A 1-D time-varying median filter for seismic random, spike-like noise elimination[a]

[a]Published in Geophysics, 74, V17-V24, (2009)

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ABSTRACT

Random noise in seismic data affects the signal-to-noise ratio, obscures details, and complicates identification of useful information. We present a new method for reducing random, spike-like noise in seismic data. The method is based on a 1-D stationary median filter (MF) – the *1-D time-varying median filter* (TVMF). We design a threshold value that controls the filter window according to characteristics of signal and random, spike-like noise. In view of the relationship between seismic data and the threshold value, we chose median filters with different time-varying filter windows to eliminate random, spike-like noise. When comparing our method with other common methods, e.g., the band-pass filter and stationary MF, we found that the TVMF strikes a balance between eliminating random noise and protecting useful information. To demonstrate the feasibility of our method in reducing seismic random, spike-like noise, we present results for one synthetic dataset. Results of applying the method to seismic land data from Texas demonstrate that the TVMF method is effective in practice.

INTRODUCTION

Random noise in prestack seismic data can come from various sources, such as wind motion, poorly planted geophones, or electrical noise, and some of this seismic random noise invariably exhibits spike-like characteristics. Although stacking can at least partly suppress random noise in prestack data, residual random noise after stacking will decrease the accuracy of final data interpretation. In recent years, several authors have developed effective methods of eliminating random noise. For example, Gülünay (2000) used the noncausal prediction filter for random-noise attenuation, Ristau and Moon (2001) compared several adaptive filters, which they applied in an attempt to reduce random noise in geophysical data. Karsli et al. (2006) applied complex-trace analysis to seismic data for random-noise suppression, recommending it for low-fold seismic data, and some transform methods were also used to eliminate seismic random noise, e.g., seislet transform (Fomel, 2006; Fomel and Liu, 2008), discrete cosine transform (Lu and Liu, 2007), and curvelet transform (Neelamani et al., 2008).

On the other hand, the median filter, a well-known method that can effectively suppress spike-like noise, refers to nonlinear signal processing. Bednar (1983) and

Duncan and Beresford (1995) found the method to be both simple and effective for seismic prospecting. More recently, new median filters have been proposed. Mi and Margrave (2000) incorporated median-filter noise reduction into standard Kirchhoff time migration. Zhang and Ulrych (2003) used a hyperbolic median filter to suppress multiples. Liu et al. (2006) advocated random-noise attenuation using the 2-D multistage median filter (MLM).

Because the median filter is a nonlinear filter, filter-window length needs to be adjusted before its characteristics can be changed. The stationary filter, on the other hand, maintains a fixed window length, retaining useful information and random noise at the same scale. An unsuitable filter-window choice would therefore end up in useful information being destroyed or noise remaining. Here we propose a time-varying median filter that adjusts to different filter-window lengths by threshold, making value judgments on useful information versus noise throughout the process. We show that the time-varying window is more powerful than the stationary window.

In this paper, a new nonlinear filter called the *time-varying median filter* (TVMF) is presented, which we designed by defining a threshold value. After adjustment of the filter window in the time domain, this filter eliminates random noise in seismic data. We compare TVMF with other common methods. We use numerical examples, along with synthetic and field data, to demonstrate the validity of the proposed method in practice.

THEORETICAL BASIS

In contrast to useful information, random, spike-like noise in seismic data is neither continuous nor correlative, and a 1-D stationary median filter, having a large filter-window length, can easily remove such noise. However, signal can be damaged by such a filter.

The 1-D TVMF is based on the 1-D stationary median filter. We propose to measure the local noise content of the data and to adjust the filter length adaptively. If a threshold value that judges random noise and estimates noise intensity can be chosen, a filter having a large filter-window length can eliminate random noise while processing useful information by using a small filter-window length. Such a filter can thus effectively eliminate random noise while maximizing preservation of a detailed structure of useful information.

1-D stationary median filter (MF)

The 2-D seismic record can be represented by the following data sequence:

$$x_{i,j} \qquad (i = 1, \cdots, m, \cdots, N_x; j = 1, \cdots, n, \cdots, N_t) \ , \tag{1}$$

where i is the spatial sample index, j is the temporal sample index, and N_x and N_t are the numbers of spatial and temporal samples. When filter-window length, C, of the stationary median filter is defined (normally C is odd), the result after filtering at the point on the m^{th} trace and the n^{th} sample can be found by

1. Setting the center point at the m^{th} trace and the n^{th} sample, and choosing C samples in the m^{th} trace,

2. Sorting the C samples from smaller to larger, and then

3. Picking the center value, after sequencing, as the output at the point on the m^{th} trace and the n^{th} sample.

Repetition of the process on all data achieves 1-D stationary median filtering of the seismic record. The 1-D stationary median filter can be expressed as $median[x_{i,j}]$.

