

A 125 GeV Higgs Boson and Muon $g - 2$ in More Generic Gauge Mediation

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Abstract

Recently, the ATLAS and CMS collaborations reported exciting hints of a Standard Model-like Higgs boson with a mass around 125 GeV. A Higgs boson this heavy is difficult to realize in conventional models of gauge mediation. Here we revisit the lightest Higgs boson mass in “more generic gauge mediation,” where the Higgs doublets mix with the messenger doublets. We show that a Higgs boson mass around 125 GeV can be realized in more generic gauge mediation models, even for a relatively light gluino mass, $m_{\text{gluino}} \sim 1 \text{ TeV}$. We also show that the muon anomalous magnetic moment can be within 1σ of the experimental value for these models, even when the Higgs boson is relatively heavy. We also discuss the LHC constraints and the prospects of discovery.

1 Introduction

The ATLAS and CMS collaborations collected almost 5 fb^{-1} data in 2011 [1, 2]. A tantalizing hint for a Standard Model-like Higgs boson has emerged from this data with a mass around 125 GeV. Although these results are not conclusive enough to claim discovery, such a relatively heavy Higgs boson would have significant impact on the supersymmetric (SUSY) Standard Model (SSM), if it is indeed confirmed by further data collection.

In particular, a lightest Higgs boson mass around 125 GeV is very problematic in the minimal SSM (MSSM). To realize such a heavy Higgs boson in the MSSM, we need either very large squark masses, $O(10 - 100)$ TeV [3, 4], or a large stop A -term with stop squark masses around a TeV (see recent discussions in Refs. [5, 6, 7]). In the former case, the search for the superparticles at the early LHC is quite difficult, even if the gauginos are within reach of the LHC as in Split-Supersymmetry [8, 9, 10] or pure gravity mediation models [11] (see also Refs. [12, 13]). In this sense, the latter case with large A -terms is more interesting for the LHC experiments where we have a better chance of discovering the superparticles in near future.

It is, however, not easy to find models with large A -terms at the low-scale. The major obstacle in creating a viable model is the suppression of the A -term during renormalization group running to the low-scale. In gravity mediation models, for example, the high-scale cutoff is rather larger, i.e. GUT scale or Plank scale. The A -terms are renormalization group evolved over many orders of magnitude. This prolonged running drastically suppresses the A -terms. To offset this suppression, the A -terms must be very large at the high-scale. Other models are worse. For example, minimal gauge mediation has nearly vanishing A -term at the messenger scale.

In a recent paper [14], three of us (J.L.E., M.I. and T.T.Y.) constructed a class of models termed “more generic gauge mediation.” In these models, the messenger doublets are mixed with the Higgs doublets without generating flavor changing neutral currents (FCNC) or rapid proton decay. It was also shown that the desired large A -terms can be generated. The suppression of the A -terms due to the renormalization group evolution in these models, however, is minimal since the messenger scale can be as low as $O(100)$ TeV. As a result, a relatively heavy Higgs boson was obtained [14].

In light of the recent results of the ATLAS and CMS collaborations, we revisit the lightest Higgs boson mass in these more generic gauge mediation models. We also show that the muon anomalous magnetic moment can be consistent with the experimental value at the 1σ level in large regions of parameter space.¹ We stress here that the field content of more generic gauge mediation is the same as in minimal gauge mediation. It is quite surprising that merely introducing mixing between the Higgs and the messenger doublets can resolve the tension in gauge mediation models, i.e. a relatively heavy Higgs boson and a large enough muon $g - 2$.

The organization of the paper is as follows. In section 2, we review more generic gauge mediation models. In section 3, we show that a relatively heavy Higgs boson and a consistent muon $g - 2$ can be simultaneously obtained in more generic gauge mediation models. In section 4, we discuss constraints and prospects of detection for the present model at the LHC experiments. The final section is devoted to our conclusions and discussions.

2 More Generic Gauge Mediation

2.1 Higgs-messenger mixing

Let us briefly review our more generic gauge mediation model which was constructed in Ref. [14]. In these more generic gauge mediation models, we allowed the Higgs doublets to mix with the doublet portion of the messenger multiplets via the superpotential couplings,

$$W_{\text{mixing}} = gZ\bar{\Phi}\Phi + g'Z\bar{\Phi}_L H_u + g''ZH_d\Phi_L, \quad (1)$$

while the unwanted flavor mixing and proton decay operators are suppressed (see Ref. [14] for detailed discussion). Here, we have assumed the messengers $(\Phi, \bar{\Phi})$ are a fundamentals and anti-fundamentals of the minimal grand unified gauge group, $SU(5)$, and we split the messengers into $\Phi = (\Phi_D, \Phi_{\bar{L}})$ and $\bar{\Phi} = (\bar{\Phi}_D, \bar{\Phi}_{\bar{L}})$ in accordance with the MSSM gauge charges. We also treat the supersymmetry breaking field Z as a spurion which breaks

¹See discussions on the simultaneous explanation of a lightest Higgs boson mass around 125 GeV and the deviation of the muon $g - 2$ in the focus point supersymmetry [15] in models with extra matter [16, 17] and in models with extended gauge interactions [18].

supersymmetry having the vacuum expectation value,

$$g \langle Z \rangle = M + F\theta^2 . \quad (2)$$

In Ref. [14], we found four possible classes of gauge mediation which are consistent with flavor constraints as well as rapid proton decay constraints;

- No mixings between the messengers and the Higgs pair (i.e. $g' = g'' = 0$).
- The messenger $\Phi_{\bar{L}}$ mixes with H_u (i.e. $g' \neq 0, g'' = 0$).
- The messenger $\bar{\Phi}_{\bar{L}}$ mixes with H_d (i.e. $g' = 0, g'' \neq 0$).
- The messengers $\Phi_{\bar{L}}$ and $\bar{\Phi}_{\bar{L}}$ mix with H_u and H_d , respectively (i.e. $g' \neq 0, g'' \neq 0$).

Each class of models can be realized with the help of a “charged” coupling constant, i.e. the SUSY-zero mechanism [14] (see also the appendix A).² The first class of models corresponds to conventional gauge mediation. As emphasized in Ref. [14], the second class of models, which was named Type-II gauge mediation, leads to a peculiar mass spectrum when compared with conventional gauge mediation. Particularly, the lightest Higgs boson mass can be rather large and a mass of 125 GeV can be easily realized even if the gluino mass is relatively light, $m_{\text{gluino}} \lesssim 2 \text{ TeV}$. In the following discussion, we concentrate on these Type-II models since we are most interested in the mass of the lightest Higgs boson. However, these other two new classes of models will have their own unique spectrum.

Before closing this section, it should be noted that more generic gauge mediation requires messengers that couple to a spurion which has both a scalar expectation value as well as an F -term expectation value as in Eq. (2). To realize such a messenger sector with a stable vacuum, the origin of the spurion field should be a secondary supersymmetry breaking field as realized in “cascade supersymmetry breaking” models [20, 21] (see earlier implementations of the cascade supersymmetry breaking [22, 23, 24, 25, 26] which revived the original ideas of the gauge mediation [27, 28, 29]). In cascade supersymmetry breaking, the size of the primary supersymmetry breaking is generally much larger than

² To realize the fourth class of model by the SUSY zero mechanism, we need at least two pairs of messengers. A similar model to the fourth class has been considered based on the framework of extra dimensions [19]. However, these models have more difficulty realizing a 125 GeV lightest Higgs boson.

the secondary breaking appearing in Eq. (2). As a result, the gravitino mass is expected to be not too light, i.e. $m_{3/2} > \mathcal{O}(100)$ keV.³

2.2 Soft parameters in Type-II gauge mediation model

Let us discuss the soft parameters peculiar to Type-II gauge mediation models where only H_u mixes with the messengers. The superpotential of Type-II gauge mediation at the messenger scale is given by

$$W = gZ\bar{\Phi}\tilde{\Phi} + g'Z\bar{\Phi}_L\tilde{H}_u + \tilde{\mu}\tilde{H}_uH_d + \tilde{y}_{Uij}\tilde{H}_uQ_{Li}\bar{U}_{Rj} , \quad (3)$$

where $\tilde{\mu}$ is a dimensionful parameter, \tilde{y}_{Uij} is the usual 3×3 Yukawa coupling matrix. We have also placed tildes on H_u and Φ_L for later purposes and have neglected the rest of the MSSM superpotential which is not relevant for our discussion. The unwanted terms such as $\Phi_L Q_L \bar{U}_R$ and $\Phi_D Q_L Q_L$ can be forbidden because of the SUSY zero mechanism [14].⁴ The explicit charge assignments for the SUSY zero mechanism are given in the appendix A.⁵

