable light. It would be a great shame if the inability of such customers to pay normal consultancy fees were to stop this from happening. Care must be taken however with such clients—as pointed out by Wright (2007), the ethical issues are not the same when carrying out work for amateurs as they are for professionals.

Another group of people without whom professional sport could not exist is the fans. Should we be giving OR advice to them? Well, I suppose we could be helping extreme anoraks decide the best way to visit all 92 league football grounds in the least time possible or at the lowest cost, but this is a tiny minority unrepresentative of real fans.

In his excellent book ‘Up Pompey’, novice football fan Chuck Culpepper (2007) is ticked off by two Newcastle United fans for saying he had almost chosen to support Newcastle; their retort was ‘you don’t choose’. He eventually comes to understand this and ends his book with the words ‘I think it’s hard being a fan’. Hard not in the OR sense of hard

decisions, but hard because being a fan is intensely visceral rather than rational. I'm not sure that OR as we know it can help the real fan.

8. Conclusion

In this paper I have tried to give a flavour of a very diverse and fascinating topic which has burgeoned greatly over the last 50 years. Although I have included a lot of references, I have not tried to give a full list of all those I have read. Moreover, there are bound to be some fascinating papers that have escaped my perusal, for which I apologise.

How about the next 50 years? Well, there will be further papers, special issues, dedicated conferences, journals, etc—there will always be enough OR researchers who love sport to make sure that these will continue. The number of possible topics is far from exhausted!

Sophisticated OR software will probably make further inroads into the scheduling of fixtures and officials (and possibly other things such as training facilities) not only for professionals but also at amateur level. Meanwhile, other academics will continue to create and solve new variants of their theoretical problems.

There is now a huge amount of data available for a growing number of sports, capturing almost every facet of a sports encounter on computer in a form susceptible to quantitative analysis. This enables sophisticated OR models, of the type pioneered by Sierksma (2006), to be of practical value to decision-makers within sport. This trend will doubtless continue, and it may well be that OR will become an accepted tool in helping players, coaches and others to plan and play their games.

As for a totally new field of endeavour for sports-related OR, how about using it in conjunction with psychology? Most sport is said to be played within the minds of its participants—could OR help fine tune those minds for better performance? Now there's a challenge!

