A Bias-Adjusted LM Test of Error Cross Section Independence*

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Abstract

This paper proposes bias-adjusted normal approximation versions of Lagrange multiplier (NLM) test of error cross section independence of Breusch and Pagan (1980) in the case of panel models with strictly exogenous regressors and normal errors. The exact mean and variance of the Lagrange multiplier (LM) test statistic are provided for the purpose of the bias-adjustments, and it is shown that the proposed tests have a standard normal distribution for the fixed time series dimension (T) as the cross section dimension (N) tends to infinity. Importantly, the proposed bias-adjusted NLM tests are consistent even when the Pesaran's (2004) CD test is inconsistent. Also alternative bias-adjusted NLM tests, which are consistent under local error cross section independence of any fixed order p, are proposed. The finite sample behavior of the proposed tests are investigated and compared to the LM, NLM, and CD tests. It is shown that the bias-adjusted NLM tests successfully control the size, maintaining satisfactory power in panel with exogenous regressors and normal errors, even when cross section mean of the factor loadings is close to zero, where the CD test has little power. However, it is also shown that the bias-adjusted NLM tests are not as robust as the CD test to non-normal errors and/or in the presence of weakly exogenous regressors.

JEL-Classification: C12, C13, C33

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1 Introduction

A number of different approaches already exist for testing cross section independence in panel data models. An early contribution is due to Moran (1948) who provides a test of spatial independence in the context of a pure cross section model. Further developments of Moran's test are reviewed in Anselin (1988, 2001). This approach depends on the choice of the spatial matrix, and may not be appropriate for many panels in economics and finance where space is not a natural metric for modelling of cross section dependence. An alternative procedure would be to use the Lagrange multiplier (LM) correlation test of Breusch and Pagan (1980), which does not require *a priori* specification of a spatial matrix. The LM test is based on the average of the squared pair-wise correlation coefficients of the residuals and is applicable in the case of panel data models where the cross section dimension (N) is small relative to the time dimension (T), and where Zellner's (1962) seemingly unrelated regression equation (SURE) method can be used.

Pesaran (2004) examines the normal approximation version of the LM test (denoted by NLM) where the mean and variance of the test indicator is approximated up to $O\left(T^{-1}\right)$. The NLM test is shown to exhibit substantial size distortions for N large and T small, a situation that can frequently arise in empirical applications. This is primarily due to the fact that for T fixed, the mean approximation of the LM statistic will not be correct, and with N large the incorrect centering of the test statistic is likely to be accentuated, resulting in size distortions that tend to get worse with N.

Frees (1995) has proposed a version of the Breusch and Pagan LM test, R^2_{AVE}, based on squared pair-wise Spearman rank correlation coefficients which is applicable to panel data models where N is large relative to T. However, Frees only provides the distribution of the test in the case of models with only one regressor (intercept), and its generalization for models with additional explanatory variables is not known. For example, the mean of R^2_{AVE} appearing in Corollary 1 and Theorem 2 of Frees (1995) may not be valid for the models with explanatory variables.[1] Frees (1995) also proposes tests based on average pair-wise sample correlations of the series across the different cross section units. His R_{AVE} test statistic is based on Spearman rank correlations, and his C_{AVE} test statistic is based on Pearson rank correlations. The latter is closely related the CD test also considered in Pesaran (2004).

Pesaran (2004) shows that unlike the LM test statistic, the CD statistic has exactly mean zero for fixed values of T and N, under a wide class of panel data models, including heterogeneous dynamic models subject to multiple breaks in their slope coefficients and error variances, so long as the unconditional means of y_{it} and $\mathbf{x}_{it}$ are time-invariant and their innovations are symmetrically distributed. However, the CD test has an important drawback; namely it will lack power in certain situations where the population average pair-wise correlations is zero, although the underlying individual population pair-wise correlations are non-zero. This could arise, for example, where under the alternative hypothesis cross dependence can be characterized as a factor model with mean zero factor loadings. See Pesaran (2004, p.14).

In this paper we propose bias-adjusted versions of the NLM tests, which use the exact mean (and variance) of the LM statistic in the case of panel data models with strictly exogenous regressors and normal errors. The adjustments are obtained using the results in Ullah (2004), so that the centering of the LM statistic is correct for fixed T and N. We consider two versions of bias-adjusted LM tests: the mean-bias-adjusted NLM test (denoted by NLM*), and mean-variance-bias-adjusted NLM test (denoted by NLM**). The NLM* test is expected to be out-performed by the NLM** test in small samples, however, its simpler computation is an advantage. Importantly, it will be shown that these bias-adjusted tests are consistent even when

[1] In fact using Monte Carlo experiments we found that the uncorrected version of the R^2_{AVE} test tends to behave similarly to the uncorrected version of the Breusch and Pagan LM test when N is large for models with explanatory variables. These results are available from the authors upon request.

the cross section mean of the factor loadings is near zero, under which Pesaran's CD test is not consistent.

In cases where the cross section units can be ordered *a priori*, as with spatial observations, the proposed LM tests might not be sufficiently powerful as they do not exploit the spatial information. To deal with this problem we also propose a generalization of the NLM* and NLM** tests, which capture the spatial patterns. We call them NLM(p)* and NLM(p)** tests, that correspond to the $CD\,(p)$ test proposed in Pesaran (2004).

The finite sample behavior of the bias-adjusted tests is investigated by means of Monte Carlo experiments, and compared to that of the (non-bias-adjusted) LM and NLM tests, as well as to the CD test. It will be shown that the bias-adjusted NLM tests successfully control the size, maintaining reasonable power in panels with exogenous regressors and normal errors, even when cross section mean of the factor loadings is close to zero, where the CD test has little power. Also their spatial versions perform similarly in the case of spatial cross section dependence. However, it is shown that the bias-adjusted NLM tests are not as robust as the CD test to non-normal errors and/or in the presence of weakly exogenous regressors.

The plan of the paper is as follows. Section 2 presents the panel data model and the existing tests of cross section independence, and formulate the bias-adjusted tests. Section 3 reports the results of the Monte Carlo experiments. Section 4 provides some concluding remarks.

2 Model and Tests

Consider the following panel data model

$$y_{it} = \boldsymbol{\beta}_i' \mathbf{x}_{it} + u_{it}, \text{ for } i = 1, 2, ..., N;\ t = 1, 2, ..., T, \tag{1}$$

where i indexes the cross section dimension and t the time series dimension, $\mathbf{x}_{it}$ is a $k \times 1$ vector of strictly exogenous regressors with unity on its first row. The coefficients, $\boldsymbol{\beta}_i$, are defined on a compact set and allowed to vary across i. For each i, $u_{it} \sim IID(0, \sigma^2_{ui})$, for all t, although they could be cross-sectionally correlated. We first provide an over-view of the alternative approaches advanced in the literature to test the cross section independence of the errors.

2.1 Breusch and Pagan's Test of Cross Section Independence

In the SURE context with N fixed and as $T \to \infty$, Breusch and Pagan (1980) proposed a Lagrange multiplier (LM) statistic for testing the null of zero cross equation error correlations which is particularly simple to compute and does not require the system estimation of the SURE model. The test is based on the following LM statistic

$$LM = T \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}^2_{ij}, \tag{2}$$

where $\hat{\rho}_{ij}$ is the sample estimate of the pair-wise correlation of the residuals. Specifically,

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^{T} e_{it} e_{jt}}{\left(\sum_{t=1}^{T} e^2_{it}\right)^{1/2} \left(\sum_{t=1}^{T} e^2_{jt}\right)^{1/2}}, \tag{3}$$

and e_{it} is the Ordinary Least Squares (OLS) estimate of u_{it} defined by

$$e_{it} = y_{it} - \hat{\boldsymbol{\beta}}_i' \mathbf{x}_{it}, \tag{4}$$

with $\hat{\boldsymbol{\beta}}_i$ being the estimates of $\boldsymbol{\beta}_i$ computed using the OLS regression of y_{it} on $\mathbf{x}_{it}$ for each i, separately. This LM test is generally applicable and does not require a particular ordering of the cross section units. However, it is valid for N relatively small and T sufficiently large. In this setting Breusch and Pagan show that under the null hypothesis specified by

$$Cov\left(u_{it}, u_{jt}\right) = 0, \text{ for all } t, \, i \neq j, \tag{5}$$

the LM statistic is asymptotically distributed as chi-squared with $N(N-1)/2$ degrees of freedom. As it stands this test is not applicable when $N \to \infty$. However, noting that under the null hypothesis, as $T \to \infty$

$$T\hat{\rho}_{ij}^2 \to_d \chi_1^2,$$

with $\hat{\rho}_{ij}^2$, $i = 1, 2, .., N-1$, $j = i+1, 2, ..., N$, being asymptotically uncorrelated, the following scaled version of the LM statistic can be considered even for N and T large:

$$NLM = \sqrt{\frac{1}{N(N-1)}} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (T\hat{\rho}_{ij}^2 - 1). \tag{6}$$

Under H_0 with $T \to \infty$ first and then $N \to \infty$ we have:[2]

$$NLM \to_d N(0, 1).$$

However, a test based on this result is likely to exhibit substantial size distortions for N large and T small, a situation that can frequently arise in empirical applications. This is primarily due to the fact that for a finite T, $E(T\hat{\rho}_{ij}^2 - 1)$ will not be correctly centered at zero, and with N large the incorrect centering of the LM statistic is likely to be accentuated, resulting in size distortions that tend to get worse with N.

Recently Ullah (2004) provides unified techniques to obtain the exact and approximate moments of econometric estimators and test statistics. We make use of this approach to correct for the small sample bias of the LM statistic.

2.2 Finite Sample Adjustments

To obtain the bias-adjusted NLM tests we make the following assumptions:

Assumption 1: For each i, the disturbances, u_{it}, are serially independent with the mean 0 and the variance, $0 < \sigma_i^2 < \infty$.

Assumption 2: Under the null hypothesis defined by $H_0 : u_{it} = \sigma_i \varepsilon_{it}$, where $\varepsilon_{it} \sim IIDN(0, 1)$ for all i and t.

Assumption 3: The regressors, $\mathbf{x}_{it}$, are strictly exogenous such that $E(u_{it}|\mathbf{X}_i) = 0$, for all i and t where $\mathbf{X}_i = (\mathbf{x}_{i1}, \ldots, \mathbf{x}_{iT})'$ is a $T \times k$ matrix, and $\mathbf{X}_i'\mathbf{X}_i$ is a positive definite matrix.

Assumption 4: $T > k$ and the OLS residuals, e_{it}, in (4), are not all zero.

Now we introduce the following idempotent matrix of rank $T - k$,

$$\mathbf{M}_i = \mathbf{I}_T - \mathbf{H}_i; \quad \mathbf{H}_i = \mathbf{X}_i(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i', \tag{7}$$

such that $Tr(\mathbf{M}_i) = T - k$, where $\mathbf{I}_T$ is an identity matrix of order T. Similarly $\mathbf{M}_j = \mathbf{I}_T - \mathbf{H}_j$ is the same as $\mathbf{M}_i$ with $\mathbf{X}_i$ replaced by $\mathbf{X}_j$, and $Tr(\mathbf{M}_j) = T - k$. Then we can state the following theorem.

Theorem 1 *: Consider the panel data model (1), and suppose that Assumptions 1-4 hold. Then the exact mean and variance of* $(T-k)\hat{\rho}_{ij}^2$ *are, respectively, given by*

$$\mu_{Tij} = E\left[(T-k)\,\hat{\rho}_{ij}^2\right] = \frac{1}{T-k} Tr\left(\mathbf{M}_i\mathbf{M}_j\right) \tag{8}$$

[2] See also Frees (1995, p.395).

and

$$\upsilon_{Tij}^2 = Var\left[(T-k)\,\hat{\rho}_{ij}^2\right] = \left[Tr\left(\mathbf{M}_i\mathbf{M}_j\right)\right]^2 a_{1T} + 2Tr\left[\left(\mathbf{M}_i\mathbf{M}_j\right)^2\right] a_{2T}, \tag{9}$$

where

$$a_{1T} = a_{2T} - \frac{1}{(T-k)^2},\ a_{2T} = 3\left[\frac{(T-k-8)(T-k+2)+24}{(T-k+2)(T-k-2)(T-k-4)}\right]^2. \tag{10}$$

Proof is given in Appendix A.2.