To elicit the important low-scale phenomenon, we change our field basis by the rotation

$$\begin{pmatrix} \tilde{\Phi}_L \\ \tilde{H}_u \end{pmatrix} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} \Phi_L \\ H_u \end{pmatrix} . \quad (4)$$

In this new basis, the superpotential becomes

$$W = \bar{g}Z\bar{\Phi}\tilde{\Phi} + \mu H_u H_d + \mu'\tilde{\Phi}_L H_d + y_{Uij}H_u Q_{Li}\bar{U}_{Rj} + y'_{Uij}\tilde{\Phi}_L Q_{Li}\bar{U}_{Rj} , \quad (5)$$

where the parameters are defined as

$$\bar{g} = \sqrt{g^2 + g'^2} , \quad \mu = \frac{g}{\sqrt{g^2 + g'^2}}\tilde{\mu} , \quad \mu' = \frac{g'}{\sqrt{g^2 + g'^2}}\tilde{\mu} ,$$

³In some models of cascade supersymmetry breaking, a light gravitino can be realized in a non-perturbative limit [20, 21].

⁴In Eq. (3), we are assuming that the possible Higgs-Messenger mixing in Kähler potential has been eliminated by appropriate field redefinitions. Such field redefinitions leads to additional terms such as $\tilde{\Phi}_L Q_L \bar{U}_R$. However, they do not cause the unwanted flavor-changing effects since their flavor structure is aligned with the Yukawa interaction $H_u Q_L \bar{U}_R$. In the following, we neglect such effects by assuming $g, g' \ll 1$, although our discussion is not changed too much even for $g, g' = \mathcal{O}(1)$.

⁵The charge assignments defined in Ref. [14] are incomplete to suppress unwanted terms in the superpotential, while the ones in the appendix A completely suppress all the unwanted terms.

$$y_{Uij} = \frac{g}{\sqrt{g^2 + g'^2}} \tilde{y}_{Uij} , \quad y'_{Uij} = \frac{g'}{\sqrt{g^2 + g'^2}} \tilde{y}_{Uij} . \quad (6)$$

This new basis is much better for calculating low scale physics because the only heavy states are clearly $\Phi, \bar{\Phi}$. Hereafter, we also change the definition of the spurion in Eq. (2) by replacing g with \bar{g} . In this basis, the mixing angle between the Higgs and the messengers doublets is suppressed by $O(\mu/M)$, as compared to $O(g'/g)$ in the original. Since we will consider $g'/g \sim 1$, this basis is better suited for physics below the messenger scale.

It should be noted that the new flavor dependent interactions,

$$W = y'_{Uij} \Phi_{\bar{L}} Q_{Li} \bar{U}_{Rj} , \quad (7)$$

are not dangerous. These new flavor dependent interactions are aligned with the MSSM Yukawa coupling, y_U , and hence, y_U and y'_U can be simultaneously diagonalized.⁶ In the following discussion, we choose the basis where \tilde{y}_U is diagonal and neglect everything except the top Yukawa coupling,

$$W = y_t H_u Q_{L3} \bar{T}_R + y'_t \Phi_{\bar{L}} Q_{L3} \bar{T}_R . \quad (8)$$

Not only are these interactions not dangerous, but it is these new interactions that give Type-II gauge mediation its unique spectrum.

As discussed in Ref. [14], the newly added interaction in Eq. (8) leads to an A -terms at the one-loop level,

$$A_t = -\frac{3}{32\pi^2} y_t^2 \frac{F}{M} \frac{1}{x} \log \left(\frac{1+x}{1-x} \right) , \quad (9)$$

for the stop and

$$A_b = -\frac{1}{32\pi^2} y_t^2 \frac{F}{M} \frac{1}{x} \log \left(\frac{1+x}{1-x} \right) , \quad (10)$$

for the sbottom. Here, we have defined $x = F/M^2$, and the above results reduce to

$$A_t \simeq -\frac{3y_t^2}{16\pi^2} \frac{F}{M} , \quad (11)$$

⁶In this sense, Type-II gauge mediation is a natural realization of the so called “minimal flavor violation” scenario (see for example Ref. [30]).

for $x \ll 1$. We see that the one-loop contribution to the A -terms can be comparable to the gauge mediated soft masses squared

$$m_{Q,T}^2 \simeq \frac{8}{3} \left(\frac{\alpha_3}{4\pi} \right)^2 \frac{F^2}{M^2}, \quad (x \ll 1). \quad (12)$$

for $y'_t \simeq 1$.

The soft masses squared of Q_{L3} and \bar{T}_R are also generated at the one-loop level, and are given by

$$\delta m_{Q_3}^2 = \frac{y_t'^2}{32\pi^2} \frac{F^2}{M^2} \left(\frac{(2+x)\log(1+x) + (2-x)\log(1-x)}{x^2} \right), \quad (13)$$

and

$$\delta m_{\bar{T}}^2 = 2 \times \delta m_{Q_3}^2. \quad (14)$$

It should be noted that these one-loop contributions to the stop squared masses are negative [14]. The negative contributions, however, are subdominant for $x \ll 1$, since they are suppressed by x^2 compared to the positive two-loop contributions of gauge mediation.

Besides these one-loop contributions, the newly added interaction leads to a sizable two-loop contribution to m_Q^2 , m_T^2 and $m_{H_u}^2$. Unlike the one-loop contributions to m_Q^2 and m_T^2 , the two-loop contributions are not suppressed in the limit of $x \ll 1$. The leading two-loop contributions can easily be extracted from the wave function renormalization by analytic continuation into superspace [31] leading to

$$\begin{aligned} \delta m_{Q_3}^2 &= \frac{y_t'^2}{128\pi^4} \left(3y_t'^2 + 3y_t^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{13}{30}g_1^2 \right) \frac{F^2}{M^2}, \\ \delta m_{\bar{T}}^2 &= \frac{y_t'^2}{128\pi^4} \left(6y_t'^2 + 6y_t^2 + y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right) \frac{F^2}{M^2}, \\ \delta m_{\bar{B}}^2 &= -\frac{y_b^2 y_t'^2}{128\pi^4} \frac{F^2}{M^2}, \\ \delta m_{H_u}^2 &= -9 \frac{y_t^2 y_t'^2}{256\pi^4} \frac{F^2}{M^2}, \\ \delta m_{H_d}^2 &= -3 \frac{y_b^2 y_t'^2}{256\pi^4} \frac{F^2}{M^2}, \end{aligned} \quad (15)$$

where y_b is the bottom Yukawa coupling constant and $m_{\bar{B}}^2$ is the soft squared mass of the right-handed sbottom. The derivation of these results are given in the appendix B.

The soft SUSY breaking squared mass for H_d also has a “tree-level” contribution due to the third term in the superpotential of Eq. (5). By integrating out the messengers, the down-type Higgs H_d gets a tree-level soft squared mass,

$$m_H^2 = -\mu'^2 \frac{F^2}{M^4 - F^2} . \quad (16)$$

Here, μ' is assumed to be of the same order of magnitude as the μ -term, for $g/g' = O(1)$. This contribution can be important in low scale gauge mediation where $F/M^2 \simeq 1$. However, as we push up the messenger scale this contribution falls off quickly. This tree-level mediation does not play an important role in most of the parameter space we are interested in.

Finally, let us summarize the parameters of type-II gauge mediation models. It should be noted that the only new interaction is the one given in Eq. (8), and hence, y'_t is the only additional parameter not present in conventional gauge mediation models. That is, type-II gauge mediation model can be parameterized by,

$$N_5, \quad \Lambda = \frac{F}{M}, \quad M, \quad \tan \beta, \quad y'_t, \quad \text{sgn}(\mu) , \quad (17)$$

where N_5 is the effective number of the messenger multiplets, and $\tan \beta$ is the ratio of the two vacuum expectation values of the Higgs doublets. As discussed in Ref. [14], the soft terms generated by this single interaction significantly change the prediction on the lightest Higgs boson mass.