Signal-to-noise ratio (SNR) estimation using the stack method

A simple definition of the SNR was introduced by Liu and Li (1997). Window D, a part of the seismic record, can be chosen for SNR analysis:

$$D = [x_{i,j}]_{M\times N} \qquad (0 < M \leq N_x, 0 < N \leq N_t) \ . \tag{2}$$

Further assumptions are: waveform, amplitude, and phase of seismic wavelet in window D keep stable in respect to distance "i"; noise is "zero mean" randomly distributed, along with survey-line direction being independent (decorrelated) of the signal, so that

$$x_{i,j} = s_j + n_{i,j} \tag{3}$$

$$\sum_{i=1}^{M} n_{i,j} = 0 \ , \tag{4}$$

where s_j is amplitude of signal, and $n_{i,j}$ is amplitude of noise. These assumptions generally imply a limitation to this method, but they can be satisfied if the local window is chosen in the stable signal region of the seismic section. So if the signal energy in the window is

$$E_S = M\sum_{j=1}^{N} s_j^2 = \frac{1}{M}\sum_{j=1}^{N}(\sum_{i=1}^{M} x_{i,j})^2 \ , \tag{5}$$

the noise energy can be calculated by

$$E_N = \sum_{j=1}^{N}\sum_{i=1}^{M} x_{i,j}^2 - E_S \ . \tag{6}$$

Finally, a decibel expression of the SNR is estimated as

$$SNR = \frac{E_S}{E_N} = 10\ \log_{10}\left(\frac{\sum_{j=1}^{N}(\sum_{i=1}^{M} x_{i,j})^2}{M\sum_{j=1}^{N}\sum_{i=1}^{M} x_{i,j}^2 - \sum_{j=1}^{N}(\sum_{i=1}^{M} x_{i,j})^2}\right) \ . \tag{7}$$

1-D time-varying median filter (TVMF)

Using the above definitions, a TVMF can be designed. We use the following three steps to determine its parameters:

1. Choose the reference median filter length.

At point $x_{m,n}$, where the filter-window length of the reference median filter is chosen as C, output can be expressed as

$$Y^C_{m,n} = median[x_{i,j}] \qquad (i = m; j = n - (C-1)/2, \cdots, n + (C-1)/2) \; , \tag{8}$$

where filter-window length C is a large odd number so that random noise could be eliminated as much as possible. C is determined by using the SNR estimation method, which will be discussed later.

2. Choose the threshold value.

Using the reference median filter with its large filter-window length, we processed the seismic data first to find $Y^C_{m,n}$. Then we applied the absolute mean value to calculate the threshold value, which is shown as

$$T = \frac{1}{N_x \times N_t} \sum_{i=1}^{N_x} \sum_{j=1}^{N_t} |Y^C_{i,j}| \; , \tag{9}$$

We can evaluate random-noise data versus useful signal data by using the threshold value. When $|Y^C_{i,j}| < T$, the point is judged to be random noise, whereas when $|Y^C_{i,j}| \geq T$, the point should be signal data. We can therefore use the threshold value as a judgment norm – data in which $|Y^C_{i,j}| \geq T$ should be processed by the median filter having windows smaller than C to protect the detailed signal structure. Data in which $|Y^C_{i,j}| < T$ should be processed by the median filter having windows larger than C to strengthen its ability to eliminate random noise.

3. Choose the time-varying filter windows.

Choices involving time-varying windows abound after the threshold value has been chosen. We can define four scales of windows. Detailed time-varying window length $C_{i,j}$ is defined as

$$C_{i,j} = \begin{cases} C + \alpha, & 0 < |Y^C_{i,j}| < T/2 \\ C + \beta, & T/2 < |Y^C_{i,j}| < T \\ C - \gamma, & T \leq |Y^C_{i,j}| < 2T \\ C - \delta, & |Y^C_{i,j}| \geq 2T \end{cases} \; , \tag{10}$$

where α, β, γ, and δ are constant even numbers, and $\alpha > \beta$ and $\delta > \gamma$. Specific values for these parameters will be discussed in the next section. Using the above definition, we distinguish between random noise and useful signal, such that we can process the seismic data using different filter scales.

SYNTHETIC DATA TESTS

To choose reference filter windows and time-varying filter windows, we analyzed the characteristics of our TVMF formulation using synthetic data. Referring to the velocity model (Table 1), note the synthetic common-shot record with a dominant frequency of 40 Hz shown in Figure 1a. A 5%-density white spike noise was then added to the model. Three different noise peaks (noise amplitude is half, one, and two times the maximum value of the reflections, and the corresponding noise intensity is 1/2, 1, and 2) were chosen to separately test the TVMF. Only noise with twice the maximum value of the reflected wave is displayed in Figure 1b. Energy attenuation has not been taken into account, nor has spherical spreading or AVO. Here, the signal has been drowned out by noise.

Stratum thickness (m)	500	1739	2069	
Stratum velocity (m/s)	2500	3162	4138	4500

Table 1: Velocity model

Definitions of signal energy (equation 5), noise energy (equation 6), and SNR (equation 7) were used to analyze characteristics of the reference median filter. We chose the stationary MF with different filter-window lengths of from 3 to 19 points, comparing various signal energies (Figure 2a), noise energies (Figure 2b), and the SNR's (Figure 2c) after filtering. To meet the assumptions of the SNR model, we chose five traces near the zero offset, where seismic events are approximately invariant across the five traces. Note that in the figures, curves of the three noise intensities (1/2, 1 and 2) appear to have similar tendencies. Both signal energy and noise energy decrease as filter-window length increases, but the SNR displays a wiggle shape. The energy levels of signals are stable after filter-window length reaches 11 points, but the energy levels of noise are stable after filter-window length reaches 15 points. And because the curves show a different rate of descent, the SNR has a different tendency. It reaches a peak at the 5-point filter window and decreases afterward because signal energy attenuates faster than noise energy. The SNR then reaches the minimum near 11 points, where the signal energy is stable, and next, the SNR improves again because the noise energy still decreases. Finally, the SNR reaches stability, however, the signal has been barely damaged.