Firstly, by using (8), the mean-bias-adjusted NLM test statistic is defined by

$$NLM^* = \sqrt{\frac{1}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left[(T-k)\hat{\rho}_{ij}^2 - \mu_{Tij}\right]. \tag{11}$$

Note that the mean of NLM* statistic is exactly zero for all T and N, and it is unlikely that the increase in N enhances the size distortion of the test. However, the variance of this test statistic is still subject to small sample bias. Therefore, under Assumptions 1-4, with $T \to \infty$ first, then $N \to \infty$, we would have (under H_0)

$$NLM^* \to_d N(0,1).$$

Next, using (8) and (9), the mean-variance-bias-adjusted NLM test statistic is defined as

$$NLM^{**} = \sqrt{\frac{2}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{(T-k)\hat{\rho}_{ij}^2 - \mu_{Tij}}{\upsilon_{Tij}}. \tag{12}$$

Under Assumptions 1-4, for all T, as $N \to \infty$ we would have (under H_0)

$$NLM^{**} \to_d N(0,1).$$

Clearly, the NLM* test is more likely to exhibit size distortions as compared to the NLM** test. However, it has the advantage of being relatively simple to compute.

2.3 Pesaran's (2004) CD Test and its Potential Inconsistency

Pesaran (2004) proposed a cross section independence test,

$$CD = \sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}\right), \tag{13}$$

and it was shown that under H_0, for T sufficiently large, as $N \to \infty$, $CD \to_d N(0,1)$.[3] Unlike the NLM test statistic, the above statistic has exactly mean zero for fixed values of T and N, under a wide class of panel data models, including heterogeneous dynamic models subject to multiple breaks in their slope coefficients and error variances, so long as the unconditional means of y_{it} and $\mathbf{x}_{it}$ are time-invariant and their innovations are symmetrically distributed. We also note that the NLM*and NLM** test statistics have exact means zero for fixed values of T and N as well, so long as Assumptions 1 to 4 hold.

However, as pointed out in Pesaran (2004), under a particular case the CD test would be inconsistent. To see this, we first specify the error structure of the model (1) under the alternative as

$$H_1 : u_{it} = \gamma_i f_t + \varepsilon_{it}, \tag{14}$$

[3]The CD test requires less restrictive version of Assumption 2, where ε_{it} are symmetrically *i.i.d.* distributed around zero with unit variance.

where $\gamma_i \sim IID\left(0, \sigma_\gamma^2\right)$, with $0 < \sigma_\gamma^2 < \infty$, are factor loadings, $f_t \sim IID\,(0,1)$ are unobserved common effects, and we assume $E\left(f_t^4\right) = \mu_{f4}$ with $0 < \mu_{f4} < \infty$, $\varepsilon_{it} \sim IIDN(0,1)$, and $E\left(\varepsilon_{it} f_s\right) = 0$ for all i, t, and s. Under H_1

$$Cov\left(u_{it}, u_{jt}\right) = E\left(\gamma_i\right) E\left(\gamma_j\right),$$

and the CD test statistic is centred at 0 if $E\left(\gamma_i\right) = 0$, even when $\gamma_i \neq 0$ for some i and $f_t \neq 0$. Therefore, under the alternatives with $E(\gamma_i) = 0$ the power of the CD test would not increase with N. But, the power of the LM type tests involves the terms

$$Cov\left(u_{it}^2, u_{jt}^2\right) = E\left(\gamma_i^2\right) E\left(\gamma_j^2\right) \mu_{f4} \tag{15}$$

which continue to differ from zero even when $E(\gamma_i) = 0$. Hence, the power of LM type tests will increase with N even under alternatives with $E(\gamma_i) = 0$. These results will continue to hold under multi-factor alternatives.

2.4 Tests for Local Cross Section Independence

As shown by Pesaran (2004), the power of the CD test is adversely affected when the dependence under the alternative hypothesis is spatial (local). The spatial dependence of the errors can be modelled using the spatial weight matrix, $\mathbf{W} = (w_{ij})$, which is applied to a particular ordering of the cross section units. It is often convenient to order the cross section units by their topological position, so that the p^{th} order neighbors of the i^{th} cross section unit can be defined as the $i + p$ and the $i - p$ cross section units.

Observing this, under the alternative hypothesis of a p^{th} order local dependence, Pesaran (2004) proposes a p^{th} order generalization of the CD test defined by

$$CD\,(p) = \sqrt{\frac{2T}{p\,(2N - p - 1)}} \left(\sum_{s=1}^{p} \sum_{i=1}^{N-s} \hat{\rho}_{i,i+s} \right). \tag{16}$$

In a similar manner, we may propose p^{th} order NLM, NLM*, and NLM** tests defined by

$$NLM(p) = \sqrt{\frac{1}{p\,(2N - p - 1)}} \sum_{s=1}^{p} \sum_{i=1}^{N-s} \left(T \hat{\rho}_{i,i+s}^2 - 1 \right), \tag{17}$$

$$NLM(p)^* = \sqrt{\frac{1}{p\,(2N - p - 1)}} \sum_{s=1}^{p} \sum_{i=1}^{N-s} \left[(T - k) \hat{\rho}_{i,i+s}^2 - \mu_{Ti,i+s} \right], \tag{18}$$

$$NLM(p)^{**} = \sqrt{\frac{2}{p\,(2N - p - 1)}} \sum_{s=1}^{p} \sum_{i=1}^{N-s} \frac{(T - k) \hat{\rho}_{i,i+s}^2 - \mu_{Ti,i+s}}{\upsilon_{Ti,i+s}}, \tag{19}$$

where

$$\mu_{Ti,i+s} = \frac{Tr\left(\mathbf{M}_i \mathbf{M}_{i+s}\right)}{T - k}, \quad \upsilon_{Ti,i+s} = \left[Tr\left(\mathbf{M}_i \mathbf{M}_{i+s}\right)\right]^2 a_{1T} + 2Tr\left[\left(\mathbf{M}_i \mathbf{M}_{i+s}\right)^2\right] a_{2T}.$$

3 Finite Sample Behavior of the Tests of Cross Section Independence

In this section we investigate the finite sample behavior of alternative tests of cross section independence by means of Monte Carlo experiments. We shall focus on our proposed bias-adjusted NLM tests, NLM* and NLM**, defined by (11) and (12), respectively, and compare their performance to the naive LM and NLM tests defined by (2) and (6), respectively, and the CD test defined by (13). Initially we consider the experiments in panels with exogenous regressors. In view of the validity of CD test in the various cases of dynamic models, we also consider experiments in the cases of stationary and unit root dynamic panels with and without parameter heterogeneity and structural breaks, under which the bias-adjusted NLM tests may not be valid. Finally, we also provide small sample evidence on alternative tests of cross section independence against spatial alternatives.

3.1 Experimental Designs

Initially, we consider the data generating process (DGP) specified as

$$y_{it} = \alpha_i + \sum_{\ell=2}^{k} x_{\ell it}\beta_i + u_{it},\ i = 1, 2, ..., N;\ t = 1, 2, ..., T, \tag{1}$$

where $\alpha_i \sim IIDN(1, 1)$, $\beta_{\ell i} \sim IIDN(1, 0.04)$. The covariates are generated as

$$x_{\ell it} = 0.6x_{\ell it-1} + v_{\ell it},\ i = 1, 2, ..., N;\ t = -50, ..., 0, ..., T;\ \ell = 2, 3, ..., k \tag{2}$$

with $x_{\ell i,-51} = 0$ where $v_{\ell it} \sim IIDN(0, \tau_{\ell i}^2/\left(1 - 0.6^2\right))$, $\tau_{\ell i}^2 \sim IID\chi^2(6)/6$. The disturbances are generated as

$$u_{it} = c_{(\gamma,k)}\left(\gamma_i f_t + \sigma_i \varepsilon_{it}\right),\ i = 1, 2, ..., N;\ t = 1, 2, ..., T,$$

where $f_t \sim IIDN(0, 1)$, and $\sigma_i^2 \sim IID\chi^2(2)/2$. The idiosyncratic errors, ε_{it}, are generated under two different schemes, (i) normal errors, $IIDN(0, 1)$, and (ii) chi-squared errors, $IID\left(\chi^2(1) - 1\right)/\sqrt{2}$. The latter is to check the robustness of the tests to non-normal errors. The values of α_i, $x_{\ell it}$, σ_i^2 are drawn for each $i = 1, 2, ..., N$, and then fixed across replications. Under the null hypothesis we have $\gamma_i = 0$ for all i, and under the alternatives we consider

$$\begin{aligned}(\text{i})\ \gamma_i &\sim IIDU[0.1, 0.3],\\ (\text{ii})\ \gamma_i &\sim IIDN(0, 0.1),\end{aligned}$$

where under (ii), the CD test is inconsistent, as shown above. In order to examine the effects of changing the number of regressors, $k = 2, 4, 6$ are considered. Meanwhile the same average population explanatory power of each cross section regression and the same degree of error cross section correlation are to be maintained for all k. To this end, $c_{(\gamma,k)}^2$ is set

$$c_{(\gamma,k)}^2 = \begin{cases} 1.04(k-1) & \text{for } \gamma_i = 0 \\ \frac{12.48}{12.13}(k-1), & \text{for } \gamma_i \sim IIDU[0.1, 0.3], \\ \frac{10.4}{11.0}(k-1), & \text{for } \gamma_i \sim IIDN(0, 0.1), \end{cases}$$

so that $\bar{R}^2 = 0.5$ across experiments, where $\bar{R}^2 = E(\sigma_{ui}^2)/\overline{Var}(y_{it})$ with $\sigma_{ui}^2 = Var(u_{it})$ and $\overline{Var}(y_{it}) = (k-1)E(\beta_{\ell i}^2) + E(\sigma_{ui}^2)$.

For examining the power of the first order cross section independence tests, the DGP defined by (1) for $k = 2$ but with spatially correlated errors are considered:

$$u_{it} = \lambda\left(0.5u_{i-1,t} + 0.5u_{i+1,t}\right) + \sigma_i\varepsilon_{it}, \tag{3}$$

with end points set at $u_{1t} = u_{2t} + \varepsilon_{1t}$ and $u_{Nt} = u_{N-1t} + \varepsilon_{Nt}$, where $\sigma_i^2 \sim IID\chi^2(2)/2$, $\varepsilon_{it} \sim IIDN(0, 1)$. For this DGP, the finite sample performance of the spatial version of the tests, defined by (16), (17), (18) and (19), are examined in the case of $p = 1$, and for the values of $\lambda = 0$, -0.1 and 0.1.

In the case of dynamic models, following Pesaran (2004), five specifications are considered. The first is the heterogeneous first order autoregressive (AR(1)) panel data model:

$$y_{it} = \mu_i (1 - \beta_i) + \beta_i y_{i,t-1} + u_{it}, \tag{4}$$

$$u_{it} = \gamma_i f_t + \sigma_{it}\varepsilon_{it}, i = 1, 2, ..., N; t = -50, -49, ..., T,$$

with $y_{i,-51} = 0$. The idiosyncratic errors, ε_{it}, are generated under two different schemes as above, (i) normal errors, $IIDN(0,1)$, and (ii) chi-squared errors, $IID\left(\chi^2(1) - 1\right)/\sqrt{2}$. Here we focus on the heterogeneous slope experiments where $\beta_i \sim IIDU[0,1)$. The fixed effects, μ_i, are drawn as $\varepsilon_{i0} + \eta_i$, with $\eta_i \sim IIDN(1,2)$, thus allowing for the possibility of correlations between fixed effects and the initial values, y_{i0}. γ_i, $\sigma_{it} = \sigma_i$, and f_t are generated in the same manner as specified for the DGP with exogenous regressors. The parameters η_i, β_i and σ_{it} are fixed across replications.