3 Higgs Boson Mass and Muon $g - 2$

3.1 Relatively heavy lightest Higgs boson in Type-II model

The ATLAS and CMS collaborations have reported interesting hints of a Higgs boson with a mass around 125 GeV [1, 2]. In conventional gauge mediation models the A -terms are quite small. To get a lightest Higgs boson of this mass requires squark masses of $O(10)$ TeV. As we have seen, however, a sizable A -term for the stop can be generated for $y'_t \simeq 1$ in the Type-II gauge mediation models. These large A -terms increase the mass of the lightest Higgs boson significantly [14].

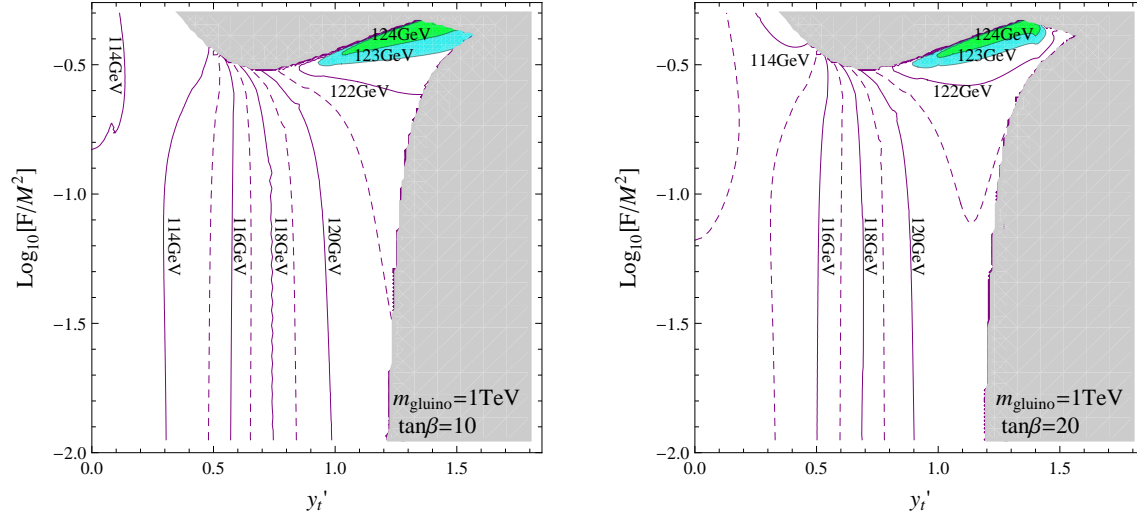


Figure 1: The contour plots of the lightest Higgs boson mass for $\tan \beta = 10$ (left) and $\tan \beta = 20$ (right). In both plots, we have taken a gluino mass of 1 TeV. The green shaded region corresponds to $m_h > 124$ GeV and the light-blue shaded region corresponds to $m_h > 123$ GeV. The gray shaded region is excluded by tachyonic superparticles.

With a relatively large A -term the lightest Higgs boson mass, which receives important SUSY breaking corrections from the top-stop loop diagrams [3, 4], is pushed up to

$$m_{h^0}^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} y_t^2 m_t^2 \sin^2 \beta \left(\log \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{A_t^2}{m_{\tilde{t}}^2} - \frac{A_t^4}{12m_{\tilde{t}}^4} \right). \quad (18)$$

Here, m_Z and m_t are the masses of the Z -boson and top quark, respectively. The above expression for the Higgs mass is maximized for an A -term of order $A_t \simeq \sqrt{6} \times m_{\tilde{t}}$ (i.e. the hmax scenario).

In Fig. 1, we show a contour plot of the lightest Higgs boson mass as a function of y_t' and F/M^2 for $\tan \beta = 10$ (left) and $\tan \beta = 20$ (right). To calculate the weak scale soft masses, we have used *SoftSusy* [32], and the lightest Higgs boson mass is calculated using *FeynHiggs* [33]. In both panels, the green region corresponds to $m_h > 124$ GeV and the light-blue shaded region corresponds to $m_h > 123$ GeV. In our analysis, we used the central values of the top quark mass $m_t = 173.2 \pm 0.9$ GeV [34] and the strong coupling constant, $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ [35]. The gray shaded region in Fig. 1 for $x \simeq 1$ is excluded due to a very light stop, while the gray shaded region for $y_t' \gtrsim 1$ and $x \ll 1$ is excluded due to a very light slepton. Within the allowed region, we find that the vacuum

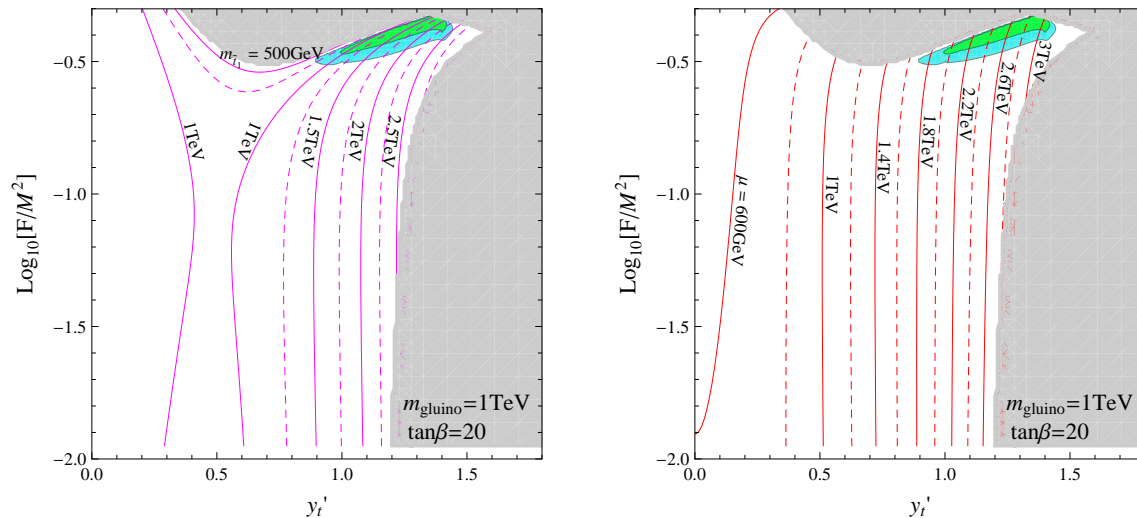


Figure 2: Contour plots of the lighter stop mass (left) and the μ -term for $\tan\beta = 20$ (right). In both plots, we have taken a gluino mass of 1 TeV. The results do not significantly depend on $\tan\beta$ as long as $\tan\beta \gtrsim 10$.

stability condition [36],

$$A_t^2 + 3\mu^2 < 7.5 (m_{t_L}^2 + m_{t_R}^2) , \quad (19)$$

is always satisfied. Thus, the relatively large A -terms do not cause vacuum instability problems in Type-II models.

These figures shows that a relatively heavy Higgs boson is obtained for $y'_t \simeq 1$. Notice that the lightest Higgs boson is heaviest in regions where $x \simeq 1$. Since $x \simeq 1$ corresponds to a low messenger scale, the suppression of the A -terms due to renormalization group evolution is less significant in this region. The figures also shows that the Higgs mass is only weakly dependent on $\tan\beta$ for much of the parameter space(see also later discussion).

Before closing this section, we also show the lighter stop mass in Fig. 2. This figure shows how the stop mass increases with y'_t . This increase in the stop masses is due to the two-loop contribution found in Eq. (15). As a result, the stop becomes heavier for the bulk of the parameter space where the Higgs boson mass is enhanced. It should be also noted that the stops can be much lighter at the corner of the parameter region for $x \simeq 1$, where the negative one-loop contributions found in Eqs. (13) and (14) are important. In particular, if the stop becomes significantly lighter than the gluino, the gluino will decay

mainly into a top and a stop, which affects the search strategies for these models at the LHC.

In Fig. 2, we also show contour plots of the μ -term. This figure shows that the μ -term is relatively large even for the regions with a light stop. This is due to the new two-loop contribution to $m_{H_u}^2$ given in Eq. (15). Because it is large and negative, a large μ -term is needed to compensate. As a result, the Higgsino masses are much heavier than the colored superparticles in the region where the Higgs boson mass is largest.