References

- Albright SC and Winston W (1978). A probabilistic model of winners' outs versus losers' outs rules in basketball. *Opns Res* **26**(6): 1010–1019.
- Annis DH (2006). Optimal end-game strategy in basketball. *J Quant Anal Sports* **2**(2).
- Armstrong J and Willis RJ (1993). Scheduling the cricket World Cup—A case study. *J Opl Res Soc* **44**(11): 1067–1072.
- Audas R, Dobson S and Goddard J (2002). The impact of managerial change on team performance in professional sports. *J Econ Bus* **54**(6): 633–650.
- Barnett T and Clarke SR (2005). Combining player statistics to predict outcomes. *IMA J Mngt Math* **16**: 113–120.
- Bartsch T, Drexl A and Kröger S (2006). Scheduling the professional soccer leagues of Austria and Germany. *Comput Opns Res* **33**(7): 1907–1937.
- Bennett PG, Dando MR and Sharp RG (1980). Using hypergames to model difficult social issues: An approach to the case of soccer hooliganism. *J Opl Res Soc* **31**(7): 621–635.
- Biajoli FL, Souza MJF, Chaves AA, Mine OM, Cabral LAF and Pontes RC (2004). Scheduling the Brazilian soccer championships: A simulated annealing approach. In: Burke EK and Trick MA (eds). *Fifth International Conference on the Practice and Theory of Automated Timetabling*. Pittsburgh, USA. Springer-Verlag: Berlin.
- Boon BH and Sierksma G (2003). Team formation: Matching quality supply and quality demand. *Eur J Opl Res* **148**(2): 277–292.
- Brimberg J, Hurley WJ and Johnson RE (1999). A punt returner location problem. *Opns Res* **47**(3): 482–487.
- Briskorn D and Drexl A (2007). A branch-and-price algorithm for scheduling sport leagues. *J Opl Res Soc* **60**: 84–93.
- Bruinshoofd A and ter Weel B (2003). Manager to go? Performance dips reconsidered with evidence from Dutch football. *Eur J Opl Res* **148**(2): 233–246.
- Buraimo B, Forrest D and Simmons R (2008). Insights for clubs from modelling match attendance in football. *J Opl Res Soc* advance online publication 9 January 2008, doi: 10.1057/palgrave.jors.2602549.
- Carter M and Guthrie G (2004). Cricket interruptus: Fairness and incentive in limited overs cricket matches. *J Opl Res Soc* **55**(8): 822–829.
- Clarke SR (1988). Dynamic programming in one-day cricket—Optimal scoring rates. *J Opl Res Soc* **39**(4): 331–337.
- Clarke SR (1993). Computer forecasting of Australian Rules football for a daily newspaper. *J Opl Res Soc* **44**(8): 753–759.
- Clarke SR and Norman JM (1978). What chance playing up to 10; they may be better if you think again. *The Squash Player*, July, pp 50–51.
- Clarke SR and Norman JM (1998). When to rush a 'behind' in Australian Rules football: A dynamic programming approach. *J Opl Res Soc* **49**(5): 530–536.
- Clarke SR and Norman JM (1999). To run or not to run? Some dynamic programming models in cricket. *J Opl Res Soc* **50**(5): 536–545.
- Clarke SR and Norman JM (2003). Dynamic programming in cricket: Choosing a night watchman. *J Opl Res Soc* **54**(8): 838–845.
- Condon EM, Golden BL and Wasil EA (1999). Predicting the success of nations at the summer Olympics using neural networks. *Comput Opns Res* **26**(13): 1243–1265.
- Cooper WW, Ruiz JL and Sirvent I (2008). Selecting non-zero weights to evaluate effectiveness of basketball players with DEA. *Eur J Opl Res* **195**(2): 563–574.
- Culpepper C (2007). *Up Pompey*. Orion Publishing Group: London.
- Davey RC, Hayes M and Norman JM (1994). Running uphill: An experimental result and its applications. *J Opl Res Soc* **45**(1): 25–29.
- della Croce F and Oliveri D (2006). Scheduling the Italian Football League: An ILP-based approach. *Comput Opns Res* **33**(7): 1963–1974.
- della Croce F, Tadei R and Asioli PS (1999). Scheduling a round robin tennis tournament under courts and players availability constraints. *Ann Opns Res* **92**: 349–361.
- Dixon MJ and Robinson ME (1998). A birth process model for association football matches. *Statistician* **47**: 523–538.
- Duarte AR, Ribeiro CC, Urrutia S and Haeusler EH (2007). Referee assignment in sports leagues. *Lect Notes Comput Sci* **3867**: 158–173.
- Duckworth FC and Lewis AJ (1998). A fair method for resetting the target in interrupted one-day cricket matches. *J Opl Res Soc* **49**(3): 220–227.
- Easton K, Nemhauser GL and Trick MA (2001). The traveling tournament problem: Description and benchmarks. *Lect Notes Comput Sci* **2239**: 580–584.
- Eiselt HA and Laporte G (1991). A combinatorial optimization problem arising in dartboard design. *J Opl Res Soc* **42**(2): 113–118.

