A Behavioral Analysis of Stochastic Reference Dependence

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Abstract

We examine the reference-dependent risk preferences of Kőszegi and Rabin (2007), focusing on their choice-acclimating personal equilibria concept. We relate their model to other existing generalizations of expected utility. We demonstrate that linear gain-loss choice-acclimating personal equilibria is equivalent to the intersection of quadratic utility and pessimistic rank-dependent utility. However, it has only a trivial intersection (i.e. expected utility) with other reference-dependent preferences. We use these relationships to extend our understanding of Kőszegi and Rabin's model: linking their functional form to behavior; demonstrating new applications; and deriving new tests and looking for support in existing experimental data. (98 words)

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1 Introduction

The notion of reference dependence was first introduced in economics by Markowitz (1952) and was formalized by Kahneman and Tversky (1979). Their reference-dependent model, prospect theory, has become popular because it accommodates common behavior that is anomalous within the expected utility framework. However, Kahneman and Tversky, in both their original formulation as well follow-up work, did not specify how the reference point was formed, making it difficult to derive general predictions and tests. Recently, Kőszegi and Rabin (2006, 2007) proposed a model of reference dependence that specifies how individuals form their reference point. In their model consumers care about consumption utility (over final wealth) as well as gain-loss utility (utility over deviations from the reference point). The reference point is determined by the probabilistic beliefs of the decision-maker about the choice sets she will face, and the decision she will make for each choice set (i.e. expectations). Since expectations determine the reference point, Kőszegi and Rabin provide a solution concept that determines expectations endogenously. Kőszegi and Rabin's model has inspired numerous applications: Heidhues and Kőszegi (2008, 2012); Sydnor (2010); Herweg, Müller and Weinschenk (2010); Abeler, Falk, Goette and Huffman (2011); Card and Dahl (2011); Crawford and Meng (2011); Pope and Schweitzer (2011); Carbajal and Ely (2012); Karle and Peitz (2012); and Eliaz and Spiegler (2013), among others.

Despite its popularity, it can be difficult to understand the implications of Kőszegi and Rabin's model for behavior, even in simple domains, due to its complicated functional form. For example, examining choice over risk, little is known about how to distinguish their theory from other models of reference dependence, such the earlier models of Gul (1991), Bell (1985) and Loomes and Sugden (1986), all which have similar formulations as Kőszegi and Rabin (2007) but specify a different process of reference point formation.[1] More generally, it also is not clear how Kőszegi and Rabin's model relates to other model of non-expected utility theory which rely on different psychological intuitions, such as rank-dependent utility (introduced by Quiggin, 1982).

We focus on preferences induced by Kőszegi and Rabin's (2007) choice-acclimating personal equilibrium, as well as those that have linear gain-loss utility, and refer to these preferences as $\mathbb{CPE}$ (we generalize our analysis in Section 6 to accommodate non-linear gain-loss utility).[2] Kőszegi and Rabin (2007) discuss how $\mathbb{CPE}$ captures the idea of a decision-maker committing to a choice long before uncertainty is resolved (as in insurance choice). Therefore, in line with the motivation

[1]Loomes and Sugden (1982), and Bell (1982) construct models of regret, which have a very different psychological motivation; comparisons are not across different outcomes for a single lottery, but across different possible lotteries.

[2]Kőszegi and Rabin (2007) use two other solution concepts. They first consider unacclimated personal equilibrium (UPE). x is a UPE if it is better than all other options when x is a reference point. The second one is a refinement of UPE which is called the preferred personal equilibrium (PPE). Unlike PPE, which can induce intransitive choice, $\mathbb{CPE}$ induces choices that satisfy transitivity, and so can be represented using a preference relation. In independent work Freeman (2012) characterizes the PPE solution concept. He mentions examples of $\mathbb{CPE}$, including quadratic utility functionals, but does not consider the implications for choice under risk that we do.

provided by Kőszegi and Rabin (2007), the results in this paper should be interpreted in the context of choice where uncertainty will not be resolved immediately, but rather in the future, so that the chosen lottery has time to become the reference point. Due to the relative tractability of $\mathbb{CPE}$ it has been widely applied and tested (e.g. Sydnor, 2010, Herweg, Müller and Weinschenk, 2010, Abeler, Falk, Goette and Huffman, 2011, Ericson and Fuster, 2011, Gill and Prowse, 2012, and Barseghyan, Molinari, O'Donoghue, and Teitelbaum, 2013).

We first investigate the relationship between $\mathbb{CPE}$ and other major non-expected utility (non-EU) theories of choice under risk. Our first observation in Section 3 is that $\mathbb{CPE}$ is a subset of the quadratic utility class introduced by Machina (1982) and Chew, Epstein and Segal (1991). $\mathbb{CPE}$ and quadratic utility representations share the feature that they represent deviations from expected utility as utility distortions with correctly assessed probabilities. Hence deviations occur not because of mistakes in calculating objective probabilities but rather because of preferences. As Kőszegi and Rabin (2007) note; "We assume that a person correctly predicts her probabilistic environment and her own behavior in that environment, so that her beliefs fully reflect the true probability distribution of outcomes." Moreover, quadratic utility functionals share a similar psychological intuition to Kőszegi and Rabin (2007), each outcome is evaluated in comparison to all other outcomes in the support of the lottery.

We next show that $\mathbb{CPE}$ is also a subset of rank-dependent utility ($\mathbb{RDU}$). This implies that there is an equivalence between correct beliefs, but non-standard utility defined over more than just final wealth à la $\mathbb{CPE}$ and a type of distorted beliefs à la rank dependent utility, but standard utility over final wealth. Moreover, the Bernoulli utility function over degenerate outcomes (the "consumption utility" of Kőszegi and Rabin, 2007) is the same in the two representations. Therefore, even if utility over degenerate outcomes can be identified from a different context, we cannot distinguish between patterns of choice generated by $\mathbb{CPE}$ and some types of rank-dependent utility. This is surprising, since looking purely at the functional form, it would seem to be the case that $\mathbb{CPE}$ should generate similar behavior to cumulative prospect theory (formalized by Tversky and Kahneman, 1992), but without the effects of probability weighting. As we make clear, in the case of linear gain-loss utility, the correct comparison is actually the opposite — $\mathbb{CPE}$ is exactly equivalent to cumulative prospect theory, but with only the probability weighting, and no gain-loss utility.

In addition to $\mathbb{CPE}$, there are other models that attempt to capture similar psychological intuitions regarding reference dependence. These models appear on the surface to be quite close in nature. Kőszegi and Rabin (2007) themselves say, "[e]xcept that we specify the reference point as a lottery's full distribution rather than its certainty equivalent, [our] concept is similar to the disappointment-aversion models of Bell (1985), Loomes and Sugden (1986), and Gul (1991)." However, an implication of the relationship of $\mathbb{CPE}$ to quadratic preferences and $\mathbb{RDU}$ is that the intersection of $\mathbb{CPE}$ and other classical models of endogenous reference points, such as the models of disappointment aversion introduced by Gul (1991), Bell (1985), and Loomes and Sugden (1986), is only expected utility. In other words, despite trying to capture the same intuition about the effect of expectations on preferences, these models do so in distinct ways.[3] In fact, if a decision-maker exhibits preferences consistent with $\mathbb{CPE}$ and either Gul's models or Bell-Loomes-Sugden's models of disappointment aversion, then they must be expected utility maximizers; in other words they must not exhibit any reference dependence at all. Intuitively, models of reference dependence capture psychological intuitions not only about loss-aversion (or first-order risk aversion) but also other behavior, such at attitude towards randomization, or the mixing of indifferent lotteries. For example, preferences in $\mathbb{CPE}$ always exhibit aversion to the mixing of indifferent lotteries. In contrast, Gul's preferences always exhibit indifference to the mixing of already indifferent lotteries. Understanding these distinctions between models of reference dependence allows us to point out directions in which to differentiate $\mathbb{CPE}$ from other models of reference dependence using choice data, which we do in Section 7.

The relationships between $\mathbb{CPE}$ and other non-EU theories can be used as a stepping stone to generate new results linking choice behavior to $\mathbb{CPE}$'s functional form (characterization) and their notions of risk and loss aversion (comparative statics), allowing us to better understand $\mathbb{CPE}$. In Section 4 we first identify the behavioral foundations of $\mathbb{CPE}$: we show that the intersection of pessimistic $\mathbb{RDU}$ and quadratic utility is exactly $\mathbb{CPE}$. Moreover, we show that $\mathbb{CPE}$ preferences have a simple graphical representation — the indifference curves take on the shape of ellipses. The location of center of the ellipses depends only on the loss aversion parameter. On the other hand, the direction and relative lengths of the axes of the ellipses is governed solely by the consumption utility function.

Our characterization allows us to identify loss aversion and consumption utility and compare them across individuals using observed choices. Although Kőszegi and Rabin (2007) discuss how to identify loss aversion and the consumption utility function from choice behavior, it relies on individuals who make choices over risk where there is immediate resolution but delayed consequences, and so individuals do not experience any gain-loss utility. In contrast, our results hold within the standard $\mathbb{CPE}$ framework. We first identify what specifications of $\mathbb{CPE}$ are consistent with classical notions of risk aversion (i.e. aversion to mean preserving spreads). Our results point to a tight linkage between whether preferences respect the orderings induced first-order stochastic dominance and mean preserving spreads. Kőszegi and Rabin (2007) note that their preferences may not respect first-order stochastic dominance. We are not only able to characterize, in terms of parameters, when

[3] The results derived in this paper consider an arbitrary number of outcomes. If we examine choice over restricted domains the relationships can differ. If choices are defined on lotteries over only 2 outcomes, $\mathbb{CPE}$, Gul's model and the Bell-Loomes-Sugden model are all subsets of RDU. For example, for $\mathbb{CPE}$ the probability weighting function is $p + (1-\lambda)(1-p)p$. For Gul the weighting function is $\frac{p(1+\beta)}{1+\beta p}$. Because many studies only look at choices on the set of lotteries defined over two degenerate outcomes they provide limited data to distinguish many non-expected utility models from one another.

such violations occur, but also show that, unless the Bernoulli utility function is linear, whenever preferences violate first-order stochastic dominance they must also violate the ordering induced by mean-preserving spreads. That is, we can find a lottery F that is a mean-preserving spread of G and where F is preferred to G by the decision-maker.

We then go on to show to separately identify within an individual, as well as compare across individuals, the coefficient of loss aversion and the curvature of the consumption (Bernoulli) utility function using observed behavior. These results are useful because they link the functional form of $\mathbb{CPE}$, which can be difficult to use with lotteries over many outcomes, to relatively simple behavioral postulates which can often be easily verified. Moreover, the identification and comparison of these components of utility can be accomplished separately. For example, we can identify and compare loss aversion across individuals without needing to know anything about their consumption utility functions.

In Section 5 we use the relationships between $\mathbb{CPE}$ and other non-EU models of behavior to apply $\mathbb{CPE}$ preferences in a variety of new environments. For example, we show that dynamic, recursive $\mathbb{CPE}$ preferences cannot accommodate a universal preference for early (late) resolution of uncertainty (as conjectured by Kreps and Porteus, 1978) or a universal preference for one-shot resolution of uncertainty (as conjectured by Dillenberger, 2010). Moreover, we show that $\mathbb{CPE}$ suffers from a very similar calibration critique to the one Rabin (2000) leveled against expected utility: plausible choices over small stakes lotteries imply implausible choices over large stakes lotteries. Thus, in order to address the Rabin critique we must look beyond linear gain-loss functions.

In Section 6, we do exactly this, considering more general forms of $\mathbb{CPE}$, where the gain-loss utility function may not be linear. We discuss whether and how our results from previous subsections extend when more general functional forms are allowed.

