# A 5-D Localization Method for a Magnetically Manipulated Untethered Robot using a 2-D Array of Hall-effect Sensors 

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#### Abstract

This paper introduces a new five-dimensional localization method for an untethered meso-scale magnetic robot, which is manipulated by a computer-controlled electromagnetic system. The developed magnetic localization setup is a two-dimensional array of mono-axial Hall-effect sensors, which measure the perpendicular magnetic fields at their given positions. We introduce two steps for localizing a magnetic robot more accurately. First, the dipole modeled magnetic field of the electromagnet is subtracted from the measured data in order to determine the robot's magnetic field. Secondly, the subtracted magnetic field is twice differentiated in the perpendicular direction of the array, so that the effect of the electromagnetic field in the localization process is minimized. Five variables regarding the position and orientation of the robot are determined by minimizing the error between the measured magnetic field and the modeled magnetic field in an optimization method. The resulting position error is $2.1 \pm 0.8 \mathrm{~mm}$ and angular error is $6.7 \pm 4.3^{\circ}$ within the applicable range ( 5 cm ) of magnetic field sensors at 200 Hz . The proposed localization method would be used for the position feedback control of untethered magnetic devices or robots for medical applications in the future.


## Index Terms

Capsule endoscopy; localization; magnetic robot; magnetic actuation

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## I. Introduction

Magnetically actuated capsule endoscopes (MACEs) provide a promising medical technology for minimally invasive diagnosis on gastrointestinal organs [1]-[7]. Hong et al. showed the feasibility of a MACE in a pig's esophagus, stomach and large intestine using a multi degrees of freedom (DOF) robotic manipulator [8]. Recently, Carpi et al. conducted animal experiments using a commercial permanent magnet-based actuation system (Niobe, Stereotaxis, Inc, USA), which is mainly used for the magnetic navigation of cardiovascular active catheters [9]. In our previous study, we proposed a new multi-functional endoscopic capsule robot and an original magnetic manipulation method [10]-[13]. Recently, Petrusuka et al. introduced an electromagnet system constructed for a MACE with direct and rapid magnetic field control without moving any parts of the setup [14].

Magnetic manipulation of MACEs assumes that their positions and orientations are wellestimated in real-time. If magnetic interactions such as magnetic force and torque are not estimated accurately, the motion of the MACE is not controlled as desired. Poor motion control of the MACE results in an imperfect stomach diagnosis.

One general localization method for magnetic capsules is to detect the magnetic field from a small permanent magnet inside the capsule using an external magnetic sensor array [15][18]. However, these methods are not applicable to MACEs because the strong magnetic field from the magnetic actuation system interferes with sensor array(s), which results in decreased accuracy or failure. Recently, Hashi et al. proposed the idea of superimposing high frequency alternating magnetic field on a low frequency manipulating magnetic field. This magnetic localization method is compatible with the external magnetic field, and shows sub-millimeter position accuracy. However, it is limited to three-dimensional (3-D) localization, and cannot determine the capsule's orientation [19]-[21].

A different strategy for localization is to use the onboard magnetic sensor(s) to calculate relative position and orientation to the external magnetic field source. By using onboard sensors, the magnetic field from the MACE's magnet is considered as a DC offset, which can be easily calibrated. Kim et al. proposed a localization method utilizing a rotating external magnetic field with onboard magnetic sensors, which gave 15 mm position error and $15^{\circ}$ orientation error [22]. Similarly, Popek et al. utilized a rotating magnetic field with 11 mm position error and $11^{\circ}$ orientation error by using onboard magnetic sensors [23]. However, these methods are only applicable for a rotating external magnetic field that limits locomotion of the MACE to only rotation. Natali et al. introduced a localization method that compares the measured sensory data with pre-calculated data of the external magnetic field. The method requires multiple magnetic sensors and an inertial sensor inside the system. Their method gave $3.4 \pm 3.2 \mathrm{~mm}$ position error and $19 \pm 50^{\circ}$ angular error within a 15 cm radius workspace [24]. However, the angular accuracy is not sufficient for a disease diagnosis.

Previously, we proposed a 3-D localization method using an internal magnetic sensor [25], which consists of three steps: the coaxial alignment stage between the MACE and the external magnet, the MACE deformation stage, and the MACE shape recovery stage. The
proposed method showed 2.1 mm resulting position error in the experiment. However, the coaxial alignment stage required careful adjusting of the external magnet. Even a small direction error could cause a large localization error as the MACE moves farther from the external magnet. Furthermore, this method required specific motions of the external magnet, which could not enable continuous real-time MACE localization. For a continuous real-time localization, a different working principle is required.

Even though most of the methods utilize the on-board magnetic sensor system, employing an external sensor system has considerable benefits. If an external sensor system is used, the number of electrical components inside the MACE is reduced. Thus, its volume and energy consumption is minimized. Furthermore, the external sensor system allows us to utilize the abundant amount of sensors without much spatial and energy restriction, which leads to better accuracy with the increased number of the sensors than the onboard sensory system.