- Elf M, Jünger M and Rinaldi G (2003). Minimizing breaks by maximizing cuts. *Opns Res Lett* **31**(5): 343–349.
- Evans JR (1988). A microcomputer-based decision support system for scheduling umpires in the American baseball league. *Interfaces* **18**(6): 42–51.
- Evans JR, Hebert JE and Deckro RF (1984). Play ball—The scheduling of sports officials. *Perspect Comput* **4**(1): 18–29.
- Fitt AD, Howls CJ and Kabelka M (2006). Valuation of soccer spread bets. *J Opl Res Soc* **57**(8): 975–985.
- Fleurent C and Ferland JA (1993). Allocating games for the NHL using integer programming. *Opns Res* **41**(4): 649–654.
- Flitman AM (2006). Towards probabilistic footy tipping: A hybrid approach utilising genetically defined neural networks and linear programming. *Comput Opns Res* **33**(7): 2003–2022.
- Freeze RA (1974). An analysis of baseball batting order by Monte Carlo simulation. *Opns Res* **22**(4): 728–735.
- Gil Lafuente J (2004). The best systems for appointing referees. In: Butenko S, Gil Lafuente J and Pardalos PM (eds). *Economics, Management and Optimization in Sports*. Springer-Verlag: Berlin, pp 101–120.
- Glickman ME (2008). Bayesian locally optimal design of knockout tournaments. *J Statist Plann Infer* **138**(7): 2117–2127.
- Golding M (2002). Tactics and planning in round the world yacht racing. Presented at the OR Society Southern OR Group One-day Event on OR in Sport. Oxford.
- Hayes M and Norman JM (1984). Dynamic programming in orienteering: Route choice and the siting of controls. *J Opl Res Soc* **35**(9): 791–796.
- Heazlewood T (2006). Prediction versus reality: The use of mathematical models to predict elite performance in swimming and athletics at the Olympic Games. *J Sports Sci Med* **5**: 541–547.
- Hersh M and Ladany SP (1989). Optimal pole-vaulting strategy. *Opns Res* **37**(1): 172–175.
- Hirotsu N and Wright MB (2002). Using a Markov process model of an association football match to determine the optimal timing of substitution and tactical decisions. *J Opl Res Soc* **53**(1): 88–96.
- Hirotsu N and Wright MB (2003). A Markov chain approach to optimal pinch hitting strategies in a designated hitter rule baseball game. *J Opns Res Soc Japan* **46**(3): 353–371.
- Hope C (2003). When should you sack a football manager? Results from a simple model applied to the English Premiership. *J Opl Res Soc* **54**(11): 1167–1176.
- Johns S (2001). Complexity in amateur sports scheduling. Paper Presented at the Operational Research Society Conference. Bath.
- Kendall GX (2008). Scheduling English football fixtures over holiday periods. *J Opl Res Soc* **59**(6): 743–755.
- Klaassen FJGM and Magnus JR (2003). Forecasting the winner of a tennis match. *Eur J Opl Res* **148**(2): 257–267.
- Köhler D (1982). Optimal strategies for the game of darts. *J Opl Res Soc* **33**(10): 871–884.
- Kostuk KJ, Willoughby KA and Saedt APH (2001). Modelling curling as a Markov process. *Eur J Opl Res* **133**(3): 557–565.
- Kuper GH and Sterken E (2004). *Do skating suits increase average skating speed?* Groningen University Working Paper, <http://www.ub.rug.nl/eldoc/ccso/200404>.
- Ladany SP (1975a). Optimal starting height for pole-vaulting. *Opns Res* **23**(5): 968–978.
- Ladany SP (1975b). Optimization of pentathlon training plans. *Mngt Sci* **21**(10): 1144–1155.
- Ladany SP and Machol RE (eds) (1977). *Optimal Strategies in Sports*. North-Holland: New York.
- Lee KT and Chin St (2004). Strategies to serve or receive the service in volleyball. *Math Meth Opns Res* **59**(1): 53–67.
- Lewis M (2003). *Moneyball: The Art of Winning an Unfair Game*. Norton: New York.
- Lilien. Optimal weightlifting. In: Ladany SP and Machol RE (eds). *Optimal Strategies in Sports*. North-Holland: New York, pp 101–112.
- Lim A, Rodrigues B and Zhang X (2006). A simulated annealing and hill-climbing algorithm for the traveling tournament problem. *Eur J Opl Res* **174**(3): 1459–1478.
- Lindsey GR (1963). An investigation of strategies in baseball. *Opns Res* **11**(4): 477–501.
- Lo VSY, Bacon-Shone J and Busche K (1995). The application of ranking probability models to racetrack betting. *Mngt Sci* **41**(6): 1048–1059.
- Machol RE and Ladany SP (1976). *Management Science in Sports*. North-Holland: New York.
- McGarry T (1998). On the design of sports tournaments. In: Bennett JM (ed). *Statistics in Sport*. Arnold: London, pp 199–217.
- Miyashiro R and Matsui T (2006). Semidefinite programming based approaches to the break minimization problem. *Comput Opns Res* **33**(7): 1975–1982.
- Mottley CM (1954). The application of operations research methods to athletic games. *J Opns Res Soc Amer* **2**(3): 335–338.
- Nemhauser GL and Trick MA (1998). Scheduling a major college basketball conference. *Opns Res* **46**(1): 1–8.
- Norman JM (1985). Dynamic programming in tennis—When to use a fast serve. *J Opl Res Soc* **36**(1): 75–77.
- Norman JM (2004). Running uphill: Energy needs and Naismith's rule. *J Opl Res Soc* **55**(3): 308–311.
- Norman JM and Clarke SR (2007). Dynamic programming in cricket: Optimizing batting order for a sticky wicket. *J Opl Res Soc* **58**(12): 1678–1682.
- Nowak M, Epelman M and Pollock SM (2006). Assignment of swimmers to dual meet events. *Comput Opns Res* **33**(7): 1951–1962.
- Percy DF (2007a). Handicapping systems for disabled Alpine skiing. In: Percy DF, Scarf PA and Robinson CL (eds). *Proceedings of the 1st International Conference on Mathematical Modelling in Sport*. Salford, pp 157–162. Salford University: Salford, UK.
- Percy DF (2007b). A mathematical analysis of badminton scoring systems. *J Opl Res Soc* **60**: 63–71.
- Philpott AB, Henderson SG and Teimey D (2004). A simulation model for predicting yacht match race outcomes. *Opns Res* **52**(1): 1–16.
- Pollock SM (1974). A model for evaluating golf handicapping. *Opns Res* **22**(5): 1040–1050.
- Rasmussen RV and Trick MA (2007). A Benders approach for the constrained minimum break problem. *Eur J Opl Res* **177**(1): 198–213.
- Ribeiro CC and Urrutia S (2007). Heuristics for the mirrored traveling tournament problem. *Eur J Opl Res* **179**(3): 775–787.
- Russell RA and Leung JMY (1994). Devising a cost effective schedule for a baseball league. *Opns Res* **42**(4): 614–625.
- Saltzman RM and Bradford RM (1996). Optimal realignments of the teams in the National Football League. *Eur J Opl Res* **93**(3): 469–475.
- Scarf P and Grehan P (2005). An empirical basis for route choice in cycling. *J Sports Sci* **23**(9): 919–925.
- Scarf P and Xin S (2008). The importance of a match in a tournament. *Comput Opns Res* **35**(7): 2406–2418.
- Schönberger J, Mattfeld DC and Kopfer H (2004). Memetic algorithm timetabling for non-commercial sport leagues. *Eur J Opl Res* **153**(1): 102–116.
- Sierksma G (2006). Computer support for coaching and scouting in football. *Sports Eng* **9**(4): 229–249.
- Silverman D and Schwartz BL (1973). How to win by losing. *Opns Res* **21**(2): 639–643.
- Spark M (1961). *The Prime of Miss Jean Brodie*. Macmillan: London.
- Sphicas GP and Ladany SP (1977). Dynamic policies in the long jump. In: Ladany SP and Machol RE (eds). *Optimal Strategies in Sports*. North-Holland: New York, pp 101–112.

