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A BASIS INDEPENDENT FORMULATION OF THE CONNECTION BETWEEN QUARK MASS MATRICES, CP VIOLATION AND EXPERIMENT

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A B S T R A C T

In the standard electroweak model, with three families, a one-to-one correspondence between certain determinants involving quark mass matrices (m and m' for charge 2/3 and -1/3 quarks respectively) and the presence/absence of CP violation is given. In an arbitrary basis for mass matrices, the quantity Im $\det[mm^+, m'm'^+]$ appropriately normalized is introduced as a measure of CP violation. By this measure, CP is not maximally violated in any transition in Nature. Finally, constraints on quark mass matrices are derived from experiment. Any model of mass matrices, with the ambition to explain Nature, must satisfy these conditions.

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1. INTRODUCTION

Recent investigations[1,2] of the systematics of the quark mass matrices, in the framework of the Standard Electroweak Model[3] with three families, have revealed some intriguing and as yet badly understood features. The derivation of these results, which are also discussed below, depended crucially on the assumption that the quark mass matrices are hermitian. Indeed, in the Standard Model it is always possible[2], by a suitable redefinition of the right-handed quark fields to go to a hermitian basis for the mass matrices. Nevertheless, one is left with a somewhat uneasy feeling. Surely, the derived systematics cannot just be an artifact of hermiticity? Therefore, it is essential to find the counterparts of these results in an arbitrary basis.

The purpose of this paper is just to examine the systematics of the quark mass matrices and the connection with CP-violation in a basis independent fashion. The organization of this paper is as follows. In Section 2 the commutators of functions of quark mass matrices and their intimate connection with CP-violation are discussed in the hermitian basis. In Section 3, it is shown that such an intimate connection exists in any basis provided appropriate commutators are considered. Basis independent constraints on mass matrices are derived, from experiment, in Section 4 and in Section 5, CP-asymmetry parameters are introduced and the question of maximal CP-violation is discussed. Finally, the conclusions are presented in Section 6.

2. THE COMMUTATOR OF THE QUARK MASS MATRICES AND CP-VIOLATION

Let m and m' denote the three by three quark mass matrices for the charge 2/3 and -1/3 quarks respectively. In the hermitian basis, $m=m^+$ and $m'=m'^+$, we have

$$[m, m'] = iC, \tag{1}$$

when C is hermitian and traceless. In Ref. 1 it was shown that the eigenvalues of C are measurable quantities and that its determinant is given by

$$\det C = -2\, T \cdot B \cdot J, \tag{2}$$

where

$$T = (m_t - m_u)(m_t - m_c)(m_c - m_u), \tag{3}$$

$$B = (m_b - m_d)(m_b - m_s)(m_s - m_d). \tag{4}$$

Here m_j refers to the mass of the quark j and the quantity J is a function of the elements of the quark mixing matrix V (the Kobayashi-Maskawa matrix[4]). J is obtained as follows. Cross out an arbitrary row r and column s of the matrix V and denote the remaining two by two submatrix by

$$\begin{pmatrix} V_{ij} & V_{ik} \\ V_{\ell j} & V_{\ell k} \end{pmatrix}.$$

Then J is given by[1]

$$J = (-1)^{r+s}\, \mathrm{Im}(V_{ij} V_{\ell k} V_{ik}^{*} V_{\ell j}^{*}) \tag{5}$$

i.e.,

$$J = \mathrm{Im}(V_{11} V_{22} V_{12}^{*} V_{21}^{*}) = \mathrm{Im}(V_{22} V_{33} V_{23}^{*} V_{32}^{*}) = \cdots$$

The essential point is that J is unique[1], in the Standard Model, as the measure of CP-violation. It is phase convention independent, i.e., it is invariant under the transformations

$$V \to \mathrm{diag}(e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3})\, V\, \mathrm{diag}(e^{i\psi_1}, e^{i\psi_2}, e^{i\psi_3}),$$

where φ and ψ denote arbitrary phases. In the Kobayashi-Maskawa parametrization one has[1)]

$$J = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta, \tag{6}$$
$$s_i \equiv \sin\theta_i, \quad c_i \equiv \cos\theta_i,$$

a quantity which is familiar to anyone who has performed explicit calculations of CP-violation effects[5)] in the Standard Model. Every such effect is proportional to J. Note that J vanishes if any of the angles in (6) assumes its maximum or minimum value, i.e., $\theta_i=0$ or $\pi/2$; $\delta=0$ or π.