This paper introduces a new real-time 5-D localization method for a MACE using an external Hall-effect sensor array and an external omnidirectional electromagnet [14]. The key point of the developed 5-D localization method is to separate the MACE's magnetic field from the actuator's magnetic field. By subtracting the electromagnet's field from the measured data, we can obtain the pure magnetic field of the MACE within the coupled magnetic fields. Additionally, the error is reduced with the second order directional differentiation by taking the advantage of the Laplacian of the magnetic field. Note that a low pass filter was applied to the magnetic field before the differentiation to prevent a significant noise increase. The proposed method is compatible with any magnetic capsule robots or magnetic microrobots, which are actuated by an external magnetic field.

This paper is organized as follows. Section II introduces the localization setup, the working principle, and the algorithms. In Section III, the proposed method is verified in experiments. Section IV discusses the effect of the inherent sensor noise on the accuracy of the method.

## II. 5-D Localization Method

## A. Setup

Figure 1(a) shows the experimental setup that consists of four main parts. The first part is the 2-D Hall-effect sensor array board. Sixty-four Hall-effect sensors on the board measure the magnetic field in the direction perpendicular to the array ( $z$-direction in Fig. 1(b)) at their positions. Increasing the number of sensors improves the accuracy of the localization results. In our setup, however, the number of analog input channels (8) of the data-acquisition (DAQ) board and the multiplexers ( 3 bit) limited the number of sensors that could be used. The second part is the omnidirectional electromagnet made of three box-shaped orthogonal coils and a soft iron core [14]. The third part is the multiplexer board connecting the Halleffect sensor outputs with the computer. Eight multiplexers on the board distribute the sensor signals to the DAQ board. The last part is the desktop computer with the DAQ board. The main algorithm and graphical user interface are implemented in Labview (National Instruments co.) with an operation frequency of 200 Hz ; the sampling rates of the data acquisition loop and the optimization algorithm loop are 1 kHz and 200 Hz , respectively. The specifications of the localization setup are presented in Table I.

## B. Working Principle

Figure 2 shows the application scenario of the developed 5-D localization method. Each dimension can be localized except the rotation axis of a magnetic moment of the robot. The goal is to estimate the position and orientation of the MACE while it is manipulated by the external magnet. We propose following two steps to decouple the effect of the external magnet at a point of interest (sensor position): 1) subtraction of a modeled magnetic field of the external magnet from measured data, and 2) second order directional differentiation to reduce the B -field error.

The magnetic sensors experience magnetic fields both from the MACE and the external magnet. Those magnetic fields are expressed as B-fields in (1):

$$
\begin{equation*}
B_{s}=B_{c}+B_{e} \tag{1}
\end{equation*}
$$

where $\mathbf{B}_{\mathrm{s}}$ is the measured B-field at a sensor, $\mathbf{B}_{\mathrm{c}}$ is the B-field of the MACE, and $\mathbf{B}_{\mathrm{e}}$ is the Bfield of the external magnet. A simple way to estimate a pure $\mathbf{B}_{c}$ is to subtract $\mathbf{B}_{\mathrm{e}}$ from $\mathbf{B}_{\mathrm{s}}$. To subtract, we should model $\mathbf{B}_{\mathrm{e}}$, and the general way to model a magnetic field is to use the magnetic dipole equation in a coordinate-free form,

$$
\begin{equation*}
\mathbf{B}_{\mathrm{dpl}}(\mathbf{r}, \mathbf{m})=\frac{\mu_{0}}{4 \pi\|\mathbf{r}\|^{3}}\left(3 \hat{\mathbf{r}} \hat{\mathbf{r}}^{\mathrm{T}}-\mathbf{I}\right) \mathbf{m} \tag{2}
\end{equation*}
$$

where $\mu_{0}$ is the permeability of free space, $\mathbf{r}$ is the position vector (with associated unit vector $\mathbf{r})$ from the magnetic source to the point of interest, $\mathbf{m}$ is the magnetic moment vector of the magnetic source, and $\mathbf{I}$ is a $3 \times 3$ identity matrix. Thus, $\mathbf{B}_{\mathrm{e}}$ can be expressed using (2) as

$$
\begin{equation*}
\mathbf{B}_{\mathrm{e}}=\mathbf{B}_{\mathrm{dpl}}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right) \tag{3}
\end{equation*}
$$

where $\mathbf{r}_{\mathrm{e}}$ is the position vector from the external magnet to the sensor position and $\mathbf{m}_{\mathrm{e}}$ is the magnetic moment vector of the external magnet. However, in actuality, we cannot measure the exact $\mathbf{r}_{\mathrm{e}}$ and $\mathbf{m}_{\mathrm{e}}$, which results in the B-field error. Also the real magnetic field includes multi-pole magnetic fields, which are not modeled in (2). Thus, (1) can be rewritten as

$$
\begin{equation*}
\mathbf{B}_{\mathrm{c}}=\mathbf{B}_{\mathrm{s}}-\left[\mathbf{B}_{\mathrm{dpl}}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)+\mathbf{B}_{\mathrm{err}}\right] . \tag{4}
\end{equation*}
$$