In Section 7 we build on our previous results to better relate Kőszegi and Rabin's model, as well as other models of non-expected utility, to experimental data. First, our results allow us to use much of the existing literature on experimental choice over risky outcomes to directly test $\mathbb{CPE}$. Thus, our results allow us to expand the set of relevant evidence that can be used to evaluate the predictions of $\mathbb{CPE}$. In looking at this evidence, we find relatively strong evidence against linear gain-loss utility, and mixed evidence for more general forms of gain-loss utility. Second, we show how to use existing experiments originally designed to test $\mathbb{CPE}$, such as Abeler, Falk, Goette and Huffman (2011), to shed light on other models of non-expected utility. Third, we discuss how the behavioral equivalence between different models of non-expected utility can influence our ability to identify preference parameters — in particular the shape of the probability weighting function and the coefficient of loss aversion. Last, we show how the relationships we develop in Section 3 provide new ways to experimentally distinguish $\mathbb{CPE}$ from other models of reference dependence. Section 8 concludes.

2 Kőszegi-Rabin Preferences

Consider an interval $[w, b] = X \subset \mathbb{R}$ of money. Let Δ_X be the set of all simple lotteries (i.e. probability measures with finite support) on X. A lottery $F \in \Delta_X$ is a function from X to $[0, 1]$ such that $\sum_{x \in X} F(x) = 1$ and the number of prizes with non-zero probability is finite. $F(x)$ represents the probability assigned to the outcome x in lottery F. For any lotteries F, G we let $\alpha F + (1-\alpha)G$ be the lottery that yields x with probability $\alpha F(x) + (1-\alpha)G(x)$. Denote by δ_x the degenerate lottery that yields x with probability 1, i.e. $\delta_x(x) = 1$. We will also refer to δ_x simply as x. We will often only consider lotteries over three outcomes. For three outcomes $x, y, z \in X$, we denote the unit simplex of possible lotteries over those three outcomes as $\Delta_{x,y,z}$, or for an arbitrary set of 3 outcomes, Δ_3. We will refer to the best outcome as $\bar{\delta}$, the worst outcome as $\underline{\delta}$ and the middle outcome as $\hat{\delta}$. $\succsim$ is a weak order over Δ_X which represents the decision-maker's preferences over lotteries.

Kőszegi and Rabin (2006), building on Bowman, Minehart and Rabin (1999), extend the idea of Tversky and Kahneman (1979) by having individuals' utility depend both on gain-loss utility (the comparison of outcomes to a reference) and consumption utility (which depends only on the absolute value of the outcomes, rather than a comparison to a referent). This formulation is applied to lotteries in Kőszegi and Rabin (2007): the utility value of a lottery F with a reference lottery F' to a decision-maker (DM) is

$$U(F|F') = \underbrace{\sum_x u(x)F(x)}_{\text{consumption utility}} + \underbrace{\sum_x \sum_y g(u(x) - u(y))\ F(x)F'(y)}_{\text{gain-loss utility}}$$

where u is a consumption (Bernoulli) utility function over final wealth which is increasing and g is a gain-loss function, which is also increasing. Like much of Kőszegi and Rabin's (2007) analysis, we, for the moment, focus on linear gain-loss functions, where g can be written as

$$g(z) = \begin{cases} z & \text{if } z \geq 0 \\ \lambda z & \text{if } z < 0 \end{cases}$$

where $\lambda \geq 1$ is the coefficient of loss aversion.[4,5] If $\lambda = 1$ the preferences reduce to expected utility.

As mentioned, $\mathbb{CPE}$ is meant to capture situation where at the time of resolution of uncertainty,

[4]Although here the gain-loss functional is linear in the difference between the consumption utility levels, we discuss more general gain-loss functionals later in the paper.

[5]Although we focus on loss averse agents, for which $\lambda \geq 1$ our results naturally extends to loss loving agents where $\lambda \leq 1$.

the choice is indeed the reference point. Hence, the value of a lottery F can be written as:

$$V_{\mathbb{CPE}}(F) = U(F|F) = \sum_x u(x)F(x) + \sum_x \sum_y g(u(x) - u(y))\ F(x)F(y)$$

In a slight abuse of notation we will use $\mathbb{CPE}$ to refer to the set of preferences that can be represented using $V_{\mathbb{CPE}}(F)$, as well as the representation itself. We refer to the set of monotone utility functionals (in the sense of respecting first-order stochastic dominance) within $\mathbb{CPE}$ as $\mathbb{CPE}_M$. Moreover, we will denote expected utility functionals as the class $\mathbb{EU}$. Our formulation of choice leads us to a definition that is analogous to that in Kőszegi and Rabin (2007):

Definition: *F is a choice-acclimating personal equilibrium (CPE) of a choice set S if $F \in S$ and $V_{\mathbb{CPE}}(F) \geq V_{\mathbb{CPE}}(G)$ for all G in S.*

3 Relation to Other Models

In this section we explore the relationship between $\mathbb{CPE}$ and other important classes of non-EU preferences.

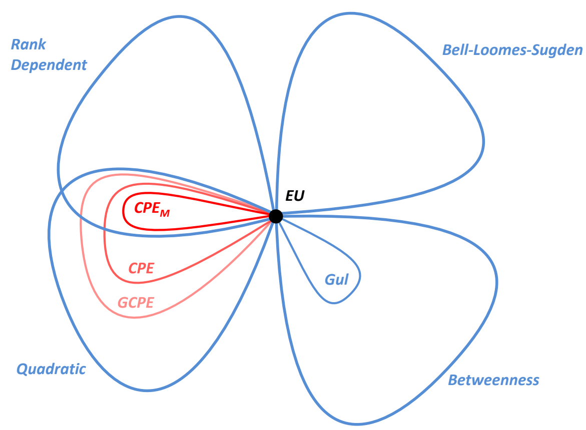

Figure 1: Summary of Relationship of Non-EU Models

We show that $\mathbb{CPE}$ is a subset of quadratic utility functionals as well as rank-dependent preferences. We also show that the intersection of $\mathbb{CPE}$ with a second major model of endogenous reference formation, Gul's (1991) disappointment aversion, is only expected utility. This intuition is quite broad — the intersection of $\mathbb{CPE}$ with two generalizations of Gul, betweenness preferences and NCI preferences (introduced by Dillenberger, 2010), is also only expected utility. Moreover, we show that $\mathbb{CPE}$ is distinct (i.e. the intersection of the two classes is only expected utility) from the disappointment aversion functionals introduced by Bell (1985) and Loomes and Sugden (1986), where the reference point is the expected Bernoulli utility of the lottery. Figure 1 summarizes many of these results.[6,7]

3.1 Quadratic Utility

A utility functional is said to be quadratic in probabilities if it can be expressed in the form

$$V_{\mathbb{Q}}(F) = \sum_x \sum_y \phi(x,y)\ F(x)F(y)$$

where $\phi : X \times X \to \mathbb{R}$ is a symmetric function (i.e. $\phi(x,y) = \phi(y,x)$ for all x, y).[8] The quadratic functional form was introduced in Machina (1982) and further developed in Chew, Epstein, Segal (1991, 1994). We denote the set of utility functions in the quadratic class as $\mathbb{Q}$. One can think of ϕ as function that compares any given outcome to any other given outcome (e.g. it gives the value of x when y is the reference point). The value of a lottery is then the average value of all of those comparisons over the outcomes with positive support. Viewed this way, the intuition for $\mathbb{Q}$ is very similar to that of $\mathbb{CPE}$. In fact, our first observation points out that $\mathbb{CPE}$ is a subset of quadratic utility.[9,10,11]

Observation 1 $\mathbb{CPE}$ *is a proper subset of* $\mathbb{Q}$*.*

To see why this is true, first observe that we can rewrite the consumption utility term:

$$\sum_x u(x)F(x) = \sum_x \sum_y \frac{u(x)+u(y)}{2}\ F(x)F(y)$$

Next, consider the gain-loss utility. Pick any two outcomes x and y. Then the sum of $g(u(x) - u(y))\ F(x)F(y)$ and $g(u(y) - u(x))\ F(y)F(x)$ is equal to $(1-\lambda)|u(x) - u(y)|\ F(x)F(y)$.[12] Thus we can write

$$\sum_x \sum_y g(u(x)-u(y))\ F(x)F(y) = \sum_x \sum_y \frac{(1-\lambda)|u(x)-u(y)|}{2}\ F(x)F(y)$$

[6] The fact that the intersection of Betweenness and Quadratic preferences, and of Betweenness and Rank-Dependent preferences, is only expected utility is a known result. The relationship between Bell-Loomes-Sugden's disappointment aversion and the other models is not previously known and shown in Masatlioglu and Raymond (2014).

[7] $\mathbb{GCPE}$ refers to preferences which do not necessarily have a linear gain-loss function (see Section 6).

[8] There is no loss of generality in restricting ϕ to be symmetric, since an arbitrary $\phi(x,y)$ can always be replaced by $\frac{\phi(x,y)+\phi(y,x)}{2}$.

[9] Freeman (2012) independently points out the same relationship.

[10] Our observation is in terms of relationships between functional forms. We can also restate it in terms of behavior. For example, suppose preferences are in $\mathbb{CPE}_M$. Then they satisfy either i) independence or ii) projective independence (Chew, Epstein and Segal, 1994) and non-betweenness.

[11] Although we present the proof for Observation 1 in the text, the proofs for the rest of our claims are in the Appendix.

[12] This simply extends the idea of Proposition 12 in Kőszegi and Rabin (2007).

Observation 1 follows from by defining ϕ as follows

$$\phi(x,y) = \frac{1}{2}(u(x)+u(y)) + \frac{1}{2}(1-\lambda)|u(x)-u(y)|$$

where λ is the coefficient of loss aversion in $V_{\mathbb{CPE}}$. So long as $1 \leq \lambda < \infty$ preferences over degenerate lotteries respect monotonicity. However, as discussed by Kőszegi and Rabin (2007), $\mathbb{CPE}$ does not necessarily respect first-order stochastic dominance. Proposition 7 of Kőszegi and Rabin (2007) points out that if loss aversion is a strong enough factor in preferences then a decision-maker will always choose the best degenerate lottery, thus violating first-order stochastic dominance. In fact, using Chew, Epstein, Segal (1991), we can clarify exactly when violations of first-order stochastic dominance occur: if and only if $1 \leq \lambda \leq 2$. In other words, when $\lambda > 2$, then there exists a non-degenerate lottery *strictly* worse than the worst degenerate lottery.[13]

Observation 2 *Assume $\succsim$ is in $\mathbb{CPE}$. Then $\succsim$ in $\mathbb{CPE}_M$ if and only if $1 \leq \lambda \leq 2$.*

3.2 Rank-Dependent Utility

A utility functional $V_{\mathbb{RDU}}$ is a rank-dependence expected utility functional if there exists a function v and a strictly increasing, continuous function $w : [0,1] \to [0,1]$, with $w(0) = 0$ and $w(1) = 1$, such that

$$V_{\mathbb{RDU}}(F) = \sum_x v(x) \left[w\left(\sum_{y \geq x} F(y)\right) - w\left(\sum_{y > x} F(y)\right)\right]$$

This form was introduced in Quiggin (1982) and has been examined by numerous authors — see Abdellaoui (2002) for a recent characterization and references to the larger literature. We will use $\mathbb{RDU}$ to denote the class of rank-dependent utility functionals and the preferences that they represent. Observe that when w is the identity function $V_{\mathbb{RDU}}$ reduces to expected utility. Otherwise, w acts to distort the decumulative distribution function associated with lottery F. $w\left(\sum_{y \geq x} F(y)\right) - w\left(\sum_{y > x} F(y)\right)$ measures the marginal probability contribution of x to the distorted decumulative distribution function.

The intuition for $\mathbb{RDU}$ goes back to the idea, first formalized by Kahneman and Tversky (1979), that individuals engage in probability weighting— they distort objectively given probabilities in a consistent fashion. $\mathbb{RDU}$ accommodates probability weighting while ensuring preferences respect first-order stochastic dominance.

[13] Many applications of reference dependence set $\lambda \geq 2$, which, in combination with $\mathbb{CPE}$, implies that preferences are not monotone. However, the studies that estimate λ do not use the functional form $V_{\mathbb{CPE}}$, and so it is not clear how to interpret their estimates of λ in the context of $\mathbb{CPE}$. See Section 7 for more detailed discussion of this issue.