From Eqs. (2)-(5) follows that the determinant of C vanishes if and only if there is no CP-violation. Therefore, in Ref. 1, it was suggested that the det[m, m'] appropriately normalized may be used to define a measure of CP-violation in the Standard Model. This point is further developed in Section 5 of this paper.

Before going to an arbitrary basis we make the following observation. In the hermitian basis, we may form the commutator of functions of mass matrices,

$$f(m) = \sum_{n=1}^{\infty} a_n m^n, \quad g(m') = \sum_{n=1}^{\infty} b_n m'^n, \tag{7}$$

where the coefficients a_n and b_n are arbitrary real numbers. Then the commutator

$$[f(m), g(m')] = i\, C(f(m), g(m')) \tag{8}$$

again defines a hermitian traceless matrix, C(f,g). The important point is that the eigenvalues of this commutator are calculable functions of the quark masses and the elements of the quark mixing matrix V. The determinant of C(f,g) is particularly simple and is given by

$$\det C(f,g) = -2\, T(f(m)) \cdot B(g(m')) \cdot J, \tag{9}$$

where

$$T(f(m)) = [f(m_t)-f(m_u)][f(m_t)-f(m_c)][f(m_c)-f(m_u)], \tag{10}$$

$$B(g(m')) = [g(m_b)-g(m_d)][g(m_b)-g(m_s)][g(m_s)-g(m_d)] \tag{11}$$

and J is as defined before, see Eq. (5).

One interesting consequence of the above result is that it clarifies a mystery in connection with Eqs. (2)-(4). We know that the sign of the mass in the Lagrangian is irrelevant. However in the quantities T and B the first power of the masses appears and thus makes the sign of the mass relevant when we discuss the presence or absence of CP-violation!

Eq. 9 provides an answer to this dilemma. Instead of taking the commutator [m,m'] we should take

$$[m^2, m'^2] = i\, C(m^2, m'^2). \tag{12}$$

From Eqs. (9)-(11) then follows that

$$\det C(m^2, m'^2) = -2\, T(m^2)\, B(m'^2) . J, \tag{13}$$

$$T(m^2) = (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2),$$

$$B(m'^2) = (m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2).$$

Indeed the determinant of $C(m^2,m'^2)$ vanishes irrespectively of the sign of the mass, if the magnitudes of the masses of two quarks with the same charge are the same.

To conclude this Section we have found that in the hermitian basis the $\det[m^2,m'^2]$ provides a measure of CP-violation because it vanishes iff there is no CP-violation. It is also not sensitive to the sign of the mass.

The results (8)-(11) of this section provide a method for a basis independent analysis of the connection between the commutators and CP-violation, as is discussed immediately below.

3. COMMUTATORS IN AN ARBITRARY BASIS

In an arbitrary basis the quark mass matrices m and m' are not necessarily hermitian and as usual one needs two different unitary matrices for the diagonalization of each of them,

$$U_L m U_R^+ = d\,, \qquad U_L' m' U_R'^+ = d' \tag{14}$$

$$d = \mathrm{diag}(m_u, m_c, m_t)\,, \quad d' = \mathrm{diag}(m_d, m_s, m_b)\,. \tag{15}$$

Here U_x and U_x' , x=L, R, are unitary matrices. Although m is not hermitian, one can form from it two hermitian matrices, namely mm^+ and m^+m. They are, as usual, diagonalized by U_L and U_R respectively,

$$U_L m m^+ U_L^+ = d^2, \qquad U_R m^+ m U_R^+ = d^2 \tag{16}$$

and similarly for the primed quantities. The quark mixing matrices are then

$$V_L = U_L U_L'^+, \qquad V_R = U_R U_R'^+, \tag{17}$$

where V_L is just the measurable quark mixing matrix V while V_R is not measureable in the Standard Model. It is however measurable in the left-right symmetric models.