Here the B-field error, $\mathbf{B}_{\text {err }}$, is specified as

$$
\begin{gather*}
\mathbf{B}_{\mathrm{err}}=\mathbf{B}_{\mathrm{s}}-\mathbf{B}_{\mathrm{c}}-\mathbf{B}_{\mathrm{dpl}}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right) \\
=\mathbf{B}_{\mathrm{dpl}}\left(\mathbf{r}_{\mathrm{c}}+\mathbf{r}_{\mathrm{crr}}, \mathbf{m}_{\mathrm{c}}+\mathbf{m}_{\mathrm{crr}}\right)+\mathbf{B}_{\mathrm{quad}}+\mathbf{B}_{\mathrm{hexa}}+\ldots-\mathbf{B}_{\mathrm{dpl}}\left(\mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right) \tag{5}
\end{gather*}
$$

where $\mathbf{r}_{\mathrm{err}}$ and $\mathbf{m}_{\text {err }}$ are a positioning error and a magnetic moment measurement error of the external magnet, respectively. In (5), those two dipole terms scale with $\left\|\mathbf{r}_{\mathrm{e}}\right\|^{-3}$ as described in (2), and the quadrupole and hexapole terms scale with $\left\|\mathbf{r}_{\mathrm{e}}\right\|^{-5}$ and $\left\|\mathbf{r e}_{\mathrm{e}}\right\|^{-7}$, respectively [26].

Because the error terms in (5) are inversely proportional to the distance, they reduce as the external magnet moves farther from the sensor array. Conversely, the B-field from the MACE increases as the MACE gets closer to the sensor array. To express this relationship, we define a new parameter, Signal Quality Ratio (SQR) in B-field, as

$$
\begin{equation*}
S Q R_{\mathrm{B}}=\frac{\left\|\mathbf{B}_{\mathrm{c}}\right\|}{\left\|\mathbf{B}_{\text {err }}+N_{\mathrm{s}}\right\|} \approx \frac{\left\|\mathbf{B}_{\mathrm{c}}\right\|}{\left\|\mathbf{B}_{\text {err }}\right\|} \propto\left(\frac{\left\|\mathbf{r}_{\mathrm{e}}\right\|}{\left\|\mathbf{r}_{\mathrm{c}}\right\|}\right)^{3} \tag{6}
\end{equation*}
$$

where $\mathbf{r}_{\mathrm{c}}$ is a position vector from the sensor array to the MACE and $N_{\mathrm{s}}$ is a noise level of the sensor. The magnetic field error due to multi-pole terms in (5) is negligible as they are much smaller than the dipole term in (6). Note that, assuming that $N_{\mathrm{s}}$ is negligible, $S Q R_{\mathrm{B}}$ is inversely proportional to the third order of the distance.

The directional differentiation of the analytical model (2) results in

$$
\begin{equation*}
\frac{\partial^{2} \mathbf{B}_{\mathrm{dpl}}(\mathbf{r}, \mathbf{m})}{\partial\|\mathbf{r}\|^{2}}=\frac{3 \mu_{0}}{\pi\|\mathbf{r}\|^{5}}\left(3 \hat{\mathbf{r}}^{\mathrm{T}}-\mathbf{I}\right) \mathbf{m} \tag{7}
\end{equation*}
$$

The SQR in the second order differentiated B-field is expressed as

$$
\begin{align*}
& S Q R_{\mathrm{r}}=\frac{\left\|\partial^{2} \mathbf{B}_{\mathrm{c}} / \partial\right\| \mathbf{r}_{\mathbf{c}}\left\|^{2}\right\|}{\left\|\partial^{2}\left(\mathbf{B}_{\mathrm{err}}+N_{\mathrm{s}}\right) / \partial \mathbf{r}_{\mathrm{e}}\right\|^{2} \|} \\
& \approx \frac{\left\|\partial^{2} \mathbf{B}_{\mathrm{c}} / \partial\right\| \mathbf{r}_{\mathrm{c}}\left\|^{2}\right\|}{\left\|\partial^{2} \mathbf{B}_{\text {crr }} / \partial\right\| \mathbf{r}_{\mathrm{c}}\left\|^{2}\right\|} \propto\left(\frac{\left\|\mathbf{r}_{\mathrm{c}}\right\|}{\left\|\mathbf{r}_{\mathrm{c}}\right\|}\right)^{5} . \tag{8}
\end{align*}
$$

Assuming that $N_{\mathrm{S}}$ is negligible, $S Q R_{\mathrm{L}}$ is inversely proportional to the fifth order of the distance ratio. Because the $\left\|\mathbf{r}_{\mathrm{e}}\right\|$ is larger than $\left\|\mathbf{r}_{\mathrm{c}}\right\|, S Q R_{\mathrm{L}}$ is always higher than $S Q R_{\mathrm{B}}$. The more the B -field is differentiated, the better SQR is achieved because of the scaling law. However, the number of the differentiation is limited by the number of the sensor elements, and the noise is magnified by the differentiation. In this paper, the second order differentiation was sufficient for the given number of the sensors and the noise level of the sensors.

