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A 10d view on the KKLT AdS vacuum and uplifting

F.F. Gautason, V. Van Hemelryck, T. Van Riet and G. Venken

*Institute of Theoretical Physics, KU Leuven,
Celestijnenlaan 200D B-3001 Leuven, Belgium*

E-mail: ffg@kuleuven.be, vincent.vanhemelryck@kuleuven.be,
thomas.vanriet@kuleuven.be, gerben.venken@kuleuven.be

ABSTRACT: We analyse the ten-dimensional Einstein equations in the KKLT setting. We verify that the quartic gaugino term is needed to remove singularities in the on-shell action as suggested by Hamada et al. We contrast two approaches that have been taken in the literature when employing the effect of gaugino condensation in the ten-dimensional equations of motion. Here we follow the proposal to insert explicit non-zero fermion bilinear vev into the localised energy-momentum tensor of the 7-branes obtained from varying the 10d off-shell action with respect to the 10d metric. Our procedure is common in semi-classical physics and is manifestly local in 10d. However, it does not lead to the KKLT effective field theory. The alternative procedure of deriving the energy momentum tensor after replacing fermion bilinears by the gaugino vev, might be less well motivated in 10d, but reproduces the results of the KKLT effective field theory.

KEYWORDS: D-branes, Flux compactifications, Superstring Vacua

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1 Introduction

Moduli stabilisation in string theory has a long history and its understanding is crucial for the study of top-down phenomenology. One of the main breakthroughs came in 2003 with the KKLT model [1] and in 2005 with the Large Volume Scenario [2]. Both methods to stabilise moduli rely on computing leading-order corrections to ten-dimensional supergravity solutions with orientifold sources and three-form fluxes in type IIB supergravity [3–5]. Despite the long history of this field there has never been any genuine top-down understanding of these mechanisms to stabilise moduli. The arguments always involved a mixture between top-down and bottom-up viewpoints. This is not a problem per se, but it can lead to a false sense of freedom to tune parameters in the bottom-up effective field theory. It is therefore desirable to find at least one concrete top-down description of moduli-stabilisation.

Recently the field of moduli stabilisation has regained attention due to a growing suspicion that string theory may fail to accommodate any de Sitter (dS) vacuum [6–8]. This suspicion is mainly based on three things. Firstly on the lack of explicit dS examples string constructions such as classical type II supergravity and tree-level heterotic string

theory [9–12] (see [7] for a review). Secondly, on non-trivial generalisations of the Dine-Seiberg argument [13] (see also [14]).¹ And thirdly, on the old debate concerning the quantum breakdown of dS space (due to backreacting particle creation) [16–22].²

This revival of interest has provided a healthy spark of ideas to construct new possible dS vacua (or more general accelerating cosmologies) [27–32] or revisit existing ones [33–36]. In this paper we direct our attention to the consistency of the KKLT model; both the construction of the AdS vacuum and its potential uplift to a meta-stable dS vacuum. In particular the uplift procedure has been heavily criticised on two fronts. First the stability of the anti-branes themselves came under intense scrutiny in [37–39]. It is our opinion that the arguments pointing towards the instability of anti-branes have been to a large extent addressed and refuted in [40–43] (see however [44, 45]). The second concern, raised in [33] (see also [46]), has to do with the interaction between the open string degrees of freedom of the anti-brane and closed string degrees of freedom of the background. This interaction is only possible in a compact model where the two effects cannot be separated indefinitely.

The authors of [33] suggested that it should be possible to study the interaction using classical ten-dimensional supergravity including the energy-momentum of a gaugino condensate on D7-branes that live in the UV.³ However, there is some confusion regarding the gravitational backreaction of the gaugino condensate in ten-dimensional supergravity. This was discussed in [33, 34] and we refer the reader to those references for further elucidations. The main observation of [33] was that the ten-dimensional energy-momentum tensor behaves such that a dS vacuum is not attainable with a single gaugino condensate. From a four-dimensional viewpoint this effect can be understood as a flattening of the potential caused by a backreaction of the supersymmetry-breaking ingredients on the volume modulus.

Later, it was shown in [34] that there is a loophole in the argument of [33] due to a subtlety in the computation. Unfortunately the issue could not be settled in [34] since some of the terms in the energy-momentum tensor were divergent and a regularisation is needed that could influence the fate of the flattening effects. Recently Hamada *et al.* [48] (see also [49]) proposed how to perform this regularisation by adding a quartic fermion term to the action. In this note we redo the computations in [33, 34] including this new term. We find that indeed the four-dimensional curvature is expressed entirely in terms of regular *background* fluxes and the gaugino condensate, but we fail to find the KKLT effective theory and explain why. The computation can rather straightforwardly be extended to include anti-branes.

Simultaneously with this work two other works appeared [50, 51] about the same problem and one of these works [51] comes to a very different conclusion than us. We discuss in section 6 on the relation between our work and these two other papers. The difference relates to manner the energy-momentum tensor is computed and the alternative method of [51] and [50] does reproduce qualitatively the KKLT effective field theory and finds no dangerous flattening effects when anti-branes are added.

¹See [15] for early suggestions of a refined form of the dS swampland bounds.

²See [23–26] for some critical remarks on these ideas.

³Other somewhat orthogonal worries about the consistency of uplifting were expressed in [36, 47].

2 The framework

The KKLT proposal [1] to stabilize moduli in a supersymmetric AdS vacuum rests on reasonable assumptions about quantum corrections to the classical flux compactifications of [3, 5]. These corrections have been argued for using four-dimensional effective field theories described within $\mathcal{N} = 1$ supergravity. The quantum correction used by KKLT is the leading non-perturbative quantum effect to the superpotential W . Perturbative corrections to W are absent because of non-renormalisation theorems whereas it is argued that (non-)perturbative corrections in the Kähler potential K can sometimes be self-consistently ignored.⁴ The non-perturbative quantum correction arises as a result of gauginos living on a stack of D7-branes condensing in a confining vacuum of the *four-dimensional* gauge theory. In what follows we assume, like KKLT, that the underlying Calabi-Yau manifold only admits a single Kähler deformation. It then follows that there is only one (holomorphic) 4-cycle wrapped by the D7-branes.

At first sight the gaugino condensation obscures a dimensional oxidation of the KKLT model. However, it has been suggested in [33] that it should nonetheless be possible using ten-dimensional supergravity with D7 probe actions included. The way the gaugino condensate couples to gravity and other closed strings could then be understood simply by keeping explicit non-zero gaugino bilinears of the fermionic part of the D7 action and compute its contribution to the ten-dimensional energy-momentum tensor [52–56].