3.2 The muon anomalous magnetic moment

Since the muon anomalous magnetic moment has been measured quite precisely, it is an important probe of new physics beyond the Standard Model. The current experimental value of the anomalous magnetic moment of the muon is [37]:

$$a_\mu^{\text{exp}} = 11659208.9(6.3) \times 10^{-10} . \quad (20)$$

The most recent calculation of the Standard Model prediction, on the other hand, is [38];

$$a_\mu^{\text{SM}} = 11659182.8(4.9) \times 10^{-10} , \quad (21)$$

which includes the updated data from $e^+e^- \rightarrow \text{hadrons}$ and the latest evaluation of the hadronic light-by-light scattering contributions. As a result, the experimental value of the muon $g - 2$ significantly deviates from the Standard Model prediction by about 3.3σ , i.e.

$$\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10} . \quad (22)$$

It is quite tantalizing that this deviation can be explained by the existence of superparticles. The supersymmetric contribution to the anomalous magnetic moment of the muon is proportional to $\tan \beta$ and is suppressed for the heavy superparticle masses. For a precise expression of the supersymmetric contribution to the muon $g - 2$, see Ref. [39]. Unfortunately, however, the relatively heavy Higgs boson hinted at by ATLAS and CMS collaborations requires rather heavy superparticle masses in most models. Therefore, it is not easy to realize both a Higgs boson mass of around 125 GeV and a consistent muon $g - 2$ simultaneously.

The above tension between a relatively heavy Higgs boson mass and a sizable supersymmetric contribution to the muon $g-2$ is eased in Type-II models. As we have seen, the relatively heavy lightest Higgs boson can be realized even when the other superparticles are relatively light. This feature is quite advantageous for simultaneously explaining both the heavy Higgs boson mass and the deviation of the muon $g-2$.

Another advantage Type-II models have is light left-handed sleptons. The rather light left-handed sleptons are due to renormalization group evolution,

$$\frac{d}{dt}m_{\text{slepton}}^2 = - \sum_{a=1,2} 8C_a \frac{g_a^2}{16\pi^2} |M_a|^2 + \frac{1}{8\pi^2} \frac{3}{5} Y g_1^2 \mathcal{S} , \quad (23)$$

where M_a denote the gaugino masses, $C_2 = 3/4$ and $Y = -1/2$ for the doublet sleptons, and $C_2 = 0$ and $Y = 1$ for the right handed sleptons. \mathcal{S} is given by,

$$\mathcal{S} = \text{tr} [Y_i m_i^2] = m_{H_u}^2 - m_{H_d}^2 + \text{tr} [m_Q^2 - m_L^2 - 2m_U^2 + m_D^2 + m_E^2] . \quad (24)$$

The purely gauge mediated contributions to the above expression cancel at the messenger scale. However, the new contributions to the soft masses that are proportional to y'_t do not cancel.⁷ As we see from Eq. (15), the two-loop contribution to $m_{Q_{3,\bar{T}}}^2$ are large and positive for $y'_t \gtrsim 1$ which leads to a negative \mathcal{S} . The negative tree-level contribution to $m_{H_d}^2$ for $x \simeq 1$ also gives a negative contribution to \mathcal{S} . Therefore, through renormalization group running, the doublet sleptons become lighter at the low energy scale, while the right-handed sleptons become heavier.⁸ This suppression of the left handed slepton mass is also important for obtaining a sizable supersymmetric contribution to the muon $g-2$ in Type-II model.

In Fig. 3, we show contour plots of the supersymmetric contribution to the muon $g-2$ as a function of y'_t and F/M^2 for both $\tan\beta = 10$ (left) and $\tan\beta = 20$ (right). Here, we have taken $\mu > 0$ so that the supersymmetric contribution shifts the muon $g-2$ in the right direction. The muon $g-2$ is calculated using *FeynHiggs*. By comparing the results for $\tan\beta = 10$ and $\tan\beta = 20$, we see that $\delta a_\mu^{\text{SUSY}}$ is proportional to $\tan\beta$ as expected.

⁷This means that the spectrum of Type-II gauge mediation deviates from the prediction in General Gauge Mediation [40].

⁸The squark masses also receive a similar, but less significant, renormalization group effect from \mathcal{S} with the signs depending on their $U(1)$ hypercharges.

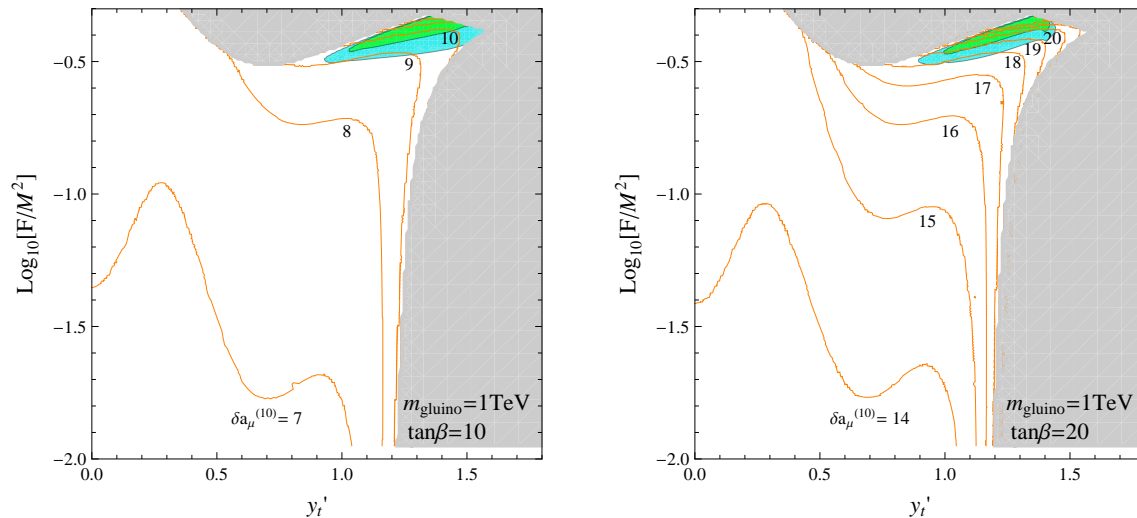


Figure 3: The contour plots of the supersymmetric contribution to the muon anomalous magnetic moment $\delta a_\mu^{\text{SUSY}} \times 10^{10}$ for $\tan \beta = 10$ (left) and $\tan \beta = 20$ (right). The green and light-blue shaded regions are same as in Fig. 1.

This figure shows that a muon $g - 2$ consistent with the experimental value at the 1σ level can be realized for $\tan \beta = 20$.

In Fig. 4, we show the $\tan \beta$ dependence of the lightest Higgs boson mass and muon $g - 2$ for $m_{\text{gluino}} = 1 \text{ TeV}$ and $y_t' = 1$. The bands on the Higgs boson mass show the uncertainties of the Higgs mass which were estimated by *FeynHiggs*. We also find that the 1σ error on the top quark mass $m_{\text{top}} = 173.2 \pm 0.9 \text{ GeV}$ lead to similar uncertainties in the lightest Higgs boson mass which we have not shown here. For $x < 0.035$, the lightest Higgs boson mass does not change if the other parameters are fixed (see also Fig. 1). This figure shows that the lightest Higgs boson mass is saturated for $\tan \beta \simeq 15 - 20$. The muon $g - 2$, on the other hand, is proportional to $\tan \beta$. Therefore, we find that $\tan \beta \simeq 20$ and $x \simeq 0.3$ is most advantageous for simultaneously explaining a relatively Heavy Higgs boson and the deviation in the muon $g - 2$.