Another advantage of using the second order directional differentiation is that we can calculate the vertical directional differentiation using a lateral 2-D array of mono-axial Halleffect sensors. Equation (9) is always valid at all positions based on Maxwell's equations in the absence of current or a changing electric field,

$$
\begin{equation*}
\nabla^{2} \mathbf{B}=\frac{\partial^{2} \mathbf{B}}{\partial x^{2}}+\frac{\partial^{2} \mathbf{B}}{\partial y^{2}}+\frac{\partial^{2} \mathbf{B}}{\partial z^{2}}=\mathbf{0} \tag{9}
\end{equation*}
$$

Equation (9) shows that the second order derivative in the $z$-direction equals a negative sum of those in the $x$ - and $y$-directions. Though Hall-effect sensors in the $X Y$-plane measure the magnetic fields in the $z$-direction, the second order derivative in the $z$-direction can be calculated without using multiple layers in the $z$-direction based on the above property. Here, we define the second order $z$-directional derivative $K$ as

$$
\begin{equation*}
K(x, y)=\frac{\partial^{2} B_{z}}{\partial z^{2}}=-\frac{\partial^{2} B_{z}}{\partial x^{2}}-\frac{\partial^{2} B_{z}}{\partial y^{2}} \tag{10}
\end{equation*}
$$

where $B_{Z}$ is a $z$-directional component of $\mathbf{B}$. In the two-dimensional sensor array, $K$ is calculated by using the magnetic field of the neighboring sensors. Using the five-point stencil finite difference method, $K$ at the sensor node $(i, j)$, or $K^{i, j}$, is expressed as

$$
\begin{equation*}
K^{i, j} \approx-\frac{B^{i, j-1}+B^{i, j+1}+B^{i-1, j}+B^{i+1, j}-4 B^{i, j}}{h^{2}} \tag{11}
\end{equation*}
$$

where $B^{i, j}$ is the $z$-directional component of the B-field measured by the sensor at the node $(i, j)$, and $h$ is the nodal distance between neighboring sensors.

We conducted experiments to compare $S Q R_{\mathrm{L}}$ with $S Q R_{\mathrm{B}}$ in order to verify the analysis performed in (6) and (8). Because our setup measures only the $z$-directional components of the B-field, we defined two new terms, $S Q R_{z, \mathrm{~B}}$ and $S Q R_{z, \mathrm{~L}}$ as (12) and (13).

$$
\begin{equation*}
S Q R_{z, \mathrm{~B}}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}, \mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)=\frac{1}{n} \sum_{i} \sum_{j} \frac{\left|B_{\mathrm{c}}^{i, j}\left(\mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)\right|}{\left|B_{\mathrm{c}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)-B_{\mathrm{e}, \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)\right|} \tag{12}
\end{equation*}
$$

where $n$ is the number of sensors, $B_{\mathrm{c}}^{i, j}\left(\mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)$ is the $z$-directional magnetic field from the MACE, and $B_{\mathrm{e}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)$ is the $z$-directional magnetic field from the electromagnet; each is measured by the sensor at node ( $i, j$ ) in the absence of the other's magnetic field.
$B_{\mathrm{e}, \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)$ is the $z$-directional magnetic field of the electromagnet assuming the dipole model.

$$
\begin{equation*}
S Q R_{z, \mathrm{~L}}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}, \mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)=\frac{1}{n} \sum_{i} \sum_{j} \frac{\left|K_{\mathrm{c}}^{i, j}\left(\mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)\right|}{\left|K_{\mathrm{c}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)-K_{\mathrm{e}, \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)\right|} \tag{13}
\end{equation*}
$$

where $K_{\mathrm{c}}^{i, j}\left(\mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)$ and $K_{\mathrm{e}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)$ are the second derivatives of the $z$-directional magnetic fields from the MACE and the external magnet, respectively. Both of these are calculated using (11). $K_{\mathrm{e}, \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)$ is the second derivative of the $z$-directional magnetic field of the external magnet assuming the dipole model, which is calculated using (7).

Although $S Q R_{Z}$ represents only the $z$-directional components of SQR , the comparison of $S Q R_{z, \mathrm{~L}}$ and $S Q R_{Z, \mathrm{~B}}$ indirectly represents the effect of the scaling law in (6) and (8) on the signal quality. In the experiments, we set $\mathbf{m}_{\mathrm{c}}$ to $(0,0,0.45) \mathrm{A} \cdot \mathrm{m}^{2}$ and $\mathbf{m}_{\mathrm{e}}$ to $(0,0,30.0)$ $\mathrm{A} \cdot \mathrm{m}^{2}$, and both $S Q R_{z, \mathrm{~B}}$ and $S Q R_{Z, \mathrm{~L}}$ are measured for 10 seconds, then averaged.