Like [33] we compute the trace over the four-dimensional part of the (trace-reversed) Einstein equation. We will perform this computation in detail in next section but here we will discuss the form of the resulting expression,

$$R_4 = \int_6 \sqrt{g_6} \mathcal{E}[F^2, \lambda^2], \tag{2.1}$$

where R_4 is the curvature scalar of the four large dimensions, $\int \sqrt{g_6}$ integration over the compact dimensions and $\mathcal{E}[F^2, \lambda^2]$ represents some function of the background fluxes (F^2) and the gaugino bilinear (λ^2). We find that our approach is consistent with an AdS spacetime but further relations between the fields are required to make a quantitative comparison with the KKLT result. Strictly speaking the method of the trace-reversed Einstein equation allows to compute only the curvature of a vacuum solution which is related to the on-shell potential. The relation between fields required to make a comparison to KKLT is only obtained from the off-shell potential which we do not derive here.

Adding anti-branes to this computation is straightforward as explained in [33, 34]. One can show it simply adds the following term to the right hand side of equation (2.1):

$$R_4 = \int_6 \sqrt{g_6} (\mathcal{E}[F^2, \lambda^2] - 2nT\delta_6), \tag{2.2}$$

where T is the warped tension of a stack of anti-D3 branes and n the number of anti-D3's. This seems a contradiction since this term contributes negatively compared to the

⁴The Large Volume Scenario [2] uses perturbative corrections to the Kähler potential to stabilize the volume modulus at exponentially large value in a non-supersymmetric vacuum.

rest, so how can it uplift? In fact [33] realised that, in the right approximation scheme, the anti-brane term is completely subdominant compared to the bulk piece \mathcal{E} and should be ignored.

This naive “paradox” arises because (2.2) computes an *on-shell* four-dimensional potential instead of a full off-shell expression.⁵ The way uplifting should work from this viewpoint is rather different. Uplifting works by a shift of the volume modulus ρ induced by the supersymmetry-breaking, altering the value of \mathcal{E} and potentially flipping its sign. To make that point very explicit let us for simplicity assume that the term \mathcal{E} has two pieces

$$\mathcal{E}[F^2, \lambda^2; \rho] = \mathcal{E}_+[F^2, \lambda^2; \rho] + \mathcal{E}_-[F^2, \lambda^2; \rho], \tag{2.3}$$

where \mathcal{E}_- is negative definite and \mathcal{E}_+ positive definite and we introduced the symbolic dependence on ρ . In the supersymmetric vacuum we obviously have $\mathcal{E}_+[\rho] < -\mathcal{E}_-[\rho]$. But when ρ shifts to $\rho + \delta\rho$ due to supersymmetry breaking we expect that the value of R_4 and therefore $\mathcal{E}[\rho + \delta\rho]$ is less negative. A dS is reached by making sure that $\mathcal{E}_+[\rho + \delta\rho] > -\mathcal{E}_-[\rho + \delta\rho]$.

Apart from the ρ -dependence the only quantities appearing in \mathcal{E} are the *background* fluxes and gaugino vev. We can assume that their values have not altered when breaking supersymmetry. This is a crucial assumption that is at the core of the KKLT model. In other words the fluxes F^2 are the ones of the GKP solution in the background. One could worry about this assumption since there are two sources for deviations from the GKP solution: 1) near the anti-D3 branes non-ISD fluxes are sourced whereas the original GKP construction only allowed for ISD fluxes [5]. But as we emphasised earlier this is a local backreaction effect, and any extra terms in the Einstein equation that are otherwise missed in the approximation will only contribute negatively. 2) Near the 7-branes also non-ISD fluxes are sourced. Even worse, these fluxes are divergent, similar to what was believed to be the case for anti-D3 branes [37, 39] but later shown not to happen because of brane polarisation [41, 42]. In fact the discussion of anti-D3 flux singularities parallels that of D7 gaugino-induced flux singularities. Whereas brane polarisation cures the singularities for anti-D3 branes, it was shown in [48] that quartic gaugino-terms, previously ignored, do the same for D7-branes.

The essence of this paper is the computation of the right hand side of equation (2.2) incorporating the quartic gaugino term of [48] and is outlined in the next section.

3 A subtlety in the semi-classical Einstein equations

At this point a note of caution is required. While this paper was finalised we learned of two other papers [50, 51] that appeared about the same problem. Their results differ from ours and the reason is rather straightforward. The difference relies in how one computes the semi-classical Einstein equation in 10-dimensions:

$$G_{\mu\nu} = \langle T_{\mu\nu} \rangle. \tag{3.1}$$

⁵See [33, 34] for a more elaborate explanation.

Our method relies upon the following procedure. We first compute the form of the tensor $T_{\mu\nu}$ in 10 dimensions by varying the action with respect to the 10d metric *keeping all other fields fixed* and keep the fermion bilinears arbitrary. Subsequently we use the 4d gauge theory results for the vevs of the fermion bilinears to fix the expression of $T_{\mu\nu}$. Notice that since the 4d vevs of the fermion bilinears have an explicit dependence on the volume of the compact space, there is an implicit dependence on the 10d metric. Hence this procedure will lead to different result than if one instead first replaces the fermion bilinears in the 10d action with the 4d vevs and *then* compute the variation of the action with respect to the volume. In section 6.2, we demonstrate that this difference in the order of variations leads to the results of Hamada et al. [51]. Note that our method gives a manifest local description in 10d whereas there seems to be no ten-dimensional, manifestly local, covariant description in the approach of [50, 51].

The main advantage of the approach followed by [51] is that it seems to directly reproduces the 4d computation done by KKLT. In this work we prefer to remain agnostic about the correctness of the 4d result for multiple reasons which we now explain. First of all the motivation of our paper is to establish (or refute) the KKLT model from a 10d perspective, before and after uplift. Hence we cannot assume the 4d EFT as a prior. Alternatively one can regard the spirit of this work to be a test for the proposal that a 10d derivation of KKLT can be found using fermion bilinear vevs in 10D as the only relevant extra contribution to the “GKP” background. Whether semi-classical 10d supergravity with localised 7-brane sources is the appropriate description of the four-dimensional gauge theory in its confining phase is an open question. In this paper we report our results assuming that for the sake of determining the vacuum, it is adequate. Secondly, the precise way in which gaugino condensates couple to gravity is not understood, even in four dimensions. KKLT makes a proposal to simply sum the background GVW superpotential with the field theory superpotential. This is reasonable, but we are not aware of a derivation of this and caveats can be around the corner. For instance, since gaugino condensation is a consequence of long-wavelength fluctuations of fields, one expects it to be sensitive to the background it is supposed to live on. Hence gaugino condensation can look different in AdS or some other curved space. In other words: how exactly gaugino condensation couples to gravity can be more complicated than simply adding it into the supergravity F-term. Furthermore, the classical gravity background breaks supersymmetry via its fluxes. This means that there is an explicit SUSY breaking occurring in theory once we incorporate the gravitational degrees of freedom and hence the assumptions of having SUSY of the off-shell Lagrangian, required for computing gaugino condensation, is not obvious. In what follows we lay out the details of the computation of the 4d piece in the 10d Einstein equation (3.1). As we will show, our results are not incompatible with an AdS solution but we are unable to determine a quantitative value for the vacuum energy that can be compared with the KKLT results. This will be discussed further in section 6.