For examining the empirical size of the tests in the case of structural break(s), two specifications are considered. The first dynamic DGP is subject to single break, specified as (4) except $\mu_i \sim IIDN(1,1)$, $\beta_{it} = \beta_t = 0.6$ for $t = -50, ..., T/2$, $\beta_t = 0.8$ for $t = T/2 + 1, ..., T$; $\sigma_{it} = \sigma_t = \sqrt{1.5}$ for $t = -50, ..., T/2$, $\sigma_t = 1$ for $t = T/2 + 1, ..., T$, and $\varepsilon_{it} \sim IIDN(0,1)$. The second dynamic DGP is subject to multiple breaks, specified as (4) except $\beta_{it} = 0.5$ for $t = -50, ..., 0$ and all i, $\beta_{it} \sim IIDU[0,1)$ for $t = 1, ..., T$, $i = 1, ..., N$; $\sigma_{it}^2 \sim IID\chi^2(2)/2$ for $t = -50, ..., T$, $i = 1, ..., N$. For both designs, the first 50 observations are discarded.

Finally, the DGP subject to unit root, which is specified as (4) except $\beta_{it} = \beta = 1$ for all i and t, $\sigma_{it}^2 \sim IID\chi^2(2)/2$, are considered.

The test statistics are computed using the OLS residuals from the individual regressions. For all experiments the combinations of sample sizes $N = 10, 20, 30, 50, 100, 200$ and $T = 20, 30, 50, 100$ are considered. The nominal size of the tests is set at the 5% significance level. All experiments are based on 2000 replications.

3.2 Monte Carlo Outcomes

Table 1 reports the size of the tests for the DGP with different number of exogenous regressors ($k = 2, 4, 6$) and normal errors. As shown in Pesaran (2004), the CD test has the correct size, and the LM and NLM tests severely over-reject the null particularly for $N \geq T$. In contrast the adjusted versions of the NLM test, particularly the NLM** version defined by (12), successfully controls the size for all combinations of N and T, except when both k and N are large and T small. However, the NLM* version of the test defined by (11), tends to under-reject for small T, and such a tendency is accentuated as k is increased. In the case of $\gamma_i \sim IIDU[0.1, 0.3]$, whose results are reported in Table 2, the bias-adjusted NLM tests and the CD test seem to have reasonable power. In the case of $\gamma_i \sim IIDN(0, 0.1)$, whose results are reported in Table 3, as theory predicts the CD test has little power. The power of CD test increases with T very slowly, but it does not increase with N for given T. This is because the sample average of factor loadings for finite N can be different from zero far enough for the test to reject the null, for some replications, and the precision of this happening increases as T rises. On the other hand, the NLM* and NLM** tests maintain reasonable power under the same design. Overall, the powers of both the CD and bias-adjusted NLM tests increase faster with N than T, and it seems that the number of regressors does not affect the power of these tests much (for the same average explanatory power and error cross section dependence).

The results for the case with $IID\left(\chi^2(1) - 1\right)/\sqrt{2}$ errors are given in Table 4, and show that the bias-adjusted NLM tests are generally not as robust to non-normal errors as the CD test. They tend to over-reject (moderately) for all combinations of N and T.

Table 5 summarizes the results of the spatial first order tests. Interestingly, the sizes of the NLM(1) test are now closer to their nominal levels, except when N is much larger than T. The NLM(1)** test defined by (19), successfully controls the size, and the NLM(1)* test defined by

(18), tends to slightly under-reject. The CD(1) test has the correct size, as shown in Pesaran (2004). All of the bias-adjusted NLM(1) tests and CD(1) test seem to have reasonable power under the alternatives defined by $\lambda = \pm 0.1$.

Tables 6 to 10 provide the results for the various dynamic DGPs. For all experiments, the CD test has the correct size and the LM and NLM tests severely over-reject the null when $N \geq T$. This is to be expected, as discussed above, since for small T relative to N, the mean approximation of $\hat{\rho}_{ij}^2 \approx 1$ will not be correct, and with N large the incorrect centering of the test indicator is likely to be accentuated, resulting in size distortions that tend to get worse with N.

Unlike in the case of DGP with exogenous regressors and normal errors, in the case of heterogenous dynamic AR(1) specifications with $IIDN\,(0,1)$ errors (Table 6), the bias-adjusted NLM tests tend to over-reject when N is much larger than T. With respect to the power, as was in the case of DGP with exogenous regressors, the CD test has little power in the case of $\gamma_j \sim IIDN\ (0, 0.1)$. The results for $IID\left(\chi^2\,(1) - 1\right)/\sqrt{2}$ errors (Table 7) is similar to those in Table 4. For the DGP with a single structural break (Table 8), the bias-adjusted NLM tests reject the null too often. For example, when $N = 200$, the estimated size of the NLM** test is 100% for all T. In the case of multiple structural breaks (Table 9), the bias-adjusted NLM tests tend to over-reject, especially for $N \geq T$. Finally, in the case of models with unit roots (Table 10) the bias-adjusted NLM tests also tend to over-reject, with the extent of over-rejection increasing with N.

4 Concluding Remarks

This paper has proposed bias-adjusted normal approximation version of the LM test (NLM) of cross section independence. For the bias-adjustment, we derived the exact mean and variance of the test indicator of the LM statistic in the case of the model with strictly exogenous regressors and normal errors, based on the work in Ullah (2004), so that the centering of the LM statistic is correct for fixed T and large N. Importantly, the proposed bias-adjusted NLM tests are consistent even when the Pesaran's (2004) CD test is inconsistent. Small sample evidence based on Monte Carlo experiments suggests that the bias-adjusted NLM tests successfully control the size, maintaining reasonable power in panels with exogenous regressors and normal errors, even when cross section mean of the factor loadings is close to zero, where the CD test has little power. Also their spatial versions perform similarly in the case of spatial cross section dependence. However, it is shown that the bias-adjusted NLM tests are not as robust as the CD test to non-normal errors and/or in the presence of weakly exogenous regressors.

Clearly, it would be worth deriving the mean and variance of the LM test statistic in the case of dynamic models, and in the case where the errors are non-normal. This will be the subject of future research.

A Appendix

A.1 Evaluation of the First Two Derivatives of $E\left(W^{-r}\right)$

Let us consider a quadratic form $W = \mathbf{u}'\mathbf{M}\mathbf{u}$, where the $T \times 1$ vector $\mathbf{u} \sim N(\boldsymbol{\mu}, \mathbf{I}_T)$ and $\mathbf{M}$ is an idempotent matrix of rank $m \leq T$. Then W is distributed as a non-central chi-squared distribution with the non-centrality parameter $\theta = \boldsymbol{\mu}'\mathbf{M}\boldsymbol{\mu}/2$. When $\boldsymbol{\mu} = \mathbf{0}$, hence $\theta = 0$, W is distributed as a central chi-square distribution. In what follows we evaluate $\mathbf{d}E\left(W^{-r}\right)$ and $\mathbf{d}\mathbf{d}'E\left(W^{-r}\right)$, where $\mathbf{d} = \boldsymbol{\mu} + \partial/\partial\boldsymbol{\mu}$ and $r = 1, 2, ...$

Now

$$\mathbf{d}'E\left(W^{-r}\right) = \left(\boldsymbol{\mu}' + \frac{\partial}{\partial\boldsymbol{\mu}'}\right)E\left(W^{-r}\right) = \boldsymbol{\mu}'E\left(W^{-r}\right) + \left[\frac{\partial}{\partial\theta}E\left(W^{-r}\right)\right]\boldsymbol{\mu}'\mathbf{M}, \tag{A.1}$$

$$\begin{aligned}\mathbf{d}\mathbf{d}'E\left(W^{-r}\right) &= \left[\boldsymbol{\mu} + \frac{\partial}{\partial\boldsymbol{\mu}}\right]\left[\boldsymbol{\mu}'E\left(W^{-r}\right) + \frac{\partial}{\partial\boldsymbol{\mu}'}E\left(W^{-r}\right)\right] \\ &= E\left(W^{-r}\right) + \boldsymbol{\mu}\boldsymbol{\mu}'E\left(W^{-r}\right) + \left[\frac{\partial}{\partial\theta}E\left(W^{-r}\right)\right]\left[\boldsymbol{\mu}\boldsymbol{\mu}'\mathbf{M} + \mathbf{M}\boldsymbol{\mu}\boldsymbol{\mu}' + \mathbf{M}\right] \\ &\quad + \left[\frac{\partial^2}{\partial\theta^2}E\left(W^{-r}\right)\right]\mathbf{M}\boldsymbol{\mu}\boldsymbol{\mu}'\mathbf{M},\end{aligned}$$

where we use

$$\begin{aligned}\frac{\partial}{\partial\boldsymbol{\mu}}E\left(W^{-r}\right) &= \left[\frac{\partial}{\partial\theta}E\left(W^{-r}\right)\right]\mathbf{M}\boldsymbol{\mu}, \\ \frac{\partial^2}{\partial\boldsymbol{\mu}\partial\boldsymbol{\mu}'}E\left(W^{-r}\right) &= \frac{\partial}{\partial\boldsymbol{\mu}}\left[\frac{\partial}{\partial\theta}E\left(W^{-r}\right)\boldsymbol{\mu}'\mathbf{M}\right] = \left[\frac{\partial^2}{\partial\theta^2}E\left(W^{-r}\right)\right]\mathbf{M}\boldsymbol{\mu}\boldsymbol{\mu}'\mathbf{M} + \left[\frac{\partial}{\partial\theta}E\left(W^{-r}\right)\right]\mathbf{M}.\end{aligned} \tag{A.2}$$

First we note from Ullah (2004, p.193) that

$$\begin{aligned}E\left(W^{-r}\right) &= \frac{1}{2^r}e^{-\theta}\sum_{i=0}^{\infty}\frac{\Gamma\left(\frac{m}{2} - r + i\right)}{\Gamma\left(\frac{m}{2} + i\right)}\frac{\theta^i}{i!}, \text{ when } \theta \neq 0 \\ &= \frac{1}{2^r}\frac{\Gamma\left(\frac{m}{2} - r\right)}{\Gamma\left(\frac{m}{2}\right)} = \frac{1}{(m-2)(m-4)...(m-2r)}, \text{ when } \theta \neq 0.\end{aligned} \tag{A.3}$$

Further, for $s = 1, 2, ...$,

$$\begin{aligned}\frac{\partial^s}{\partial\theta^s}EW^{-r} &= (-1)^s\frac{1}{2^r}\frac{\Gamma(r+s)}{\Gamma(r)}e^{-\theta}\sum_{i=0}^{\infty}\frac{\Gamma\left(\frac{m}{2} - r + i\right)}{\Gamma\left(\frac{m}{2} + 1s + i\right)}\frac{\theta^i}{i!}, \text{ when } \theta \neq 0 \\ &= (-1)^s\frac{1}{2^r}\frac{\Gamma(r+s)}{\Gamma(r)}\frac{\Gamma\left(\frac{m}{2} - r\right)}{\Gamma\left(\frac{m}{2} + s\right)}, \text{ when } \theta = 0.\end{aligned} \tag{A.4}$$

Substituting $r = 1, 2$ and $s = 1, 2$ in (A.3) and (A.4), and using these results in (A.1) we get $\mathbf{d}'E\left(W^{-r}\right)$and $\mathbf{d}\mathbf{d}'E\left(W^{-r}\right)$ for $\theta \neq 0$. When $\boldsymbol{\mu} = \mathbf{0}$, hence $\theta = 0$, we obtain

$$\mathbf{d}'E\left(W^{-r}\right) = \mathbf{0}', \quad \mathbf{d}\mathbf{d}'E\left(W^{-r}\right) = E\left(W^{-r}\right) + \left[\frac{\partial}{\partial\theta}E\left(W^{-r}\right)\right]\mathbf{M}, \tag{A.5}$$

where $\partial E\left(W^{-r}\right)/\partial\theta$ is given by (A.4) for $s = 1$ and $\theta = 0$.