In Fig. 5, we show the predicted value of the lightest Higgs boson mass and the muon $g - 2$ for $y_t' = 1$ (red) and $y_t' = 0$ (blue). We have fixed $\tan \beta = 20$ and $x = 0.35$. The oval regions correspond to $\chi < 1$ (green) and $\chi < 2$ (blue), respectively, where χ is defined by,

$$\chi = \left(\frac{(m_h - 125 \text{ GeV})^2}{\sigma_h^2} + \frac{(\delta a_\mu^{\text{SUSY}} - 26.1 \times 10^{-10})^2}{(8.0 \times 10^{-10})^2} \right)^{1/2}. \quad (25)$$

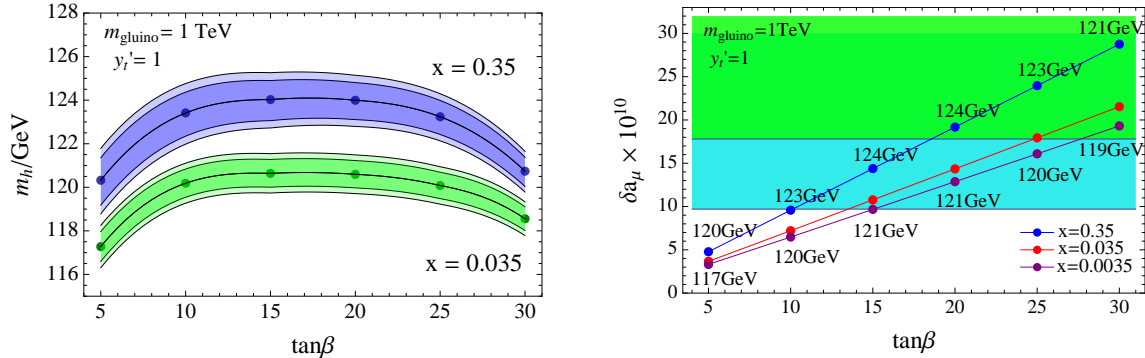


Figure 4: (Left) The $\tan\beta$ dependence of the lightest Higgs boson mass for $m_{\text{gluino}} = 1 \text{ TeV}$. (Right) The $\tan\beta$ dependence of the muon $g-2$ for $m_{\text{gluino}} = 1 \text{ TeV}$. The green shaded region corresponds to the muon $g-2$ consistent with the experimental value at the 1σ level. We also show the lightest Higgs boson mass for each parameter point. The lightest Higgs boson masses are degenerate for $x = 0.035$ and $x = 0.0035$.

Here, we have used $\sigma_h = 1 \text{ GeV}$ for illustrative purpose. This figure shows that the relatively heavy Higgs boson mass and a consistent muon $g-2$ at the 1σ level can be realized simultaneously for $m_{\text{gluino}} \simeq 1 \text{ TeV}$.

So far, we have considered a minimal messenger sector, i.e. $N_5 = 1$. For $N_5 > 1$, the gauginos become relatively heavier for the same squark/slepton masses. Thus, the muon $g-2$ can be explained for a relatively heavier gluino mass compared to the models with $N_5 = 1$. In the right panel of Fig. 5, we show the lightest Higgs boson mass and the muon $g-2$ for $N_5 = 2$. As expected, this figure shows that muon $g-2$ can be explained even for a heavier gluino mass. As has been mentioned above, the left handed sleptons are light in the regions where the lightest Higgs mass is enhanced. Thus, we find that the next lightest superparticles (NLSP) is a stau if $N_5 > 1$. As we will see in the next section, a stable stau NLSP scenario can be easily excluded by the LHC experiments in the near future if we require the muon $g-2$ be consistent with the experimental value at the 1σ level.

3.3 Perturbativity of \tilde{y}_U

In the above analysis, we have taken $y'_t = 1$ as a bench mark point to explain both the Higgs mass around 125 GeV and the observed muon $g-2$. Such a relatively large Yukawa

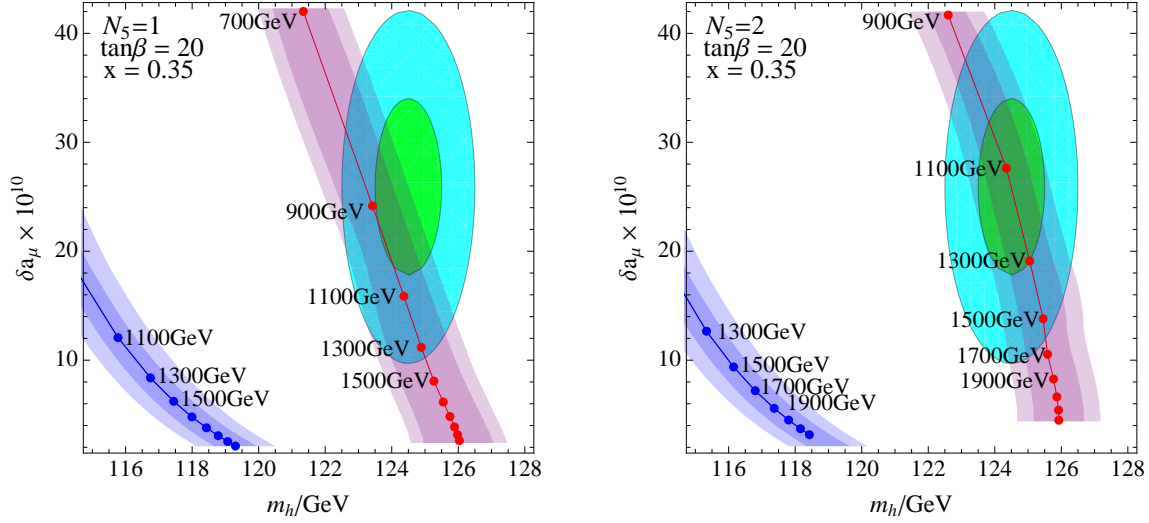


Figure 5: The correlation between the lightest Higgs boson mass and the muon $g - 2$ for $y'_t = 1$ (red) and $y'_t = 0$ (blue). Each band corresponds to uncertainties of the Higgs mass estimated by FeynHiggs. The definition of the oval region is given in the text. The gluino masses for a each point are also shown.

coupling constant, however, often has a Landau pole below the GUT scale, ruining one of the important motivation for the SSM. In this subsection, we discuss the constraints on the Yukawa coupling constant

Type-II models above the messenger scale are well described by the superpotential given in Eq. (3). Therefore, in the high energy theory we need to consider the perturbativity of \tilde{y}_t . This coupling is related to our low-scale parameters at the messenger scale through the expression

$$\tilde{y}_t = (y_t^2 + y_t'^2)^{1/2} \quad (26)$$

Therefore, the perturbativity constraint on \tilde{y}_t is more stringent than the usual constraint on y_t in the MSSM, since \tilde{y}_t is larger than y_t at the messenger scale.

Assuming $g, g' \ll O(1)$, the renormalization group equation of \tilde{y}_t is identical to that of y_t which is given by,

$$\frac{d}{dt} \tilde{y}_t = \frac{\tilde{y}_t}{16\pi^2} \left(6\tilde{y}_t^2 + y_b^2 + y_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right), \quad (27)$$

and all other parameters appearing here are the standard couplings of the MSSM. In

Fig. 6, we show the scale M_* for which y'_t becomes non-perturbative, i.e. $y'_t(M_*) \simeq 4\pi$. Notice that the scale M_* is not sensitive to $\tan\beta$ or deviations in $m_{\text{gluino}} = O(1)$ TeV.

Fig. 6 shows y'_t needs to be $\lesssim 0.75$ for the theory to be perturbative up to the GUT scale. Thus, the bench mark point we have taken in the previous section is plagued by a Landau pole below the GUT scale. In the right panel of Fig. 6, we replot Fig. 5 but take $y'_t = 0.75$. The figure shows that even in this case, the muon $g - 2$ can be within 1σ of the experimental value for $m_h \simeq 123$ GeV and be within 2σ for $m_h \simeq 125$ GeV.

One simple way to make the theory perturbative up to the GUT scale for $\tilde{y}_t = 1$ is to make the MSSM gauge interactions more asymptotic non-free.⁹ The larger gauge coupling constants at the higher energy suppress \tilde{y}_t at the higher energy via the renormalization group equation in Eq. (27). In Fig. 6, we show the scale M_* in the presence of N_E extra matter multiplets each with a mass around 1 TeV.¹⁰ The figure shows that three pairs of $\mathbf{5} + \mathbf{5}^*$ (or a pair of $\mathbf{10} + \mathbf{10}^*$) is enough to make the bench mark point $y'_t = 1$ perturbative up to the GUT scale¹¹.