4 The four-dimensional cosmological constant

In this section we utilise the approach of [5] to compute the four-dimensional curvature when compactifying ten-dimensional type IIB supergravity with fluxes and D7-brane

sources with non-vanishing gaugino bilinear. In doing so, we assume a maximally symmetric four-dimensional spacetime. The ten-dimensional metric reads⁶

$$ds_{10}^2 = e^{2A} ds_4^2 + e^{-2A} ds_6^2, \tag{4.1}$$

where ds_4^2 is a maximally symmetric on a four-dimensional spacetime, with a constant curvature scalar R_4 . The warp factor A only depends on the coordinates of the transverse manifold M_6 equipped with the metric $e^{-2A} ds_6^2$. Maximal symmetry demands that the ten-dimensional axion-dilaton τ and 3-form field strength G_3 only have legs and dependence along M_6 . The type IIB 5-form F_5 is self-dual and given by

$$F_5 = (1 + \star_{10}) dC_4, \quad C_4 = \alpha \text{vol}_4, \tag{4.2}$$

where α is a function on M_6 . The inclusion of three-form fluxes induces three-brane charge that contributes to the tadpole which must be cancelled by the inclusion of local sources such as O3/O7 planes. Taking these ingredients into account, [5] found a consistency condition demanded by the trace-reversed Einstein equation that takes the form⁷

$$\Delta_6 \Phi^- = R_4 + \frac{e^{2A}}{\text{Im} \tau} |G_3^-|^2 + e^{-6A} |d\Phi^-|^2, \tag{4.3}$$

where

$$\Phi^\pm = e^{4A} \pm \alpha, \quad G_3^\pm = \frac{1}{2} (\star_6 \pm i) G_3. \tag{4.4}$$

This equation can also be derived directly from the type IIB action (together with O3/O7 source terms) upon varying with respect to the warp factor A (see appendix B). The power of this equation becomes apparent when one integrates (4.3) over M_6 . The left-hand-side integrates to zero which implies that the four-dimensional curvature scalar R_4 is non-positive. The GKP vacua are the ones with $R_4 = G_3^- = \Phi^- = 0$.

We now derive the equation corresponding to (4.3) when $N > 1$ D7-branes are included with non-vanishing fermion bilinear. At this stage we will be agnostic about the value of the fermion-bilinear, and assume that it will take some non-vanishing value at low energies where the gauge group condenses. In the procedure outlined we combine the ten-dimensional type IIB action with the effective action of D7-branes. The bosonic D7-brane action

$$S_{D7} = -2\pi N \int_{\Sigma_8} d^8x (\text{Im} \tau)^{-1} \sqrt{|P[g]|} + 2\pi N \int_{\Sigma_8} P[C_8], \tag{4.5}$$

does not contribute to (4.3) since “it is BPS” with respect to the background.⁸ One can verify this claim by using the ansatz (4.1) and noticing that the warp factor drops out. Even though the bosonic action does not play an important role in our discussion, the fermion

⁶Note that the four-dimensional metric ds_4^2 is not in four-dimensional Einstein frame because the volume modulus has not been properly factored out. This does not cause any issue with our computation in this section since we assume that all moduli are stabilised by the gaugino condensate. We will return to this issue when we make an explicit comparison to KKLT in section 5.

⁷Here $\Delta_6 \Phi^- = -\star_6 d\star_6 d\Phi^-$ and all inner products of forms are ten-dimensional, i.e. they include warp factors.

⁸BPS is used in a loose form since fluxes can already break supersymmetry of the GKP background.

terms do. Indeed the gauge theory living on the D7-brane world volume descends to an $\mathcal{N} = 1$ gauge theory in four dimensions with various matter couplings. In particular the eight-dimensional world-volume fermions give rise to the $\mathcal{N} = 1$ gaugini in four dimensions.⁹ The fermionic D7-brane action contains an interaction term between the gaugino bilinear $\lambda\lambda \equiv \text{tr}(\lambda^{\dot{\alpha}}\lambda_{\dot{\alpha}})$ and the three-form G_3 [57] (see [58] for an early discussion)

$$S_{D7}^{\text{ferm}} = \pi \int \star_{10} (G_3 \cdot I + \overline{G_3} \cdot \bar{I}), \quad I = \frac{e^{-4A}}{\sqrt{\text{Im } \tau}} \frac{\bar{\lambda}\lambda}{16\pi^2} \Omega \delta(\Sigma_8). \quad (4.6)$$

The contribution of the fermionic terms to (4.3) turns out to be singular due to the backreaction of the fermion bilinears on the three-form fluxes [33, 34]

$$G_3 \sim (\text{Im } \tau) \bar{I}, \quad (4.7)$$

where I is the source appearing in (4.6) and carries an explicit delta function. In fact, even the on-shell action is UV divergent due to this singular backreaction. In [48] it was suggested that the singular backreaction is ultimately a result of omitting a four-fermion term in the D7-brane action. Currently the four-fermion terms are not known for the D-brane action and so [48] suggested to fix their form such that the singularities would be cancelled. The suggestion of [48] was inspired by a similar problem appearing in Hořava-Witten theory [59–61] which was resolved by a four-fermion term that appears together with the M-theory 4-form in a perfect square. This was in direct analogy with a similar square structure that appears in heterotic supergravity [49, 62]. It is therefore reasonable to assume that the fermionic D7-brane coupling (4.6) together with the kinetic terms for G_3 should be combined with the four-fermion terms to form a perfect square [48, 49]. Indeed this goes a long way to prevent singular on-shell action as a result of backreaction the fermion bilinear.