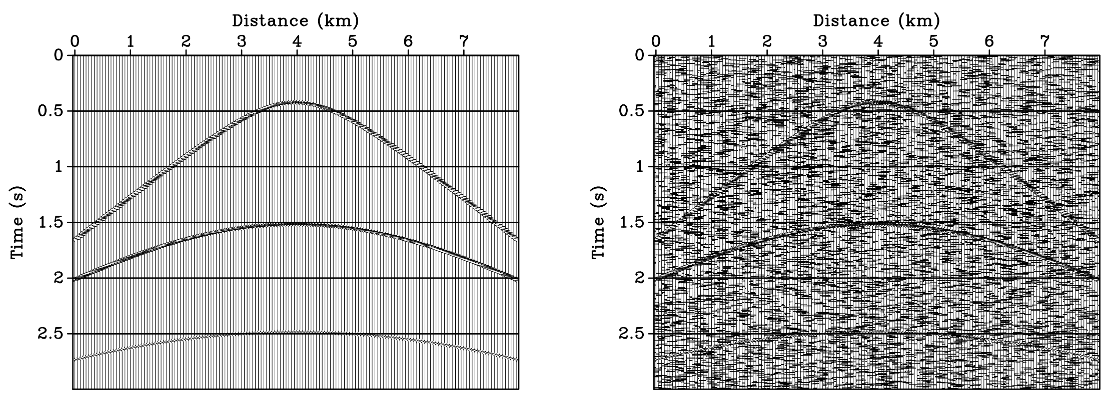
In the noisy model (Figure 1b), the SNR is -10.7 dB. After stationary MF filtering, even the minimum SNR remains much larger than -10.7 dB, illustrating that all stationary median filters can improve the SNR in the noisy model. We can select large filter windows of reference median filters to keep noise energy to a minimum when threshold T is well able to separate signal from noise, but at the same time we can define time-varying filter windows on the basis of reference filter windows so that the reference filter window will be limited. Three basic principles should be satisfied:

1. The reference filter window can be chosen as a large value to separate signal from noise when noise energy is small.

2. Time-varying filter windows at signal positions should be small values to protect the signal when removing noise.

3. Time-varying filter windows at noise positions should be large values to attenuate noise energy.

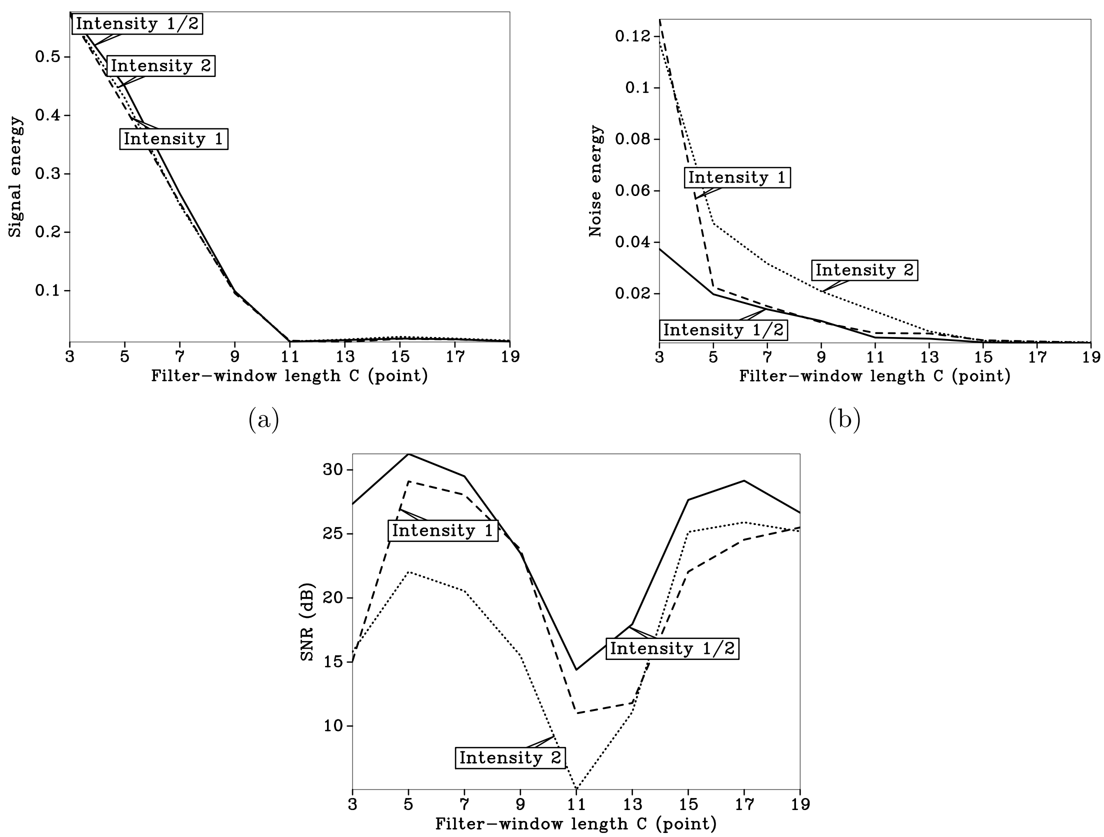
In Figure 2, reference filter window C should be larger than the stable signal point (11 points), and time-varying filter windows $C-\gamma$ and $C-\delta$ at signal positions should be in the range of from 5 to 7 points in order to preserve signal energy. Time-varying filter windows $C+\alpha$ and $C+\beta$ should be limited in range from 11 to 13 points in order to attenuate noise energy and save calculation time. To meet all principles, C can be 11 points, with $\alpha = 2$, $\beta = 0$, $\gamma = 4$, and $\delta = 6$. After more tests on synthetic and real data it became clear that these filter parameter choices work for most real data.