A.2 Proof of Theorem 1

From (3) and (4)

$$\hat{\rho}_{ij}^2 = \frac{\left(\mathbf{u}_i'\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j\right)^2}{\left(\mathbf{u}_i'\mathbf{M}_i\mathbf{u}_i\right)\left(\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j\right)} \tag{A.6}$$

$$\begin{aligned}&= \frac{\mathbf{u}_i'\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j\mathbf{u}_j'\mathbf{M}_j\mathbf{M}_i\mathbf{u}_i}{\left(\mathbf{u}_i'\mathbf{M}_i\mathbf{u}_i\right)\left(\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j\right)} \\ &= \frac{\mathbf{u}_i'\mathbf{A}_{ij}\mathbf{u}_i}{\mathbf{u}_i'\mathbf{M}_i\mathbf{u}_i} \text{ with } \mathbf{A}_{ij} = \frac{\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j\mathbf{u}_j'\mathbf{M}_j\mathbf{M}_i}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j},\end{aligned}$$

where $\mathbf{M}_j$ and $\mathbf{M}_i$ are as defined in (7), $\mathbf{u}_i \sim N(\mathbf{0}, \mathbf{I}_T), \mathbf{u}_j \sim N(\mathbf{0}, \mathbf{I}_T)$. Taking the expectations on both sides of (A.6) we can write

$$E\left(\hat{\rho}_{ij}^2\right) = E_{u_j}\left[E\left(\hat{\rho}_{ij}^2|\mathbf{u}_j\right)\right],$$

in which

$$
\begin{aligned}
E\left(\hat{\rho}_{ij}^2|\mathbf{u}_j\right) &= E\left(\frac{\mathbf{u}_i'\mathbf{A}_{ij}\mathbf{u}_i}{\mathbf{u}_i'\mathbf{M}_i\mathbf{u}_i}\right) \qquad (A.7)\\
&= E\left[\left(\mathbf{u}_i'\mathbf{A}_{ij}\mathbf{u}_i\right)W_i^{-1}\right]\\
&= \left(\mathbf{d}_i'\mathbf{A}_{ij}\mathbf{d}_i\right)E\left(W_i^{-1}\right) = Tr\left\{\mathbf{A}_{ij}\left[\mathbf{d}_i\mathbf{d}_i'E\left(W_i^{-1}\right)\right]\right\}\\
&= Tr\left(\mathbf{A}_{ij}\right)E\left(W_i^{-1}\right) + \left(\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i\right)E\left(W_i^{-1}\right) + \left[\frac{\partial}{\partial\theta_i}E\left(W_i^{-1}\right)\right]\times\\
&\quad \left[\boldsymbol{\mu}_i'\mathbf{M}_i\mathbf{A}_{ij}\boldsymbol{\mu}_i + \boldsymbol{\mu}_i'\mathbf{A}_{ij}\mathbf{M}_i\boldsymbol{\mu}_i + Tr\left(\mathbf{A}_{ij}\mathbf{M}_i\right)\right] + \left[\frac{\partial^2}{\partial\theta_i^2}E\left(W^{-1}\right)\right]\boldsymbol{\mu}_i'\mathbf{M}_i\mathbf{A}_{ij}\mathbf{M}_i\boldsymbol{\mu}_i,
\end{aligned}
$$

where we use (A.1), $W_i = \mathbf{u}_i'\mathbf{M}_i\mathbf{u}_i$, $\mathbf{d}_i = \boldsymbol{\mu}_i + \partial/\partial\boldsymbol{\mu}_i$, the third equality follows by using the results in Ullah (2004, (2.28)), and $\theta_i = \boldsymbol{\mu}_i'\mathbf{M}_i\boldsymbol{\mu}_i/2$. Now

$$
\begin{aligned}
E\left(\hat{\rho}_{ij}^2\right) &= E\left(W_i^{-1}\right)E\left(\frac{\mathbf{u}_j'\mathbf{M}_j\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j}\right) + \left(\boldsymbol{\mu}_i'E\left(\mathbf{A}_{ij}\right)\boldsymbol{\mu}_i\right)E\left(W_i^{-1}\right) \qquad (A.8)\\
&\quad + \left[\frac{\partial}{\partial\theta_i}E\left(W_i^{-1}\right)\right]\left[2\boldsymbol{\mu}_i'E\left(\mathbf{A}_{ij}\right)\boldsymbol{\mu}_i + E\left(\frac{\mathbf{u}_j'\mathbf{M}_j\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j}\right)\right]\\
&\quad + \left[\frac{\partial^2}{\partial\theta_i^2}E\left(W_i^{-1}\right)\right]\boldsymbol{\mu}_i'E\left(\mathbf{A}_{ij}\right)\boldsymbol{\mu}_i.
\end{aligned}
$$

But for $\boldsymbol{\mu}_i = \mathbf{0}$

$$
\begin{aligned}
E\left(\hat{\rho}_{ij}^2\right) &= E\left(W_i^{-1}\right)E\left(\frac{\mathbf{u}_j'\mathbf{M}_j\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j}\right) + \left[\frac{\partial}{\partial\theta_i}E\left(W_i^{-1}\right)\right]E\left(\frac{\mathbf{u}_j'\mathbf{M}_j\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j}\right) \qquad (A.9)\\
&= \left[E\left(W_i^{-1}\right) + \frac{\partial}{\partial\theta_i}E\left(W_i^{-1}\right)\right]E\left(\frac{\mathbf{u}_j'\mathbf{M}_j\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j}\right).
\end{aligned}
$$

Now, writing $\mathbf{B}_{ij} = \mathbf{M}_j\mathbf{M}_i\mathbf{M}_j$,$W_j = \mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j$, $\theta_j = \boldsymbol{\mu}_j'\mathbf{M}_j\boldsymbol{\mu}_j/2$ and using (A.7),

$$
\begin{aligned}
E\left(\frac{\mathbf{u}_j'\mathbf{B}_{ij}\mathbf{u}_j}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j}\right) &= Tr\left(\mathbf{B}_{ij}\right)E\left(W_j^{-1}\right) + \left(\boldsymbol{\mu}_j'\mathbf{B}_{ij}\boldsymbol{\mu}_j\right)E\left(W_j^{-1}\right) \qquad (A.10)\\
&\quad + \left[\frac{\partial}{\partial\theta_j}E\left(W_j^{-1}\right)\right]\left[2\boldsymbol{\mu}_j'\mathbf{M}_j\mathbf{B}_{ij}\boldsymbol{\mu}_j + Tr\left(\mathbf{B}_{ij}\mathbf{M}_j\right)\right]\\
&\quad + \left[\frac{\partial^2}{\partial\theta_j^2}E\left(W^{-1}\right)\right]\boldsymbol{\mu}_j'\mathbf{M}_j\mathbf{B}_{ij}\mathbf{M}_j\boldsymbol{\mu}_j,
\end{aligned}
$$

which can be written for $\boldsymbol{\mu}_j = \mathbf{0}$ as

$$
E\left(\frac{\mathbf{u}_j'\mathbf{B}_{ij}\mathbf{u}_j}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j}\right) = \left[E\left(W_j^{-1}\right) + \frac{\partial}{\partial\theta_j}E\left(W_j^{-1}\right)\right]Tr\left(\mathbf{B}_{ij}\right),\ Tr\left(\mathbf{B}_{ij}\right) = Tr\left(\mathbf{M}_i\mathbf{M}_j\right). \qquad (A.11)
$$

Substituting (A.11) in (A.9) we get

$$
\begin{aligned}
E\left(\hat{\rho}_{ij}^2\right) &= \left[E\left(W_i^{-1}\right) + \frac{\partial}{\partial\theta_i}E\left(W_i^{-1}\right)\right]\left[E\left(W_j^{-1}\right) + \frac{\partial}{\partial\theta_j}E\left(W_j^{-1}\right)\right]Tr\left(\mathbf{M}_i\mathbf{M}_j\right) \qquad (A.12)\\
&= \left[\frac{1}{m-2} - \frac{2}{m\left(m-2\right)}\right]^2 Tr\left(\mathbf{M}_i\mathbf{M}_j\right)\\
&= \frac{1}{m^2}Tr\left(\mathbf{M}_i\mathbf{M}_j\right),
\end{aligned}
$$

and

$$
E\left(m\hat{\rho}_{ij}^2\right) = \frac{1}{m}Tr\left(\mathbf{M}_i\mathbf{M}_j\right), \qquad (A.13)
$$

where $m = T - k$ and we use the results (A.3) and (A.4) for $r = 1$ and $s = 1$.

Next we consider

$$
\hat{\rho}_{ij}^4 = \left(\mathbf{u}_i'\mathbf{A}_{ij}\mathbf{u}_i\right)^2 / \left(\mathbf{u}_i'\mathbf{M}_i\mathbf{u}_i\right)^2, \qquad (A.14)
$$

and taking the expectations on both sides of (A.14) we get

$$
E\left(\hat{\rho}_{ij}^4\right) = E_{u_j}\left[E\left(\hat{\rho}_{ij}^4|\mathbf{u}_j\right)\right],
$$

in which using Ullah (2004, (2.28))

$$
E\left(\hat{\rho}_{ij}^4|\mathbf{u}_j\right) = E\left[\left(\mathbf{u}_i'\mathbf{A}_{ij}\mathbf{u}_i\right)^2 W_i^{-2}\right] \qquad (A.15)
$$

$$
\begin{aligned}
&= \left(\mathbf{d}_i'\mathbf{A}_{ij}\mathbf{d}_i\right)^2 E\left(W_i^{-2}\right) \\
&= \left(\mathbf{d}_i'\mathbf{A}_{ij}\mathbf{d}_i\right)\left[\left(\mathbf{d}_i'\mathbf{A}_{ij}\mathbf{d}_i\right) E\left(W_i^{-2}\right)\right] \\
&= \left(\mathbf{d}_i'\mathbf{A}_{ij}\mathbf{d}_i\right) Tr\left[\mathbf{A}_{ij}\mathbf{d}_i\mathbf{d}_i' E\left(W_i^{-2}\right)\right] \\
&= Tr\left[\mathbf{A}_{ij}\mathbf{d}_i\mathbf{d}_i' c(\boldsymbol{\mu}_i)\right],
\end{aligned}
$$

where $c(\boldsymbol{\mu}_i) = Tr\left[\mathbf{A}_{ij}\mathbf{d}_i\mathbf{d}_i' E\left(W_i^{-2}\right)\right]$, and using (A.7) (see also Ullah (2004, Chapter 2)),

$$
\begin{aligned}
c(\boldsymbol{\mu}_i) &= Tr\left(\mathbf{A}_{ij}\right) E\left(W_i^{-2}\right) + \left(\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i\right) E\left(W_i^{-2}\right) + \left[\frac{\partial}{\partial\theta_i} E\left(W_i^{-2}\right)\right]\left[2\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i + Tr\left(\mathbf{A}_{ij}\right)\right] \\
&\quad + \left[\frac{\partial^2}{\partial\theta_i^2} E\left(W_i^{-2}\right)\right]\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i.
\end{aligned}
\tag{A.16}
$$