Our next result points out that $\mathbb{CPE}_M$ is a subset of $\mathbb{RDU}$. Moreover, this relationships is strict as $\mathbb{RDU}$ respect first-order stochastic dominance. Not only do $\mathbb{CPE}_M$ preferences have an equivalent $\mathbb{RDU}$ representation, it is the case that the distortions (relative to expected utility) for a given consumption utility u introduced by gain-loss comparisons is equivalent to distortions induced by a probability weighting function, with the same u.

Proposition 1 *For any preference $\succsim$ in $\mathbb{CPE}_M$ with a representation (u, λ) there exists a function w_λ such that (u, w_λ) is a $\mathbb{RDU}$ representation of $\succsim$. Moreover, $w_\lambda(z) = (2-\lambda)z + (\lambda-1)z^2$.*

For all $\lambda \in [1, 2]$, w_λ is a convex function, which means that preferences are "pessimistic"— worse outcomes are overweighted. Therefore, Proposition 1 implies that loss aversion can be considered a type of pessimism, in that loss averse individuals overweight bad outcomes.

3.3 Other Endogenous Reference-Dependent Models

Gul's Disappointment Aversion

An important model of endogenous reference formation is Gul's (1991) model of disappointment aversion. It was developed to accommodate behavior such as the Allais paradox while still maintaining much of the structure and tractability of expected utility theory. A utility functional V is said to be in the class of Gul's disappointment aversion models $\mathbb{G}$ if it can be expressed in the form

$$V_{\mathbb{G}}(F) = \sum_x u(x)F(x) + \beta \sum_{x \leq u^{-1}(V_{\mathbb{G}}(F))} \left(u(x) - V_{\mathbb{G}}(F)\right) F(x)$$

This model imposes a penalty, proportional to β, on disappointing outcomes — those that lie below the threshold of $u^{-1}(V_{\mathbb{G}}(F))$, the certainty equivalent of the lottery. When $\beta = 0$, this model is identical to $\mathbb{EU}$ (i.e. there is no penalty for disappointing outcomes). If $\beta > 0$ then the individual is disappointment averse, and if $\beta < 0$ she is disappointment loving.

$V_{\mathbb{G}}$ resembles $V_{\mathbb{CPE}}$. They both feature consumption utility and comparative utility with respect to a reference point. Nevertheless, there are two key differences. The first is that in Gul's model there is only loss utility but no gain utility. The second is that the reference point is not a distribution but rather the certainty equivalent of the lottery.

A commonly used generalization of $\mathbb{G}$ are betweenness preferences. Betweenness preferences, $\mathbb{B}$, were introduced by Chew (1983) and Dekel (1986). Betweenness functionals have the form

$$V_{\mathbb{B}}(F) = \sum_x \nu(x, V_{\mathbb{B}}(F))F(x)$$

where ν is continuous and an increasing function of its first argument. The axiom that characterizes these preferences, Betweenness, is a weakening of independence. It says that if two lotteries F and G are indifferent, any mixture of F and G must be indifferent to F.

Betweenness: If $F \sim G$, then $\alpha F + (1-\alpha)G \sim F$ for all $\alpha \in [0,1]$.

Denote the class of preferences, and their associated functionals, that satisfy betweenness, monotonicity, and the standard technical assumptions as $\mathbb{B}$.

A second generalization of $\mathbb{G}$ was introduced by Dillenberger (2010) and discussed in more detail in Cerreia-Vioglio, Dillenberger and Ortoleva (2013). Dillenberger (2010) formulates 'negative certainty independence' (NCI), which captures a certainty effect. The idea is that an individual has a premium for certainty. Therefore, if a degenerate lottery is worse than some other lottery, then mixing both with a third lottery in equal proportions cannot reverse that preference.

Negative Certainty Independence: If $\delta_x \precsim G$, then $\alpha\delta_x + (1-\alpha)F \precsim \alpha G + (1-\alpha)F$ for all $\alpha \in [0,1]$.

Call the class of preferences (and their associated functionals) that satisfy negative certainty independence the standard technical assumptions and respect first-order stochastic dominance $\mathbb{NCI}$. $\mathbb{NCI}$ preferences have a the following representation:

$$V_{\mathbb{NCI}}(F) = \inf\nolimits_{v \in W} \; v^{-1}\left(\sum_x v(x)F(x)\right)$$

where W is a set of strictly increasing utility functions over outcomes.

It turns out that despite the similarities between $\mathbb{CPE}$ and $\mathbb{G}$, the two models are conceptually distinct relaxations of the expected utility. Moreover, $\mathbb{CPE}$ is distinct not only from Gul's model but also from the two aforementioned generalizations of it.

Observation 3 $\mathbb{CPE} \cap \mathbb{B} = \mathbb{CPE} \cap \mathbb{NCI} = \mathbb{CPE} \cap \mathbb{G} = \mathbb{EU}$.

Disappointment aversion, along with its generalizations, and $\mathbb{CPE}$ are capturing endogenous loss aversion in distinct ways. An preferences that are fully consistent with $\mathbb{CPE}$'s notion of reference-dependence, as well as $\mathbb{G}$'s model of reference dependence (or either $\mathbb{B}$ or $\mathbb{NCI}$) must not exhibit any reference dependence at all — they must be $\mathbb{EU}$.

These distinctions point out that models of reference dependence capture intuitions not only about loss aversion or first-order risk aversion, but also about other important behavior, such as attitudes towards randomization. Consider two lotteries F and G that a decision-maker is indifferent between. If her preferences are in $\mathbb{B}$, then she must also be indifferent between F and any mixture of F and G. If her preferences are in $\mathbb{NCI}$ then she must weakly prefer any mixture of F and G to F. And, as we will see in Section 4, if her preferences are in $\mathbb{CPE}$, she must weakly prefer F to any mixture of F and G. These distinctions can allow us to distinguish between models of reference dependence, a theme we return to in Section 7.

Bell-Loomes-Sugden's Disappointment Aversion

Predating both Kőszegi and Rabin (2007) and Gul (1991) is the model of disappointment aversion introduced by Bell (1985) and Loomes and Sugden (1986). This model has proven a quite popular class of models in applications, as the reference point is neither stochastic nor recursively defined, but is instead simply the expected consumption (Bernoulli) utility of the lottery. Given a function u, denote the expected value of u given lottery F as $E_u(F)$. The value of a lottery is then:

$$V_{\mathbb{BLS}}(F) = \sum_x u(x)F(x) + \sum_x g(u(x) - E_u(F))F(x)$$

where g is a piece-wise linear function with $g(0) = 0$ (as in $V_{\mathbb{CPE}}$).

We denote the this set of functionals, and their associated preferences, as $\mathbb{BLS}$.[14] This model also resembles $\mathbb{CPE}$. Again, it is based consumption utility plus comparative utility. However, here the reference point is the expected consumption utility of the lottery as opposed to the distribution of utilities. Despite these similarities, but like the result in the previous subsection, we find that $\mathbb{BLS}$ and $\mathbb{CPE}$ are capturing endogenous loss aversion in distinct ways. As with $\mathbb{CPE}$'s relationship to $\mathbb{G}$, although $\mathbb{CPE}$ and $\mathbb{BLS}$ capture similar attitudes towards loss aversion, they have distinct implications about attitudes towards other phenomena, such as the mixing of indifferent lotteries, which serve to distinguish them behaviorally.

Observation 4 $\mathbb{CPE} \cap \mathbb{BLS} = \mathbb{EU}$.

4 A Characterization and Comparative Statics

4.1 Shapes of Indifference Curves

In order to better understand $\mathbb{CPE}$ we first investigate the structure of the indifference curves induced by $\mathbb{CPE}$. Quadratic functionals have simple graphical representations in the Marschak-Machina triangle; their indifference curves are conic sections (i.e. ellipses, parabolas or hyperbolas). Therefore, preferences in $\mathbb{CPE}$ must have indifference curves that are conic sections; in fact, they are always sections of concentric ellipses.

[14] The original papers introducing this model do not require g to be piecewise linear, but we make this restriction in order to make the model as comparable to $\mathbb{CPE}$ as possible.

Observation 5 *For any Δ_3, $\mathbb{CPE}$ preferences generate indifference curves that are concentric ellipses. The shared center of the ellipses lies on a line that passes through the best and worst degenerate outcomes. Moreover, the axes of ellipses are oriented parallel to the best-medium outcome edge and the worst-medium outcome edge.*

The following figure demonstrates what the indifference curves appear like in the Marschak-Machina triangle for $\lambda = 2$ and when the utilities of three outcomes, b (best), m (middle), and w (worst), are equally spaced, showing the center of the concentric ellipses (C), and how the indifference curves extend beyond the unit simplex.

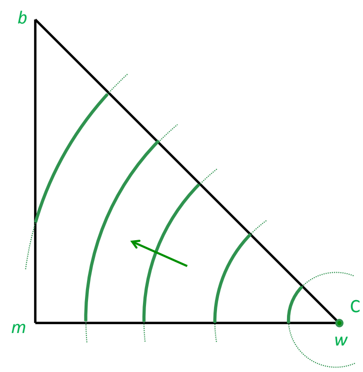

Figure 2: $\mathbb{CPE}$ Indifference Curves when $u(b) - u(m) = u(m) - u(w)$

$\mathbb{CPE}$ functionals have two factors that influence how lotteries are valued: the consumption (Bernoulli) utility function u and the coefficient of loss aversion λ. Each of these determines a particular part of the structure of the ellipse. First, the center of the ellipses is always along the best to worst outcome line; moreover, the center's location along the best to worst outcome line varies with λ, but not with u. For example, if gain-loss utility is a strong component of $V_{\mathbb{CPE}}$ ($\lambda > 2$) so that preferences are non-monotone, then the center is within the unit simplex and is the worst lottery. As the loss aversion coefficient falls to 1, the center shifts down and to the right. $\mathbb{EU}$ is when the center is infinitely far from the best outcome. Figure 3 demonstrates how the center changes with λ, with C_i being the center of the concentric ellipses for individual i, where $i = A, B$, and C. It is the case that individual A is more loss averse than individual B, who is more loss averse than individual C: $\lambda_A > \lambda_B = 2 > \lambda_C$.

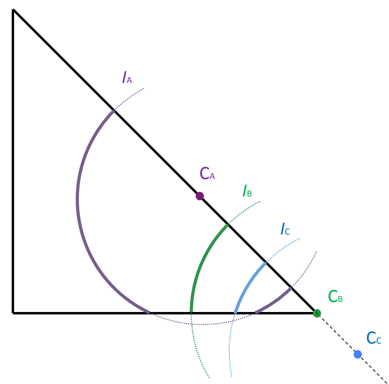

Figure 3: Changing Loss Aversion Coefficient (λ): $\lambda_A > \lambda_B = 2 > \lambda_C$

While λ governs the location of the center of the ellipses, it does not affect their orientation (i.e. the orientation and relative length of the axes of the ellipses). Instead, this is governed solely by the consumption utility functional u. The two axes of the ellipses are always aligned with the horizontal and vertical axes of the unit simplex — the edge connecting the medium and worst outcomes and the edge connecting the medium and best outcome respectively. If the individual is consumption risk neutral (i.e. u is linear) and the outcomes are equally spaced (i.e. $w = 0$, $m = 1$ and $b = 2$), then their indifference curves are circles. As the individual becomes more consumption risk averse, i.e. u becomes more concave, the vertical axis becomes relatively longer than the horizontal axis (and vice versa for less consumption risk averse). Figure 4 demonstrates what happens as consumption risk aversion changes. Individual B is more consumption risk averse than individuals A — u_B is more concave than u_A.

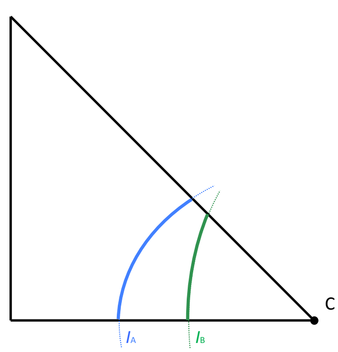

Figure 4: Changing Consumption Risk Aversion (u)

Later in this section we formalize these graphical intuitions and relate the constituent parts of $\mathbb{CPE}$ functionals, u and λ, to observable preferences.