(a) (b)

Figure 1: Synthetic model (a) and white-noise model (b).

We used the TVMF, with defined reference and time-varying filter windows, to process the noise model (Figure 1b), the result of which is shown in Figure 3a. When assumptions can be met, the SNR estimation using the stack method can be used to compare results. After TVMF processing, the SNR is 19.7 dB. To compare, we also used the low-pass filter for preserving the signal in the dominant frequency band (Figure 3b, the SNR is -7.4 dB). After TVMF processing, white spike noise attenuated well, but the result after low-pass filtering became a signal with band-limited noise, and a great deal of low-frequency noise remained. Next, we also used the TVMF to process the band-limited noise model (Figure 3b). SNR analysis shows that the parameters of TVMF only change a little, so the same parameters can be used for processing. The TVMF cannot eliminate all band-limited noise, like white noise, but the energy of the noise has been degraded. At the same time, the TVMF introduces a few high-frequency noise components having low energy, but these can be easily removed by using a high-cut filter. Figure 4a shows the result after TVMF and high-cut filtering; the SNR is 0.5 dB. The low-pass filter does nothing about band-limited noise. The 11-point stationary MF (Figure 4b, the SNR is -8.4 dB) can be used

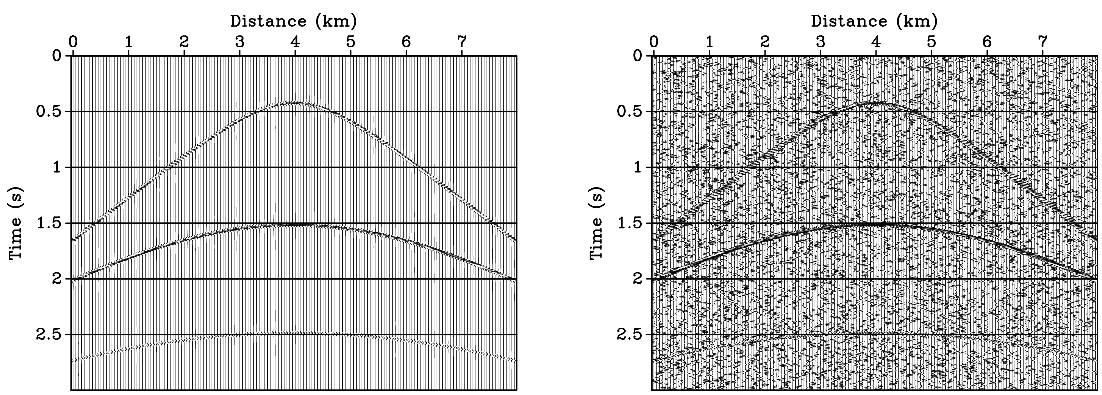


(c)

Figure 2: Comparison of different noise levels (Intensity (1/2, 1, and 2) is the amplitude ratio between noise and reflections). Signal energy (a), noise energy (b), and SNR (c).

to compare with the TVMF. The stationary MF can remove most of the noise, but useful information is also destroyed. After comparing the result of the TVMF with those of the stationary MF and the band-pass filter, we conclude that the TVMF is superior when processing random, spike-like noise.

We can compare results of using different methods by analyzing their spectra as well. We chose spectra of the trace at a distance of 4 km, corresponding to the pertinent parts of Figures 1, 3, and 4. Results in Figure 5 show that spectral values of random noise are larger than those of the signal at every frequency and that reflected waves have been masked by random noise (Figure 5b). The low-pass filter can eliminate noise in the high-frequency band, but it does nothing in the dominant-frequency band (Figure 5c). After the band-limited model is processed by the TVMF using an 11-point reference filter window, most spectral components in the dominant-frequency band can be recovered. An additional high-cut filter was used to remove high-frequency noise introduced by the TVMF (Figure 5d). The corresponding time profile is shown in Figure 4a.



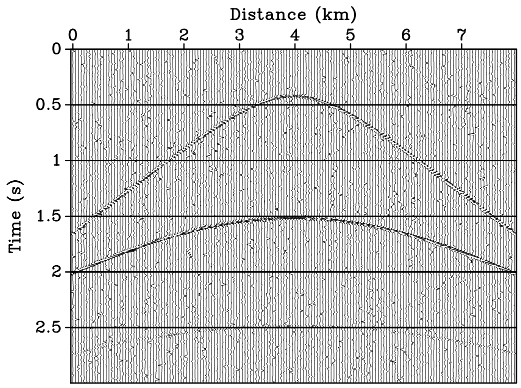
(a) (b)

Figure 3: Denoised result after 11-point TVMF filtering (a) and denoised result (band-limited noise model) after low-pass filtering (b).

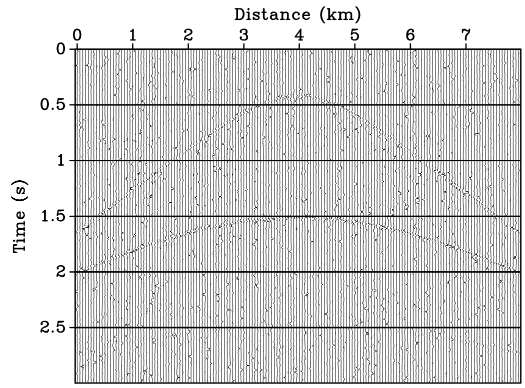
Given the result of model filtering, the TVMF can easily remove spike-like noise, especially noise having a white spectrum. When the TVMF is used to attenuate band-limited, spike-like noise, its filter ability decreases, although it can still work better than other common methods.