In order to evaluate the term in the last equalities of (A.15) we note that

$$
\mathbf{d}_i' c\left(\boldsymbol{\mu}_i\right) = \left(\boldsymbol{\mu}_i' + \frac{\partial}{\partial\boldsymbol{\mu}_i'}\right) c\left(\boldsymbol{\mu}_i\right),
\tag{A.17}
$$

$$
\begin{aligned}
\mathbf{d}_i\mathbf{d}_i' c\left(\boldsymbol{\mu}_i\right) &= \left(\boldsymbol{\mu}_i + \frac{\partial}{\partial\boldsymbol{\mu}_i}\right)\left[\boldsymbol{\mu}_i' c(\boldsymbol{\mu}_i) + \frac{\partial}{\partial\boldsymbol{\mu}_i'} c\left(\boldsymbol{\mu}_i\right)\right] \\
&= \boldsymbol{\mu}_i\boldsymbol{\mu}_i' c\left(\boldsymbol{\mu}_i\right) + \boldsymbol{\mu}_i\frac{\partial}{\partial\boldsymbol{\mu}_i'} c\left(\boldsymbol{\mu}_i\right) + c\left(\boldsymbol{\mu}_i\right) + \left[\frac{\partial}{\partial\boldsymbol{\mu}_i} c\left(\boldsymbol{\mu}_i\right)\right]\boldsymbol{\mu}_i' + \frac{\partial^2}{\partial\boldsymbol{\mu}_i\partial\boldsymbol{\mu}_i'} c\left(\boldsymbol{\mu}_i\right).
\end{aligned}
\tag{A.18}
$$

Using (A.18) in (A.15) we then get

$$
\begin{aligned}
Tr\left[\mathbf{A}_{ij}\mathbf{d}_i\mathbf{d}_i' c(\boldsymbol{\mu}_i)\right] &= \left(\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i\right) c\left(\boldsymbol{\mu}_i\right) + Tr\left(\mathbf{A}_{ij}\right) c\left(\boldsymbol{\mu}_i\right) \\
&\quad + \left\{Tr\left[\mathbf{A}_{ij}\boldsymbol{\mu}_i\frac{\partial}{\partial\boldsymbol{\mu}_i'} c\left(\boldsymbol{\mu}_i\right)\right] + Tr\left[\mathbf{A}_{ij}\frac{\partial}{\partial\boldsymbol{\mu}_i} c\left(\boldsymbol{\mu}_i\right)\boldsymbol{\mu}_i'\right]\right\} \\
&\quad + Tr\left[\mathbf{A}_{ij}\frac{\partial^2}{\partial\boldsymbol{\mu}_i\partial\boldsymbol{\mu}_i'} c\left(\boldsymbol{\mu}_i\right)\right],
\end{aligned}
\tag{A.19}
$$

in which, using (A.16), we can verify that

$$
\begin{aligned}
\frac{\partial c(\boldsymbol{\mu}_i)}{\partial\boldsymbol{\mu}_i} &= Tr\left(\mathbf{A}_{ij}\right)\left[\frac{\partial}{\partial\theta_i} E\left(W_i^{-2}\right)\right]\mathbf{M}_i\boldsymbol{\mu}_i + 2\mathbf{A}_{ij}\boldsymbol{\mu}_i E W_i^{-2} + \left[\frac{\partial}{\partial\theta_i} E\left(W_i^{-2}\right)\right]\mathbf{M}_i\boldsymbol{\mu}_i\left(\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i\right) \\
&\quad + \left[\frac{\partial^2}{\partial\theta_i^2} E\left(W_i^{-2}\right)\right]\mathbf{M}_i\boldsymbol{\mu}_i\left[2\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i + Tr\left(\mathbf{A}_{ij}\right)\right] \\
&\quad + \left[\frac{\partial}{\partial\theta_i} E\left(W_i^{-2}\right)\right]\left(4\mathbf{A}_{ij}\boldsymbol{\mu}_i\right) + \left[\frac{\partial^3}{\partial\theta_i^3} E\left(W_i^{-2}\right)\right]\mathbf{M}_i\boldsymbol{\mu}_i\left(\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i\right) \\
&\quad + \left(2\mathbf{A}_{ij}\boldsymbol{\mu}_i\right)\left[\frac{\partial^2}{\partial\theta_i^2} E\left(W_i^{-2}\right)\right] \\
&= a_1(\theta)\left(2\mathbf{A}_{ij}\boldsymbol{\mu}_i\right) + a_2(\theta)\left(\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i\right)\mathbf{M}_i\boldsymbol{\mu}_i + a_3(\theta) Tr\left(\mathbf{A}_{ij}\right)\mathbf{M}_i\boldsymbol{\mu}_i,
\end{aligned}
\tag{A.20}
$$

where

$$
\begin{aligned}
a_1(\theta) &= E\left(W_i^{-2}\right) + 2\frac{\partial}{\partial\theta_i} E\left(W_i^{-2}\right) + \frac{\partial^2}{\partial\theta_i^2} E\left(W_i^{-2}\right), \\
a_2(\theta) &= \frac{\partial}{\partial\theta_i} E\left(W_i^{-2}\right) + 2\frac{\partial^2}{\partial\theta_i^2} E\left(W_i^{-2}\right) + \frac{\partial^3}{\partial\theta_i^3} E\left(W_i^{-2}\right), \\
a_3(\theta) &= \frac{\partial}{\partial\theta_i} E\left(W_i^{-2}\right) + \frac{\partial^2}{\partial\theta_i^2} E\left(W_i^{-2}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial^2}{\partial\boldsymbol{\mu}_i\partial\boldsymbol{\mu}_i'} c\left(\boldsymbol{\mu}_i\right) &= 2\mathbf{A}_{ij} a_1\left(\theta\right) + 2\mathbf{M}_i\boldsymbol{\mu}_i\boldsymbol{\mu}_i'\mathbf{A}_{ij} a_2\left(\theta\right) \\
&\quad + \left\{2\mathbf{A}_{ij}\boldsymbol{\mu}_i\boldsymbol{\mu}_i'\mathbf{M}_i + \boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i\mathbf{M}_i\right\} a_2\left(\theta\right) \\
&\quad + \left(\boldsymbol{\mu}_i'\mathbf{A}_{ij}\boldsymbol{\mu}_i\right)\mathbf{M}_i\boldsymbol{\mu}_i\boldsymbol{\mu}_i'\mathbf{M}_i a_4\left(\theta\right) \\
&\quad + Tr\left(\mathbf{A}_{ij}\right)\mathbf{M}_i a_3\left(\theta\right) + \mathbf{M}_i\boldsymbol{\mu}\boldsymbol{\mu}_i'\mathbf{M}_i Tr\left(\mathbf{A}_{ij}\right) a_5(\theta),
\end{aligned}
\tag{A.21}
$$

where

$$
\begin{aligned}
a_4(\theta) &= \frac{\partial^2}{\partial\theta_i^2} E\left(W_i^{-2}\right) + 2\frac{\partial^3}{\partial\theta_i^3} E\left(W_i^{-2}\right) + \frac{\partial^4}{\partial\theta_i^4} E\left(W_i^{-2}\right), \\
a_5(\theta) &= \frac{\partial^2}{\partial\theta_i^2} E\left(W_i^{-2}\right) + \frac{\partial^3}{\partial\theta_i^3} E\left(W_i^{-2}\right).
\end{aligned}
$$

In order to evaluate (A.19) for $\boldsymbol{\mu}_i = \mathbf{0}$ we first note that

$$\begin{aligned} c(\mathbf{0}) &= Tr(\mathbf{A}_{ij}) \left[E\left(W_i^{-2}\right) + \frac{\partial}{\partial \theta_i} E\left(W_i^{-2}\right) \right], \\ \frac{\partial}{\partial \boldsymbol{\mu}_i} c(\boldsymbol{\mu}_i) &= \mathbf{0}, \\ \frac{\partial^2}{\partial \boldsymbol{\mu}_i \partial \boldsymbol{\mu}_i'} c(\boldsymbol{\mu}_i) &= 2\mathbf{A}_{ij} \left[E\left(W_i^{-2}\right) + 2\frac{\partial}{\partial \theta_i} E\left(W_i^{-2}\right) + \frac{\partial^2}{\partial \theta_i^2} E\left(W_i^{-2}\right) \right] \\ &\quad + Tr(\mathbf{A}_{ij}) \mathbf{M}_i \left[\frac{\partial}{\partial \theta_i} E\left(W_i^{-2}\right) + \frac{\partial^2}{\partial \theta_i^2} E\left(W_i^{-2}\right) \right]. \end{aligned} \tag{A.22}$$

Therefore, for $\boldsymbol{\mu}_i = \mathbf{0}$, we can simplify

$$\begin{aligned} Tr\left[\mathbf{A}_{ij} \mathbf{d}_i \mathbf{d}_i' c(\boldsymbol{\mu}_i)\right] &= \left[[Tr(\mathbf{A}_{ij})]^2 + 2Tr\left(\mathbf{A}_{ij}^2\right) \right] \left[E\left(W_i^{-2}\right) + 2\frac{\partial}{\partial \theta_i} E\left(W_i^{-2}\right) + \frac{\partial^2}{\partial \theta_i^2} E\left(W_i^{-2}\right) \right] \\ &= 3\left[Tr(\mathbf{A}_{ij})\right]^2 \left[E\left(W_i^{-2}\right) + 2\frac{\partial}{\partial \theta_i} E\left(W_i^{-2}\right) + \frac{\partial^2}{\partial \theta_i^2} E\left(W_i^{-2}\right) \right], \end{aligned} \tag{A.23}$$

because $[Tr(\mathbf{A}_{ij})]^2 = Tr\left(\mathbf{A}_{ij}^2\right)$. Further, using (A.23), (A.3) and (A.4) in (A.15) we get, for $\boldsymbol{\mu}_i = \mathbf{0}$,

$$\begin{aligned} E\left(\hat{\rho}_{ij}^4\right) &= 3E\left\{[Tr(\mathbf{A}_{ij})]^2\right\} a_{1m} \\ &= 3\left\{[Tr(\mathbf{M}_i\mathbf{M}_j)]^2 + 2Tr\left[(\mathbf{M}_i\mathbf{M}_j)^2\right]\right\} a_{1m}^2, \end{aligned} \tag{A.24}$$

where

$$\begin{aligned} E\left\{[Tr(\mathbf{A}_{ij})]^2\right\} &= E\left[\left(\frac{\mathbf{u}_j'\mathbf{M}_j\mathbf{M}_i\mathbf{M}_j\mathbf{u}_j}{\mathbf{u}_j'\mathbf{M}_j\mathbf{u}_j}\right)^2\right] \\ &= \left\{[Tr(\mathbf{M}_i\mathbf{M}_j)]^2 + 2Tr\left[(\mathbf{M}_i\mathbf{M}_j)^2\right]\right\} a_{1m}, \end{aligned}$$

and

$$a_{1m} = \frac{(m-8)(m+2)+24}{(m+2)\,m\,(m-2)\,(m-4)}.$$

Next, using (A.12) and (A.24), we get

$$\begin{aligned} V\left(\hat{\rho}_{ij}^2\right) &= E\left(\hat{\rho}_{ij}^4\right) - \left[E\left(\hat{\rho}_{ij}^2\right)\right]^2 \\ &= [Tr(\mathbf{M}_i\mathbf{M}_j)]^2 \left(3a_{1m}^2 - \frac{1}{m^4}\right) + 6Tr\left[(\mathbf{M}_i\mathbf{M}_j)^2\right] a_{1m}^2, \end{aligned} \tag{A.25}$$

which gives

$$V\left(m\hat{\rho}_{ij}^2\right) = [Tr(\mathbf{M}_i\mathbf{M}_j)]^2 \left(b_m - \frac{1}{m^2}\right) + 2b_m Tr\left[(\mathbf{M}_i\mathbf{M}_j)^2\right],$$

where

$$b_m = 3\left[\frac{(m-8)(m+2)+24}{(m+2)\,(m-2)\,(m-4)}\right]^2 = 3m^2 a_{1m}^2.$$

References

Anselin, L., (1988). *Spatial Econometrics: Methods and Models.* Dorddrecht: Kluwer Academic Publishers.