The perfect square replaces the original action for G_3 and the fermionic coupling by

$$S_3 = -\pi \int \star_{10} \frac{|G_3 - (\text{Im } \tau) \mathcal{P}(\bar{I})|^2}{(\text{Im } \tau)}, \quad (4.8)$$

The projector \mathcal{P} is defined to eliminate the coexact piece of the form it acts on. It does not affect the G_3 equation of motion but is the final piece of the puzzle to ensure that the on-shell action is regular [48]. Upon expanding the square the action consists of three terms:

$$S_3 = -\pi \int \star_{10} \frac{|G_3|^2}{(\text{Im } \tau)} + \pi \int \star_{10} (G_3 \cdot \mathcal{P}(I) + \overline{G_3} \cdot \mathcal{P}(\bar{I})) - \pi \int \star_{10} (\text{Im } \tau) |\mathcal{P}(I)|^2. \quad (4.9)$$

The first term is the standard bulk term for G_3 , the second term is the fermionic action of the D7-brane considered before in [33, 34, 54], with the exception that the three-form I is now projected to the set of closed forms. The last term is the new quartic fermion term. We are now ready to re-derive the equation (4.3) with the D7-brane fermion contribution. The explicit computation is carried out in appendix B and the result is

$$\Delta_6 \Phi^- = R_4 + e^{-6A} |d\Phi^-|^2 + e^{2A} \left(\frac{|G_3^-|^2}{\text{Im } \tau} - \frac{3}{4} \text{Re} (G_3 \cdot \mathcal{P}(I)) + \frac{\text{Im } \tau}{4} |\mathcal{P}(I)|^2 \right). \quad (4.10)$$

⁹We refer to [33, 34] for an explicit map between the eight-dimensional fermions and the gaugino.

This expression is exact and despite its appearance is completely regular.¹⁰ In order to demonstrate this and further analyse equation (4.10) we must now make simplifying approximations.

First we rewrite the delta function appearing in the source term I in terms of a Green's function. Schematically we write

$$\frac{\partial^2 G}{\partial z \partial \bar{z}} = \delta(\Sigma_8) - \frac{1}{\mathcal{V}_2}, \quad (4.11)$$

where z and \bar{z} are complex coordinates on the transverse two-cycle and \mathcal{V}_2 , a volume factor of that two-cycle, is introduced to make this equation consistent. We see that the 3-form $I \sim \delta(\Sigma_8)\Omega$ is not closed and so the role of the projector \mathcal{P} in this case is to add terms of the form $(\partial_z \partial_{\bar{z}} G)\Omega$ such that the resulting expression is closed. Explicitly we have [48]

$$\mathcal{P}(\bar{I}) = \frac{e^{-4A}}{\sqrt{\text{Im } \tau}} \frac{\lambda \lambda}{16\pi^2} \left(d \frac{\partial G}{\partial z} \wedge \bar{\Omega}_2 + \frac{1}{\mathcal{V}_2} \bar{\Omega} \right), \quad (4.12)$$

where Ω_2 is the holomorphic two-form on the holomorphic four-cycle. Notice that we have made a crucial assumption here which we will continue making in the following. Namely we have assumed that close to the seven-branes where most of our analysis takes place, we can treat the warp factor as constant. Furthermore we assume the so-called Sen limit [63] where the stack of 7-branes consists of an O7 with 4 parallel D7-branes on top, in this case the 7-brane stack does not source a gradient for τ and so it can be taken to be a constant.¹¹ These assumptions allow us to determine the backreacted three-forms in the same spirit as [54]. For notational simplicity we write

$$\mathcal{P}(I) = I^{\text{sing}} + I^{\text{reg}}, \quad (4.13)$$

where I^{sing} is the exact part of $\mathcal{P}(I)$ in (4.12) which is also singular, and I^{reg} is the harmonic part which is regular. Finally in order to solve for G_3 we assume that the background close to the seven-branes is only slightly perturbed from the standard GKP background for which $\Phi^- = e^{4A} - \alpha = 0$. By our previous assumption that e^{4A} is constant, this implies that the Chern-Simons terms in the action does not enter in the equations of motion for G_3 . With these approximations, the solution to the equation of motion for G_3 derived from action eq. (4.8) takes a simple form¹²

$$G_3 = (\text{Im } \tau) \overline{I^{\text{sing}}} + G_3^0, \quad (4.14)$$

¹⁰Equation (4.10) does have delta-function sources on the right-hand-side but is regular upon integration. This should be compared to the situation without quartic fermion terms where even the integrated expression was singular due to δ^2 terms appearing.

¹¹One could wonder whether in this case the orientifold projection does not eliminate the backreaction of the gaugino bilinears on the three-forms. We do not address this question here, but argue that our setup is sufficiently close to the Sen limit such that the gradients of τ do not enter without eliminating the three-form backreaction altogether.

¹²Under these assumptions, the Chern-Simons action does not contribute to the G_3 equations of motion. The details of the fate of the Chern-Simons action in our set-up are discussed in appendix C. Note that since I^{reg} is harmonic with these assumptions, it can always be combined with an arbitrary harmonic contribution that can be added to a solution to the equations of motion to produce another solution to the equations of motion and we denote the entire harmonic contribution as G_3^0 .

where G_3^0 is harmonic and quantised. We can now evaluate the expression (4.10) which constrains the four-dimensional cosmological constant. Substituting the solution of the G_3 equation of motion (4.14), and after some algebra outlined in appendix B we find

$$0 = \int \star_6 \left(R_4 + \frac{e^{2A}}{\text{Im } \tau} |G_3^{0,-}|^2 - \frac{3e^{2A}}{4} \text{Re} (G_3^0 \cdot I^{\text{reg}}) + \frac{e^{2A}}{4} (\text{Im } \tau) |I^{\text{reg}}|^2 \right), \quad (4.15)$$

The equation only contains the background flux G_3^0 and the fermion condensate through the regular part I^{reg} . We notice that all singular terms have been eliminated from the expression as a result of our approximations. We expect that more generally the expression (4.10) will be regular also when the approximations are relaxed.

Equation (4.15) is the main result of our paper. From it we can extract the four-dimensional curvature R_4 . However we notice that as it stands the curvature is not restricted to be negative for all possible configurations of the fields. In the next section we discuss a schematic comparison to the four-dimensional KKLT vacuum before any anti-D3 branes are included.