Figure 3 shows that $S Q R_{\mathrm{L}}$ is higher than $S Q R_{\mathrm{B}}$ where the distance ratio $\left(\left\|\mathbf{r}_{\mathrm{e}}\right\| / / / \mathbf{r}_{\mathrm{c}} \|\right)$ is larger than 1. This means that $S Q R_{\mathrm{L}}$ becomes a clearer standard than $S Q R_{\mathrm{B}}$ does. Especially, as $\mathbf{r}_{\mathrm{c}}$ becomes smaller and $\mathbf{r}_{\mathrm{e}}$ becomes larger, $S Q R_{\mathrm{L}}$ increases exponentially, whereas $S Q R_{\mathrm{B}}$ stays in low level. These experimental results show that the proposed method gives better signal information than the B-field subtraction method.

## C. Algorithm

The developed algorithm finds the optimal $\mathbf{r}_{\mathrm{c}}$ and $\mathbf{m}_{\mathrm{c}}$ by minimizing a cost function using the Levenberg-Marquardt Algorithm (LMA). LMA is a trust region based optimization method that uses the steepest descent method for global convergence and Newton's method (quadratic method) for local convergence in a way that gives smooth transition between them [27]. This optimization solver is known as the efficient and effective solution for a magnetic marker localization problem [28]. The cost function is defined as

$$
\begin{equation*}
c=\sum_{i} \sum_{j}\left(K^{i, j}-K_{\mathrm{e}, \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)-K_{\mathrm{c}, \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)\right)^{2} \tag{14}
\end{equation*}
$$

where $K^{i, j}$ is the second $z$-directional derivative of the B-field based on the measured data at the sensor node number $(i, j)$ and $K_{\mathrm{c} . \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)$ is the MACE's modeled second $z$ directional derivative of the B-field at the sensor node number (i,j). $K_{\mathrm{e}, \mathrm{dpl}}^{i, j}$ and $K_{\mathrm{c}, \mathrm{dpl}}^{i, j}$ are computed by using the following analytical equation, which is derived from (2),

$$
\begin{equation*}
K_{\mathrm{dpl}}^{i, j}(\mathbf{r}, \mathbf{m})=\frac{\partial^{2} B_{\mathrm{z}}}{\partial z^{2}}=\frac{\mu_{0}}{4 \pi}\left(\frac{9 m_{\mathrm{z}}}{\|\mathbf{r}\|^{5}}-\frac{45 r_{\mathrm{z}}\left(\mathbf{m} \cdot \mathbf{r}+m_{\mathrm{z}} r_{\mathrm{z}}\right)}{\|\mathbf{r}\|^{7}}+\frac{105 r_{\mathrm{Z}}^{3}(\mathbf{m} \cdot \mathbf{r})}{\|\mathbf{r}\|^{9}}\right) \tag{15}
\end{equation*}
$$

where $m_{\mathrm{Z}}$ is the $z$-directional component of $\mathbf{m}$ and $r_{\mathrm{Z}}$ is the $z$-directional component of $\mathbf{r}$.

Fig. 4 shows a flow chart of the developed algorithm, and describes how each term of the cost function is calculated. The goal of the algorithm is to estimate the optimal $\mathbf{r}_{c}$ and $\mathbf{m}_{c}$ minimizing the cost function, (14). First, the measured magnetic field ( $B^{i, j}$ ) is transformed to the second derivative ( $K^{i, j}$ ) by using (11). Second, the electromagnet's input current ( $\mathbf{I}_{\mathrm{e}}$ ) gives the estimate of its magnetic moment ( $\mathbf{m}_{e}$ ). Using the calculated $\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}$, and (15), we can obtain the second derivative of the electromagnet's B-field ( $K_{\mathrm{e} . \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{e}}, \mathbf{m}_{\mathrm{e}}\right)$ ). The key of the cost function is the last term ( $K_{\mathrm{c}, \mathrm{dpl}}^{i, j}\left(\mathbf{r}_{\mathrm{c}}, \mathbf{m}_{\mathrm{c}}\right)$ ). The optimal $\mathbf{r}_{\mathrm{c}}$ and $\mathbf{m}_{\mathrm{c}}$ of the previous iteration become the initial conditions for the current $\mathbf{r}_{c}$ and $\mathbf{m}_{c}$. The terms are calculated by (15), and iteratively updated by the optimization. The new optimal $\mathbf{r}_{c}$ and $\mathbf{m}_{c}$ that minimize the cost function become the current position and orientation of the capsule.

## III. Experiments

We conducted experiments to evaluate the accuracy and reliability of the developed localization methods. Since water and biological tissue do not affect the low frequency magnetic field, our simple and magnetically transparent experimental setup is applicable to the magnetic capsule endoscopy. As a single magnet works as a magnetic source for both actuation and localization in the proposed method, The MACE was represented by a box shaped $\left(6.4 \times 6.4 \times 12.8 \mathrm{~mm}^{3}\right) \mathrm{NeFeB}$ magnet with $0.45 \mathrm{~A} \cdot \mathrm{~m}^{2}$ magnetic moment. The work space for the MACE was given as $70(\mathrm{w}) \times 70(\mathrm{~d}) \times 50(\mathrm{~h}) \mathrm{mm}^{3}$ below the sensor array. A plane with a slope was used for the working surface in the experiment (Fig. 5(b)). The external magnet, which was positioned 20 cm below the array, generated a 2.5 mT rotating magnetic field at the center of the MACE for climbing rolling locomotion. While it is rolling from the initial position, $(20,-20,-35) \mathrm{mm}$, to the final position, $(-18,18,-20) \mathrm{mm}$, the proposed method ran in real-time at 200 Hz (limited by LMA loop speed) to track the position and orientation of the MACE. The B-field subtraction method ran in parallel with the proposed method for comparison. For the ground truth position and orientation, two video cameras recorded the MACE with visual markers (Fig. 5(a)). 5-D visual reference data was extracted using an image processing software [29].