Anselin, L., (2001). Spatial econometrics, In: B. Baltagi (ed.) *A companion to Theoretical Econometrics*, Oxford: Blackwell.

Breusch, T.S., Pagan, A.R., (1980). The Lagrange multiplier test and its applications to model specification tests in econometrics. *Review of Economic Studies* 47, 239-53.

Frees, E.W., (1995). Assessing cross-sectional correlation in panel data. *Journal of Econometrics* 69, 393-414.

Moran, P.A.P., (1948). The interpretation of statistical maps. *Biometrika* 35, 255-60.

Pesaran, M.H., (2004). General diagnostic tests for cross section dependence in panels. *CESifo Working Papers* No.1233.

Ullah, A., (2004). *Finite Sample Econometrics*. New York: Oxford University Press.

Zellner, A. (1962). An efficiency method of estimating seemingly unrelated regression equations and tests for aggregation bias. *Journal of the American Statistical Association* 57, 348-368.

Table 1
Size of Cross Section Independence Tests with Exogenous Regressors, $\gamma_i = \gamma = 0$
Normal Errors with Different Number of Regressors (k)

	$k=2$						$k=4$						$k=6$					
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**
LM																		
20	7.50	11.65	17.85	35.90	85.70	100.00	8.65	15.65	25.80	50.20	96.95	100.00	10.85	22.85	42.15	71.35	99.95	100.00
30	5.40	8.05	11.90	19.70	54.25	97.70	7.60	11.10	14.85	25.35	67.25	99.45	7.80	12.45	21.35	38.60	87.45	100.00
50	5.40	6.35	8.95	11.65	26.25	66.15	6.35	8.30	9.60	14.00	32.40	77.35	5.05	8.00	11.75	17.85	43.40	90.20
100	4.80	5.90	6.10	8.80	11.20	26.95	5.50	5.05	6.15	8.65	11.60	27.55	4.75	5.55	6.95	9.70	15.40	34.15
NLM																		
20	5.75	8.00	12.75	25.60	78.20	100.00	6.15	10.90	18.15	39.45	94.20	100.00	8.25	17.20	32.45	60.70	99.80	100.00
30	4.35	6.20	7.95	13.25	41.15	95.30	5.55	7.65	10.15	17.60	56.15	99.10	5.90	8.90	14.50	30.35	80.10	100.00
50	5.25	4.70	5.90	7.30	17.60	54.30	5.40	5.90	6.85	8.65	22.15	66.45	4.40	6.35	8.20	12.25	32.00	84.25
100	4.45	5.20	5.45	6.05	6.85	18.15	4.75	4.55	4.60	6.40	7.50	18.35	4.75	5.70	5.70	6.60	9.90	22.25
NLM*																		
20	2.95	2.40	2.35	2.25	2.70	2.45	1.60	1.30	1.65	1.70	2.30	3.05	1.05	0.85	1.05	1.65	1.45	2.25
30	2.85	3.40	3.65	3.55	3.15	3.10	2.45	2.25	2.45	2.40	2.95	3.45	1.90	1.60	1.90	1.55	1.75	2.55
50	4.05	3.40	4.00	3.70	3.55	3.85	3.25	3.55	3.55	3.60	3.10	3.35	1.95	2.70	2.75	2.60	2.45	3.65
100	3.95	5.10	4.10	4.90	3.65	4.45	3.75	3.35	3.65	4.35	3.80	4.25	3.60	4.65	3.40	4.10	3.45	4.40
NLM**																		
20	5.15	4.60	4.75	5.05	4.80	5.05	4.90	5.35	4.90	5.25	6.65	7.80	5.40	5.70	5.25	7.00	8.20	10.50
30	4.25	5.15	5.20	5.10	4.40	4.95	4.95	5.20	5.55	5.10	5.55	7.00	4.55	4.70	5.60	4.80	6.15	6.95
50	4.80	4.15	4.85	4.60	4.30	4.85	5.15	5.15	5.45	5.50	4.95	5.10	4.05	5.15	5.25	4.80	4.90	7.05
100	4.60	5.50	4.60	5.05	4.20	5.40	5.00	4.35	4.20	5.60	4.70	5.15	4.70	5.75	4.50	5.80	4.65	5.75
CD																		
20	4.70	5.05	5.35	4.10	4.80	4.90	4.50	5.75	4.60	4.05	6.40	5.50	4.95	5.60	4.60	5.50	5.30	5.65
30	4.75	5.75	4.55	5.10	5.35	5.10	5.20	5.00	4.30	5.40	5.50	4.20	5.05	5.85	5.55	5.35	5.50	5.90
50	4.45	5.00	5.15	5.45	4.85	5.15	5.50	5.30	5.25	5.00	4.95	5.75	5.20	5.20	4.75	5.85	4.90	4.65
100	5.05	5.25	5.30	4.85	4.70	6.10	5.10	4.90	5.15	4.90	4.80	5.25	4.95	4.15	4.65	5.20	4.30	4.75

Notes: Data are generated as $y_{it} = \alpha_i + \sum_{\ell=2}^{k} x_{\ell it}\beta_{\ell i} + u_{it}$, $u_{it} = c_{(\gamma,k)}(\gamma_i f_t + \sigma_i \varepsilon_{it})$, $i = 1, 2, ..., N$, $t = 1, 2, ..., T$, where $\alpha_i \sim IIDN(1,1)$, with $x_{\ell it} = 0.6x_{\ell it-1} + v_{\ell it}$, $\ell = 2, 3, ..., k$, $i = 1, 2, ..., N$, $t = -50, -49, ..., T$, $x_{\ell i,-51} = 0$, where $v_{\ell it} \sim IIDN(0, \tau_{\ell i}^2/\left(1 - 0.6^2\right))$, $\tau_{\ell i}^2 \sim IID\chi^2(6)/6$. $\beta_{\ell i} \sim IIDN(1, 0.04)$, $f_t \sim IIDN(0,1)$, $\sigma_i^2 \sim IID\chi^2(2)/2$, and $\varepsilon_{it} \sim IIDN(0,1)$. α_i, $x_{\ell it}$, σ_i^2 are fixed across replications. $c_{(\gamma,k)}^2$ is chosen so that $\bar{R}^2 = E(\sigma_{ui}^2)/\overline{Var}(y_{it}) = 0.5$ with $\sigma_{ui}^2 = Var(u_{it})$ and $\overline{Var}(y_{it}) = (k-1)E(\beta_{\ell i}^2) + E(\sigma_{ui}^2)$. LM is Breusch-Pagan (1980) LM test, NLM is normal approximation version of LM test, NLM* and NLM** are mean-adjusted and mean-variance-adjusted LM tests which are proposed, respectively, CD is Pesaran's (2004) CD test, LM test is based on $\chi^2_{N(N-1)/2}$ distribution. NLM, NLM*,NLM** and CD tests are based on two-sided $N(0,1)$ test. All tests are conducted at 5% nominal level. All experiments are based on 2,000 replications.

Table 2
Power of Cross Section Independence Tests with Exogenous Regressors, $\gamma_i \sim IIDU[0.1, 0.3]$
Normal Errors with Different Number of Regressors (k)

	$k=2$						$k=4$						$k=6$					
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**
LM																		
20	27.75	57.30	79.15	95.70	100.00	100.00	26.75	55.80	75.60	95.70	100.00	100.00	27.35	54.95	78.95	97.80	100.00	100.00
30	40.35	72.15	88.40	98.45	100.00	100.00	35.30	69.60	86.30	98.40	100.00	100.00	34.35	65.95	83.55	98.10	100.00	100.00
50	53.10	86.60	96.55	99.65	100.00	100.00	54.85	83.95	95.90	99.65	100.00	100.00	50.95	83.25	95.95	99.65	100.00	100.00
100	76.75	96.70	99.90	100.00	100.00	100.00	74.40	96.45	99.60	100.00	100.00	100.00	72.90	97.10	99.80	100.00	100.00	100.00
NLM																		
20	24.05	50.95	74.20	94.10	99.95	100.00	22.05	49.10	69.60	93.75	100.00	100.00	23.70	48.05	73.20	95.90	100.00	100.00
30	36.45	67.40	84.95	97.85	99.95	100.00	32.00	64.25	82.30	97.15	100.00	100.00	30.90	60.50	80.15	97.05	100.00	100.00
50	49.25	84.05	95.65	99.40	100.00	100.00	51.50	81.45	94.50	99.60	100.00	100.00	47.15	80.35	94.15	99.60	100.00	100.00
100	74.20	95.70	99.85	100.00	100.00	100.00	71.85	95.70	99.55	100.00	100.00	100.00	70.50	96.60	99.70	100.00	100.00	100.00
NLM*																		
20	16.45	35.80	53.75	77.90	94.95	99.65	11.25	25.65	39.70	63.95	89.40	98.70	7.55	16.10	26.45	47.95	78.00	95.45
30	30.40	58.10	76.95	93.40	99.65	100.00	23.75	49.40	67.00	89.50	99.30	100.00	19.20	40.40	59.40	84.60	97.80	99.85
50	45.95	81.05	94.10	99.10	100.00	100.00	46.25	76.20	91.50	99.15	100.00	100.00	40.40	73.35	88.75	98.45	100.00	100.00
100	73.20	95.00	99.85	100.00	100.00	100.00	70.40	95.25	99.40	100.00	100.00	100.00	67.75	95.85	99.50	100.00	100.00	100.00
NLM**																		
20	20.35	40.95	59.00	81.55	96.30	99.70	17.80	34.05	49.15	72.10	92.45	99.30	16.85	27.40	39.50	61.35	87.30	97.45
30	33.60	61.35	79.05	94.35	99.75	100.00	29.75	56.20	72.65	92.00	99.55	100.00	26.15	49.30	67.05	88.40	98.75	99.95
50	47.50	82.05	94.60	99.30	100.00	100.00	49.75	79.15	92.95	99.30	100.00	100.00	44.40	76.75	91.05	98.95	100.00	100.00
100	73.75	95.15	99.85	100.00	100.00	100.00	71.50	95.35	99.45	100.00	100.00	100.00	70.25	96.25	99.55	100.00	100.00	100.00
CD																		
20	50.15	85.40	95.95	99.80	100.00	100.00	47.65	80.85	94.00	99.30	100.00	100.00	43.15	77.20	90.55	98.70	100.00	100.00
30	65.05	93.50	99.20	100.00	100.00	100.00	62.00	92.20	98.60	100.00	100.00	100.00	58.65	89.95	98.10	100.00	100.00	100.00
50	78.55	98.35	100.00	99.95	100.00	100.00	77.05	98.15	99.95	100.00	100.00	100.00	72.60	97.90	100.00	100.00	100.00	100.00
100	92.55	99.95	100.00	100.00	100.00	100.00	91.55	99.95	100.00	100.00	100.00	100.00	89.85	99.90	100.00	100.00	100.00	100.00

Notes: The design is the same as that of Table 1 except $\gamma_i \sim IIDU[0.1, 0.3]$.