4 Constraints and Prospects at the LHC

In this section, we discuss the constraints and prospect of discovery for the present model at the LHC. At the LHC, the production cross section of SUSY particles is tightly linked to the first family squark masses and gaugino masses. As we have discussed, the stops are rather heavier in the bulk of the parameter region which realizes a rather heavy Higgs particle $m_h \simeq 125$ GeV (see Fig. 2). Thus, the LHC signature of our model is very similar to the usual minimal gauge mediation models for most cases.

As was shown above, $m_{\text{gluino}} \simeq 1$ TeV and $x = \mathcal{O}(0.1)$ are best suited to explain the Higgs mass and the deviations in the muon $g - 2$. Gauge mediation with this messenger scale can be tested at the LHC rather easily. The LHC signatures, however, are strongly dependent on the gravitino mass. As we discussed above, the gravitino mass tends to be

⁹Another simple way to ameliorate the Landau problem is to introduce additional $U(1)$ gauge symmetry. This choice would alter the beta function of \tilde{y}_t making it more asymptotically free.

¹⁰Here, $N_E = 1$ corresponds to an extra matter multiplet of $\mathbf{5} + \mathbf{5}^*$.

¹¹The bench mark point with $y'_t = 1$ and no extra matter multiplets ($N_E = 0$) has a Landau pole at around 10^{11} GeV. This may be regarded as an indication of a rather intriguing possibility that the Higgs and top quarks are composite states of some new strongly interacting theory at 10^{11} GeV [44, 45]

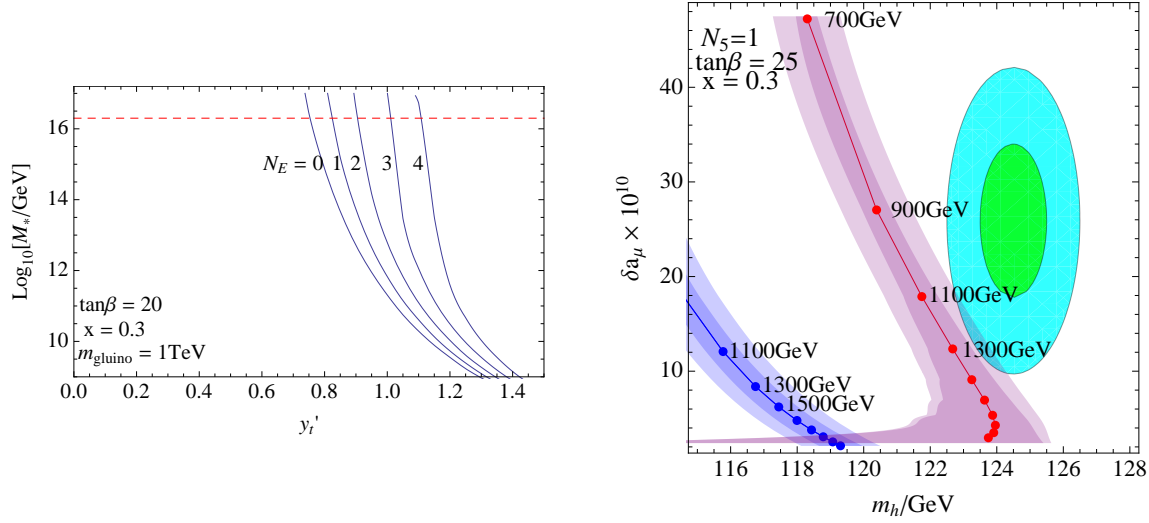
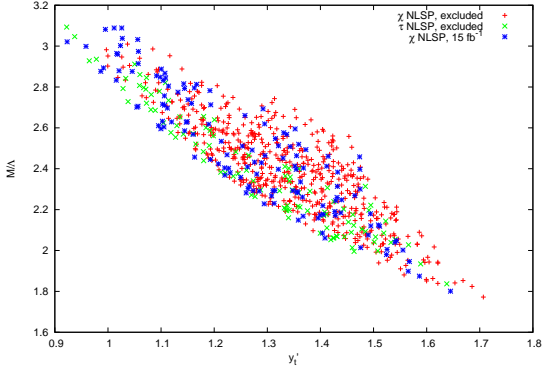


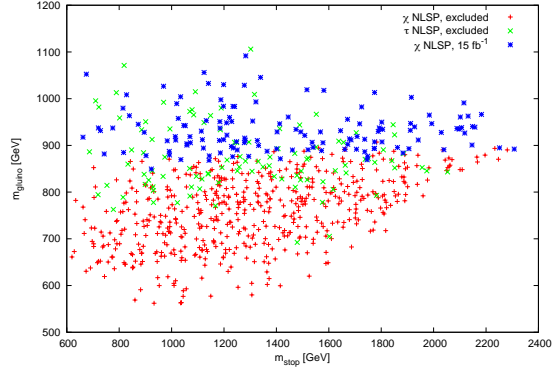
Figure 6: (Left) The scale M_* where \tilde{y}_t becomes non-perturbative for $N_E = 0 - 4$. For $N_E > 0$, we assumed that the mass of the extra vector-like multiplet is 1 TeV. The dashed line denotes the GUT scale about 2×10^{16} GeV. (Right) The correlation between the lightest Higgs boson mass and the muon $g - 2$ for $y_t' = 0.75$ (red) where the \tilde{y}_t is perturbative up to the GUT scale.

not very light in Type-II gauge mediation. Thus, we assume that the NLSP is stable inside the detectors of the LHC experiments. In the present model, the possible NLSP are the lightest neutralino ($\tilde{\chi}_1^0$) or the stau ($\tilde{\tau}_1$). In the former case, the key signature is high p_T jets plus large missing energy. This signature is a very similar signature to conventional gravity mediation models. In the latter case, the stau penetrates the detectors and heavy charged tracks would be observed.

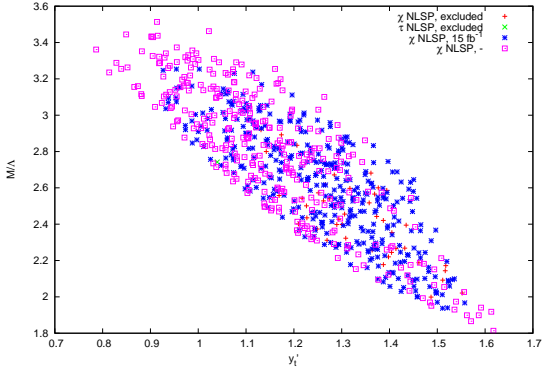
In Figs. 7, we show the results of a parameter scan over the inputs of the present model. We also require $m_h > 123.5$ GeV and a_μ within 1-2 σ of the experimental value. To estimate the LHC constraints, we have used the data of Refs.[41, 42, 43] and the program ISAJET 7.72 [46] to generate the MSSM mass spectrum and decay tables and Herwig 6.510 [47, 48, 49, 48] to generate SUSY events at the LHC. For the detector simulation, we have used AcerDet 1.0[50] which was slightly modified by the authors. We also show the prospects of excluding these models for higher integrated luminosity, 15 fb $^{-1}$. We have used the following cuts, 4-jets mode: at least four jets, with $p_T > 100$ GeV. $E_T^{\text{miss}} > 200$ GeV, $M_{\text{eff}} > 1200$ GeV and $E_T^{\text{miss}}/M_{\text{eff}} > 0.3$, or a 2-jets mode with the leading jet having $p_T > 300$ GeV, and the other jet having $p_T > 200$ GeV, and $E_T^{\text{miss}} > 400$ GeV,



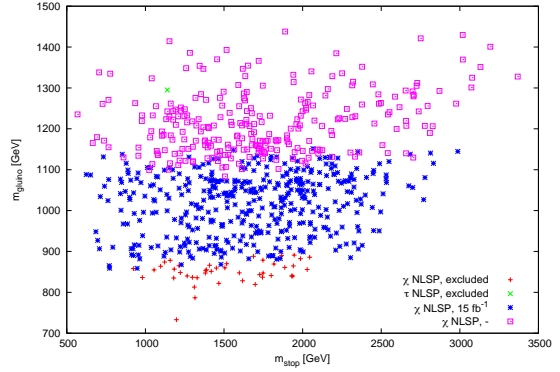
(a) y'_t vs $1/x$ for $|a_\mu - a_\mu^{\text{exp}}| < 1\sigma$.