PROCESSING OF FIELD DATA

A real-data example involves land prestack data from Texas (Figure 6a), from which we show one common midpoint gather that has 59 traces and an average group interval of 42 m. The sample frequency is 4 ms, and the sample number is 500. In Figure 6a, noise is mainly random noise caused by the surface condition of this area; the 60-Hz electrical interference with high amplitude is especially serious. According

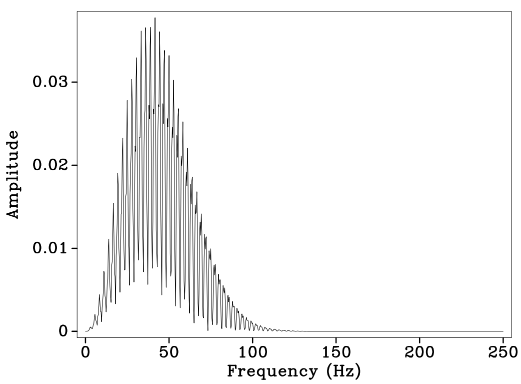


(a)

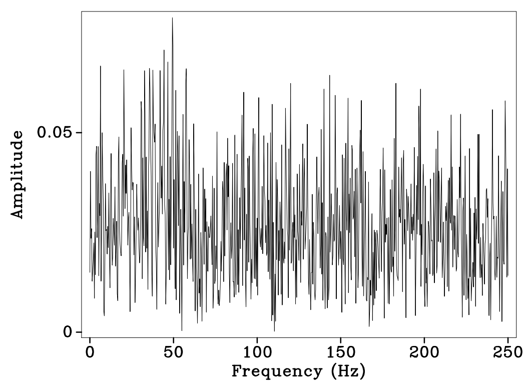


(b)

Figure 4: Denoised band-limited noise model using different methods (the result of Figure 3b was input to these tests); 11-point TVMF followed by a high-cut filter (a) and 11-point stationary MF (b).



(a)



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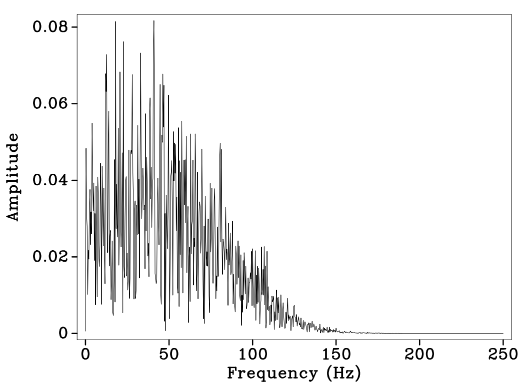
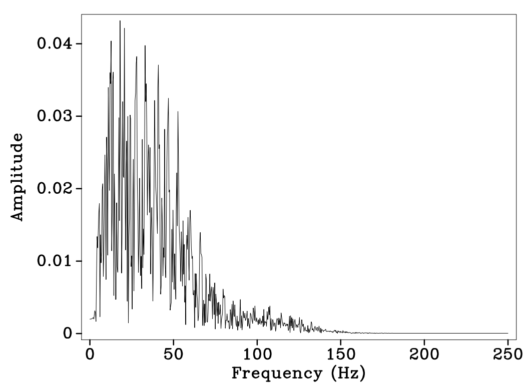


(c)



(d)

Figure 5: Comparison of amplitude spectra (trace at distance of 4 km) before and after processing. Amplitude spectra in Figure 1a (a), corresponding spectra in Figure 1b (b), corresponding spectra in Figure 3b (c), and corresponding spectra in Figure 4a (d).

to the amplitude spectra, the dominant frequency of the real data is about 25 Hz; the 60-Hz electrical interference thus exhibits high-frequency, spike-like characteristics when compared with the useful signal. In the time domain (Figure 6a, especially at locations “A,” “B,” and “C”), this spike-like characteristic of electrical interference can also be observed. To meet the assumptions of SNR estimation using the stack method, a rectangle region “D” that has 9 traces and 125 samples is chosen, and, after calculating, the SNR in the original data is found to be -7.3 dB. First, we apply a low-pass filter to remove the random noise (Figure 6b); the frequency is limited to 60 Hz. The low-pass filter can partly eliminate random noise, and the corresponding SNR is improved to 0.4 dB, but it cannot attenuate the noise in the dominant-frequency band. We can thus still see lots of random noise after filtering. The 11-point stationary MF is also used to compare with the TVMF, and the result is shown in Figure 6c. After processing, there is still some spike-like noise left, and part of the signal has been attenuated. The SNR is 1.9 dB, and a larger filter window will further destroy the useful signal. By using the SNR analysis introduced in the theory section, we are choosing the parameters about the TVMF in window “D.” The TVMF with the same parameters as for the synthetic examples, $C = 11$, $\alpha = 2$, $\beta = 0$, $\gamma = 4$, and $\delta = 6$, is applied to the prestack data (Figure 6d). Figure 6e shows the difference section, in which the processed data using the TVMF have been subtracted from the original data. After TVMF processing, continuity of events and information of the reflected waves are all enhanced, and there is little noise left. Note that the 60-Hz electrical interference in particular can be completely removed. The SNR reaches 4.7 dB. In the difference section it can be seen that the TVMF eliminates major spike-like noise and that there is little useful information beyond the spike-like noise, showing that the TVMF can effectively eliminate spike-like noise and protect useful information at the same time. Because the TVMF is based on the stationary MF that can filter noise spikes, the TVMF can effectively eliminate spike-like noise. And because the TVMF can separate useful information from spike-like noise, it can protect useful information and perform better overall than other potential methods in spike-like noise attenuation.