Next we construct the commutators

$$[mm^+,\ m'm'^+] = i K_L, \tag{18}$$

$$[m^+m,\ m'^+m'] = i K_R, \tag{19}$$

where K_x, x=L, R are hermitian and traceless matrices. They are related to measurable quantities by

$$iK_X = U_X^+ [d^2, V_X d'^2 V_X^+] U_X, \quad X=L,R. \tag{20}$$

Comparing Eqs. (20) and (12) yields that the eigenvalues of K_L are identical to those of $C(m^2, m'^2)$. Thus in a general basis mm^+ plays the role played by m^2 in the hermitian basis. We have

$$\det K_X = -2\, T(m^2) \cdot B(m'^2) \cdot J_X, \quad X=L,R, \tag{21}$$

where $T(m^2)$ and $B(m'^2)$ are as given in Eq. (13) and

$$J_X = \mathrm{Im}(V_{11} V_{22} V_{12}^* V_{21}^*)\Big|_{V=V_X} \tag{22}$$

For the general construction of J_x see Eq. (5).

Thus our previous result, in the hermitian basis, that in the Standard Model with three families, the determinant of $[m^2, m'^2]$ vanishes iff there is no CP-violation is now replaced by the basis independent statement that the determinant of $[mm^+, m'm'^+]$ vanishes iff there is no CP-violation.

The analogy with the hermitian case can be carried out further if we compute the commutator of functions $f(mm^+)$ and $g(m'm'^+)$, see Eqs. (7) and (8). The relation (8) is then replaced by

$$[f(mm^+), g(m'm'^+)] = i\, \hat{C}(f,g), \tag{23}$$

where the determinant of $\hat{C}$ is simply obtained from Eqs. (9)-(11) by just replacing everywhere m_j by m_j^2. This result establishes the correspondence between the two bases, viz.

hermitian basis		nonhermitian basis
m^2	$\rightarrow$	mm^+
m'^2	$\rightarrow$	$m'm'^+$

(24)

In the Standard Model there are no right-handed currents and therefore V_R is not measurable. In the left-right symmetric models, however, V_R is just as fundamental and measurable as V. Then the eigenvalues of the matrix K_R are also measurable quantities, see Eqs. (19)-(21). As an application of the above results we consider the so-called pseudo-manifest left-right symmetric models[6) where the mass matrices are symmetric but not necessarily hermitian. Then putting m=S and m'=S', where S(S'), denotes the appropriate three-by-three symmetric matrix, we have, from Eqs. (18) and (19)

$$K_R = - K_L^* . \tag{25}$$

Here we have used that

$$S^+ = S^*, \qquad S'^+ = S'^* . \tag{26}$$

Thus

$$\det K_R = - \det K_L \tag{27}$$

where we have used the fact that K_L is hermitian and thus its determinant is real. Hence the vanishing of det K_L would imply that also det K_R vanishes. This result shows that the one-to-one correspondence between the vanishing of the determinant of the commutator and the absence of CP-violation is a special feature of the Standard Model; the vanishing of the determinant is, of course, a necessary condition for the absence of CP-violation in the left-right symmetric models also. But this condition is not a sufficient condition.

4. BASIS INDEPENDENT CONSTRAINTS ON QUARK MASS MATRICES FROM EXPERIMENT

In the hermitian basis it is convenient to normalize the mass matrices by defining[2)]

$$M = \frac{m}{m_t}, \qquad M' = \frac{m'}{m_b}, \tag{28}$$

such that the largest eigenvalue equals unity. Then the difference matrix

$$\Delta = M - M' \tag{29}$$

is given by

$$\Delta = U^{+}(D - V D' V^{+}) U, \tag{30}$$

where

$$D = \mathrm{diag}(m_u/m_t\,,\ m_c/m_t\,,\ 1), \tag{31}$$

$$D' = \mathrm{diag}(m_d/m_b\,,\ m_s/m_b\,,\ 1). \tag{32}$$

Furthermore V is the quark mixing matrix and U is a unitary matrix. Eq. (30) shows that the eigenvalues of the difference matrix Δ are observable quantities. In order to compute them it is convenient to use a parametrization of V due to Wolfenstein[7)],

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{33}$$

where $V_{12}=V_{us}\equiv\lambda\approx 0.23$. A, ρ and η are real, $A\approx 1$, $\rho^2+\eta^2\leq(0.6)^2$. We shall also use the empirical information that m_s/m_b, in Eq. (32), is of order λ^2 and m_c/m_t is at most of the same order.