(b) m_{stop} vs m_{gluino} for $|a_\mu - a_\mu^{\text{exp}}| < 1\sigma$.



(c) y'_t vs $1/x$ for $1\sigma < |a_\mu - a_\mu^{\text{exp}}| < 2\sigma$.



(d) m_{stop} vs m_{gluino} for $1\sigma < |a_\mu - a_\mu^{\text{exp}}| < 2\sigma$.

Figure 7: Scatter plots of the model parameters and the MSSM soft masses showing collider constraints and prospects for each point. Red crosses are the regions where the neutralino is the NLSP and has already been excluded at the LHC, green crosses have the stau NLSP and are also already excluded, blue stars have a neutralino NLSP and will be excluded by the 7 TeV run at the LHC, and purple squares are neutralino NLSP and cannot be excluded at LHC with a 7 TeV center of mass energy. The sup mass is well fitted by $m_{\text{sup}} \simeq 1.25 m_{\text{gluino}}$.

$M_{\text{eff}} > 1200 \text{ GeV}$ and $E_{\text{T}}^{\text{miss}}/M_{\text{eff}} > 0.35$. To estimate the Standard model background, we have used the programs MC@NLO 3.42 [51] (for $t\bar{t}, WW, WZ$ and ZZ), Alpgen 2.13 [52] (for Wj, Zj and $W/Z + b\bar{b}/t\bar{t}$).

As pointed out in Ref. [43], the long-lived stau mass is strongly constrained, $m_{\tilde{\tau}_1} \gtrsim 290 \text{ GeV}$. This constraint is inconsistent with a large $\delta a_{\mu}^{\text{SUSY}}$ for the points in the stau NLSP region. For larger values of N_5 , the stau tends to be the NLSP. This makes it difficult to find models that are consistent with both the muon $g - 2$ measurements and LHC collider phenomenally.

In the case of a neutralino NLSP, the LHC is less constraining and there are regions that are consistent with a large $\delta a_{\mu}^{\text{SUSY}}$. The present constraints on the gluino mass are $m_{\text{gluino}} \gtrsim 900 \text{ GeV}$. After the complete $\sqrt{s} = 7 \text{ TeV}$ LHC run, the constraints on the gluino mass could be pushed to $m_{\tilde{g}} \gtrsim 1100 \text{ GeV}$. Once the LHC is upgraded to $\sqrt{s} = 8 \text{ TeV}$, its reach could be extended to $m_{\tilde{g}} \sim 1200 \text{ GeV}$. Requiring a_{μ} within 1σ , we can see that almost all regions can be tested at the LHC with an integrated luminosity of 15 fb^{-1} . For $1\sigma < |a_{\mu} - a_{\mu}^{\text{exp}}| < 2\sigma$, the soft mass can be larger by about a factor of 1.4. Because of the larger superparticle masses, it is unlikely that the 7 TeV or 8 TeV LHC run will be capable of detecting the superparticles for this parameter space. However, the 14 TeV LHC can still easily exclude this region or possibly make a discover.

5 Conclusions and Discussions

In this paper, we have revisited the lightest Higgs boson mass in Type-II gauge mediation. We have shown that a Higgs boson mass around 125 GeV can be realized even for a gluino mass as light $m_{\text{gluino}} \sim 1 \text{ TeV}$. Interestingly, we have also found that the muon anomalous magnetic moment can be consistent with the experimental value, at the 1σ level, even for this relatively heavy Higgs boson. It was also shown that much of the parameter space can be checked at the LHC experiments in the near future.

We emphasize again that the field content of this more generic gauge mediation is the same as minimal gauge mediation, and the only real difference is the newly added interaction Eq.(8). It is surprising that such a small extension of minimal gauge mediation can resolve many of its difficulties, i.e. a relatively heavy Higgs boson and the deviation

of the muon $g - 2$.

Finally, let us comment on possible dark matter candidates for these models. As we mentioned above, the gravitino is expected to be heavier. Thus, it can be the dark matter candidate if the reheat temperature is appropriately chosen [53]. Furthermore, a 1 GeV gravitino dark matter can be consistent with thermal leptogenesis [54].¹²

A Realization of Type-II Gauge Mediation

In this appendix, we give the symmetries which realize the Type-II gauge mediation. In Table 1, we show the R -symmetries and $U(1)$ symmetry, which is only by the positively charged spurion ϕ_+ . Under these charge assignments, the generic superpotential at the renormalizable level is given by,

$$W = Z\tilde{\Phi}\bar{\Phi} + \phi_+ Z\tilde{H}_u\bar{\Phi} + Z^3 + \tilde{\mu}\tilde{H}_u H_d + (\text{MSSM Yukawa interactions}) . \quad (28)$$

Here, we have omitted the coupling constants for convenience. The cubic term for Z is important for cascade supersymmetry breaking [20, 21], although, the Type-II mechanism can be applied to other models which have supersymmetry breaking spurions with both an A -term and F -term expectation value.

For example, the unwanted term

$$W = \tilde{\Phi} \mathbf{10} \mathbf{10} , \quad (29)$$

which could cause FCNC or rapid proton decay is forbidden by the holomorphic property of the superpotential, i.e. the SUSY zero mechanism. The other problematic term,

$$W = \bar{\Phi} \mathbf{10} \mathbf{5}^* , \quad (30)$$

is forbidden by the R -symmetry. Furthermore, the above symmetries also forbid unnecessary mass terms,

$$W = \tilde{\Phi}_{\bar{L}} H_d + \bar{\Phi}_{\bar{L}} H_u . \quad (31)$$

¹²The detailed analysis of the gravitino dark matter including the thermal history of the messenger/supersymmetry breaking sectors will be discussed elsewhere. We also mention that the gravitino dark matter scenario with mass lighter than 1 GeV can also be consistent with thermal leptogenesis if there is sufficient entropy production [26, 55].

Table 1: The charge assignments for the broken $U(1)$ symmetry are presented here. We have used $SU(5)$ GUT representations for the MSSM matter fields, i.e. $\mathbf{10} = (Q_L, \bar{U}_R, \bar{E}_R)$ and $\mathbf{5}^* = (\bar{D}_R, L_L)$. We also show the charge of the right-handed neutrinos \bar{N}_R , which is need for the see-saw mechanism [56].

	ϕ_+	\tilde{H}_u	H_d	$\mathbf{10}$	$\mathbf{5}^*$	\bar{N}_R	$\tilde{\Phi}$	$\bar{\Phi}$	Z
R	0	4/5	6/5	3/5	1/5	1	4/5	8/15	2/3
$U(1)$	+1	0	0	0	0	0	+1	-1	0

B Two-loop Contributions to Scalar Masses

Here, we show a few details of the calculation for the two-loop scalar masses. This calculation proceeds roughly the same as the case with $y'_t = 0$. The wave function renormalization is analytically continued into superspace so that it is a function of the spurion $X = M + \theta^2 F$,

$$\mathcal{L} = \int d^4\theta \mathcal{Z}(|X|) \Psi^\dagger \Psi. \quad (32)$$

The wave function renormalization can then be expanded in θ to give

$$\mathcal{Z}(|X|) = Z(M) + \left(\frac{\partial Z(M)}{\partial M} F \theta^2 + \frac{\partial Z(M)}{\partial M^\dagger} F^\dagger \bar{\theta}^2 \right) + \frac{\partial^2 Z(M)}{\partial M \partial M^\dagger} F F^\dagger \theta^4 \quad (33)$$

After the field rotation

$$\Psi \rightarrow Z^{-1/2} \left(1 - Z^{-1} \frac{\partial Z(M)}{\partial M} \right) \Psi', \quad (34)$$

we have the following Lagrangian

$$\mathcal{L} = \int d^4\theta (\mathcal{Z}(|M|) - \theta^4 m_\Psi^2) \Psi^\dagger \Psi, \quad (35)$$

where

$$m_\Psi^2 = \frac{1}{4} \left(\left(\frac{\partial Z}{\partial \ln M} \frac{1}{Z} \right)^2 - \frac{1}{Z} \frac{\partial^2 Z}{\partial^2 \ln M} \right) \frac{F F^\dagger}{M M^\dagger}, \quad (36)$$

and we have used

$$\frac{\partial f(|M|)}{\partial \ln M} = \frac{1}{2} \frac{\partial f(|M|)}{\partial \ln |M|}. \quad (37)$$

