# A BIFURCATION THEOREM FOR CRITICAL POINTS OF VARIATIONAL PROBLEMS

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# A Bifurcation Theorem for Critical Points of Variational Problems

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# A Bifurcation Theorem for Critical Points of Variational Problems

## §1 Introduction and main theorem

In this note we consider the problem

$$\min_{x \in E} F(x,\lambda)$$

where $E$ is a real Hilbert space, $\lambda \in \mathbb{R}$ and

(1.1) $$F: E \times \mathbb{R} \to \mathbb{R}$$

is a $C^2$-functional

(1.2) $$f(x,\lambda) = D_x F(x,\lambda)$$

be the gradient of $F$, which by the Riesz representation theorem may be identified with an element of $E$. An element $(x_0,\lambda_0) \in E \times \mathbb{R}$ such that

(1.3) $$f(x_0, \lambda_0) = 0$$

is called a critical point of $F$. We assume that a $C^2$-smooth curve of critical points of $F$ is given, and that it may be parameterized

(1.4) $$x = x(\lambda).$$

Without loss of generality, we may assume that $x(\lambda) \equiv 0$, i.e. $(0,\lambda)$ is a critical point of $F$ for all $\lambda \in \mathbb{R}$. We want to investigate the branching of further critical points from this curve, which we shall call the trivial solution (of (1.3)). Let $D_x f(0,\lambda) = A(\lambda)$ and assume

(1.5) $$0 < n = \dim \ker A(0) < \infty.$$

Moreover, assume that $0$ is isolated in the spectrum $\sigma(A(0))$. Then, for small $|\lambda| \neq 0$, there exist $n$ eigenvalues near zero, we assume that none of these is zero, i.e. there exists a number $\varepsilon_0 > 0$ , such that

$$\ker A(\lambda) = \{0\} \quad \text{for} \quad 0 < |\lambda| < \varepsilon_0.$$

Let $eig_0(A(\lambda))$ be the set of eigenvalues of $A(\lambda)$, which approach $0$ as $\lambda \to 0$, in Kato's terminology, the 0-group [3]. Let $r(A(\lambda))$ be the number of elements in

$$eig_0\,(A(\lambda)) \cap \{t \in R \mid t<0\}$$

Assume

$$r^+_{A(\lambda)} = \lim_{\substack{\lambda\to 0\\ \lambda>0}} r(A(\lambda)) \tag{1.6}$$

and

$$r^-_{A(\lambda)} = \lim_{\substack{\lambda\to 0\\ \lambda<0}} r(A(\lambda) \tag{1.7}$$

exist. Then we have

<u>Theorem</u> Let $F: E \times \mathbb{R} \to \mathbb{R}$ be $C^2$ and $f(x,\lambda) = D_x F(x,\lambda)$. Suppose that $f(0,\lambda) = 0$ for all $\lambda \in \mathbb{R}$ and $0$ is an eigenvalue of $D_x f(0,0) = A(0)$. If

$$r^+_{A(\lambda)} - r^-_{A(\lambda)} \neq 0,$$

then $(0,0)$ is a bifurcation point of the equation $f(x,\lambda) = 0$.

<u>Remark 1</u> In applications, we consider partial differential equations expressed in terms of integral equations. For example, many boundary value problems may be written as an operator equation

$$(\lambda I - L)x + N(x,\lambda) = 0$$

where $I$ is the identity, $L : E \to E$ is linear, compact and selfadjoint, and $N = o(|x|)$ as $|x| \to 0$ uniformly for bounded $\lambda$. Thus, our theorem is applicable if $f(x,\lambda) = (\lambda I-L)x + N(x,\lambda)$.

Remark 2: This theorem generalizes earlier results by Rabinowitz [5,8], Clark [9], Fadell & Rabinowitz [10], Böhme [1], Marino [4], Berger [11] and Takens [12]. The major difference between these earlier results and ours is that we allow an arbitrary dependence of $F$ on $\lambda$, while the other authors required that $F$ depends linearly on $\lambda$.

Remark 3: The major difference between our proof and the proof given by Rabinowitz [5] is that he uses the Ljapunov-Schmidt method, while we use center manifold theory. The difficulty using Ljapunov-Schmidt method arises from the fact that the bifurcation equation might not possess a variational structure, compare also Chow & Hale [2].