In Ref. 2 it was shown that the order λ term in Δ vanishes and that Δ is at most of order λ^2. It was subsequently shown, in Ref. 1, that the order λ^2 term in Δ cannot vanish due to $A\approx 1$. Thus we obtain the empirical relation

$$M=M'+\mathcal{O}(\lambda^2), \tag{34}$$

which all models with hermitian mass matrices must satisfy in order to agree with data. The question is then what is the counterpart of relation (34) for nonhermitian mass matrices? From the analysis presented in Section 3 it is evident that the relevant experimentally accessible quantities in the Standard Model are MM^+ and $M'M'^+$, see Eq. (24), which can be related to each other in any arbitrary basis. In general we have

$$f(MM^+)-g(M'M'^+)=U_L^+\left[f(D^2)-Vg(D'^2)V^+\right]U_L, \tag{35}$$

where $f(x)=\sum_{n=1}^{\infty}a_n x^n$, $g(x)=\sum_{n=1}^{\infty}b_n x^n$ with a_n and b_n being arbitrary coefficients. Taking $f(x)=x$ and $g(x)=x$ gives

$$MM^+-M'M'^+=U_L^+RU_L, \tag{36}$$

where

$$R_{jk}=D_j^2\delta_{jk}-\sum_{\ell}V_{j\ell}V_{k\ell}^*D_\ell'^2. \tag{37}$$

All the elements of the matrix R are measurable quantities. Hence, the eigenvalues of the matrix $MM^+-M'M'^+$ are observables. Using Eqs. (31)-(33) we find that the three eigenvalues of this matrix are given by

$$r_{1,2} = \pm A\lambda^2 + \frac{1}{2}\left[\left(\frac{m_c}{m_t}\right)^2 - \left(\frac{m_s}{m_b}\right)^2\right] + O(\lambda^5), \qquad (38)$$
$$r_3 = O(\lambda^6).$$

Thus the relation (34) has as its basis independent counterpart the constraint

$$MM^+ - M'M'^+ = O(\lambda^2). \qquad (39)$$

Furthermore, from Eq. (38) follows that the $O(\lambda^2)$ term on the RHS of Eq. (39) is nonzero, because the eigenvalues of R, to this order are 0 and $\pm A\lambda^2$ and thus cannot all three vanish, if $A\approx 1$.

In the left-right symmetric models one could have got constraints from the relation

$$f(M^+M) - g(M'^+M') = U_R^+\left[f(D^2) - V_R\, g(D'^2)V_R^+\right]U_R, \qquad (40)$$

if V_R had been known from data.

In Ref. 2 a couple of applications of the relation (34) were given for models with hermitian mass matrices. As an application of the result (39) consider, for example, a model by Ellis et al.[8] where the mass matrices are given by

$$M = \begin{pmatrix} \theta_1^2\theta_2^2 & \theta_1\theta_2^2 & \theta_1\theta_2 \\ \theta_1\theta_2^2 & \theta_2^2 & \theta_2 \\ \theta_1\theta_2 & \theta_2 & 1 \end{pmatrix}, \quad M' = \begin{pmatrix} \epsilon_1\epsilon_2\theta_1\theta_2 & \epsilon_1\epsilon_2\theta_2 & \epsilon_1\epsilon_2 \\ \epsilon_2\theta_1\theta_2 & \epsilon_2\theta_2 & \epsilon_2 \\ \theta_1\theta_2 & \theta_2 & 1 \end{pmatrix} \qquad (41)$$

Note that M' is manifestly nonhermitian. The constraint relation (39) then immediately gives that $\theta_2 = \varepsilon_2 + O(\lambda^2)$.

To conclude this section, we have found basis independent restrictions, Eqs. (35)-(39), on mass matrices which all successful mass matrices must satisfy. The restrictions in Eqs. (30) and (34) valid in the hermitian basis are special cases of the general results.

As constructing successful mass matrices for quarks is a central issue in particle physics pursued by many people[9] the results obtained in this section should be useful for at least quickly checking if specific models have a chance of being empirically successful or not.

5. CP-VIOLATION ASYMMETRIES IN THE STANDARD MODEL

Before discussing CP-violation it is instructive to recapitulate the situation concerning parity violation in the Standard Model. The lesson learned from parity may then be used to define CP-violation asymmetries in the Standard Model.