5 Comparing to KKLT

We now compare our results with the four-dimensional effective theory considered by KKLT [1]. This theory is specified in terms of a Kähler potential K and superpotential W , given by

$$K = -3 \log(2\text{Im } \rho) - \log(2\text{Im } \tau), \quad W = W_0 + \mathcal{A} \exp(2\pi i \rho / N), \quad (5.1)$$

where ρ is the complex volume modulus, related to the volume of the internal manifold by $\text{Im } \rho \sim \mathcal{V}_6^{2/3}$ and W_0 is the constant Gukov-Vafa-Witten superpotential (after integrating out the complex-structure moduli). In order to compare our ten-dimensional result (4.15) with the effective theory (5.1) we translate two results from the four-dimensional effective field theory back to ten dimensions. In particular we view the fluxes G_3^0 as the ISD ones, before the inclusion of the non-perturbative effects. Therefore $G_3^{0,-}$ vanishes and $G_3^0 \cdot \Omega$ can be related to W_0 ¹³

$$\int \star_6 (G_3^0 \cdot \Omega) = W_0. \quad (5.2)$$

In KKLT the gaugino condensate $\lambda\lambda$ is related to the non-perturbative effects appearing in the four-dimensional superpotential (5.1). We use the standard gauge theory expression for the fermion bilinear in the confining vacuum (here in string units) [64]

$$\langle \lambda\lambda \rangle = 16\pi i e^{K/2} \partial_\rho W = -\frac{32\pi^2}{N} \frac{\mathcal{A} e^{2\pi i \rho / N}}{(2 \text{Im } \rho)^{\frac{3}{2}}} (\text{Im } \tau)^{-\frac{1}{2}}, \quad (5.3)$$

where, like KKLT [1] we used that the complex volume modulus ρ equals the complex gauge coupling of the four-dimensional gauge theory. We also rescaled with the string

¹³We would like to thank Pablo Soler for correcting a phase factor convention.

coupling as in [51]. We note that \mathcal{A} is determined by the dynamically generated scale of the IR theory and is left unspecified.

We emphasise here that the exponential dependence of the volume modulus is crucial in the 4d EFT and its off-shell scalar potential. When balanced with the small flux contribution W_0 , this exponential stabilises the Kähler modulus at a somewhat large volume and exponentially small values of the 4d scalar potential. The 10d supergravity analysis performed here, where we opted for a local description of the gaugino condensate, does not reproduce this exponential behaviour. This is reminiscent from the fact that this procedure requires to plug in the vev after varying the action with the metric (which only renders polynomial dependences). Nevertheless we proceed in further rewriting eq. (4.15).¹⁴ Using the map between the four-dimensional theory and the ten-dimensional quantities we find the following result

$$\int \star_6 \text{Re} (G_3^0 \cdot I^{\text{reg}}) = \frac{e^{-4A} \text{Re} \left(W_0 \bar{\lambda} \bar{\lambda} (\text{Im} \rho)^{\frac{3}{2}} \right)}{16\pi^2 \mathcal{V}_2 \sqrt{\text{Im} \tau}}, \quad (5.4)$$

where we made use of eq. (5.2).

More explicitly, we can insert the expression for I^{reg} in (4.12) and make use of the rescalings of R_4 and $\lambda\lambda$ as above to find

$$R_4 = -\frac{2}{\mathcal{V}_2^2} (\text{Im} \rho)^3 \left| \frac{\lambda\lambda}{16\pi^2} \right|^2 + \frac{3}{4} \frac{1}{\mathcal{V}_2 \sqrt{\text{Im} \tau}} \text{Re} \left(\bar{W}_0 \frac{\lambda\lambda}{16\pi^2} \right). \quad (5.5)$$

We will also extract the volume dependence out of $\mathcal{V}_2 = \tilde{\mathcal{V}}_2 (\text{Im} \rho)^{\frac{1}{2}}$ so that we can rewrite eq. (5.5) with the explicit volume dependence

$$R_4 = -\frac{2}{\tilde{\mathcal{V}}_2^2} (\text{Im} \rho)^2 \left| \frac{\lambda\lambda}{16\pi^2} \right|^2 + \frac{3}{4} \frac{1}{\tilde{\mathcal{V}}_2 \sqrt{(\text{Im} \rho)(\text{Im} \tau)}} \text{Re} \left(\bar{W}_0 \frac{\lambda\lambda}{16\pi^2} \right). \quad (5.6)$$

Without further information we are unable to relate the two terms in (5.6). Based on the four-dimensional effective theory, we expect the two terms to be related to each other on shell but without solving the equations of motion we are unable to verify this. In particular, this equation does not directly reveal the sign of the curvature. A negatively curved AdS solution is however certainly not ruled out. One way to progress would be to perform the supersymmetry analysis directly in ten dimensions but this is beyond the scope of our work. For comparison we give here the EFT value of the curvature obtained from the four-dimensional KKLT model before anti-D3 branes are included (in Planck units):

$$R_4^{\text{KKLT}} = 4V_{\text{KKLT}} = -12e^K |W|^2 = -\frac{16\pi^2}{3} (\text{Im} \rho)^2 \left| \frac{\lambda\lambda}{16\pi^2} \right|^2. \quad (5.7)$$

¹⁴It is important to note the λ appearing in equation (5.3) is a canonically normalised gaugino in Einstein frame. As mentioned in footnote 6, the 4d metric must be rescaled in order to go to proper 4d Einstein frame. This amounts to a rescaling of the metric $g_4 \rightarrow (\text{Im} \rho)^{-3/2} g_4$ which leads to the rescaling of the curvature $R_4 \rightarrow (\text{Im} \rho)^{3/2} R_4$. In this new frame the fermion gaugino term is not canonical and so we must also rescale the gaugino $\lambda \rightarrow (\text{Im} \rho)^{9/8} \lambda$ such that $\lambda\lambda \rightarrow (\text{Im} \rho)^{9/4} \lambda\lambda$. It also amounts to rescaling $\Omega \rightarrow (\text{Im} \rho)^{\frac{3}{4}} \Omega$ and $A \cdot B \rightarrow (\text{Im} \rho)^{-\frac{3}{2}} A \cdot B$ where A, B are three-forms.

Finally, let us recall that the anti-D3 brane tension contributes only negatively to R_4 . An uplifting to dS is supposed to occur through the mediation of gaugino terms that contribute positively to R_4 . Such positive terms can only arise from the term $\text{Re}(G_3^0 \cdot I^{\text{reg}})$ being non-zero in a supersymmetry breaking vacuum.

6 Discussion

6.1 Summary and discussion of results

In this paper, we have revised the computations of [34] taking into account the flux renormalisation mechanism provided by the quartic gaugino term on D7-branes, as proposed in [14]. We must rely on a number of assumptions necessary to directly use the techniques of [14], namely constant axion-dilaton, warping, and C_4 -potential, and most importantly we used that the harmonic 3-form flux is the GKP flux. Consistent with the result of [34] we are unable to establish the sign of the four-dimensional curvature from ten-dimensional analysis alone. We explicitly showed that within our assumptions the quartic gaugino terms proposed by [48] are sufficient to regularize the singularities encountered in [33, 34].