Table 3

Power of Cross Section Independence Tests with Exogenous Regressors, $\gamma_i \sim IIDN(0, 0.1)$

Normal Errors with Different Number of Regressors (k)

	$k=2$						$k=4$						$k=6$					
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**
LM																		
20	45.10	77.80	92.25	99.30	100.00	100.00	37.95	73.20	90.45	99.40	100.00	100.00	36.60	71.95	90.35	99.25	100.00	100.00
30	56.05	86.15	96.95	99.85	100.00	100.00	54.40	85.40	95.85	99.85	100.00	100.00	51.05	83.80	95.45	99.80	100.00	100.00
50	72.70	95.85	99.55	100.00	100.00	100.00	71.85	94.90	99.40	100.00	100.00	100.00	69.40	94.90	99.35	100.00	100.00	100.00
100	88.60	99.35	99.95	100.00	100.00	100.00	87.55	99.10	100.00	100.00	100.00	100.00	87.50	99.00	99.90	100.00	100.00	100.00
NLM																		
20	41.10	73.15	89.60	98.95	100.00	100.00	33.60	68.40	87.40	98.95	100.00	100.00	31.95	66.80	86.65	98.65	100.00	100.00
30	52.45	83.55	95.85	99.80	100.00	100.00	50.85	82.55	94.70	99.70	100.00	100.00	46.80	80.10	94.15	99.75	100.00	100.00
50	70.70	94.55	99.45	100.00	100.00	100.00	68.40	93.45	99.25	100.00	100.00	100.00	65.85	93.05	99.00	100.00	100.00	100.00
100	87.25	99.15	99.95	100.00	100.00	100.00	85.95	98.95	100.00	100.00	100.00	100.00	86.45	98.80	99.85	100.00	100.00	100.00
NLM*																		
20	33.10	59.90	78.80	93.65	99.60	100.00	21.45	47.80	66.55	88.25	98.85	100.00	14.25	32.50	50.70	76.30	95.95	99.65
30	47.70	78.10	93.10	99.25	100.00	100.00	41.20	73.55	89.30	98.30	100.00	100.00	34.85	64.25	84.75	96.90	99.75	100.00
50	68.30	93.35	99.05	100.00	100.00	100.00	63.95	90.90	98.80	99.95	100.00	100.00	60.30	89.80	98.20	100.00	100.00	100.00
100	86.15	98.95	99.95	100.00	100.00	100.00	85.10	98.80	100.00	100.00	100.00	100.00	84.80	98.50	99.85	100.00	100.00	100.00
NLM**																		
20	37.15	64.40	81.50	94.65	99.70	100.00	29.40	55.95	74.85	92.20	99.30	100.00	23.95	46.45	64.60	85.25	97.55	99.85
30	50.25	80.25	93.60	99.35	100.00	100.00	46.80	77.75	91.35	98.90	100.00	100.00	42.15	71.35	88.35	97.75	99.95	100.00
50	69.50	93.75	99.35	100.00	100.00	100.00	66.65	92.15	98.95	100.00	100.00	100.00	63.65	91.50	98.50	100.00	100.00	100.00
100	86.70	99.10	99.95	100.00	100.00	100.00	85.75	98.85	100.00	100.00	100.00	100.00	86.15	98.70	99.85	100.00	100.00	100.00
CD																		
20	7.85	8.00	7.40	7.60	7.25	8.05	6.95	7.70	6.00	6.80	6.65	6.40	7.05	6.45	7.00	6.95	7.60	6.50
30	10.75	9.10	9.45	9.00	9.10	9.20	9.60	9.40	8.55	8.85	7.80	8.60	7.90	8.05	9.00	8.90	8.90	8.20
50	11.55	12.30	13.10	12.45	11.35	11.55	11.65	10.60	11.10	12.10	11.45	11.50	11.95	11.45	12.60	11.85	11.25	11.55
100	20.00	19.80	17.80	17.00	17.15	18.50	18.65	19.05	18.50	17.05	18.05	17.45	17.65	18.00	17.85	16.95	20.35	18.50

Notes: The design is the same as that of Table 1 except $\gamma_i \sim IIDN(0, 0.1)$.

Table 4
Size of Cross Section Independence Tests with Exogenous Regressors, $\gamma_i = \gamma = 0$
Non-normal Errors with Different Number of Regressors (k)

	$k=2$						$k=4$						$k=6$					
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**
LM																		
20	8.85	12.10	22.40	37.65	82.05	100.00	9.70	16.80	27.95	48.05	95.95	100.00	9.65	24.45	41.10	70.05	99.80	100.00
30	9.75	11.00	15.30	22.10	51.50	95.45	8.60	12.40	17.45	27.90	65.40	99.45	9.35	14.20	23.10	39.15	85.05	100.00
50	7.95	9.75	12.50	15.00	28.30	63.10	9.60	9.70	13.75	15.85	35.05	73.95	7.90	10.65	13.45	19.70	43.25	87.60
100	8.15	8.95	8.25	10.85	16.50	30.10	8.35	7.90	8.95	10.10	16.40	30.10	8.25	9.00	8.85	10.90	17.85	37.55
NLM																		
20	7.60	8.70	15.95	28.40	73.55	100.00	7.60	12.25	21.45	38.25	92.80	100.00	7.70	17.35	32.40	60.15	99.70	100.00
30	8.05	8.75	11.65	15.35	42.20	91.95	7.50	9.45	12.35	20.70	53.65	98.65	8.15	10.55	17.15	29.70	76.15	100.00
50	7.70	8.30	10.40	10.45	20.00	51.90	8.65	8.15	9.90	11.45	26.45	64.80	6.85	8.70	10.10	14.20	32.85	80.90
100	7.95	8.75	8.45	8.90	10.70	21.55	7.75	7.65	8.45	8.60	12.20	22.15	7.90	8.75	8.00	9.30	12.20	27.65
NLM*																		
20	4.45	3.80	5.15	4.80	5.10	4.10	2.10	2.35	2.60	3.15	3.25	3.45	1.25	0.80	1.30	1.20	2.20	3.40
30	5.85	6.00	6.95	6.05	6.30	6.75	4.00	3.55	4.30	4.55	4.10	5.35	2.80	2.30	2.85	2.55	3.60	3.65
50	6.55	6.55	6.95	7.30	7.15	8.50	5.65	5.70	6.05	5.55	5.60	5.50	3.95	4.20	4.60	4.50	4.35	5.00
100	6.90	8.75	7.70	7.95	8.15	8.30	6.50	6.70	7.10	6.60	7.10	6.30	6.75	6.80	6.50	5.35	6.10	6.80
NLM**																		
20	7.65	7.40	8.15	7.85	8.25	7.35	5.85	6.20	7.20	7.75	8.95	10.60	5.50	5.40	6.15	6.75	9.95	11.65
30	8.00	8.25	9.70	8.70	8.90	9.75	7.40	6.70	8.55	7.85	8.25	9.25	6.85	6.15	7.25	7.55	8.45	8.55
50	8.10	7.75	8.70	8.45	8.45	10.75	8.45	8.20	8.60	7.90	8.00	8.00	6.55	7.85	7.40	7.85	7.25	8.55
100	7.80	9.35	8.55	8.80	8.80	9.05	7.65	7.90	8.05	8.15	8.70	7.35	8.10	8.70	8.10	7.55	7.35	8.45
CD																		
20	5.15	4.85	4.45	4.70	5.00	5.85	6.15	5.00	5.30	5.05	4.05	5.40	5.50	5.20	6.65	5.85	5.35	4.90
30	4.70	4.30	4.85	4.60	5.20	4.75	4.60	5.65	4.60	5.60	5.20	4.85	5.35	4.65	5.70	4.30	6.05	5.10
50	5.60	4.85	4.70	5.05	4.75	5.00	5.75	5.95	5.45	5.85	4.90	5.20	5.25	4.90	5.50	5.00	4.80	5.65
100	4.70	5.30	4.35	5.00	4.95	5.30	5.50	5.55	5.05	4.95	5.80	4.60	4.70	4.55	4.30	4.85	4.65	4.25

Notes: The design is the same as that of Table 1 except $\varepsilon_{it} \sim IID\left[\chi^2(1) - 1\right]/\sqrt{2}$.

Table 5
Size and Power of First Order Cross Section Independence Tests with Exogenous Regressors, Spatially Correlated Errors

	$\lambda = 0$						$\lambda = 0.1$						$\lambda = -0.1$					
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**
NLM(1)																		
20	5.15	5.70	4.65	5.65	7.15	9.40	10.75	13.20	15.65	20.95	34.90	54.75	11.00	15.35	16.15	21.45	34.80	55.30
30	4.25	4.65	4.80	5.60	5.85	5.95	15.75	19.85	25.75	35.00	52.85	78.75	14.95	21.95	24.10	34.45	52.15	79.40
50	4.70	4.60	3.85	5.50	5.10	5.20	28.55	39.10	48.35	63.60	86.95	98.60	28.25	37.35	45.95	60.35	86.55	98.70
100	3.65	4.15	4.85	4.80	4.90	5.80	60.15	78.75	88.80	96.45	99.90	100.00	58.70	78.50	88.05	96.55	99.90	100.00
NLM(1)*																		
20	2.80	2.80	2.45	2.40	2.95	3.15	7.20	8.30	8.85	11.70	19.30	30.15	7.20	9.60	8.95	11.20	17.65	29.55
30	3.15	2.55	3.35	3.55	3.15	3.70	12.00	14.90	19.10	25.85	39.20	64.95	12.40	17.15	18.85	24.40	40.40	64.75
50	3.70	3.60	3.10	4.45	3.70	3.70	25.45	34.45	43.60	58.05	82.80	97.70	26.00	33.25	42.05	54.40	82.70	97.55
100	3.30	3.90	4.20	4.30	4.60	5.15	59.25	77.35	87.95	96.10	99.85	100.00	56.85	77.40	86.70	96.15	99.90	100.00
NLM(1)**																		
20	4.60	4.90	4.20	4.45	5.35	5.65	9.80	12.05	13.00	16.00	24.45	37.80	9.90	13.35	13.05	16.45	25.20	37.45
30	3.85	4.45	4.90	5.25	4.85	5.35	14.70	18.25	22.80	30.10	44.80	69.85	14.60	20.50	21.70	29.75	45.80	70.05
50	4.45	4.40	3.85	5.20	4.55	4.55	27.45	37.45	47.05	60.85	84.80	98.20	27.90	35.80	44.25	57.40	84.95	97.95
100	3.65	4.10	5.00	4.65	5.15	5.50	59.90	78.20	88.60	96.35	99.85	100.00	58.25	78.30	87.45	96.40	99.90	100.00
CD(1)																		
20	5.20	5.55	5.35	6.50	4.95	5.75	28.80	46.70	63.75	83.25	98.55	100.00	28.60	49.30	64.35	83.90	98.35	100.00
30	5.30	5.05	5.75	5.30	4.65	5.10	42.05	66.15	82.55	95.50	99.90	100.00	40.90	67.30	81.65	96.15	99.95	100.00
50	4.95	4.35	4.30	4.80	5.05	5.65	64.05	88.70	97.05	99.80	100.00	100.00	62.20	88.50	96.60	99.80	100.00	100.00
100	4.55	4.95	5.20	5.25	5.20	4.85	90.80	99.40	99.90	100.00	100.00	100.00	90.30	99.55	100.00	100.00	100.00	100.00

Notes: The design is the same as that of Table 1 for $k = 2$, except errors are spatially correlated such that $u_{it} = \lambda(0.5u_{i-1,t} + 0.5u_{i+1,t}) + \sigma_i\varepsilon_{it}$, with end points set at $u_{1t} = u_{2t} + \varepsilon_{1t}$ and $u_{Nt} = u_{N-1t} + \varepsilon_{Nt}$.