In the Standard Model the origin of all parity violation lies in the fundamental subprocesses $W \to f\bar{f}'$ and $Z \to f\bar{f}$, where f and f' denote appropriate quark or lepton pairs. The measure of parity violation in these processes is given by the asymmetry parameter a_p defined by

$$a_p = \frac{2\mathrm{Re}(va^*)}{|v|^2+|a|^2}, \qquad -1 \leq a_p \leq 1, \tag{42}$$

where v and a denote respectively the vector and axial vector coupling constants in the fermion current (viz, $\gamma_\lambda(v+a\gamma_5)$). For W we have $v=-a=1$ i.e., $a_p=-1$ for all pairs but for processes $Z \to f\bar{f}$ the coupling constants depend on the quantum numbers of the fermion f,

$$\begin{aligned} v &= 2I_{3L} - 4Q\sin^2\theta_W, \\ a &= -2I_{3L}, \end{aligned} \tag{43}$$

where Q, and I_{3L} denote the electric charge and the third component of the weak isospin. Hence parity is maximally violated in $W \to f\bar{f}'$ subprocesses but in $Z \to f\bar{f}$ there are four fundamental asymmetry parameters, i.e., two for quarks (with Q=2/3 and Q=-1/3) and two for leptons (Q=-1 and Q=0). The asymmetry is given by

$$a_p(Z) = \frac{-2I_{3L}(I_{3L}-2Q\sin^2\theta_W)}{(I_{3L})^2+(I_{3L}-2Q\sin^2\theta_W)^2} \tag{44}$$

Thus parity is maximally violated only for neutrinos. In the remaining three cases there is a deviation from maximality due to unification with electromagnetism, $\sin^2\theta_W \neq 0$ and the nonvanishing of the electric charge, $Q \neq 0$. As far as parity is concerned I believe that there is a consensus of opinion that one should define the measure for parity violation à la Eq. (42).

Recently, a number of authors[10)] have asked the question what is meant by maximal violation of CP and is CP maximally violated? In order to answer this question one must first provide a measure for CP-violation. For example, it has been argued[10)] that a phase convention independent phase Φ in the matrix V provides such a measure, i.e., $|\sin\Phi|=1$ corresponds to maximal CP-violation.

In Ref. 1 a new definition of a measure for CP-violation was given. Indeed if one follows the lesson learned from parity violation there is not much choice in introducing the CP-asymmetry parameter a_{CP}. One must then, just like the case of parity, isolate the simplest subprocesses which exhibit CP-violation in the Standard Model and introduce an appropriate set of parameters a_{cp}, $-1 \leq a_{CP} \leq 1$, for these processes. Then all CP-violation in Nature would, in principle, be expressible in terms of the fundamental a_{cp}-parameters. CP is maximally violated in a fundamental process iff $a_{cp} = \pm 1$. If so one would actually measure maximal CP-violation in that process if the experiment could be done. This definition is very different from those previously given in the literature. For example $|\sin\Phi|=1$ does not correspond to maximal CP-violation in any physical transition.

In the Standard Model, as we have seen in Section 3 all CP-violation is related to $\mathrm{Im}\det[mm^+, m'm'^+]$, i.e., to the quantity J in Eq.(5). Evidently the simplest subprocesses which exhibit CP-violation involve four different quarks two (i and k) with charges 2/3 and two (j and ℓ) with charges $-1/3$. Thus the quantity J, appropriately normalized is the measure we are looking for. Actually, there are three ways of normalizing this quantity. These lead to three classes of CP-violation parameters, which I shall refer to as diagonal, horizontal and vertical CP-asymmetries:

a) Diagonal CP asymmetry parameters.

Consider the quark mixing matrix with one row and one column crossed out, i.e., the matrix

$$\begin{pmatrix} V_{ij} & V_{il} \\ V_{kj} & V_{kl} \end{pmatrix}. \tag{45}$$

The diagonal CP-asymmetry parameters are defined by

$$a_{CP} = \frac{2\,\mathrm{Im}\,(\alpha\beta^*)}{|\alpha|^2+|\beta|^2}\;, \qquad -1 \leq a_{CP} \leq 1 \tag{46}$$

where

$$\alpha = V_{ij} V_{kl}\;, \qquad \beta = V_{kj} V_{il}\;. \tag{47}$$

Note that the numerator in Eq.(45) is just the quantity J, up to a sign (see Eq. 5). There are 9 such diagonal CP-asymmetry parameters as there are nine different ways of crossing out one row and one column in the three-by-three matrix V. These parameters enter in 4 quark transitions involving two W bosons. They are the simplest analogs of the parity violating asymmetries, Eq.(42), when instead of one W and two quarks for CP we take two W bosons and 4 quarks. In Ref. 1 only this class of CP violating parameters was introduced.

b) Horizontal CP-asymmetry parameters.