We chose a trace from the original data and corresponding traces in processed data and got the amplitude spectra of four traces (Figure 7). After low-pass filtering, high-frequency noise can be removed thoroughly, but some noise in the dominant-frequency band can remain; electrical interference in particular must be eliminated by other methods (Figure 7b). Stationary MF can eliminate enough noise, but spectra of useful signals have been distorted (Figure 7c). In Figure 7d, the TVMF can attenuate noise in the entire frequency band and protect the useful-information frequency components.

Next we obtained stacked sections corresponding to Figure 6b and 6d; the stacking fold is 59. A rectangular region, “H,” was chosen to calculate the local SNR. Events of the stacked section using low-pass processing were discontinuous (Figure 8a), the SNR is -7.8 dB, and the continuity of events in the stacked section using TVMF processing was improved, especially at locations “E,” “F,” and “G” (Figure 8b), and the corresponding SNR is -3.1 dB. Because the TVMF can eliminate spike-like noise

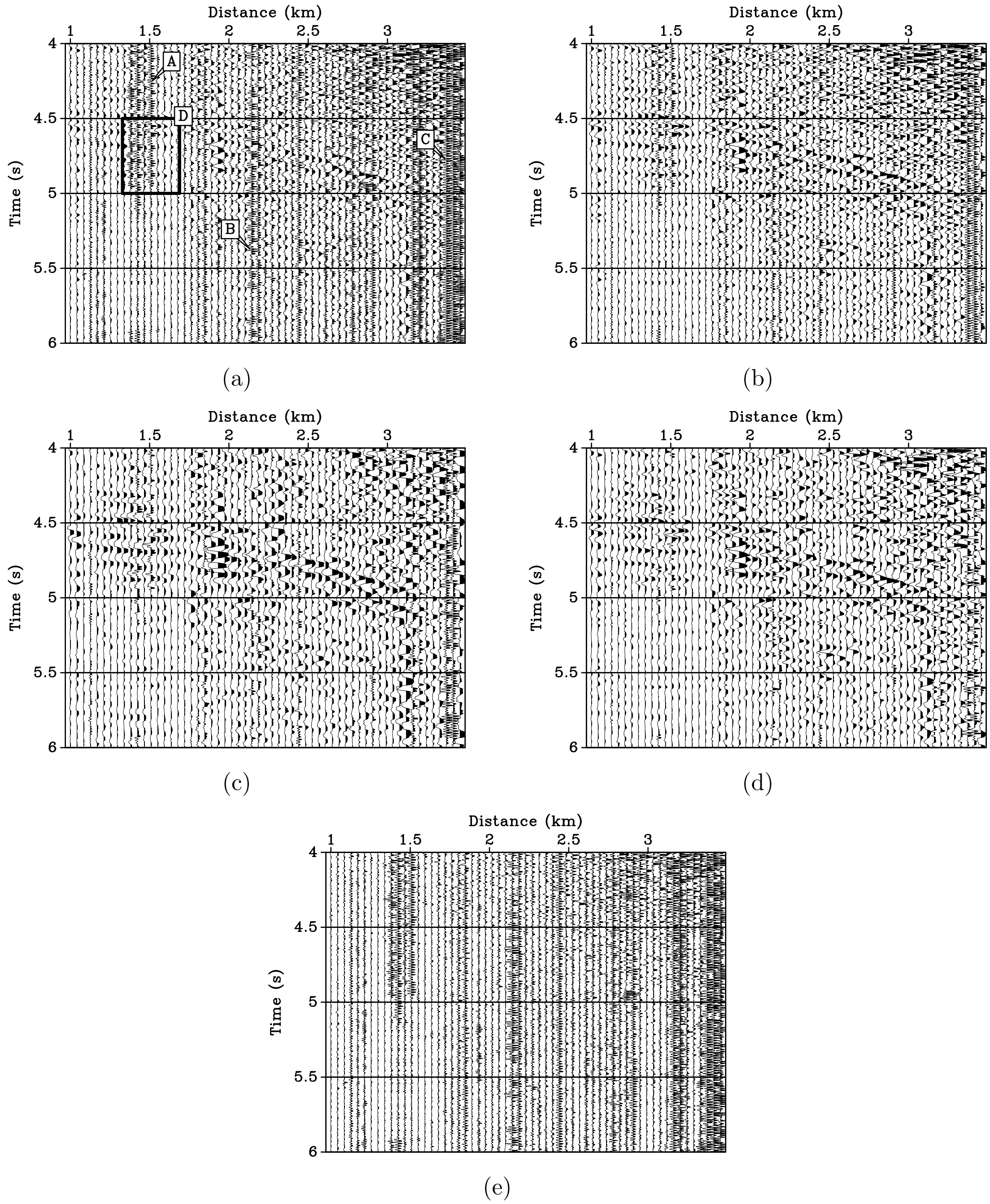


Figure 6: Real land data (a), denoised result after low-pass filtering (b), denoised result after stationary MF filter (c), denoised result after TVMF filtering (d), and difference between original data and result after TVMF processing (e).