To this point, this calculation is the same as the standard calculation. For our model, however, this calculation is complicated by the one-loop kinetic mixing of the Higgs and messenger fields. These generated off-diagonal terms must be canceled by the wave-function renormalization. To simplify this calculation we initially break up our wave-function renormalization into two pieces: the one presented in Eq. (34) and the part that removes the kinetic mixing. To begin our discussion on the kinetic mixing, we show the generic form of the one-loop Kähler potential for the Higgs and messenger fields,

$$K = \begin{pmatrix} H_u^\dagger & \Phi^\dagger \end{pmatrix} \begin{pmatrix} 1 + \delta Z_{H_u} & \delta Z_{H\Phi} \\ \delta Z_{H_u\Phi} & 1 + \delta Z_\Phi \end{pmatrix} \begin{pmatrix} H_u \\ \Phi \end{pmatrix}, \quad (38)$$

where $\mathcal{Z}(|X|) = 1 + \delta Z_X$. This matrix can be put in a diagonal, but not canonical, form by the field redefinition

$$\begin{pmatrix} H'_u \\ \Phi' \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2}\delta Z_{H_u\Phi} \\ -\frac{1}{2}\delta Z_{H_u\Phi} & 1 \end{pmatrix} \begin{pmatrix} H_u \\ \Phi \end{pmatrix} \quad (39)$$

which gives

$$K = \begin{pmatrix} H_u^\dagger & \Phi^\dagger \end{pmatrix} \begin{pmatrix} 1 + \delta Z_{H_u} & 0 \\ 0 & 1 + \delta Z_\Phi \end{pmatrix} \begin{pmatrix} H_u \\ \Phi \end{pmatrix} \quad (40)$$

to leading order. With this form of the Kahler potential, we can easily apply the techniques discussed above to calculate the two-loop contribution to the scalar masses. However, the rotation in Eq. (39) regenerates Higgs messenger mixing in the superpotential

$$\bar{g}Z\Phi\bar{\Phi} \rightarrow \bar{g}Z\Phi\bar{\Phi} - \bar{g}\frac{1}{2}\delta Z_{H_u\Phi}ZH_u\bar{\Phi}. \quad (41)$$

If the field redefinition in Eq. (39) is chosen to remove the kinetic mixing, applying the expression in Eq. (36) is complicated. To return to a basis where these formulas can be simply applied, we make an additional unitary transformation of the Higgs and messenger superfields

$$\begin{pmatrix} \hat{H}_u \\ \hat{\Phi} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2}\delta Z_{H_u\Phi} \\ \frac{1}{2}\delta Z_{H_u\Phi} & 1 \end{pmatrix} \begin{pmatrix} H'_u \\ \Phi' \end{pmatrix} = \begin{pmatrix} 1 & -\delta Z_{H_u\Phi} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} H_u \\ \Phi \end{pmatrix}. \quad (42)$$

If this new rotation is used to diagonalizes Z , we also find Eq. (40). The advantage of this rotation is it does not regenerate the problematic operator in Eq. (41). If we now combine the wave-function renormalization of Eq. (34) with that of Eq. (42), we have

$$\mathcal{Z}_{tot}^{1/2} = \begin{pmatrix} Z_{H_u}^{-1/2} \left(1 - Z_{H_u}^{-1} \frac{\partial Z_{H_u}(M)}{\partial M} \right) & -\delta Z_{H_u \Phi} \\ 0 & Z_{\Phi}^{-1/2} \left(1 - Z_{\Phi}^{-1} \frac{\partial Z_{\Phi}(M)}{\partial M} \right) \end{pmatrix} \quad (43)$$

for the wave-function renormalization of the Higgs and messenger fields. Since we will only consider contributions from the third generation, the wave-function renormalization for the other fields will be of the form in Eq. (34) and will not have any mixing.

Now we outline our procedure for calculating the derivatives of the wave-function renormalization. First, the wave-function renormalization is formally solved for,

$$Z(M, \mu_R) = \int_{\ln M}^{\ln \mu_R} d \ln \mu \gamma_L(\mu_R, M) Z(\mu_R, M) + \int_{\ln \Lambda}^{\ln M} d \ln \mu \gamma_H(\mu_R) Z(\mu_R) . \quad (44)$$

This expression is then differentiated. The derivatives are simplified using the expressions for the beta functions and we find

$$m_{\Psi}^2 = \frac{1}{4} \left(\frac{\partial \gamma_{\Psi}^L}{\partial \lambda^a} \Delta \beta_{\lambda^a} - \frac{\partial \Delta \gamma_{\Psi}}{\partial \lambda^a} \beta_{\lambda^a}^H \right) \frac{|F|^2}{M^2} \quad (45)$$

where

$$\Delta \beta_{\lambda^a} = \beta_{\lambda^a}^H - \beta_{\lambda^a}^L , \quad \Delta \gamma_{\Psi} = \gamma_{\Psi}^H - \gamma_{\Psi}^L . \quad (46)$$

To find the sfermion masses in terms of our theories parameters, we need to determine the relevant beta functions and anomalous dimensions. The anomalous dimensions for the squarks, Higgs and messenger fields are

$$\begin{aligned} \gamma_{H_u}^H &= \frac{1}{32\pi^2} \left(6y_t^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right) , \\ \gamma_{H_d}^H &= \frac{1}{32\pi^2} \left(6y_b^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right) , \\ \gamma_{\Phi}^H &= \frac{1}{32\pi^2} \left(6y_t'^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right) , \end{aligned}$$

$$\begin{aligned}
\gamma_{Q_3}^H &= \frac{1}{32\pi^2} \left(2y_t'^2 + 2y_t^2 + 2y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{1}{15}g_1^2 \right), \\
\gamma_T^H &= \frac{1}{32\pi^2} \left(4y_t'^2 + 4y_t^2 - \frac{16}{3}g_3^2 - \frac{16}{15}g_1^2 \right), \\
\gamma_B^H &= \frac{1}{32\pi^2} \left(4y_b^2 - \frac{16}{3}g_3^2 - \frac{4}{15}g_1^2 \right), \\
\gamma_{H\Phi}^H &= \frac{6y_t y_t'}{32\pi^2}.
\end{aligned} \tag{47}$$

The low scale anomalous dimensions can be found by taking $y_t' \rightarrow 0$.

The high-scale beta functions are found by applying

$$\beta_\lambda = \lambda(\gamma_a + \gamma_b + \gamma_c) \tag{48}$$

where $\gamma_{(a,b,c)}$ are the anomalous dimensions of the fields for an interaction of the type $\lambda Q_a Q_b Q_c$. Using this expression, we find the following beta functions

$$\begin{aligned}
\beta_{y_t} &= \frac{y_t}{16\pi} \left(6y_t'^2 + 3y_t'^2 + y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right), \\
\beta_{y_b} &= \frac{y_b}{16\pi} \left(6y_b^2 + y_t'^2 + y_t^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right), \\
\beta_{y_t'} &= \frac{y_t'}{16\pi} \left(6y_t'^2 + 9y_t'^2 + y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right),
\end{aligned} \tag{49}$$

where again the the low scale beta functions can be found by taking $y_t' \rightarrow 0$. Combing the results of Eq. (45), Eq. (47), and Eq. (49) we find the masses in Eq. (15),

$$\begin{aligned}
\delta m_{Q_3}^2 &= \frac{y_t'^2}{128\pi^4} \left(3y_t'^2 + 3y_t^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{13}{30}g_1^2 \right) \frac{F^2}{M^2}, \\
\delta m_{\bar{T}}^2 &= \frac{y_t'^2}{128\pi^4} \left(6y_t'^2 + 6y_t^2 + y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right) \frac{F^2}{M^2}, \\
\delta m_{\bar{B}}^2 &= -\frac{y_b^2 y_t'^2}{128\pi^4} \frac{F^2}{M^2}, \\
\delta m_{H_u}^2 &= -9 \frac{y_t^2 y_t'^2}{256\pi^4} \frac{F^2}{M^2}, \\
\delta m_{H_d}^2 &= -3 \frac{y_b^2 y_t'^2}{256\pi^4} \frac{F^2}{M^2}.
\end{aligned}$$

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