Consider the matrix (45) again. Clearly one may also define the CP-asymmetry as in Eq.(46) but where

$$\alpha = V_{ij} V^*_{il}\;, \qquad \beta = V_{kj} V^*_{kl} \tag{48}$$

c) Vertical CP-asymmetrniy parameters.

Here again the parameters are defined as in Eq.(46) but where

$$\alpha = V_{ij} V^*_{kj}\;, \qquad \beta = V_{il} V^*_{kl}\;. \tag{49}$$

The horizontal and vertical asymmetry parameters enter in four quark transitions involving one virtual W boson. These are the parameters which enter when one, for example, compares[12] the rates of conjugate charged B-meson or D-meson decays.

In conclusion there are 27 simplest subprocesses and thus 27 CP-asymmetry parameters in the hadronic sector of the Standard Model. There are 9 diagonal, 9 horizontal and 9 vertical asymmetry parameters. In the case of parity, in the hadronic sector, there are, of course, 9 fundamental W transitions and 6 Z transitions. However, exactly as in the case of parity the different CP-asymmetries are not independent. They are all functions of four parameters, e.g. the Kobayashi-Maskawa parameters θ_1, θ_2, θ_3 and δ. Furthermore the CP-asymmetry in a subprocess is maximal if and only if in Eq.(46)

$$\alpha = \pm i \beta, \tag{50}$$

for that process. This relation puts a very strong constraint on the parameters, just like requiring maximal parity violation in $Z \to f\bar{f}$ would give constraints on I_{3L} and $Q \sin^2\theta_W$, see Eq.(44). In general only 2 of the 27 a_{cp}-parameters can be simultaneously maximal because each maximality requirement gives two constraints and we have only 4 parameters. Using the Wolfenstein parametrization, Eq.(33), we find that in Nature none of the fundamental subprocesses violates CP maximally.

This result is perhaps not so surprising. After all we are not bothered by nonmaximality of parity violation, in all fundamental transitions $Z \to f\bar{f}$, which is due to unification of weak interactions with electromagnetism. The CP-asymmetry parameters, Eqs.(46)-(49), are function of the quantities V_{ij} which are intimately related to quark mass matrices. This sector of the Standard Model, in contradistinction to its gauge sector, is very poorly understood. What is worse is that we cannot even exclude the possibility that the quark mixing matrix V is real. Then all the CP-asymmetry parameters defined in this paper would vanish and one must go beyond the Standard Model to explain CP-violation.

The moral of the above analysis is nevertheless that given a model of

CP-violation one could introduce a set of CP-asymmetry parameters, for the simplest subprocesses which exhibit CP-violation. In general then all CP-violation in Nature would be due to these subprocesses. Another important point is that the asymmetry parameters should be normalized, $-1 \leq a_{cp} \leq 1$ such that $a_{cp}=\pm 1$ corresponds to maximal CP-violation. Then, if CP is maximal in a transition the experiment (perhaps a gedanken one) would actually measure maximal violation in the sense that it will find that the CP-image of the process in question is a completely forbidden process.

6. CONCLUSIONS

In this paper two issues have been studied:

1. It has been shown that in the Standard Model, with three families there is a one-to-one correspondence between the determinant of certain commutators involving mass matrices (m and m' for charge 2/3 and -1/3 quarks respectively) and the presence/absence of CP-violation. In an arbitrary nonhermitian basis the simplest such quantity is $\mathrm{Im}(\det[mm^+, m'm'^+])$ which vanishes if and only if there is no CP-violation. Thus one may use this quantity, appropriately normalized, to define a measure of CP-violation in the Standard Model. By this measure CP is not maximally violated in any fundamental transition in Nature. Furthermore the definition of CP-asymmetries is independent of the particular choice made, i.e., $\mathrm{Im}(\det[f(mm^+), g(m'm'^+)])$, with arbitrary functions f and g as defined in Section 3 would lead to the same definition of asymmetry parameters.

2. Restrictions on mass matrices are obtained without assuming that mass matrices are hermitian. It is shown that experiments impose severe conditions, see Eqs. (34)-(39), which any model of mass matrices with the ambition to agree with data must satisfy. Earlier results[1,2], valid in the hermitian basis are special cases of the general results presented in this paper. Extension of some of the results of this paper to left-right symmetric models was also briefly discussed.

Finally, it is hoped that the results obtained in this paper may provide a hint for further work and eventually a better understanding of the mass problem.

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