- Swartz TB, Gill PS, Beaudoin D and deSilva BM (2006). Optimal batting orders in one-day cricket. *Comput Opns Res* **33**(7): 1939–1950.
- Taylor BW and Keown AJ (1978). Planning urban recreational facilities with integer goal programming. *J Opl Res Soc* **29**(8): 751–758.
- Tena JD and Forrest D (2007). Within-season dismissal of football coaches: Statistical analysis of causes and consequences. *Eur J Opl Res* **181**(1): 362–373.
- Thompson J (1999). Kicking timetabling problems into touch. *OR Insight* **12**(3): 7–15.
- Urban TL and Russell RA (2003). Scheduling sports competitions on multiple venues. *Eur J Opl Res* **148**(2): 302–311.
- van Voorhis T (2002). Highly constrained college basketball scheduling. *J Opl Res Soc* **53**(6): 603–609.
- Vasko FJ (2003). Play ball—Equally: Math programming lends a hand to little league baseball. *OR Insight* **16**(2): 16–19.
- Washburn A (1991). Still more on pulling the goalie. *Interfaces* **21**: 59–64.
- Willis RJ and Terrill BJ (1994). Scheduling the Australian state cricket season using simulated annealing. *J Opl Res Soc* **45**(3): 276–280.
- Wright MB (1988). Probabilities and decision rules for the game of squash rackets. *J Opl Res Soc* **39**(1): 91–99.
- Wright MB (1991). Scheduling English cricket umpires. *J Opl Res Soc* **42**(6): 447–452.
- Wright MB (1992). A fair allocation of county cricket opponents. *J Opl Res Soc* **43**(3): 195–201.
- Wright MB (1994). Timetabling county cricket fixtures using a form of tabu search. *J Opl Res Soc* **45**(7): 758–770.
- Wright MB (2005). Scheduling fixtures for New Zealand Cricket. *IMA J Mngt Math* **16**(2): 99–112.
- Wright MB (2006). Scheduling fixtures for Basketball New Zealand. *Comput Opns Res* **33**(7): 1875–1893.
- Wright MB (2007). Case study: Problem formulation and solution for a real-world sports scheduling problem. *J Opl Res Soc* **58**(4): 439–445.
- Wright MB and Hirotsu N (2003). The professional foul in football—Tactics and deterrents. *J Opl Res Soc* **54**(3): 213–221.
- Yavuz M, İnan UH and Fiğlalı A (2008). Fair referee assignments for professional football leagues. *Comput Opns Res* **35**(9): 2937–2951.
- Zakarya Z, Hertz A and de Werra D (1989). *Un système informatique pour les calendriers d'arbitrage d'une association sportive*. Report of the Département de mathématiques, École Polytechnique Fédérale de Lausanne, Switzerland.

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