Remark 4: Below we shall indicate a slight generalization to those real Banach spaces E which admit a bounded linear map

(1.8) $$K: H \to E$$

with range $R(K)$ dense E, where $H$ is a real Hilbert space. Let $J:H \to H^*$ be the isomorphism given by the Riesz representation theorem, if $H \overset{K}{\to} E$ then

(1.9) $$E \leftarrow H \to H^* \leftarrow E^* \text{ , i.e.}$$

(1.10) $$\tilde{K} = K\, J^{-1}\, K^*$$

is a bounded linear operator, which in fact is injective, since $K$ has dense range.

Remark 5: The construction (1.9) to (1.10) applies to all Sobolev spaces $W^{m,p}(\Omega)$ for bounded domains $\Omega$ $R^N$ and $1 < p < \infty$.
If $1 < p < 2$ then

(1.11): $$W^{m,2}(\Omega) \overset{K}{\to} W^{m,p}(\Omega)$$

and if $p > 2$ then

$$W^{m',2}(\Omega) \overset{K}{\to} W^{m,p}(\Omega)$$

where $m'$ satisfies

$$\frac{1}{p} = \frac{1}{2} - \frac{m'-m}{N} .$$

Remark 6: Problems in nonlinear elasticity lead to variational problems in $W^{1,p}(\Omega)$ for $\Omega \subset R^N$, $N = 1,2,3$. Compare Marsden and Hughes [14]. Our more general formulation given below applies to the problems in hyperelasticity which have a convex strain energy function.

## II. Proof of the theorem

We consider the differential equation

$$\dot{x} = - f(x,\lambda) \tag{2.1}$$

$$\dot{\lambda} = 0 \tag{2.2}$$

where the dot refers to differentiation with respect to time. Since $F \in C^2(E \times \mathbb{R}, \mathbb{R})$ the hypotheses of the center manifold theorem (Henry [6], p. 169, see also Bates and Jones [13]) are satisfied for (2.1), (2.2) at $(x,\lambda) = (0,0)$. This implies that there exists a $(n+1)$ - dimensional manifold $M \subset E \times \mathbb{R}$ which is

(i) locally invariant under the flow generated by (2.1), (2.2) and such that

(ii) there exists a $U \subset E \times \mathbb{R}$ of $(0,0)$ such that any solution $(x(t),\lambda)$ of (2.1), (2.2) with $(x(t), \lambda) \in U$ for all $t \in \mathbb{R}$ lies on $M$ $U$.

(iii) $M$ is tangent to $\ker A(0) \times R$ at $(0,0)$. We remark that each slice $M_\lambda$ defined by

$M_\lambda = \{(x,\lambda) \mid x \in E, \lambda \in R, |\lambda|$ sufficently small$\}$ is locally invariant under the flow generated by (2.1). The center manifold $M$ is defined by a map

$$G : (\ker A \times \mathbb{R}) \cap U \to (E \times \mathbb{R}) \cap W \tag{2.3}$$

such that

$$M = \{(v + G(v,\lambda),\lambda) \mid (v,\lambda) \in (\ker A(0) \times \mathbb{R}) \cap U\}$$

where $W$ is a neighborhood of $(0,0) \in E \times \mathbb{R}$ defined by

$$W = \{(v,\lambda) \in \ker A(0) \times \mathbb{R} \mid \|v\|_E < a_1, \ |\lambda| < a_2\}.$$

for a pair of sufficiently small real numbers $a_1$, $a_2$. For fixed $\lambda$, the flow $M_\lambda$ is given by

$$\dot{v} = - Pf(v + G(v,\lambda),\lambda) \tag{2.4}$$

where $P = \frac{1}{2\pi}\int_\Gamma R(z,A(0)\, dz$, with $R(z,A(0))$ is the resolvent of the linear operator $A(0)$ at $z \in \Gamma$ and $\Gamma$ is a closed curve in $C$, such that $0$ is the only point in the spectrum lying in the bounded region defined by $\Gamma$. Write (2.4) as

$$v = -B(\lambda)v+g(v,\lambda) \tag{2.5}$$

with $g(v,\lambda) = O(\|v\|)$.