A worthwhile direction for future research would be to see if one can still obtain a definite result after when it comes to de Sitter solution using anti-D3 brane uplift terms. It is tempting to speculate what happens. For that we go back to equation (4.10). Note that this equation was derived without any of the simplifying approximations and is hence valid in general circumstances. We have argued that the renormalisation due to the four-fermion term is such that the only 3-form fluxes appearing in (4.10) are the *background* fluxes, ie those of the classical GKP solution. This means we can put $|G_3^{0,-}|^2 = 0$ since the flux appearing will be ISD. This is expected to be broken by the gaugino condensate but the deviations from the ISD background are mostly “self-energy” that has been removed by the renormalisation procedure. Once anti-branes are added they will also create some local non-ISD fluxes down the throat. But we will neglect these since they anyway come with a negative contribution to R_4 and so will not help in getting de Sitter. The main question then is whether the only positive contribution to the vacuum energy

$$\int \star_6 \text{Re}(G_3 \cdot \mathcal{P}(I)), \tag{6.1}$$

can dominate over the rest. This question seems to be very difficult to answer, and most likely requires a full solution the entire system of the ten-dimensional equations of motion which surely is out of reach.

One way to approach this problem is make a precise analysis of the term (6.1) for a supersymmetric configuration. This could be done by analysing the ten-dimensional supersymmetry variations in presence of the gaugino bilinear. This is a very interesting exercise which we plan to carry out in the future.

Finally we wish to iterate the statements in [33, 34]: to have certainty that uplifting AdS vacua with small supersymmetry-breaking ingredients does not lead to runaways instead of meta-stable dS vacua, one should achieve the following parametric scaling:

$$m^2 L^2 \gg 1. \tag{6.2}$$

In this equation m^2 is the mass squared of the lightest modulus in the AdS vacuum and L the AdS length. Interestingly not a single clear top-down AdS vacuum in string theory achieves this. The only cases known to us are “stringy-inspired” like racetrack fine-tuning [65] or non-geometric fluxes [66, 67]. It was conjectured in [35] that racetrack fine-tuning is in the Swampland¹⁵ whereas reference [34] conjectured this to be true for all vacua obeying (6.2). Since the conjecture of [34] forbidding (6.2) is a statement about AdS vacua and not dS vacua we believe it should be easier to verify than the actual no-dS conjecture of [6–8], and it would count as non-trivial evidence in support of the no-dS conjecture.

6.2 Comparison with recent papers

Simultaneous with the appearance of this work, two other papers appeared discussing the same problem [50, 51]. Reference [51] set out to do similar computations as done here but came to a different conclusion. We differ from [51] in the way the energy-momentum tensor in ten dimensions is computed. This is also discussed in reference [50].

We demonstrate the difference in the language of [51]. There the trace reversed Einstein equations are computed with variations w.r.t. the volume instead of the warp factor. Below we mention why this choice is important to reach the results presented in [51]. Concretely, the 4d curvature can be computed by using the following equation [51]:¹⁶

$$2\pi\mathcal{V}_6\mathcal{V}_4R_4 = - \left((\text{Im } \rho) \frac{\delta}{\delta(\text{Im } \rho)} - 1 \right) S. \quad (6.3)$$

We only look at the terms in the action involving the gaugino condensate where we integrated over 6D space:

$$S_{\lambda\lambda} = 2\pi \int \star_4 \frac{1}{2} \left[-8(\text{Im } \rho)^{\frac{1}{2}} \left| \frac{\lambda\lambda}{16\pi^2} \right|^2 + (\text{Im } \tau)^{-\frac{1}{2}} \left((\text{Im } \rho)^{\frac{1}{4}} \frac{\bar{\lambda}\bar{\lambda}}{16\pi^2} W_0 + \text{c.c.} \right) \right] \quad (6.4)$$

Here we have written the volume dependence coming from the 6D metric explicitly and ignored the warping dependence.

As we have mentioned, our approach is as follows: we vary the D7 brane action to obtain its energy-momentum tensor. *Subsequently* we fill in the vev of the gaugino condensate $\langle \lambda\lambda \rangle$, whose value is set by non-perturbative physics. This is how the semi-classical limit is usually defined: one inserts the quantum vevs of operators into the classical EM tensor. Following this approach, one finds by evaluating (6.3) and (6.3) that the 4d curvature satisfies:

$$\mathcal{V}_6R_4 = -2(\text{Im } \rho)^{\frac{1}{2}} \left| \frac{\lambda\lambda}{16\pi^2} \right|^2 + \frac{3}{4}(\text{Im } \tau)^{-\frac{1}{2}} \text{Re} \left((\text{Im } \rho)^{\frac{1}{4}} \frac{\bar{\lambda}\bar{\lambda}}{16\pi^2} W_0 \right) \quad (6.5)$$

This result agrees with (4.15).

However a subtlety arises which relates to the dependence of the gaugino vev on the gauge coupling (5.3). Since the coupling equals the inverse volume of the 4-cycle wrapped

¹⁵See [26, 68] for some criticism on this.

¹⁶We use a different convention for the Einstein-Hilbert action than [51]. This amounts for a difference of a factor 2 in the r.h.s. of (6.3).

by the D7 branes, a dependence of the vev $\langle \lambda\lambda \rangle$ on the ten-dimensional metric is induced. This dependence cannot be understood classically in four or ten dimensions. One could imagine reversing the order and plug the metric dependence of the gaugino condensate into the ten-dimensional action *before* computing the energy-momentum tensor. This introduces extra terms due to derivatives on the condensate, i.e. this approach would result in a 4d curvature of the form.

$$\begin{aligned} \mathcal{V}_6 R_4 = & -2(\text{Im } \rho)^{\frac{1}{2}} \left| \frac{\lambda\lambda}{16\pi^2} \right|^2 + \frac{3}{4}(\text{Im } \tau)^{-\frac{1}{2}} \text{Re} \left((\text{Im } \rho)^{\frac{1}{4}} \frac{\bar{\lambda}\bar{\lambda}}{16\pi^2} W_0 \right) \\ & + 4(\text{Im } \rho)^{\frac{3}{2}} \frac{\partial}{\partial \text{Im } \rho} \left| \frac{\lambda\lambda}{16\pi^2} \right|^2 - (\text{Im } \tau)^{-\frac{1}{2}} \text{Re} \left((\text{Im } \rho)^{\frac{5}{4}} \left(\frac{\partial}{\partial \text{Im } \rho} \frac{\bar{\lambda}\bar{\lambda}}{16\pi^2} \right) W_0 \right) \end{aligned} \quad (6.6)$$

Working with this equation, reference [51] finds a qualitative match with the four-dimensional KKLT potential and further concludes there is no obstruction to uplifting to meta-stable dS vacua. We want to emphasize that it is not clear how the semi-classical method really works in this case.