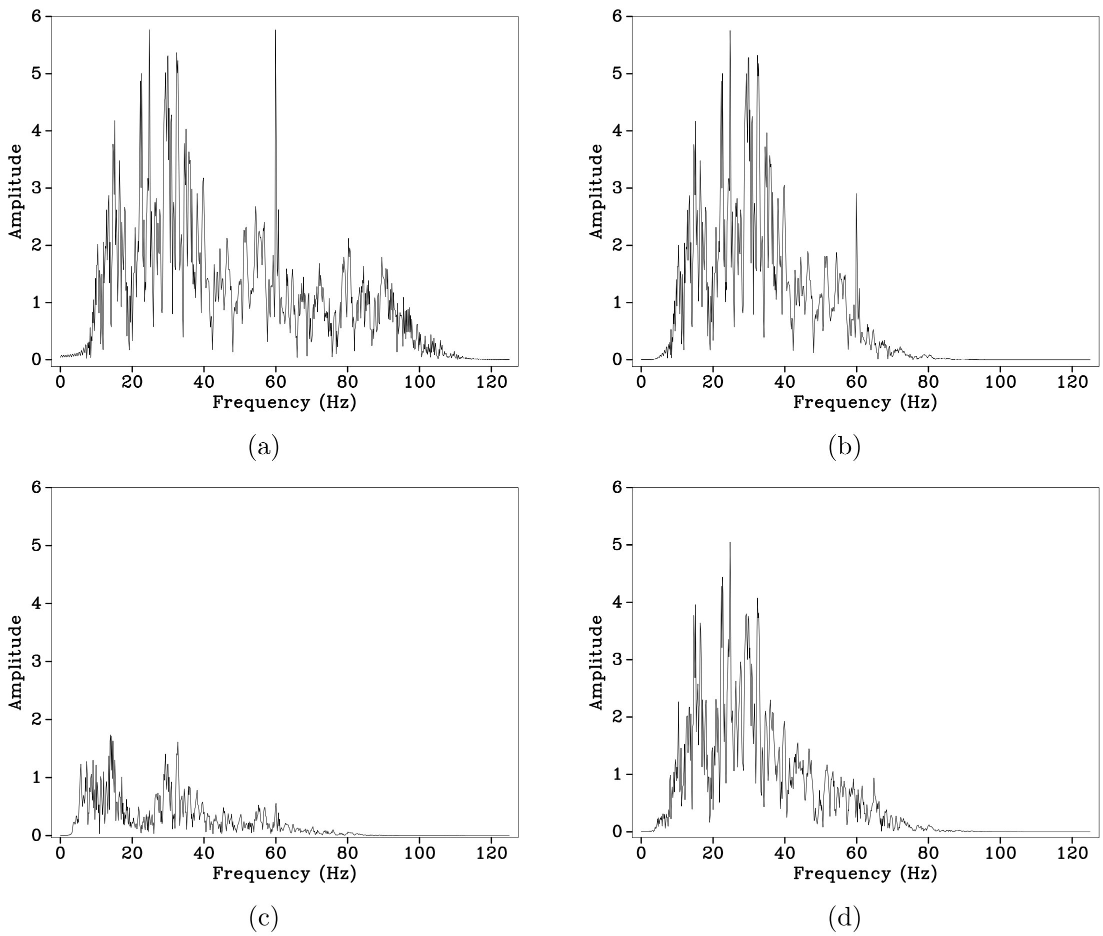


Figure 7: Comparison of amplitude spectra (trace at distance of 1.8 km) before and after processing. Amplitude spectra in Figure 6a (a), corresponding spectra in Figure 6b (b), corresponding spectra in Figure 6c (c), and corresponding spectra in Figure 6d (d).

in prestack data, it is advantageous to apply to such data.

CONCLUSION

We have proposed and tested the time-varying median filter (TVMF) for attenuating prestack random, spike-like noise. We used a signal-to-noise ratio estimation to design the reference filter window and time-varying filter windows, in which the filter can adapt filter-window length to meet the intensity of the random, spike-like noise. The ability of this method to eliminate random noise while protecting desired signal further attests to the strength of the method. Our experiments show that in field data the TVMF can eliminate random noise enough to enhance the continuity of events. Spectral analysis also shows that the TVMF can effectively suppress random, spike-like noise in the whole frequency band. Comparison of different methods shows that the TVMF is more effective than the stationary median filter for eliminating spike-like noise, and it can enhance results of band-pass filtering by attenuating spike-like noise within the pass band.

ACKNOWLEDGMENTS

The authors gratefully acknowledge Sergey B. Fomel for his suggestions and help. We thank Douglas W. McCowan for fruitful discussions. We sincerely thank assistant editor Vladimir Grechka, associate editor Eric Verschuur, and three anonymous reviewers for their constructive comments and suggestions. We also thank one anonymous reviewer for his suggestion about the possibility of our method on marine swell-noise attenuation. Thanks also to BGP International for financial support of this work. This work is partly supported by The National Basic Research Program of China (973 Program: 2009CB219301). Publication authorized by the Director, Bureau of Economic Geology.