A total of 10 experiments were conducted, and each experiment took approximately 6 seconds for the MACE to traverse the surface. Each trial had the same initial condition and planned trajectory. All the manipulation and localization were done autonomously by the pre-programmed codes in LabView (National Instruments co.). Distance errors were measured by Euclidean distance and angular errors were measured by orientation vector difference using visual reference data.

Table II shows the summarized experimental results. Overall, the proposed method is more accurate than the B-field subtraction method. Its total average errors were $2.1 \pm 0.8 \mathrm{~mm}$ (distance) and $6.7 \pm 4.3^{\circ}$ (angular), respectively, while the errors of the B-field subtraction method were $2.6 \pm 1.3 \mathrm{~mm}$ and $8.3 \pm 6.5^{\circ}$, respectively. The fact that its maximum errors ( 4.7 mm and $30^{\circ}$ ) were much smaller than the others ( 10.5 mm and $50.3^{\circ}$ ) means that the proposed method is more stable. Figure 6(a)-(c) show the worst case error of the experimental trials. The proposed method more closely and more stably tracked the ground
truth position than the B-field subtraction method did (see the abruptly increasing position errors near the initial position in Fig. 6(a)).

Error reduction by the differentiation explains the improved accuracy. In the B-field subtraction method, the position and orientation of the MACE fluctuate because of the rotating external magnetic field. This error is due to the magnetic field error in the analytical magnetic model in (5). This factor is still non-negligible and results in poor accuracy. However, by taking the proposed second order differentiation, those fluctuations are significantly reduced as shown in the plot. This method improves $z$-directional localization accuracy significantly, although it does not impact the $x$ - and $y$-position accuracy (Fig. 6(b)) as it is applied in $z$-direction.

The distance ratio, $\left\|\mathbf{r}_{\mathrm{e}}\right\| / /\left\|\mathbf{r}_{\mathrm{c}}\right\|$, is critial to localization accuracy. Although our nonlinear optimization method does not show an explicit relationship between SQR values and localization error, Fig. 6(c) shows that larger distance ratios correspond to smaller localization errors, and that the second derivative method yields smaller errors. These results are consistent with the theoretical analysis in (6) and (8).

## IV. Discussion

The developed real-time localization method gives accurate estimations of the position and orientation of a magnetically manipulated robot. This method does not require internal sensors, and allows to remain the mechanical and electrical configuration of the robot simple, which is useful for an untethered magnetic robot for medical applications where optical tracking is not possible. Table III shows the detailed comparison with other magnetic localization methods. Even though the effective distances in the experiments are different, the proposed localization method shows the smallest position error and the fastest speed for controlling a capsule robot in real-time compared to the other magnetic localization methods.

However, the proposed localization method would have a limited clinical application because of the short effective distance (<50 mm). Beyond 50 mm , SQR values drop below 5 dB (see Fig. 3) even with the second derivative method. This means the noise and error terms occupy more than $36 \%$ of the whole measured signal. With such poor signal conditioning, the nonlinear optimization algorithm tends to either diverge or give unreasonable estimations.

The effective distance can be increased by using lower noise Hall-effect sensors and a bigger magnet. In the experiments, we used a small magnet $\left(6.4 \times 6.4 \times 12.8=524.3 \mathrm{~mm}^{3}, \mathrm{NdFeB}\right.$, $\left.0.45 \mathrm{~A} \cdot \mathrm{~m}^{2}\right)$, but doubling the volume of a cylindrical shape, $\left(\varphi 11 \times 11=1,045 \mathrm{~mm}^{3}, \mathrm{NdFeB}\right.$, $0.90 \mathrm{~A} \cdot \mathrm{~m}^{2}$ ), would still be within the limits for a swallowable capsule endoscope (diameter $<12 \mathrm{~mm}$, length $<30 \mathrm{~mm}$ ). We simulated the effective distance as a function of the inherent sensor noise. We assumed that 20 dB is the minimum SQR level for quality localization (same as $10 \%$ error) to determine the effective distance, $S_{\text {eff. }}$. In the simulation, the original equations, including noise terms from (6) and (8), were used. Figure 7 shows the relationship between the inherent sensor noise and $S_{\text {eff }}$. Both the B-field subtraction method
and the proposed method have increased effective distances as the inherent sensor noise decreases. While the B-field subtraction method saturates at 13 cm , the second derivative method shows a maximum of 23 cm effective distance without saturation using currently existing sensors (e.g., MMC3316xMT, MESMIC co, RMS noise: $0.2 \mu \mathrm{~T}$ ). Additionally, an extremely low inherent noise sensor, such as $0.02 \mu \mathrm{~T}$ RMS noise level sensors, would give nearly 30 cm of $S_{\text {eff }}$, which satisfies the effective range guideline of magnetic capsule endoscopy.