In a rather different wording reference [50] warned for this subtlety and concludes that perhaps the ten-dimensional approach is not attainable after all. We remain agnostic about that point but emphasize the following; *if* a ten-dimensional viewpoint is to make sense then we want to emphasize the following two things: 1) Inserting the four-dimensional information about the gauge coupling dependence into the ten-dimensional action implies one is not following a genuine ten-dimensional and local approach after all. 2) A semi-classical limit should be something universal. In other words when one couples classical gravity to quantum matter then the approach is that the quantum vevs are inserted into the classical EM tensor. The latter tensor is a universal object in the theory and how to compute the tensor does not depend on the quantum effects of the matter sector.

This does not imply we disagree with the outcome of the results in [51], but we want to emphasize our motivations for following the other path and what the consequences are.

Finally note that, if the second part of reference [50] turns out to be correct, then dS uplifts are questionable already for a simpler reason related to having controlled uplifts and throat volumes that fit into the compact CY space.

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A Conventions

The type IIB supergravity action (in units where $2\pi\ell_s = 1$) is

$$S = 2\pi \int \star_{10} \left(R_{10} - \frac{d\tau \cdot d\bar{\tau}}{2(\text{Im } \tau)^2} - \frac{G_3 \cdot \overline{G_3}}{2(\text{Im } \tau)} - \frac{1}{4}|F_5|^2 \right) - \frac{\pi}{2i} \int \frac{C_4 \wedge G_3 \wedge \overline{G_3}}{\text{Im } \tau}, \quad (\text{A.1})$$

where R_{10} is the ten-dimensional Ricci scalar calculated using the metric g_{MN} with determinant g_{10} . The axion-dilaton is denoted by $\tau = C_0 + ie^{-\phi}$ and the NSNS and RR 3-form field strengths have been combined into a single complex 3-form

$$G_3 = F_3 - ie^{-\phi}H = dC_2 - \tau dB_2, \quad H = dB_2, \quad F_3 = dC_2 - HC_0. \quad (\text{A.2})$$

Throughout the paper we use short-hand notation to denote form contractions, let ω_p and ψ_p denote two p -forms, then

$$\star_{10} \omega_p \wedge \psi_p = \omega_p \cdot \psi_p \star_{10} 1, \quad \omega_p \cdot \psi_p = \frac{1}{p!} \omega_{M_1 M_2 \dots M_p} \psi^{M_1 M_2 \dots M_p}, \quad |\omega_p|^2 = \overline{\omega}_p \cdot \omega_p. \quad (\text{A.3})$$

The complex-conjugation in the last expression is to allow for the possibility that ω_p is a complex p -form.

B Variation of the action

In this appendix we derive eq. (4.10) step-by-step. We also fill in some of the gaps of the subsequent calculations in section 4. First we show that we can obtain equation (4.3) directly by varying the ten-dimensional type IIB supergravity action with respect to the warp factor A , as defined in (4.1). After establishing this, we include the D7-branes and redo the same computation.

The ten-dimensional action for the Ansatz (4.1), (4.2) can be found to be:

$$S_{\text{IIB}} = 2\pi \int d^{10}x \sqrt{-g_4} \sqrt{g_6} e^{-2A} \mathcal{I}, \quad (\text{B.1})$$

where

$$\mathcal{I} = e^{-2A} R_4 + e^{2A} (R_6 + 2\Delta_6 A - 8|dA|_6^2) - e^{2A} \frac{|d\tau|_6^2}{2(\text{Im } \tau)^2} - e^{6A} \frac{|G_3|_6^2}{2\text{Im } \tau} + e^{-6A} \frac{|d\alpha|_6^2}{2}. \quad (\text{B.2})$$

The $|\dots|_6^2$ denote metric contractions using purely the six-dimensional metric ds_6^2 without the warp factor. We now compute a variation with respect to A . We find:

$$0 = R_4 - \Delta_6 e^{4A} + e^{-6A} |de^{4A}|^2 + e^{2A} \frac{|G_3|_6^2}{2\text{Im } \tau} + e^{-6A} |d\alpha|^2. \quad (\text{B.3})$$

This is simply the trace-reversed ten-dimensional Einstein-equation traced over the four-dimensional indices. Adding the Bianchi identity, we recover equation (4.3):

$$\Delta_6 \Phi^- = R_4 + e^{2A} \frac{|G_3^-|_6^2}{\text{Im } \tau} + e^{-6A} |d\Phi^-|^2. \quad (\text{B.4})$$

We did not include any sources, but including O7/O3 planes does not modify the final result. Now we would now like to add D7-branes to the configuration. As explained in the main text, the inclusion of the D7-brane implies that we should perform the replacement (4.8) and vary that with respect to the warp factor. We have two crucial remarks. The careful reader will notice that I as defined in (4.6) contains a warp factor dependence of e^{-4A} . This factor should *not* be varied when deriving the equivalent to (B.3), since it does not arise from a metric in the action. On the other hand, the delta function and the holomorphic three-form do have a warp factor dependence. The holomorphic three-form goes like e^{-3A} and the delta function like e^{2A} . We can therefore write the perfect square action as

$$S_3 = -2\pi \int d^{10}x \sqrt{-g_4} \sqrt{g_6} \frac{e^{4A}}{2\text{Im } \tau} \left| G_3 - e^{-A} (\text{Im } \tau) \tilde{\mathcal{P}}(\bar{I}) \right|_6^2, \quad (\text{B.5})$$

where we have made the warp-factor dependence explicit by momentarily writing $\mathcal{P}(I) = e^{-A} \tilde{\mathcal{P}}(\bar{I})$. Using this we find

$$-\frac{e^{4A}}{8\pi \sqrt{-g_4} \sqrt{g_6}} \frac{\delta S_3}{\delta A} = \frac{e^{2A}}{2\text{Im } \tau} |G_3 - (\text{Im } \tau) \mathcal{P}(\bar{I})|^2 + \frac{e^{2A}}{4} \text{Re} [(G_3 - (\text{Im } \tau) \mathcal{P}(\bar{I})) \cdot \mathcal{P}(I)] \quad (\text{B.6})$$

$$= e^{2A} \left(\frac{|G_3|^2}{2\text{Im } \tau} - \frac{3}{4} \text{Re} (G_3 \cdot \mathcal{P}(I)) + \frac{\text{Im } \tau}{4} |\mathcal{P}(I)|^2 \right). \quad (\text{B.7})$$