4.2 Characterization

In the previous section we observed that $\mathbb{CPE}_M$ is a subset of both $\mathbb{Q}$ and $\mathbb{RDU}$. We now discuss what additional restrictions are needed within the set $\mathbb{Q} \cap \mathbb{RDU}$ to fully identify $\mathbb{CPE}_M$. These additional restrictions provide insight regarding the behavioral implications of $\mathbb{CPE}_M$.

A well known property in the literature on non-EU is mixture aversion. Preferences satisfy mixture aversion if it is the case when two lotteries are indifferent, then any mixture of them must be worse than the original lotteries (mixture loving can be defined analogously).

Mixture Aversion (MA): If $F \sim G$, then $\alpha F + (1-\alpha)G \precsim F$ for all $\alpha \in [0,1]$

Mixture aversion is not a necessary condition of either $\mathbb{Q}$ or $\mathbb{RDU}$. However, $\mathbb{CPE}$ preferences are always mixture averse.[15] For monotone preferences, this observation is a corollary of Wakker (1994), who showed that for preferences in $\mathbb{RDU}$, pessimism is equivalent to mixture aversion.

Observation 6 *Every preference in* $\mathbb{CPE}$ *satisfies mixture aversion.*

The restrictions on preferences implied by $\mathbb{Q}$ and $\mathbb{RDU}$, along with mixture aversion, are enough to characterize $\mathbb{CPE}_M$.[16]

Theorem 1 $\succsim$ *satisfies MA and is in* $\mathbb{Q} \cap \mathbb{RDU}$ *if and only if it is in* $\mathbb{CPE}_M$*.*

Theorem 1 completely characterizes $\mathbb{CPE}$ preferences. Moreover, we can readily translate theorem into observable preferences. $\mathbb{Q}$ has been characterized using preferences in Chew, Epstein and Segal (1991, 1994). There exist numerous characterizations of $\mathbb{RDU}$ using preferences; a recent one is Abdellouai (2002). One way to interpret Theorem 1 is to recall that pessimistic rank-dependent preferences are exactly those rank-dependent preferences that satisfy MA. Thus $\mathbb{CPE}_M$ is equivalent to the intersection of $\mathbb{Q}$ and pessimistic $\mathbb{RDU}$.

In order relate u and λ to behavior, as we do in the following subsections, we first need to know to what extent u and λ are uniquely identified from behavior.

Proposition 2 *For any preference in* $\mathbb{CPE}$*, u is unique up to affine transformation and λ is unique.*

Proposition 2 demonstrates that u has the standard property of being unique up to affine transformations. In addition, the coefficient of loss aversion λ is uniquely determined by behavior.

[15] Freeman (2012) looks at mixture aversion in the context of other solution concepts of Kőszegi and Rabin (2007), but his results are formally unrelated to ours.

[16] If we keep the functional form of $\mathbb{CPE}$ but allow λ to take on any value between 0 and 2, and call such preferences $\hat{\mathbb{CPE}}$, then an extension of Theorem 1 shows that $\hat{\mathbb{CPE}} = \mathbb{Q} \cap \mathbb{RDU}$.

4.3 Comparative Classical Risk Aversion

Given the uniqueness of the representation, we can analyze how behavior relates u and λ (i.e. risk and loss aversion). First, we examine when individuals' observed preferences are in accordance with the classical notion of risk aversion — aversion to mean preserving spreads.[17]

Definition $\succsim$ *is risk averse if whenever G differs from F by a mean preserving spread, $F \succsim G$.*

Intuitively it seems to be the case that loss aversion should enhance any aversion to mean preserving spreads that u alone induces. This is true if u is linear. However, more generally it is not the case that there is a 'trade-off' between risk and loss aversion in terms of observed behavior. Instead both a concave u and loss aversion are necessary conditions for an individual to be risk averse.[18] These conditions are not sufficient though. An individual who has a non-linear u, and is also so loss averse so that their preferences are no longer monotone, will not always be averse to mean preserving spreads.

Proposition 3

1. *If $\succsim$ is in $\mathbb{CPE}_M$ then $\succsim$ is risk averse if and only if u is concave.*
2. *If $\succsim$ is in $\mathbb{CPE} \backslash \mathbb{CPE}_M$ then $\succsim$ is risk averse if and only if u is linear.*

Kőszegi and Rabin (2007) develops intuitions relating riskiness of a lottery to preferences when u is linear (e.g. their Proposition 13): adding mean-preserving risk to a degenerate lottery reduces the value of the lottery for preferences in $\mathbb{CPE}$. However, Proposition 3 implies that these intuitions relating riskiness to preferences do not generalize when preferences are non-monotone and u is non-linear. Not only do all non-monotone preferences violate first-order stochastic dominance, but when u is non-linear they also violate second order stochastic dominance. Thus, respecting the ordering induced by first-order stochastic dominance is a necessary condition for preferences to respect the ordering induced by mean preserving spreads whenever an individual's Bernoulli utility function is not linear. This implies that intuitions developed around increases in risk for $\mathbb{CPE}_M$ will generally not extend to $\mathbb{CPE} \backslash \mathbb{CPE}_M$. Moreover, Kőszegi and Rabin (2007) mention that violations of first-order stochastic dominance could be interpreted as a form of risk aversion: "[w]e also feel that the preference for a stochastically dominated lottery captures in extreme form the strong risk aversion consumers display." In fact, as Proposition 3 points out, unless u is linear, violations of first-order stochastic dominance are inconsistent with standard notions of risk aversion.

[17] An alternative way of defining risk aversion is that the certainty equivalent of a lottery is less than the expected value of that lottery. $\mathbb{CPE}_M$ are risk averse in this sense if and only if u is concave. But in the case of non-monotone $\mathbb{CPE}$ a certainty equivalent is not always well defined.

[18] A simple extension of Proposition 3 shows that an individual who is a loss lover cannot be risk averse.

We can extend Proposition 3 in order to compare risk aversion across individuals. Following Chew, Karni and Safra (1987), we define comparative risk aversion.

Definition: *Individual A is more risk averse than B if $G \succsim_A F$ whenever $F \sim_B G$ and there exists an $x_0 \in X$ such that $F(x) \geq G(x)$ for all $x < x_0$ and $F(x) \leq G(x)$ for all $x \geq x_0$.*

Given the results of Proposition 3 we will restrict ourselves to considering preferences in $\mathbb{CPE}_M$. As Proposition 3 would suggest, it is the case that the relative curvature of u and value of λ jointly determine comparative risk aversion.

Proposition 4 *Let $\succsim_i$ be in $\mathbb{CPE}_M$ for $i \in \{A, B\}$ and represented by (u_i, λ_i). Then individual A is more risk averse than individual B if and only if u_A is a concave transformation of u_B and $2 \geq \lambda_A \geq \lambda_B \geq 1$.*

Again, it is not the case that there is a tradeoff between the concavity of u and the amount of loss aversion. Proposition 4 tells us that for A to be more risk averse than B it must be the case that u_A is more concave than u_B and A is more loss averse than B. Regardless of how much more concave u_A is than u_B, if B is even slightly more loss averse than A, then A cannot be more risk averse than B.

4.4 Identifying and Comparing Loss Aversion

We now turn to examining the relationship between behavior and the coefficient of loss aversion λ. Given the results of the previous sub-section we will focus on the case of monotone preferences.[19] Many people intuitively interpret loss aversion, as a separate phenomenon from aversion to risk, as implying that individuals should be averse to small-stakes lotteries — for example, Bowman, Minehart and Rabin (1999). In line with this Kőszegi and Rabin's (2007) specifically describe $\lambda > 1$ as capturing loss aversion over small stakes. Extending this, we will demonstrate how to both identify λ from choice behavior and compare λ across individuals using behavior. In order to do so we will first define comparative loss aversion between two individuals using the relative sizes of λ.

Definition: *Let $\succsim_i$ be in $\mathbb{CPE}_M$ for $i \in \{A, B\}$ and represented by (u_i, λ_i). Individual A is more loss averse than individual B if $1 \leq \lambda_B \leq \lambda_A \leq 2$.*

This definition is in terms of a parameter of the model, rather than behavior. We first relate the parameter λ to risk preferences over small-stakes lotteries, showing that the intuitive relationship between λ and small-stakes risk preferences holds. However, this relationship is not directly testable in terms of behavior, because it relies on a result that holds only at stakes get arbitrarily small.

[19]The results can easily be extended to include non-monotone preferences.

Therefore, we go on to develop new tools that characterize comparative loss aversion using directly testable behavior.

In order to relate λ to small-stakes risk preferences we will rely on Segal and Spivak's (1990) analysis of first-order risk aversion. Individuals who are first-order risk averse display an aversion to small-stakes lotteries. As in Segal and Spivak (1990) we will measure the extent to which individuals dislike small-stakes lotteries using the notion of risk premium, i.e. the difference between the expected value and certainty equivalent of a lottery. Denote $\pi(F)$ as the risk premium of the lottery F. An individual is first-order risk averse if the derivative of the risk premium of a fair lottery does not goes to zero as the stakes in the lottery become arbitrarily small. We will focus on looking situations where individuals have a wealth level w and are facing a lottery ϵF, where ϵ is a scalar that multiplies the the sizes of all the outcomes in lottery F. We denote this situation as $w + \epsilon F$.

Definition: *Preferences exhibit first-order risk aversion at wealth level w if for all $F \neq \delta_0$, where $E(F) = 0$, $\frac{\partial \pi(w+\epsilon F)}{\partial \epsilon}|_{\epsilon=0^+} \neq 0$.*

If $\frac{\partial \pi(w+\epsilon F)}{\partial \epsilon}|_{\epsilon=0^+} \neq 0$ then the individual not risk neutral over arbitrarily small lotteries. Moreover, as Segal and Spivak (1990) observe, if $\frac{\partial \pi(w+\epsilon F)}{\partial \epsilon}|_{\epsilon=0^+} < 0$ then the individual will refuse all better than fair sufficiently small lotteries. Thus, in a similar exercise to Bowman, Minehart and Rabin (1999), we can link λ to behavior (refusing small-stakes lotteries). First, it is the case that $\mathbb{CPE}$ preferences can exhibit first-order risk aversion, and whether they do or not is governed entirely by λ. In order to simplify our statements, we will make the assumption that u is differentiable everywhere on it's domain (which we refer to as 'everywhere differentiable').[20]

Proposition 5 *Let $\succsim$ be in $\mathbb{CPE}_M$ and represented by (u, λ) with u everywhere differentiable. Then $\succsim$ is first-order risk averse at all wealth levels if and only if $\lambda > 1$.*

Moreover, we can extend previous analyses by ordering individuals' degree of first-order risk aversion by the absolute size of $\frac{\partial \pi(w+\epsilon F)}{\partial \epsilon}|_{\epsilon=0^+}$.

Definition: *Individual A is more first-order risk averse than individual B at wealth level w if $\frac{\partial \pi(w+\epsilon F)}{\partial \epsilon}|_{\epsilon=0^+} \leq \frac{\partial \pi(w+\epsilon F)}{\partial \epsilon}|_{\epsilon=0^+}$.*

This definition allows us to relate λ to preferences over small stakes lotteries (as captured by the risk premia attached to those lotteries).

Proposition 6 *Suppose $\succsim_i$ is in $\mathbb{CPE}_M$ for $i \in \{A, B\}$ and represented by (u_i, λ_i) with u_i everywhere differentiable. Then individual A is more loss averse than individual B if and only if A is more first-order risk averse than B at all wealth levels.*

[20] If u is not differentiable, then because it is monotonically increasing it must be the case that it is differentiable almost everywhere and the definitions and propositions can be appropriately modified.