## V. Conclusion

In this paper, we introduced a new real-time 5-D localization method for an untethered meso-scale magnetic robot, which is manipulated by a computer-controlled external electromagnetic system. The developed magnetic localization setup is a 2-D array of monoaxial Hall-effect sensors, which measure the perpendicular magnetic fields at their positions. We propose two steps for localizing the magnetic robot more accurately. First, the dipole modeled magnetic field of the electromagnet is subtracted from the measured data in order to determine the pure magnetic field from the magnetic robot. Next, the subtracted magnetic field is twice differentiated in the perpendicular direction of the array, so that the effect of the electromagnetic field in the localization process is minimized. Five variables regarding the position and orientation of the magnetic robot are determined by minimizing the error between the measured magnetic field and the modeled magnetic field in an optimization method. The resulting position error is $2.1 \pm 0.8 \mathrm{~mm}$ and angular error is $6.7 \pm 4.3^{\circ}$ within the applicable range ( 5 cm ) of magnetic field sensors at 200 Hz . The proposed localization method would be used for the position feedback control of untethered magnetic devices or robots for medical applications in the future.

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## Biographies



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Metin Sitti (S'94-M'00-SM'08-F'14) received the B.Sc. and M.Sc. degrees in electrical and electronics engineering from Bogazici University, Istanbul, Turkey, in 1992 and 1994, respectively, and the Ph.D. degree in electrical engineering from the University of Tokyo, Tokyo, Japan, in 1999. He was a research scientist with the University of California at Berkeley, Berkeley, CA, USA, during 1999-2002. He is currently a director in Max-Planck Institute for Intelligent Systems and a professor in Department of Mechanical Engineering and Robotics Institute at Carnegie Mellon University. His research interests include smallscale physical intelligence, mobile milli- and microrobots, medical miniature robots, bioinspired materials and locomotion, and micro/nanomanipulation.

Dr. Sitti is an IEEE Fellow. He received the SPIE Nanoengineering Pioneer Award in 2011 and NSF CAREER Award in 2005. He received the IEEE/ASME Best Mechatronics Paper Award in 2014, the Best Poster Award in the Adhesion Conference in 2014, the Best Paper Award in the IEEE/RSJ International Conference on Intelligent Robots and Systems in 2009 and 1998, the first prize in the World RoboCup Micro-Robotics Competition in 2012 and 2013, the Best Biomimetics Paper Award in the IEEE Robotics and Biomimetics Conference in 2004, and the Best Video Award in the IEEE Robotics and Automation Conference in 2002. He is the editor-in-chief of Journal of Micro-Bio Robotics.


Fig. 1.
Photographs of the five-dimensional magnetic localization setup. (a) Overview; A: Omnidirectional electromagnet; B: Arena of the MACE; C: Two-dimensional mono-axial Hall-effect sensor array; D: Multiplexer board; E: Current sensors and current amplifier; F: Data-acquisition (DAQ) board; G: Desktop computer, its monitor and Labview-based graphical user interface. (b) Close-up view of the Hall-effect sensor array. The $z$-directional Hall-effect sensors are located in the two-dimensional array with a nodal distance of 10 mm . H: MACE with visual markers on its surface. The size of the MACE is $6.4 \times 6.4 \times 12.8 \mathrm{~mm}^{3}$, its material is NdFeB , and its magnetic moment is $0.45 \mathrm{~A} \cdot \mathrm{~m}^{2}$. (c) Overall signal flow of the system. Measured sensory data from Hall-effect sensor array and current sensors are fed to the computer through the DAQ board. The omnidirectional electromagnet is driven by current drivers. The multiplexer board is omitted in the diagram.


Fig. 2.
Schematic drawing of the application scenario. A MACE is manipulated by an external magnet in the 3-D space. The objective of this paper is to estimate the position ( $\mathbf{r}_{\mathrm{c}}$ ) and orientation $\left(\mathbf{m}_{c}\right)$ of the MACE under the effect of the external magnetic field.


Fig. 3.
Experimental comparison of $S Q R_{Z, \mathrm{~B}}$ and $S Q R_{Z, \mathrm{~L}} \cdot S Q R_{Z, \mathrm{~L}}$ is higher than $S Q R_{Z, \mathrm{~B}}$ in all regions. The $z$-directional distance from the MACE and the external magnet were set from 30 mm to 75 mm and 160 mm to 230 mm with 5 mm increments, respectively. The electromagnet generated 1 mT B-field at the center of the array in the $+z$-direction. The measurement was done for 10 seconds.


## Fig. 4.

Block diagram of the main localization algorithm. The proposed algorithm minimizes the cost function to find the position $\left(\mathbf{r}_{c}\right)$ and orientation $\left(\mathbf{m}_{c}\right)$ of the MACE. Initial position and orientation of the MACE are continuously updated by the algorithm for the real-time tracking. Measured B-field's second derivative is calculated using Laplacian of the B-field, and the estimated second derivative of the B-field is calculated using the second derivative form of the dipole equation.