We should replace the $|G_3|^2$ -term in (B.3) by (B.7). This modifies (B.4) to exactly (4.10) which reads:

$$\Delta_6 \Phi^- = R_4 + e^{-6A} |d\Phi^-|^2 + e^{2A} \left(\frac{|G_3^-|^2}{\text{Im } \tau} - \frac{3}{4} \text{Re} (G_3 \cdot \mathcal{P}(I)) + \frac{\text{Im } \tau}{4} |\mathcal{P}(I)|^2 \right).$$

If one now writes $\mathcal{P}(I) = I^{\text{sing}} + I^{\text{reg}}$ as in (4.13) and substitutes the solution to the equation of motion $G_3 = (\text{Im } \tau) \overline{I^{\text{sing}}} + G_3^0$ (while making the approximation that $\Phi^- = 0$ and warping, C_4 -potential and axion-dilaton are constant) into (B.6), one finds together with the other terms in the IIB action and the Bianchi identity the following equation:

$$0 = R_4 + \frac{e^{2A}}{2\text{Im } \tau} |G_3^0 - (\text{Im } \tau) \overline{I^{\text{reg}}}|^2 + \frac{e^{2A}}{4\text{Im } \tau} \text{Re} \left[(G_3^0 - (\text{Im } \tau) \overline{I^{\text{reg}}}) \cdot (\text{Im } \tau) (I^{\text{sing}} + I^{\text{reg}}) \right] + \frac{e^{2A}}{2\text{Im } \tau} \left(|G_3^{0,-}|^2 + 2\text{Re} (G_3^{0,-} \cdot \overline{I^{\text{sing},-}}) - |G_3^{0,+}|^2 - 2\text{Re} (G_3^{0,+} \cdot \overline{I^{\text{sing},+}}) \right) \quad (\text{B.8})$$

The substitution already took into account that the Bianchi identity does not contribute δ^2 singularities, which is discussed in appendix C. Recall that I^{sing} is by definition exact, so its inner product with a harmonic form (G_3^0 or I^{reg}) is therefore zero when the expression is integrated over (the (A)ISD-parts vanish in the same fashion). In this way, we arrive at

$$0 = \int \star_6 \left(R_4 + \frac{e^{2A}}{\text{Im } \tau} |G_3^{0,-}|^2 - \frac{3e^{2A}}{4} \text{Re} (G_3^0 \cdot I^{\text{reg}}) + \frac{e^{2A}}{4} (\text{Im } \tau) |I^{\text{reg}}|^2 \right). \quad (\text{B.9})$$

C (Non)renormalisation of the Chern-Simons action

The effective bulk action of IIB string theory contains a Chern-Simons term,

$$S_{\text{CS}} = -\frac{\pi}{2i} \int \frac{C_4 \wedge G_3 \wedge \overline{G_3}}{\text{Im } \tau}. \quad (\text{C.1})$$

This term is topological and does not affect the Einstein equation directly. Still one can be concerned and wonder whether we should renormalise the Chern-Simons action or not. If we should, the renormalisation could introduce metric dependence and affect our results.

Under the assumptions of constant C_4 and axion-dilaton made in the main body of the text, we should not renormalise the Chern-Simons term. Not renormalising produces no disasters and thus there is perhaps no obvious need for renormalisation. Let us check this.

At first, it may appear that filling the solution for G_3 into the Chern-Simons action will produce a badly divergent action. However, this is not the case. Filling the on-shell value of G_3 into the Chern-Simons action yields

$$S_{\text{CS,on-shell}} = -\frac{\pi g_s}{2i} \int \left\{ C_4 \wedge G_3^0 \wedge \overline{G_3^0} + (\text{Im } \tau) C_4 \wedge G_3^0 \wedge \overline{I^{\text{sing}}} + (\text{Im } \tau) C_4 \wedge I^{\text{sing}} \wedge \overline{G_3^0} \right\} \\ - \frac{\pi g_s}{2} \int (|I^{\text{sing}-}|^2 - |I^{\text{sing}+}|^2) (\text{Im } \tau)^2 C_4 \wedge \tilde{\star}_6 1, \quad (\text{C.2})$$

where we introduced the symbols $I^{\text{sing}\pm} = \frac{1}{2}(\star_6 \pm i)I^{\text{sing}}$. The last two terms in this expression seem divergent while all other terms are clearly finite upon evaluation of the integral. Interestingly the two divergent terms exactly cancel against each other and the on-shell action is well-behaved. To see this, note that near a D7-brane we can locally write the internal manifold as $\mathcal{M} = \Sigma_2 \times \Sigma_4$, with Σ_2 transverse and Σ_4 parallel to the D7-brane. We then have

$$\overline{I^{\text{sing}}} \sim d \left(\frac{\partial G}{\partial z} \right) \wedge \bar{\Omega}_2 \\ = \left(\frac{\partial^2 G}{\partial z \partial z} \right) dz \wedge \bar{\Omega}_2 + \left(\frac{\partial^2 G}{\partial z \partial \bar{z}} \right) d\bar{z} \wedge \bar{\Omega}_2 \quad (\text{C.3})$$

with G the Green's function, which varies transverse to the D7-brane, and dz and $d\bar{z}$ are defined on Σ_2 . It is clear that the first of these terms is $(1, 2)$ while the second is $(0, 3)$. First we notice that the two terms have the same magnitude:

$$\int \star_6 \left| \left(\frac{\partial^2 G}{\partial z \partial z} \right) dz \wedge \bar{\Omega}_2 \right|^2 = \int \star_6 \left| \left(\frac{\partial^2 G}{\partial z \partial \bar{z}} \right) d\bar{z} \wedge \bar{\Omega}_2 \right|^2, \quad (\text{C.4})$$

which follows from integration by parts and the reality of G . Then we use that the first term in (C.3) corresponds to the IASD part of $\overline{I^{\text{sing}}}$, while the second term corresponds to the ISD part. It is immediate that the $(0, 3)$ term is ISD as the unique $(0, 3)$ form at our disposal is $\bar{\Omega}$ and this is ISD in our convention. That the $(1, 2)$ form is IASD follows from the fact that it is primitive and primitive $(1, 2)$ forms are IASD in our convention. The primitivity can be seen by decomposing J as

$$J = J_{\Sigma_2} + J_{\Sigma_4}, \quad (\text{C.5})$$

with the two terms defined in the obvious way. Using this, evaluation yields

$$\left(\frac{\partial^2 G}{\partial z \partial \bar{z}}\right) dz \wedge \bar{\Omega}_2 \wedge J = 0. \tag{C.6}$$

We thus see that $I^{\text{sing}+}$ and $I^{\text{sing}-}$ have the same magnitude. Therefore, the two problematic terms in (C.2) cancel against each other and the on-shell Chern-Simons action is finite without the introduction of any renormalisation counterterms.

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