Although the definition of first-order risk aversion, and its associated behavior of refusing small-stakes better than fair lotteries, capture an intuitive description of what we think loss aversion should be, they are not behaviorally verifiable — it is impossible to know whether the lotteries we are examining are of small enough scale in order to generate behavior that is dependent only on λ. However, we know that loss aversion does have behavioral content — if preferences are in $\mathbb{CPE}_M$ then they are loss averse if and only if the preferences violate the independence axiom. We can extend these intuitions to show that comparative loss aversion also has behaviorally verifiable content.

A key factor in understanding the behavioral content of loss aversion is the fact that observed choices over lotteries (i.e. observed risk aversion) is generated by both λ and u. We want to observe choices that relate only to the value of λ and not to u. In order to understand what choices these might be we first must develop intuition regarding the relaxation of Independence that holds for $\mathbb{CPE}$ preferences. This will be useful not only for understanding the results of this subsection but also the next.

In order develop intuition we will revisit an insight from Chew, Epstein and Segal (1991, 1994). They consider the 'expansion paths' of the indifference curves. Two points F and G lie on the same expansion path if there is a common sub-gradient to the indifference curves at F and G. Chew, Epstein and Segal (1991) show that for quadratic utility functionals, all expansion paths are linear and perspective (they share a single point of intersection). Moreover, Independence holds along the expansion paths. In other words mixing along expansion paths does not affect the direction of preferences.[21]

In order to understand this formulation more clearly, we will construct expansion paths within the interior of the Marschak-Machina triangle. Consider a $F_\alpha = \alpha\bar{\delta} + (1-\alpha)\underline{\delta}$ and a slope $\sigma > 0$. Denote the set of lotteries (a 'budget constraint') that lie on a line of slope σ and that pass through F as $\gamma(F, \sigma)$. We can define an upper linear budget set using $\gamma(F, \sigma)$: $B_{(F,\sigma)} = \{G | G \in \Delta_3 \text{ and } G \text{ lies above } \gamma(F, \sigma) \text{ in } \Delta_3\}$. For a given F_α and σ we can pick out the $\succsim$-worst element of $B_{(F_\alpha,\sigma)}$ and denote it $w_{\alpha,\sigma}$. For a given σ we can trace out the set of strictly interior $w_{\alpha,\sigma}$. These form an expansion path.

Recall from Observation 5 that the center of the ellipses which define the indifference curves must lie on the line that connects the best and the worst outcome for any Δ_3. Therefore, an expansion path must lie on the best to worst outcome edge if preferences are in $\mathbb{CPE}$. The construction we use to behaviorally identify loss aversion relies on this fact. In the Marschak-Machina triangle the expansions paths for individual i all intersect at $(\frac{\lambda_i}{2(\lambda_i-1)}, \frac{2-\lambda_i}{2(1-\lambda_i)})$. Since one expansion path is the line connecting the best to worst outcomes, only a second expansion path is needed to locate the center of the indifference curves — which is the single common point of intersection of the

[21] However, in general, quadratic functionals, including $\mathbb{CPE}$, will violate Independence.

expansion paths. This identifies λ_i. Of course, if preferences are in $\mathbb{EU}$ then all points lie on a single expansion path (because all indifference curves have the same slope). In this case indifference curves are linear, and although the intuition in the paragraph fails, it is easy to identify that $\lambda = 1$ in this scenario (and similarly, if $\lambda > 1$ then indifference curves cannot be linear).

We can use this construction to compare loss aversion across individuals. Individual A is more loss averse than B if the center of A's indifference curves is closer to the best degenerate outcome. Because the center of A's indifference curves is closer to the best degenerate outcome than the center of B's indifference curves, for any point F, A's expansion path through F will be steeper than B's.

Consider a DM i whose preferences over Δ_3 are in $\mathbb{CPE}_M$. Let $F' = \alpha_1\bar{\delta} + (1-\alpha_1)\underline{\delta}$, and $G = \alpha_2\hat{\delta} + (1-\alpha_2)\underline{\delta}$, for some $\alpha_1, \alpha_2 \in (0,1)$ so that $\alpha_1 > \alpha_2$. Denote $G'_i = \alpha'_i\bar{\delta} + (1-\alpha'_i)\underline{\delta}$, so that $G'_i \sim_i G$. G'_i will always exist since the i has monotone preferences.

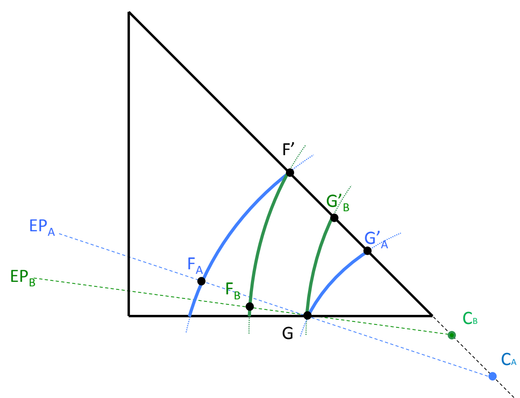

Figure 5: Constructing Expansion Paths

For some combinations of F' and G we can define an F_i such that $F_i \sim_i F'$ and $\beta_i F_i + (1-\beta_i)G \sim \beta_i F' + (1-\beta_i)G'_i$ for all $\beta_i \in (0,1)$. Denote the weight applied to the high outcome in F_i as h_i and to the middle outcome as m_i. Importantly, so long as both DMs' preferences are in $\mathbb{CPE}_M \backslash \mathbb{EU}$, then if $\frac{h_A}{m_A} \geq \frac{h_B}{m_B}$ for a particular F', G combination that define an F_i, then the inequality will also hold for all F', G combinations. Figure 5 demonstrates the construction of F', G', G_i and F_i, along with the steepness of the respective expansion paths EP_i. Importantly, because λ alone governs the steepness of the expansion paths, if A and B both have $\mathbb{CPE}_M$ preferences and A exhibits steeper expansion paths than B in a single Δ_3 then A exhibits steeper expansion paths than B in all Δ_3. Because full independence holds if preferences are in $\mathbb{EU}$, the procedure we just defined is not well defined. Thus, we will define the ratio $\frac{h_i}{m_i}$ to be equal to ∞ if $\succsim_i \in \mathbb{EU}$

Definition: *Individual A has steeper expansion paths than individual B if $\frac{h_A}{m_A} \geq \frac{h_B}{m_B}$ for an F, G that define an F_i for a given Δ_3.*

The steepness of the expansions paths reflects the location of the center of the ellipses that define the indifference curves. Therefore, the steepness of the expansion paths characterize the loss aversion of an individual.

Proposition 7 *Let $\succsim_i$ be in $\mathbb{CPE}_M$ for $i \in \{A, B\}$ and represented by (u_i, λ_i). Then individual A is less loss averse than individual B if and only if A has steeper expansion paths than B for all Δ_3.*

4.5 Comparative Consumption Risk Aversion

The other component of $\mathbb{CPE}_M$ preferences is u. We now discuss the behavioral implications of the curvature of u both in terms of comparing curvature across individuals and identifying curvature within an individual. We will define the consumption risk aversion of an individual using the curvature of u.[22]

Definition: *Let $\succsim_i$ be in $\mathbb{CPE}_M$ for $i \in \{A, B\}$ and represented by (u_i, λ_i). Individual A is more consumption risk averse than individual B if u_A is a concave transformation of u_B.*

To gain intuition for our next results we will fix three outcomes and look at the induced Marschak-Machina triangle. Consider some lottery $F = \alpha\bar{\delta} + (1-\alpha)\underline{\delta}$ and a slope $\sigma \geq 0$. Replicating the discussion in the previous subsection, denote the set of lotteries (a 'budget constraint') that lie on a line of slope σ and that pass through F as $\gamma(F, \sigma)$. We can define an upper linear budget set using $\gamma(F, \sigma)$: $B_{F,\sigma} = \{G' | G' \in \Delta_3 \text{ and } G' \text{ lies above } \gamma(F, \sigma) \text{ in } \Delta_3\}$.

For a given $F = \alpha\bar{\delta} + (1-\alpha)\underline{\delta}$, and individual i we can find a σ such that F is the $\succeq_i$ worst element in $B_{F,\sigma}$. Denote this slope as $\sigma_{i,F}$. Moreover, denote $G_{i,F}$ as the binary lottery that is on $\gamma(F, \sigma)$ and places some weight on the middle outcome. We can compare the slopes of the $\sigma_{i,F}$ across different individuals by comparing $G_{i,F}$. Figure 6 demonstrates the construction of F, $B_{i,F}$ and $G_{i,F}$.

Definition: *Individual A has steeper budget constraints than B if $G_{B,F}$ first-order stochastically dominates $G_{A,F}$ for all F for a given Δ_3.*

The slope $\sigma_{i,F}$ is determined by the tangency of the indifference curves of i passing through the best to worst outcome line. Moreover, the center of the ellipses that define the indifference curves are also always on this line. Because the center of the ellipses is determined only by λ and the shape of the ellipses only by u, the tangency condition on this edge is determined only by u, not λ. In the Marschak-Machina triangle the slope of $\sigma_{i,F}$ is simply $\frac{1-u_i(\hat{\delta})}{u_i(\hat{\delta})}$. Thus we can easily recover the utility value of $\hat{\delta}$ for any individual (after normalizing $u_i(\bar{\delta}) = 1$ and $u_i(\underline{\delta}) = 0$). Thus

[22] Again, the analysis easily extends to non-monotone preferences because the center of the elliptical indifference curves is in Δ_3. Therefore, it becomes easy to identify the relative length of the axes of the ellipse.

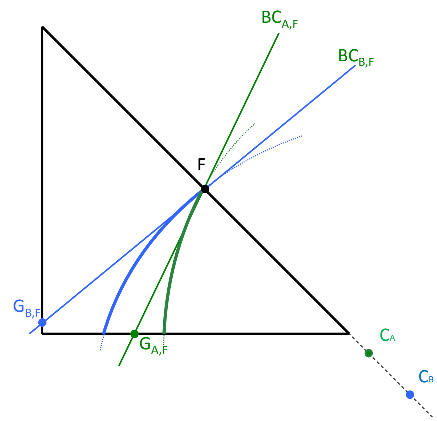

Figure 6: Constructing Upper Linear Budget Constraints

changing the center of an ellipse, but not the relative length of its axes, will leave the tangency condition unchanged. This allows us to also comparative the curvature of u across individuals. The next proposition formalizes the intuition that the steepness of the budget constraints characterizes consumption risk aversion.

Proposition 8 *Let $\succsim_i$ be in $\mathbb{CPE}_M$ for $i \in \{A, B\}$ and represented by (u_i, λ_i). Then individual A is more consumption risk averse than individual B if and only if A has steeper budget constraints than B in all Δ_3.*

5 Applications

5.1 Compound Lotteries and Information

Kőszegi and Rabin (2009), Kreps and Porteus (1978) and Dillenberger (2010), among others, discuss preferences for the resolution of information in dynamic settings. We will show that natural dynamic, recursive extension of $\mathbb{CPE}_M$ preferences are not flexible enough to accommodate a preference for early (late) resolution of uncertainty or a preference for one-shot resolution of uncertainty. We focus on recursive dynamic $\mathbb{CPE}_M$ models because recursive models are extremely tractable and typically used in macroeconomic models.[23]

In order to provide results on preferences for information we will extend our domain of analysis

[23] Although our dynamic model differs from Kőszegi and Rabin (2009) (because it, unlike theirs, is recursive) it shares the essential features of being loss averse over beliefs.

to two-stage compound lotteries, and denote the preferences over these lotteries as $\hat{\succsim}$.[24]

Two-stage compound lotteries are lotteries over lotteries. Imagine, for example, that there are two simple (one-stage) lotteries F and F'. Let F give an outcome of 1 with probability $\frac{1}{4}$ and 0 with probability $\frac{3}{4}$ and F' give an outcome of 1 with probability $\frac{3}{4}$ and 0 with probability $\frac{1}{4}$. An example of a compound lottery P is a lottery that in the first stage, with some probability $P(F)$ gives lottery F and with some probability $P(F')$ give lottery F' (since we have only two lotteries, in our case $P(F') = 1 - P(F)$. In the second stage the the decision-maker faces either F or F', which gives either 1 or 0 with the appropriate probability (see Figure 7).