Table 6
Size and Power of Cross Section Independence Tests, Heterogeneous AR(1) Specification with Normal Errors

	$\gamma_i = \gamma = 0$						$\gamma_i \sim IIDU[0.1, 0.3]$						$\gamma_i \sim IIDN(0, 0.1)$					
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**	**10**	**20**	**30**	**50**	**100**	**200**
LM																		
20	9.05	14.45	22.70	45.90	95.55	100.00	11.60	19.45	36.30	69.00	98.95	100.00	17.50	41.10	62.90	90.20	99.90	100.00
30	6.95	9.90	14.55	24.90	65.20	99.65	9.75	20.20	33.75	60.35	96.15	100.00	21.75	49.60	71.95	92.30	99.85	100.00
50	5.65	7.85	9.15	14.30	30.50	75.95	12.30	25.10	39.75	69.30	96.35	99.95	33.95	66.45	86.05	98.10	99.95	100.00
100	5.90	6.40	6.70	8.55	14.10	26.35	21.60	46.25	67.75	92.40	99.90	100.00	57.20	90.00	97.65	100.00	100.00	100.00
NLM																		
20	6.40	9.70	15.70	35.65	90.65	100.00	8.60	14.80	27.60	58.15	97.90	100.00	13.55	34.65	55.00	85.40	99.80	100.00
30	5.60	7.10	10.35	17.15	53.15	99.05	7.75	14.70	26.00	50.90	93.40	100.00	17.85	43.40	65.80	89.35	99.80	100.00
50	4.40	6.50	6.30	8.95	21.05	65.05	10.05	19.50	31.25	62.35	94.60	99.90	30.30	61.10	82.45	97.35	99.95	100.00
100	4.80	5.55	5.10	6.40	9.00	18.40	17.25	39.50	60.30	89.25	99.75	100.00	52.75	87.35	96.90	100.00	100.00	100.00
NLM*																		
20	3.10	3.15	3.15	3.00	5.95	16.15	3.90	5.65	7.95	14.60	37.00	76.45	8.25	18.80	31.60	55.00	84.75	98.45
30	3.50	4.00	3.65	3.25	4.65	7.70	4.95	7.90	12.45	22.65	51.20	87.55	13.60	33.20	50.50	74.95	96.80	99.70
50	3.25	4.10	3.85	4.85	3.80	4.80	8.20	14.25	23.40	46.85	81.60	98.25	27.45	55.45	77.15	94.85	99.65	100.00
100	4.20	4.60	4.05	4.35	4.00	4.55	15.35	35.55	55.00	85.20	99.40	100.00	51.35	86.00	96.50	100.00	100.00	100.00
NLM**																		
20	5.10	5.65	5.00	4.85	7.65	14.55	5.80	7.85	11.20	18.20	39.45	75.25	10.65	21.85	35.70	58.80	85.35	98.35
30	5.05	5.80	5.25	5.15	6.00	8.10	6.85	10.15	15.00	25.40	54.20	87.85	16.20	36.45	54.20	76.90	96.90	99.70
50	4.25	5.50	4.75	5.85	4.50	5.55	9.10	15.95	25.15	48.50	82.60	98.35	28.80	57.25	78.30	95.10	99.65	100.00
100	4.70	5.15	4.60	4.95	4.60	4.85	16.70	37.15	56.35	86.05	99.45	100.00	52.15	86.30	96.55	100.00	100.00	100.00
CD																		
20	5.80	4.80	5.25	4.80	5.50	5.10	22.85	49.25	71.90	90.90	99.55	100.00	5.75	5.85	5.00	6.45	5.60	4.95
30	5.40	5.45	4.45	5.50	5.20	5.00	28.95	63.20	84.60	97.40	99.95	100.00	6.25	6.35	6.90	5.80	7.35	5.95
50	5.45	5.15	5.25	5.15	6.05	4.90	42.10	83.15	96.65	99.90	100.00	100.00	7.10	8.55	7.25	7.90	7.00	7.10
100	4.90	5.15	4.95	4.45	4.20	4.45	63.05	97.25	100.00	100.00	100.00	100.00	9.50	9.15	8.70	9.65	10.05	9.70

Notes: See notes to Table1. The DGP is specified as $y_{it} = \mu_i (1 - \beta_i) + \beta_i y_{i,t-1} + u_{it}$, $u_{it} = \gamma_i f_t + \sigma_{it}\varepsilon_{it}$, $i = 1, 2, ..., N$; $t = -50, ..., T$, where $\beta_i \sim IIDU[0, 1)$, $\mu_i \sim \varepsilon_{i0} + \eta_i$, $\eta_i \sim IIDN(1, 2)$, $f_i \sim IIDN(0, 1)$, $\sigma_{it}^2 = \sigma_i^2 \sim IID\chi^2(2)/2$, and $\varepsilon_{it} \sim IIDN(0, 1)$. η_i, β_i and σ_{it} are fixed across replications. $y_{i,-51} = 0$ and the first 50 observations are discarded.

Table 7
Size of Cross Section Independence Tests
Heterogeneous AR(1) with Non-normal Errors

$\gamma_i = \gamma = 0$						
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**
LM						
20	11.05	17.15	26.30	45.80	93.30	100.00
30	9.15	12.55	17.20	27.30	62.00	99.00
50	8.35	10.50	13.00	17.35	34.35	70.30
100	7.60	10.20	9.30	10.45	14.85	30.85
NLM						
20	9.45	12.95	19.50	34.50	87.80	100.00
30	8.15	9.85	13.15	19.55	51.00	97.60
50	8.05	8.95	10.40	12.35	24.75	60.40
100	7.45	9.65	8.30	9.05	10.60	23.60
NLM*						
20	4.85	5.45	6.25	6.15	9.50	16.75
30	6.30	6.25	6.95	6.75	7.50	9.85
50	6.75	6.65	7.40	7.70	8.15	9.20
100	6.80	8.90	7.25	7.60	7.90	8.10
NLM**						
20	7.85	8.05	8.65	8.75	11.10	14.55
30	8.05	7.85	8.90	9.55	9.85	10.30
50	8.10	8.30	8.50	9.35	10.00	10.45
100	7.45	9.85	7.95	8.45	8.70	8.80
CD						
20	5.75	5.15	4.85	4.35	3.90	5.15
30	4.70	5.35	4.45	4.35	5.20	5.30
50	5.00	5.40	5.55	4.25	4.85	5.65
100	4.70	5.45	5.25	4.70	4.85	5.15

Notes: See the notes to Table 6. The design is the same as that of Table 6 except $\varepsilon_{it} \sim IID\left[\chi^2(1)-1\right]/\sqrt{2}$.

Table 8
Size of Cross Section Independence Tests
DGP Subject to a Single Break

$\gamma_i = \gamma = 0$						
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**
LM						
20	12.10	29.15	51.55	89.90	100.00	100.00
30	10.90	23.25	40.60	74.30	99.85	100.00
50	10.45	20.90	34.15	64.55	99.50	100.00
100	9.65	19.65	30.80	59.80	97.65	100.00
NLM						
20	9.00	21.65	41.35	83.25	100.00	100.00
30	8.45	16.60	30.55	65.55	99.55	100.00
50	8.15	15.10	25.95	54.55	98.30	100.00
100	7.50	13.95	22.60	49.20	95.60	100.00
NLM*						
20	4.45	8.25	14.90	30.70	80.55	100.00
30	5.20	8.45	14.05	29.60	80.55	100.00
50	6.45	10.60	17.45	32.80	84.00	99.95
100	7.10	11.35	18.75	39.35	89.80	100.00
NLM**						
20	6.65	11.10	18.75	35.80	82.05	100.00
30	7.15	10.45	16.40	33.65	82.35	100.00
50	7.70	11.80	19.20	35.35	85.25	100.00
100	7.45	12.50	19.80	40.50	90.55	100.00
CD						
20	6.15	4.80	5.75	5.80	5.80	4.70
30	5.10	5.30	6.10	4.70	6.10	5.75
50	5.50	6.10	5.95	5.90	5.40	5.70
100	6.50	5.15	6.75	5.25	6.05	4.25

Notes: The DGP is specified as $y_{it} - \mu_i = \beta_{it}\left(y_{it-1} - \mu_i\right) + u_{it}$, $u_{it} = \gamma_i f_t + \sigma_{it}\varepsilon_{it}$, where $\mu_i \sim IIDN\left(1,1\right)$; $\beta_{it} = \beta_t = 0.6$ and $\sigma_{it} = \sigma_t = \sqrt{1.5}$ for $t = -50, ..., T/2$; $\beta_t = 0.8$ and $\sigma_t = 1$ for $t = T/2+1, ..., T$; $\varepsilon_{it} \sim IIDN\left(0,1\right)$. $y_{i,-51} = 0$ and the first 50 observations are discarded. See also the notes to Table 6.

Table 9
Size of Cross Section Independence Tests
DGP Subject to Multiple Structural Break

	$\gamma_i = \gamma = 0$					
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**
LM						
20	9.25	15.40	34.65	46.60	95.30	100.00
30	8.60	15.95	14.20	26.95	66.70	99.80
50	5.80	7.85	9.55	18.60	30.30	74.60
100	5.10	6.50	8.35	7.35	13.25	32.60
NLM						
20	7.10	11.35	25.60	35.10	91.60	100.00
30	7.10	11.90	9.05	18.85	55.25	99.00
50	4.50	6.65	7.05	12.95	20.70	65.20
100	5.30	5.50	5.85	5.40	8.05	21.65
NLM*						
20	4.15	4.35	5.75	4.50	7.55	25.20
30	4.85	5.90	3.40	4.55	5.25	8.60
50	3.50	4.60	5.35	5.95	4.20	5.35
100	4.45	4.75	4.50	4.50	4.90	5.05
NLM**						
20	6.05	6.55	8.85	6.75	9.05	21.25
30	6.35	8.15	4.45	5.75	6.90	8.15
50	4.05	5.40	6.20	7.20	5.05	6.00
100	5.10	5.40	5.00	4.90	5.60	5.60
CD						
20	5.05	5.45	5.65	4.65	5.30	4.95
30	5.10	5.15	4.55	5.05	5.50	4.75
50	5.45	5.85	4.60	5.10	4.30	5.25
100	4.45	5.25	5.40	4.70	5.00	4.55

Notes: The design is the same as that of Table 8, except $\beta_{it} = 0.5$ for $t = -50, ..., 0$ and $\beta_{it} \sim IIDU[0,1)$ for $t = 1, ..., T$, $i = 1, ..., N$; $\sigma_{it}^2 \sim IID\chi^2(2)/2$ for $t = -50, ..., T$, $i = 1, ..., N$. See also the notes to Table 6.

Table 10
Size of Cross Section Independence Tests
DGP Subject to Unit Root

	$\gamma_i = \gamma = 0$					
(T,N)	**10**	**20**	**30**	**50**	**100**	**200**
LM						
20	9.70	15.80	37.75	55.80	96.45	100.00
30	9.45	16.85	18.40	33.90	78.55	100.00
50	6.75	8.65	13.15	22.10	43.70	89.65
100	5.40	7.50	9.60	9.20	23.20	50.15
NLM						
20	7.10	10.90	28.55	43.05	93.00	100.00
30	7.35	12.95	13.35	25.00	69.15	99.90
50	5.65	6.75	8.80	15.00	32.95	83.60
100	5.05	5.45	7.85	7.10	14.75	38.40
NLM*						
20	3.15	3.40	6.85	4.50	5.50	16.40
30	4.40	5.60	4.55	5.30	5.95	15.35
50	3.95	5.05	5.30	5.55	5.50	7.60
100	4.60	4.90	6.40	5.15	5.75	6.00
NLM**						
20	4.95	5.10	9.00	7.00	7.90	16.90
30	5.95	7.75	6.20	7.00	7.15	16.35
50	5.15	6.15	6.60	6.60	6.60	8.45
100	5.10	5.45	6.75	5.45	6.30	6.55
CD						
20	4.95	4.55	5.15	6.00	5.45	5.15
30	5.05	5.00	4.55	4.40	5.25	4.80
50	5.90	5.40	5.10	5.40	5.70	4.35
100	5.60	5.60	5.00	5.65	4.80	5.05

Notes: The DGP is the same as that of Table 10 except $\beta_{it} = \beta = 1$ for all i and t. See also the notes to Table 6.