Fig. 5.
Experimental setup and the dynamic motion of the MACE. (a) The MACE has markers on its surface. We reconstructed the position and orientation of the moving robot using the markers in images. (b) The MACE traversed the slope with the external magnetic actuation that gave rolling locomotion to the MACE.

(a)

(b)

(c)

Fig. 6.
Result of the dynamic motion tracking experiment (the worst case). (a) The proposed method tracked the MACE in real-time with the external magnetic actuation. (b) While the B-field subtraction method had a significant loss of track near the starting point and errors in the middle of the track, the proposed method tracked the MACE's motion through the whole path with the minor error. (c) As the distance ratio, $\left\|\mathbf{r}_{\mathrm{e}}\right\| /\left\|\mathbf{r}_{\mathrm{c}}\right\|$, increases, the localization error shows a decreasing trend. The proposed method shows less positioning error than the B-field subtraction method in almost all ranges. The error fluctuates because of the rotating magnetic field of the MACE.


Fig. 7.
Simulated effective localization range ( $S_{\text {eff }}$ ) as a function of the inherent sensor noise. With the $(1,1,1) \mathrm{mm}$ position misalignment and $1 \mathrm{~A} \cdot \mathrm{~m}^{2}$ magnetic moment error from the external magnet, it is shown that the $S_{\text {eff }}$ can be extended to 23 cm with the currently existing sensors. As inherent sensor noise gets smaller, it is preferable to use the second order derivative for better accuracy and long effective localization distance. The external magnetic field on the MACE was $(0,0,3) \mathrm{mT}$.

## TABLE I

Specification of the Localization Setup

|  | Value |
| :--- | :--- |
| Hall－effect sensors，A1389（Allegro） | 64 counts |
| －Nodal distance $(x$－and $y$－direction） | 10 mm |
| －Sensitivity | $9 \mathrm{mV} / \mathrm{G}$ |
| －Noise level | $15 \mathrm{mV}(=1.667 \mathrm{G})$ |
| －Measurement range | $\pm 278 \mathrm{G}$ |
| The omnidirectional electromagnet | $193 \times 200 \times 200 \mathrm{~mm}^{3}$ |
| －Size | $30.31,30.22,34.12 \mathrm{~A} \cdot \mathrm{~m}^{2} / \mathrm{A}$ |
| －Magnetic moment $(x$－，$y$－，$z$－direction） |  |
| MACE | $6.4 \times 6.4 \times 12.8 \mathrm{~mm}^{3}$ |
| －Dimensions | $0.45 \mathrm{~A} \cdot \mathrm{~m}^{2}$ |

3－bit multiplexer（74HC／HCT4051，Phillips Semiconductors）
Data－acquisition－board（NI USB 6343，National Instruments）

Current driver（SyRen 25，Dimension Engineering）
Current sensor（ACS714，Pololu Corporation）
łd!ı0snuew גOYłn $\forall$

Dynamic Motion Tracking Experiment Results

| Number of trials=10* | Position error (mm) |  |  | Orientation error $\left(^{\circ}\right)$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Avg. $\pm$ Std. ${ }^{* *}$ | Min | Max | Avg. $\pm$ Std. ${ }^{* *}$ | Min | Max |
| Proposed method | $2.1 \pm 0.8$ | 0.07 | 4.7 | $6.7 \pm 4.3$ | 0.03 | 30.0 |
| B-field subtraction method | $2.6 \pm 1.3$ | 0.4 | 10.5 | $8.3 \pm 6.5$ | 0.07 | 50.3 |

* Each trial had the same initial condition and trajectory.
** The total average of the 10 experiments.
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Comparison of the Magnetic Localization Methods

|  | Popek et al. [23] | Di Natali et al. [24] | Yim et al. [25] | The proposed method | Than et al. ${ }^{*}$ [30] [31] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Internal Sensor(s) | 6 Hall-effect sensors | 6 Hall-effect sensors + 1 tri-axial accelerometer | 1 Hall-effect sensor | None | None |
| External Sensor(s) | None | None | None | 64 Hall-effect sensors | 2 pairs of gamma ray detectors |
| Position Error (mm) | 11 | $3.4 \pm 3.2$ | 2.0 | $2.1 \pm 0.8$ | 0.4 |
| Orientation Error ( ${ }^{\circ}$ ) | 11 | $19 \pm 50$ | $5 \pm 1.2$ | $6.7 \pm 4.3$ | 2 |
| Real-time (Loop speed) | No | Yes (14 ms) | No | Yes (5 ms) | Yes (2-3 ms) |
| Effective localization Range (mm) | $136.0-144.0$ ** | 0-150 | 44.2-57.2 | 5-50 | 200-400 |

[^1]
[^0]:    The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health.

[^1]:    Non-magnetic localization method (positron emission marker dectection).
    Range in the experiment. Effective localization range is not shown explicitly in the paper.