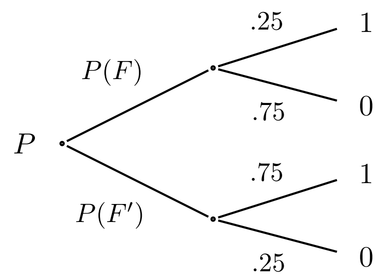

Figure 7: A Two-stage Compound Lottery

Following the notation of Dillenberger (2010) and Segal (1990), particularly important subsets of two-stage compound lotteries are:

- Γ: the the set of late resolving lotteries and
- Λ: the set of early resolving lotteries.

Early resolving lotteries have all uncertainty resolved in the first stage and so the second stage lotteries are degenerate. In contrast, late resolving lotteries have all uncertainty resolved in the second stage and so their first stage is degenerate. Figure 8 provides an example of both early and late resolving compound lotteries. In both cases the outcomes of 1 and 0 are realized with equal probability. The early resolving lottery P_E, has, in its second stage, two degenerate lotteries δ_0 and δ_1. In its first stage it gives δ_0 with probability 0.5 and δ_1 with probability 0.5. The late resolving lottery P_L has, in its second stage, a single lottery F, which gives 1 with probability 0.5 and 0 with probability 0.5, and in its first stage gives F with probability 1. We will assume that the restriction of $\hat{\succsim}$ to either Γ or Λ is in the class $\mathbb{CPE}_M$ (we denote those preferences $\hat{\succsim}_\Gamma$ and $\hat{\succsim}_\Lambda$).

We will assume that $\hat{\succsim}$ satisfies the standard assumption of recursivity (see Segal, 1990). Recursivity is useful because decision-makers with recursive preferences evaluate compound lotteries using a folding-back procedure — preferences over two stage lotteries can be evaluated using preferences over one stage lotteries. Decision-makers replace the second stage of any given compound

[24]We will define concepts informally in the text of this subsection. Formal definitions are provided in the proof of Observations 7 and 8.

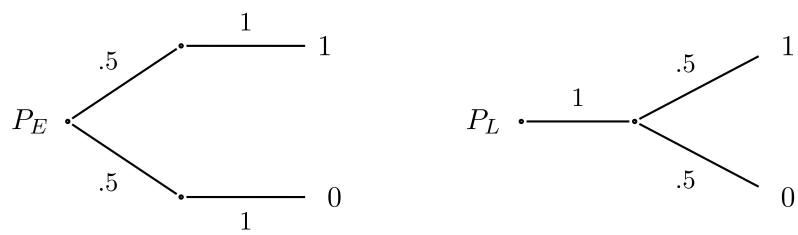

Figure 8: Early versus Late Resolving Compound Lotteries

lottery by the certainty equivalent generated by $\hat{\succsim}_\Gamma$. The resulting lottery is evaluated using $\hat{\succsim}_\Lambda$. For example, denote the utility function used to evaluate second-stage (first-stage) lotteries as u_Γ (u_Λ). In order to calculate the value of P the decision-maker first evaluates the possible second stage lotteries separately. Thus, she evaluates F according to u_Γ and finds the certainty equivalent $u_\Gamma^{-1}(0.25u_\Gamma(1)+0.75u_\Gamma(0))$. She also evaluates F' according to u_Γ and finds the certainty equivalent $u_\Gamma^{-1}(0.75u_\Gamma(1)+0.25u_\Gamma(0))$. In order to evaluate P, she substitutes for F and F' their respective certainty equivalents. This generates a one-stage lottery that with probability 0.5 gives outcome $u_\Gamma^{-1}(0.25u_\Gamma(1)+0.75u_\Gamma(0))$ and with probability 0.5 gives outcome $u_\Gamma^{-1}(0.75u_\Gamma(1)+0.25u_\Gamma(0))$. She then evaluates this lottery using u_Λ.

A large number of papers, beginning with Kreps and Porteus (1978), have discussed the importance of a preference for early resolution of information. Individuals have a preference for early resolution of uncertainty if, given two lotteries which generate the same reduced form probabilities over the same outcomes, they always prefer a compound lottery which is more Blackwell informative in the first stage. We show here that recursive $\mathbb{CPE}_M$ preferences are limited in how they can address resolution of uncertainty while maintaining first-order loss aversion.

Observation 7 *Suppose $\hat{\succsim}_\Gamma$ and $\hat{\succsim}_\Lambda$ are in $\mathbb{CPE}_M$. If $\hat{\succsim}_\Lambda$ (respectively $\hat{\succsim}_\Gamma$) satisfy risk aversion and $\hat{\succsim}$ always exhibits a preference for early (respectively late) resolution of information, then $\hat{\succsim}_\Lambda$ (respectively $\hat{\succsim}_\Gamma$) must be in $\mathbb{EU}$.*

Observation 7 is a corollary of Grant, Kajii and Polak (2000). It says that if an individual always has a preference for early or late resolution of uncertainty in a dynamic setting, and they are risk averse over early-resolving lotteries, their static preferences cannot both be strictly loss-averse and in $\mathbb{CPE}_M$. Similarly, if static preferences are always in $\mathbb{CPE}_M$, risk averse and strictly loss-averse, then they cannot have a uniform attitude towards early and late resolution of information.

An important intuition regarding dynamic loss-aversion is that loss aversion makes an individual averse to receiving information piecemeal — i.e. individuals should exhibit a preference for one-shot resolution of uncertainty or clumping of information (see Kőszegi and Rabin (2009) for an example of this). To simplify the analysis, we will assume (like Dillenberger, 2010) that preferences are time neutral — individuals are indifferent between a lottery in Λ and a lottery in Γ which give the same reduced form probabilities over the same outcomes. Dillenberger (2010) says a preference relation exhibits a preference for one-shot resolution of uncertainty (PORU) if any lottery in Γ or Λ is weakly preferred to any other compound lottery that gives the same reduced form probabilities over the same outcomes. Unlike the intuition in Kőszegi and Rabin (2009), we show that an individual with $\mathbb{CPE}$ preferences cannot always exhibit a preference for one-shot resolution of uncertainty.

Observation 8 *Suppose* $\hat{\succsim}_\Gamma$ *and* $\hat{\succsim}_\Lambda$ *are in* $\mathbb{CPE}_M \backslash \mathbb{EU}$ *and preferences are time neutral. Then preferences cannot exhibit a preference for one-shot resolution of uncertainty.*

Observation 8 is a simple application of Dillenberger (2010). It implies that although the intuition that dynamic loss aversion implies a preference for clumping of information seems very strong in simple environments, it is not an intuition that applies more broadly.[25]

5.2 Small and Large Stake Choices

An important argument against the plausibility of expected utility is the Rabin (2000) critique — if a decision-maker has preferences in $\mathbb{EU}$ then one cannot construct preferences that generate plausible behavior over both small and large stakes lotteries. Rabin's calibration result and Safra and Segal's (2005) extension show how local behavior relates to global behavior. Many authors (e.g. Cox and Sadiraj, 2006) have discussed how utility being defined over gains and losses may help avoid the calibration critique. However, even when utility functions are defined over gains versus losses, local behavior may imply strong restriction on global behavior. Safra and Segal (2005) point this out in the case of disappointment aversion.

As we will show, $\mathbb{CPE}_M$ also suffers from a modified version of Rabin's critique. Because $\mathbb{CPE}_M \subset \mathbb{RDU}$, we know that $\mathbb{CPE}_M$ preferences also suffer from the calibration critique of Theorem 5 in Safra and Segal (2005).[26] Safra and Segal (2005) show that if (i) preferences are in $\mathbb{RDU}$, (ii) u is either decreasing absolute risk aversion everywhere or increasing absolute risk aversion, and (iii) the decision maker plausible rejects small lotteries when added to any gamble defined over relevant wealth levels then she should also (implausibly) reject very attractive large stakes lotteries. For example, assume u exhibits decreasing absolute risk aversion. Then if the decision maker rejects a lottery that gives -100 with probability 0.5 and 110 with probability 0.5 when added to all gambles defined over a large enough wealth level with a lower bound of w then she will reject a lottery that gives $-20,000$ with probability 0.0054, and $100,000 - \zeta$ with probability .9946 for a sequence of ζ converging to 0 at wealth level w. See Figure 10 for an illustration of this example (for simplicity we depict the lotteries as over changes in wealth, not final wealth).

[25]However, whether the critique here extends to Kőszegi and Rabin (2009) is an open question, since their dynamic model is not recursive.

[26]Neilson (2001) makes a related calibrational critique of $\mathbb{RDU}$.

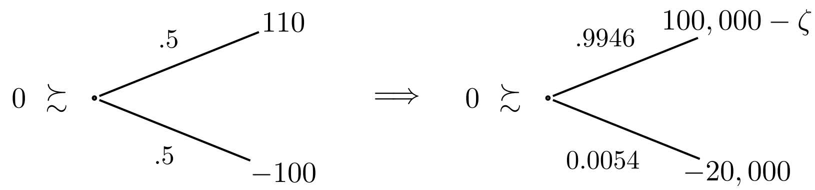

Figure 9: Small versus Large Stake Choices

Our observation shows that $\mathbb{CPE}_M$ suffer from similar problems. Of course, the conclusions of the theorem depend on the rejection of small-stakes lotteries when added to all gambles defined over a large wealth level (a stronger requirement than Rabin's original requirement that lotteries be rejected at all degenerate gambles over a large wealth level). As Freeman (2013) notes, if individuals do not reduce compound lotteries, and interpret combination of background risk and any additional lottery as a compound lottery, then Safra and Segal's critique will not hold. This is similar in spirit to assuming that individuals narrowly bracket risky choices. Thus, understanding how individuals perceive these situations is essential in interpreting these results.

Cox and Sadiraj (2006) provide an example of a gain-loss function that does not generate calibration problems in the context of the reference point being current wealth. Similarly, if the restriction on g (the gain-loss function) to be linear is relaxed, it is possible to generate plausible small and large stakes risk aversion. For example, Kőszegi and Rabin's (2007) Table 1 does exactly this. Our result points out that the most tractable form of Kőszegi and Rabin's (2007) model — that with linear gain-loss utility, cannot avoid an extension of the Rabin critique. Thus, in order to model individuals who exhibit plausible behavior over both small and large stakes lotteries, we must turn to non-linear gain-loss functionals.

5.3 Other Applications

The relationships we describe can also be used to understand $\mathbb{CPE}$ in a variety of other environments. Rank-dependent models have been extensively analyzed in the literature, and results regarding pessimistic rank-dependent utility functionals can be naturally extended to $\mathbb{CPE}_M$. There is a large literature analyzing rank-dependent utility functionals in the context of insurance design, including papers such as Konrad and Skaperdas (1993), Schlesinger (1997) and Gollier and Schlesinger (1996). Bernard, He, Yan and Zhou (2012) have a simple summary of the results of this literature. For example, they discuss how the optimal insurance indemnity is a deductible for rank-dependent utility functions with a convex probability weighting function (and so for $\mathbb{CPE}_M$ preferences). The effects of $\mathbb{RDU}$ preferences have also been looked at in the context of macroeconomics: Bleichrodt and Eeckhoudt (2005) consider savings problems, Xia and Zhou (2013) examine the existence and structure of Arrow-Debreu equilibria and Xu and Zhou (2013) look at optimal stopping.