REFERENCES

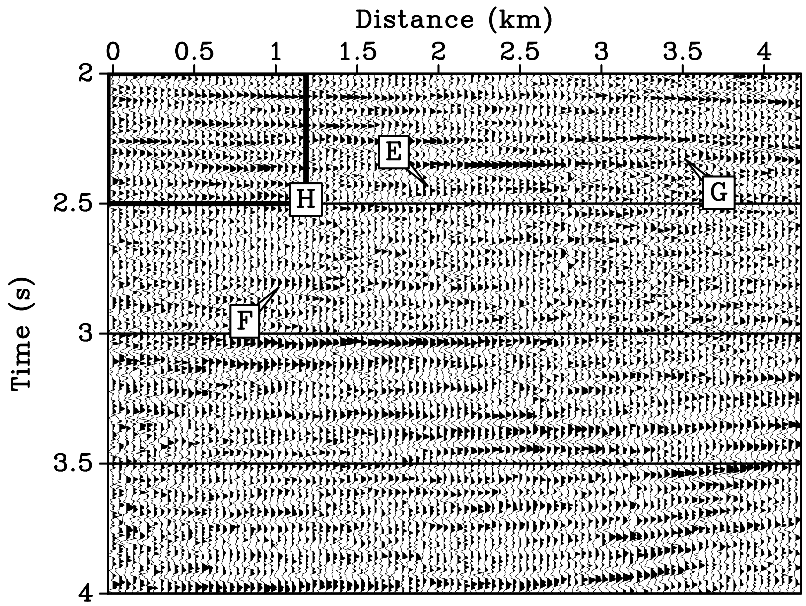
Bednar, J. B., 1983, Applications of median filtering to deconvolution, pulse estimation, and statistical editing of seismic data: Geophysics, **48**, 1598–1610.

Duncan, G., and G. Beresford, 1995, Some analyses of 2-D median f-k filters: Geophysics, **60**, 1157–1168.

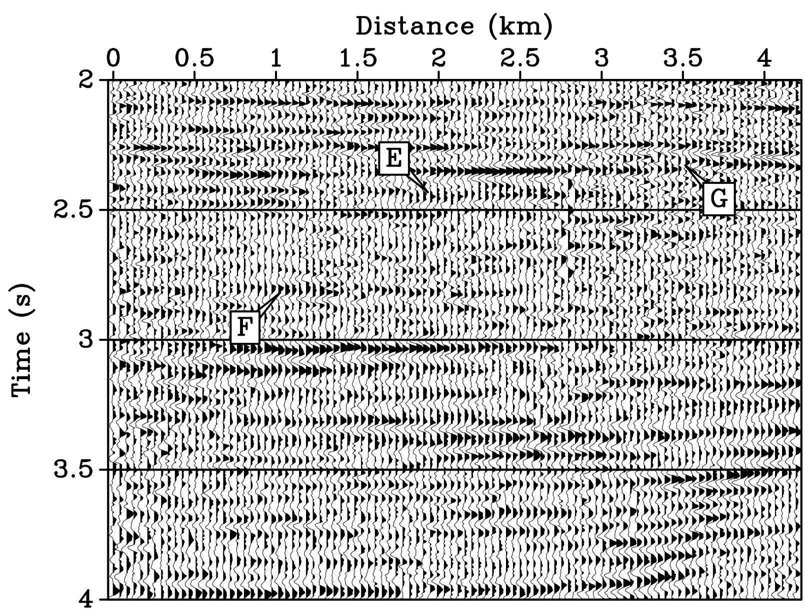
Fomel, S., 2006, Towards the seislet transform: 76th Annual International Meeting, SEG, Expanded Abstracts, 2847–2851.

Fomel, S., and Y. Liu, 2008, Seismic data analysis with one-dimensional seislet frame: 78th Annual International Meeting, SEG, Expanded Abstracts, 2581–2585.

Gülünay, N., 2000, Noncausal spatial prediction filtering for random noise reduction on 3-D poststack data: Geophysics, **65**, 1641–1653.



(a)



(b)

Figure 8: Stacked section of prestack data after low-pass filtering (a) and corresponding section after TVMF processing (b).

Karsli, H., D. Dondurur, and G. Çifçi, 2006, Application of complex-trace analysis to seismic data for random-noise suppression and temporal resolution improvement: Geophysics, **71**, V79–V86.

Liu, C., Y. Liu, B. Yang, D. Wang, and J. Sun, 2006, A 2D multistage median filter to reduce random seismic noise: Geophysics, **71**, V105–V110.

Liu, Y., and C. Li, 1997, Some methods for estimating the signal/noise ratio of seismic data: Oil Geophysical Prospecting, **32**, 257–262 (In Chinese with English abstract).

Lu, W., and J. Liu, 2007, Random noise suppression based on discrete cosine transform: 77th Annual International Meeting, SEG, Expanded Abstracts, 2668–2672.

Mi, Y., and G. F. Margrave, 2000, Median filtering in Kirchhoff migration for noisy data: 70th Annual International Meeting, SEG, Expanded Abstracts, 822–825.

Neelamani, R., A. I. Baumstein, D. G. Gillard, M. T. Hadidi, and W. I. Soroka, 2008, Coherent and random noise attenuation using the curvelet transform: The Leading Edge, **27**, 240–248.

Ristau, J. P., and W. M. Moon, 2001, Adaptive filtering of random noise in 2-D geophysical data: Geophysics, **66**, 342–349.

Zhang, R., and T. J. Ulrych, 2003, Multiple suppression based on the migration operator and a hyperbolic median filter: 73th Annual International Meeting, SEG, Expanded Abstracts, 1949–1952.