The main observation for using the center manifold in bifurcation theory is, that

$$r(A(\lambda)) = r(-B(\lambda))$$

and

$$r^+_{A(\lambda)} - r^-_{A(\lambda)} = r^+_{-B(\lambda)} - r^-_{-B(\lambda)}.$$

Therefore the Morse index of the trivial solution changes. For the moment we assume that our theorem is not true. Then $0$ will be an isolated critical point of $F(\cdot,0)$, i.e. there exists a neighborhood $V$ of $x = 0$ in $E$, such that $F(\cdot,0)$ has no other critical point in $V$. From the fact that $\dot{x} = -f(x,0)$ defines a gradient flow, it follows that $\{x=0\}$ is an isolated invariant set for (2.1) at $\lambda=0$. Therefore there exists an isolating neighborhood $N \subset V \times \{0\}$ for $(x,\lambda) = (0,0)$. This isolating neighborhood $N$ continues for small $|\lambda|$, let us say $|\lambda| < \varepsilon_2$. Again from the fact that (2.1) defines a gradient flow and the assumption that our theorem is not true it follows that $\{x = 0\}$ is an isolated invariant set for $|\lambda| < \varepsilon_2$ with isolating neighborhood $N \times \{\lambda\}$. This gives a continuation connecting two maximal invariant, isolated sets (our critical point 0) having different Morse index. This contradicts Conley's continuation theorem (Conley [7]). Therefore our theorem is proved. $\square$

## III. A Generalization

There are two directions in which one could try to generalize our theorem:

(a) more general spaces

(b) less smoothness assumptions on F.

For example, if one wants to prove a similar theorem which is applicable to problems in elasticity one needs a generalization in both directions. The simplest problems in hyperelasticity as for example the buckling of a rod lead to variational problems as

$$(3.1) \qquad \min_{u \in W^{m,p}([0,1])} \int_0^1 W(u_x, \lambda)dx$$

where $W^{m,p}([0,1])$ is the Sobolev space of functions in $L^p([0,1])$ having weak derivatives in $L^p([0,1])$. We note that "real problems" are formulated over domains $\Omega \subset R^3$. We want to assume $p>1$ although in applications this is not always the case. The functional.

$$(3.2) \qquad F = \int_0^1 W(u_x, \lambda)dx$$

is not $C^2$ and if $p \neq 2$, $W = W^{m,p}_{(\Omega)}$ is not a Hilbert space. If $p<2$, and $\Omega \subset R^N$ is bounded, then the construction given in remark 4 above applies with $H = W^{m,2}(\Omega)$ if $p>2$, take $m'$ satisfying

$$\frac{1}{p} = \frac{1}{2} - \frac{m' - m}{N}$$

and $H = W^{m',2}(\Omega)$. In these cases we look at the differential equation

(3.3) $$X = -Kf(x,\lambda).$$

If $F$ is differentiable (in some sense) we get for a solution of (3.3) ($\langle .,. \rangle_{(E^*,E)}$ refers to the pairing $E^*,E$, while $\langle .,. \rangle_H$ denote the inner product in $H$)

$$\frac{d}{dt}F(x(t),\lambda) = - \langle f(x,\lambda),\ x)_{(E^*,E)}$$

$$= - \langle f(x,\lambda)Kf(x_1\lambda)\rangle_{(E^*,E)}$$

$$= - \langle f(x,\lambda),K^*J^{-1}Kf(x,\lambda)\rangle_{(E^*,E)}$$

$$= - \langle Kf(x,\lambda),\ J^{-1}\ Kf(x,\lambda)\rangle_H < 0\ .$$

By the injectivity of $K$ we conclude that any point $x_0$ in the (strong) $\omega$-limit set of $x(t)$ is an equilibrium for (3.2) and a critical point of $F$. The main ingredient in our proof is the center manifold theorem, which holds under more general hypotheses, then we needed above (Henry [6]). In elasticity one looks at strain energy function $W(u_x,\lambda)$ where $u$ is the deformation, $u_x$ the deformation gradient and the problem is to minimize the function

(3.3) $$F = \int_0^1 W(u_x,\lambda') + b(u,x)dx$$

over $W^{m,p}([0,1])$, where $b$ incorporates external forces.

We assume

(i) $W \in C^2(R,R)$

(ii) $\frac{\partial^2}{\partial z^2}W(z,\lambda) > 0$, i.e. $W(\cdot,\lambda)$ is convex for each $\lambda$.

The Euler Lagrange equation of (3.3) is quasilinear, namely

(3.4) $$W''(u_x,\lambda)u_{xx} + b(u,x) = 0$$

We look at

(3.5) $$\frac{\partial u}{\partial t} = + (W''(u_x,\lambda)\, u_{xx} + b(u,x))$$

Changing the time variable as in Henry [G], p. 59, we obtain the semilinear parabolic equation

$$\frac{\partial u}{\partial t} = + u_{xx} + \frac{1}{W''(u_x,\lambda)} b(u,x)$$

which may be treated by our theory.

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