6 Non-Linear Gain-Loss Functionals

Up until this point we have focused on the case of linear gain-loss utility. However, there is no reason to assume that this is always the case. In fact, our analysis of choice over small and large stakes lotteries suggests that researchers may want to focus on non-linear gain-loss utility in order to better match real-world behavior. In this section we consider a more general structure, where the gain-loss function does not have to be linear. This can be thought of as a way of allowing the degree of exhibited loss aversion to be stake-dependent. We define the general $\mathbb{CPE}$ ($\mathbb{GCPE}$) functional as:

$$V_{\mathbb{GCPE}}(F) = \sum_{x \in F} u(x)F(x) + \sum_{x \in F} \sum_{y \in F} g(u(x) - u(y))F(x)F(y)$$

where

$$g(z) = \begin{cases} f(z) & \text{if } z \geq 0 \\ -\lambda f(-z) & \text{if } z < 0 \end{cases}$$

where λ, the loss aversion parameter, is greater than 1. Moreover, f is a continuous strictly increasing function that maps from the positive reals to the positive reals, $f(0) = 0$ and f is differentiable everywhere on its domain. In line with the literature we will also focus on the case where f exhibits diminishing sensitivity — f is concave. This class of preferences nests those considered in the earlier sections of the paper.[27]

Many of the relationships between $\mathbb{CPE}$ and other non-$\mathbb{EU}$ models of choice discussed in Section 3 extend easily. However, $\mathbb{GCPE}$ is not a subset of $\mathbb{RDU}$. This is true even if we restrict ourselves to monotone preferences within $\mathbb{GCPE}$.

Observation 9 *$\mathbb{GCPE} \subset \mathbb{Q}$.*

Observation 9 immediately implies that $\mathbb{GCPE} \cap \mathbb{B} = \mathbb{GCPE} \cap \mathbb{NCI} = \mathbb{GCPE} \cap \mathbb{G} = \mathbb{EU}$, and so there is still no common set of preferences, outside of $\mathbb{EU}$, which accommodate both $\mathbb{GCPE}$ and other models of reference dependence and the certainty effect such as $\mathbb{G}$ and $\mathbb{NCI}$. Despite the fact we lose the ability to use the $\mathbb{RDU}$ toolkit, we can still use methods developed for $\mathbb{Q}$ to make statement about $\mathbb{GCPE}$.

In a similar vein, we can use similar techniques as before in order to understand what preferences look like in the Marschak-Machina triangle. Because $\mathbb{GCPE} \subset \mathbb{Q}$ we can again think about the shapes of the indifference curves by using commonly known properties of conic sections.[28]

[27] Although we restrict our formal results to these assumptions, similar results apply to situations when f is not necessarily concave, when λ is not necessarily greater than 1, or when g is not necessarily symmetric other than by scalar multiplication around 0.

[28] Similar results extend to more general formulations. For example, f is more concave than x^2 if and only if the

Observation 10 *If* $\succsim$ *is in* $\mathbb{GCPE}$*, then indifference curves are ellipses for any* Δ_3.

Although the indifference curves are ellipses there are two key differences — the center of the elliptical indifference curves may not lie on a line connecting the best to worst degenerate outcomes, and the axes of the ellipses may not be vertically and horizontally oriented.

An immediate implication of the previous observation is that preferences in $\mathbb{GCPE}$ are always mixture averse.[29] This provide a powerful test of the predictions of $\mathbb{CPE}$ preferences.

Observation 11 *If* $\succsim$ *is in* $\mathbb{GCPE}$*, then* $\succsim$ *satisfy MA.*

We next turn to understanding when $\mathbb{GCPE}$ preferences respect first-order stochastic dominance and when they are risk averse. As is fairly clear, because $\mathbb{GCPE}$ models have different gain-loss utility functions f we must interpret the weighting of gains relative to losses (i.e. λ) different in terms of behavior.[30]

Observation 12 *Suppose* $\succsim$ *is in* $\mathbb{GCPE}$*. Then* $\succsim$ *respects first-order stochastic dominance if and only if* $\frac{1}{\lambda-1} \geq f'(0)$.

Observe that this condition says that in order for preferences to respect first-order stochastic dominance there must be a relationship between the value of λ and the value of $f'(z)$.[31] In particular, in the case of linear gain-loss utility, where $f'(z) = 1$ for all $z \geq 0$, this condition reduces to $1 \leq \lambda \leq 2$, exactly what we derived previously.[32]

Next we consider when preferences are classically risk averse — preferences respect the ordering of mean preserving spreads. An immediate implication of Chew, Epstein and Segal (1991) is that $\mathbb{GCPE}$ preferences are risk averse if and only if $u(x) + u(y) + (1 - \lambda)f(|u(x) - u(y)|)$ is concave in x for all y. For example, for linear utility, this reduces down to checking if $x + y + (1 - \lambda)f(|x - y|)$ is concave.

indifference curves are ellipses for any Δ_3, while f is equal to x^2 if and only if the indifference curves are parabolas for any Δ_3, and f is more convex than x^2 if and only if the indifference curves are hyperbolas for any Δ_3.

[29] Although this result is straightforward when f exhibits decreasing marginal sensitivity, the intuition extends more broadly. In fact, if we keep the functional form of $\mathbb{GCPE}$ but relax the assumption of decreasing marginal sensitivity, it is still the case that preferences satisfy mixture aversion. Even more generally, if we relax the definition of g, then $V_{\mathbb{GCPE}}$ satisfies mixture aversion if and only if the functional $V_g = \sum_{x \in F} \sum_{y \in F} g(u(x) - u(y))F(x)F(y)$ satisfies mixture aversion.

[30] This raises an important point for calibration exercises — that one must be careful about taking estimates of λ derived from one model and applying them to a second with a different f, as the same λ can generate quite different behavior. For a related discussion, see Section 7.2.

[31] Because f and $\lambda - 1$ are unique up to joint multiplicative transformations αf and $\frac{\lambda-1}{\alpha}$ for $\alpha > 0$ this condition is well-defined.

[32] More generally, preferences respect first-order stochastic dominance if and only if $u(x) + u(y) + g(u(x) - u(y))$ is increasing in x for all y.

Although we have focused our attention on situations where f exhibits decreasing marginal sensitivity, we would like show that at least one important class of preferences is equivalent to $\mathbb{GCPE}$ when f exhibits increasing marginal sensitivity. If $f(z) = z^2$, then we obtain mean-variance preferences.

Observation 13 *If $f(z) = z^2$, then $V_{\mathbb{GCPE}}(F) = \sum_x u(x)F(x) + (1-\lambda)(Var_F(u(x)))$.*

7 Experimental Evidence

The relationships developed in Section 3 provide new opportunities to relate theory to data. First, they allow us to relate experimental evidence to models in two new ways: we can test $\mathbb{CPE}$ using existing data originally designed to test other models of choice under risk (e.g. $\mathbb{RDU}$) and we can take existing experiments that test $\mathbb{CPE}$ and use them to test $\mathbb{RDU}$ and $\mathbb{Q}$. Thus, subject to the experimental design correctly capturing the psychology underlying $\mathbb{CPE}$, our results allows us to bring over twenty years of existing experimental evidence to bear on $\mathbb{CPE}$. Second, they raise issues about model misspecification — in particular how well identified preference parameters related to probability weighting and loss aversion might be. Third, the relationships can help us design new experiments that can distinguish $\mathbb{CPE}$ from other non-EU models.

7.1 Using Existing Evidence

In interpreting existing evidence with respect to $\mathbb{CPE}$ (and $\mathbb{GCPE}$) we want to highlight the fact that interpreting choices as being driven by $\mathbb{CPE}$ requires an assumption about how quickly individuals update their reference point. As we previously noted, Kőszegi and Rabin (2007) interpret choice-acclimating personal equilibria as capturing situations where the resolution of uncertainty will be delayed, so that individual's choice will be their reference point when uncertainty is resolved. To the extent that experimental settings do not allow for enough delay between choice and the resolution of uncertainty, we should be cautious in interpreting these results. However, conditional on this requirement being met, existing experiments originally designed to test other theories of non-expected utility can now be used to test $\mathbb{CPE}$ and $\mathbb{GCPE}$.

Survey evidence indicates some support for preferences being consistent with the predictions of $\mathbb{CPE}$. Hey and Orme (1994) find that of the non-$\mathbb{EU}$ models they consider, rank dependent and quadratic functionals fit the data best. Moreover, surveying the literature, Starmer (2000) indicates that the theories most consistent with existing experimental evidence are those in the class $\mathbb{RDU}$. In contrast, in a different survey, Harless and Camerer (1994), find weak support for all theories of choice under risk, although $\mathbb{RDU}$ and $\mathbb{Q}$ are not dominated in the data by other models. In order to carefully evaluate how well the predictions of $\mathbb{CPE}$ fit existing data, we will revisit much of the experimental literature testing choice over lotteries, focusing on tests of $\mathbb{RDU}$ and mixture aversion. We find that, for the most part, the existing experimental evidence is not strongly supportive of $\mathbb{CPE}$ nor $\mathbb{GCPE}$.

First, we consider tests of $\mathbb{RDU}$. Since $\mathbb{CPE}_M \subset \mathbb{RDU}$ these experiments can serve to evaluate $\mathbb{CPE}_M$ as well. Although Starmer (2000) cites strong support for $\mathbb{RDU}$ preference, he finds that those studies that assume rank-dependence and then attempt to ascertain the shape of the probability weighting function generally do not find that it is convex (as implied by $\mathbb{CPE}_M$). Instead, Tversky and Kahneman (1992), Camerer and Ho (1994), Wu and Gonzalez (1996), Gonzalez and Wu (1999), Abdellaoui (2000) and Bleichrodt and Pinto (2000) find evidence for an inverse S weighting function, which is neither convex nor concave. Despite Starmer's (2000) overall finding of support for the predictions of $\mathbb{RDU}$, those studies that specifically test whether subjects obey the axioms of $\mathbb{RDU}$, such as Wu (1994) and Wakker, Erev and Weber (1994), find very limited support for preferences satisfying the conditions of $\mathbb{RDU}$. Thus, on the whole, the evidence seems to be against preferences, if they are even in the class $\mathbb{RDU}$, having a strictly convex weighting function, and so against $\mathbb{CPE}_M$.

However, linear gain-loss utility is just one possible specification within $\mathbb{GCPE}$. Thus, we should also look to experiments that test more general properties of $\mathbb{GCPE}$. For example, we can examine evidence for mixture aversion in choice, which we show is a property satisfied by all preferences in $\mathbb{GCPE}$. Although a variety of studies have found systematic violations of betweenness, including Andreoni and Sprenger (2011), Coombs and Huang (1976), Chew and Waller (1986), Camerer (1989), Sopher and Gigliotti (1993), Prelec (1990) and Camerer and Ho (1994), the violations have been mixed between mixture aversion and mixture loving (although more often individuals are mixture averse). In addition, many studies, including Andreoni and Sprenger (2012), Coombs and Huang (1976), Camerer (1992), Starmer (1992) and Gigliotti and Sopher (1993) have found that violations of betweenness are dramatically reduced when considering lotteries in the interior of the unit simplex. Thus, in contrast to the predictions of $\mathbb{GCPE}$, behavior does not seem to be universally mixture averse.

Up until now we have described how our results allow us to use existing experiments originally designed to test other models of choice under risk to also test $\mathbb{CPE}$. Our equivalence results also allow us to apply insights from experiments designed to test $\mathbb{CPE}$ to other non-expected utility models.

We use as an example a well known experiment on labor supply by Abeler, Falk, Goette and Huffman (2011), who provide evidence that they interpret in support of $\mathbb{CPE}$ (similar exercises could be done on other papers such as Gill and Prowse, 2012). In Abeler, Falk, Goette and Huffman (2011) subjects exert effort in a counting task (output of which is measured in pages counted). After counting they have a $\frac{1}{2}$ chance of being provided with an 'outside' payment τ that does not depend on their actions. With $\frac{1}{2}$ chance they receive a payment that depends in a linear fashion on the number of pages that were counted (i.e. they are paid a piece rate w per page). There are main two treatments, which differ in the the size of the outside payment.[33]

Abeler, Falk, Goette and Huffman (2011) find that as the outside option increases from low to high the number of pages subjects count increases. This is inconsistent with preferences in $\mathbb{EU}$ but is rationalizable by preferences in $\mathbb{CPE}$. However, our results indicate that there also exists convex rank-dependent utility functionals that can rationalize the observed behavior (similarly there exist quadratic functionals that rationalize the behavior). In particular, any rank dependent model where $w(.5) < .5$ can rationalize the behavior in Abeler, Falk, Goette and Huffman (2011). Moreover, looking across subjects' behavior as the probability of receiving the outside payment varies can allow us to distinguish between different probability weighting functions. For example, under $\mathbb{CPE}$, regardless of the probability assigned to receiving the outside payment, subjects' effort must be higher as the outside payment goes from low to high. In contrast, for more general rank-dependent preferences, if the probability assigned to the outside option is p, then subjects' effort should increase between the low to high treatment if and only if $1 - w(p) \geq w(p)$.

We can also look at what happens to effort as the amount of the outside payment varies. For all rank-dependent preferences effort will only increase if the low (high) treatment gives an outside payment less (more) than the amount that depends on their work. Otherwise effort should not change. In contrast, this is not necessarily true for quadratic preferences. There exist quadratic preferences that can both rationalize the data in Abeler, Falk, Goette and Huffman (2011) and generate behavior where effort is a smooth function of the size of the outside payment. Thus, examining how effort varies both across different p's and different outside payment amounts can help identify more precisely what class of model is driving behavior in Abeler, Falk, Goette and Huffman (2011).[34]

7.2 Model Misspecification

Because loss aversion generates the same behavior as a class of pessimistic rank-dependent preferences it seems natural to ask how this relationship influences researchers ability to estimate probability weighting functions. For example, many papers estimate a probability weighting function

[33] There were also three additional treaments, but as these were designed to ensure that the observed behavior across the two main treatments was being driven by non-$\mathbb{EU}$ preferences, they are not relevant for our discussion.

[34] Abeler, Falk, Goette and Huffman (2011) also find that individuals who are more loss-averse, in the sense they exhibit a larger risk premia for small-stakes lotteries, also tend to exert effort that makes their earnings that depend on effort closer to their outside payment. They show that this is also in line with the predictions of $\mathbb{CPE}$, since the value of λ controls both behaviors. Our results imply this observed correlation is also consistent with individuals whose preferences are in $\mathbb{RDU}$. In this case, individuals who are more pessimistic will exhibit both larger risk premia for small-stakes lotteries and also try to better equalize their earnings.

either in the context of cumulative prospect theory or rank-dependent utility.[35]

This raises the question of whether the typically estimated inverse-S shaped probability function for rank-dependent preferences could be generated by model mis-specification. For example, because $\mathbb{CPE}$ is equivalent to a convex weighting function, it could not generate behavior equivalent to an inverse-S shaped weighting function. But perhaps $\mathbb{CPE}$ in conjunction with inverse-S shaped probability probability weighting could generate behavior equivalent to that generated by inverse-S shaped $\mathbb{RDU}$ preferences. In fact, given that the data is estimated using small stakes lotteries the answer is no. This implies that the existing evidence on inverse-S shaped probability weighting functions is robust to the particular form of model misspecification that we consider.

This can be illustrated in a simple example that corresponds to standard experimental elicitation of risk preferences. Suppose researchers are eliciting certainty equivalents over simple binary lotteries over a fixed low and high outcomes (we will normalize the utility of the outcomes to 0 and 1 respectively). Given that lotteries are typically small stakes, we will assume that u is linear (and in fact it is generally close to linear in the estimates — see Bruhin, Fehr Duda and Epper, 2010 for a recent study). Given a weighting function w, and individuals with rank-dependent preferences has a certainty equivalent for a lottery that gives the high outcome with probability p of $w(p)$. An inverse-S shaped weighting function will generate a certainty equivalent function that is initially concave and then later convex (and in fact there is a direct mapping between the weighting function and the certainty equivalent function) — in other words if the second derivative of the certainty equivalent function is initially negative and then positive.

In comparison, suppose that an individual experiences $\mathbb{CPE}$ references-dependence and also engages in probability weighting. We assume that the individual has a functional form as in $\mathbb{CPE}$ but replaces p with $w(p)$. The certainty equivalent (i.e. utility) function is then $w(p) + (1 - \lambda)w(p)(1 - w(p))$. We will assume that $1 \leq \lambda \leq 2$ so that individuals are both loss averse and have monotone certainty equivalent functions. Understanding the curvature of the certainty equivalent function necessitates examining its second derivative:

$$(2 - \lambda)w''(p) + 2(\lambda - 1)w''(p)w(p) + 2(\lambda - 1)w'(p)w'(p)).$$

We can now ask if a strictly convex, strictly concave, or S shaped probability weighting funtion, in conjunction with $\mathbb{CPE}$, can generate a certainty equivalent function that is initially concave and then convex.

If $w'' > 0$ then the certainty equivalent function is convex everywhere. If instead $w'' < 0$ the first term is negative, the second term is negative and the third term is positive. If the

[35] In the case of cumulative prospect theory, most papers assume lotteries are composed solely of gains or losses relative to the status-quo, as in Bruhin, Fehr Duda and Epper (2010). This implies that the estimation procedure is equivalent to estimating two separate rank-dependent functionals, one for the domain of gains and one for the domain of losses. Importantly though, the reference point is assumed to the be status-quo, rather than expectations.

certainty equivalent function is initially concave and then convex (so that the second derivative of the certainty equivalent function is initially negative and then positive)) then it must be the case that $w'(p)$ is small for small p's and then is large for large p's. But this implies $w''(p) > 0$ at some point, a contradiction. Similarly, if w is initially convex, then the certainty equivalent function must initially be convex. Thus an individual who both experiences $\mathbb{CPE}$ references-dependence and probability weights but does not have a weighting function that is inverse-S shaped cannot not generate behavior that is qualitatively similar to the behavior generated by preferences in $\mathbb{RDU}$ with an inverse-S shaped weighting function.

We might also worry about model misspecification affecting attempts to estimate the degree of loss aversion. There have been a variety of attempts to estimate the degree of loss aversion, but many of the estimates use different underlying models. This raises the question of how portable these estimates are between model for example, can estimates obtained from a $\mathbb{BLS}$ model generate similar behavior when put into a $\mathbb{CPE}$ model. The answer, of course, depends on the environment used to estimate the parameters.

We will focus on the three models of reference dependence discussed earlier — $\mathbb{G}, \mathbb{BLS}$ and $\mathbb{CPE}$ — as well as the oft-used formulation of prospect theory (without probability weighting) where the reference point is the status quo (i.e. current wealth level). If lotteries are over only two outcomes $\mathbb{BLS}$ and $\mathbb{CPE}$ generate the same certainty equivalent for the same coefficient of loss aversion. Moreover, if there are only two outcomes and they are symmetrically defined around the status quo (i.e. if the status quo is w and the lotteries of the form .5 chance of $w + \epsilon$, .5 chance of $w - \epsilon$), status quo reference dependent models give the same certainty equivalents as $\mathbb{CPE}$ for equivalent coefficients of loss aversion.

However, more generally, the estimates will not be portable across models. If lotteries are not symmetrically defined around the status quo then status quo reference dependence generates different behavior than $\mathbb{BLS}$ and $\mathbb{CPE}$, even for lotteries defined over two outcomes. Moreover, as soon as lotteries have positive support over more than 3 outcomes then the coefficient of loss aversion has a different relationship to behavior depending on whether the model is $\mathbb{BLS}$ or $\mathbb{CPE}$. Similarly, $\mathbb{G}$ has a completely different relationship between β and certainty equivalents compared to $\lambda_{\mathbb{CPE}}$.[36] Thus, other than in very particular domains researchers need to be careful about applying estimates of loss aversion from a single model to other models.

7.3 Distinguishing between Models

The relationships developed in Section 3 can also help guide future experimental research that could distinguish between models of reference dependence. As discussed previously, different models of

[36]This last comparison should not be surprising since high degrees of loss aversion in $\mathbb{CPE}$ are associated with non-monotone preferences, but even an infinite degree of disappointment aversion in $\mathbb{G}$ (i.e. $\beta = \infty$) is still associated with preferences that respect first-order stochastic dominance.

reference dependence have distinct implications for attitude towards mixing and randomization. For example, we show that $\mathbb{CPE}$ preferences are mixture averse. In contrast, Dillenberger (2010) demonstrates that $\mathbb{NCI}$ preferences are mixture loving. Moreover, it is well known that $\mathbb{G}$ and $\mathbb{B}$ preferences satisfy Betweenness (the combination of mixture loving and mixture averse). Furthermore, although $\mathbb{BLS}$ are not as well studied, Masatlioglu and Raymond (2014) discuss how they must exhibit both mixture loving and mixture aversion (depending on the domain). Thus, tests of attitudes towards the mixing of indifferent lotteries can serve to distinguish between these different notions of reference dependence.

Moreover, testing the Allais paradox with more than three outcomes can serve to distinguish models of reference dependence from one another. An implication of Observation 3 is that when there are more than 3 outcomes there will always be examples where $\mathbb{CPE}_M$ preferences induce behavior that violate Negative Certainty Independence, unlike $\mathbb{NCI}$ preferences (which, recall, include $\mathbb{G}$). This provides a potential avenue for testing $\mathbb{NCI}$ preferences against $\mathbb{CPE}_M$.

8 Conclusion

This paper contributes to understanding behavior under loss aversion and endogenous reference point formation. In particular, we feel that understanding where $\mathbb{CPE}$ fits within the taxonomy of non-EU theory can be extremely helpful for both theoretical and empirical researchers. It allows researchers to both make use of a larger toolkit of methods and to better understand how to distinguish models of reference dependence from one another. As our results make clear, the driving mechanism behind reference dependence in risky choice requires looking not just at first-order risk aversion, but other factors, such as attitudes towards randomization.

Our approach helps formalize why reference point formation is the desiderata of reference dependence, rather than the shape of the gain-loss utility function. As we observe in this paper, changing the reference point formation process from that of $\mathbb{CPE}$ to $\mathbb{G}$ to $\mathbb{BLS}$ generates very different behavior. In contrast, changing the gain-loss utility functional (e.g. changing g from linear to concave) while leaving the reference point formation process untouched preserves an important feature of the models: in particular, the weakening of the Independence axiom consistent with the preferences. For example, $\mathbb{GCPE}$ preferences satisfy the Projective Independence axiom of Chew, Epstein and Segal (1994) regardless of the gain-loss functional. In contrast, if the reference point is the certainty equivalent, as in $\mathbb{G}$ or $\mathbb{B}$, then preferences satisfy Betweenness.

As a final thought, it is important to discuss how our results fit into the larger set of solution concepts considered by Kőszegi and Rabin (2007). In this paper we have focused on Kőszegi and Rabin's choice-acclimating personal equilibria solution concept. As mentioned previously, Kőszegi and Rabin (2007) also consider other solution concepts, including preferred personal equilibrium

(PPE). A natural question to ask is how our results shed light on PPE. Unlike $\mathbb{CPE}$, PPE can generate intransitive choice patterns. This is because PPE is a two-stage choice procedure. Given a choice set S, in the first stage an individual determines whether any given lottery $f \in S$ is better than all other lotteries $g \in S$ given f as a reference point. Intuitively, fixing a reference point f the first stage indifference curves must be parallel and linear, and their slope is determined by f. Thus, in the first stage, an individual is checking whether f lies on the highest linear indifference curve, where the slope of the indifference curves depends on f. Those lotteries that survive the first stage are evaluated according to the second stage criterion, which is $\mathbb{CPE}$. Those that are also optimal in the second stage are the PPE of S. In recent work Freeman (2012) characterized a general form of PPE using choice data. We can use our analysis in this paper to shed additional light on the specific form of PPE used by Kőszegi and Rabin (2007). Differences between $\mathbb{CPE}$ and PPE, including intransitivity, occur when the first stage and second stage criteria differ, or when there are 'gaps' between the indifference curves generated by the first and second stage criteria. Using the results in this paper, it is evident that this reduces to comparing the ranking generated by linear indifference curves to that generated by elliptical indifference curves. In ongoing work we are exploring this relationship in order to better understand when the results from this paper extend to PPE and to characterize when transitivity is violated.

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