December 2008 *RR*-02.08

Lucio Bianco, Massimiliano Caramia, Stefano Giordani

A Bilevel Flow Model for HazMat Transportation Network Design

A Bilevel Flow Model for HazMat Transportation Network Design

Lucio Bianco, *Massimiliano Caramia*, *Stefano Giordani*[♣]

Dipartimento di Ingegneria dell'Impresa
Università di Roma "Tor Vergata"
Via del Politecnico 1, I-00133 Roma, Italy
{bianco, caramia, giordani}@disp.uniroma2.it

Abstract

In this work we consider the following hazmat transportation network design problem. A given set of hazmat shipments has to be shipped over a road transportation network in order to transport a given amount of hazardous materials from specific origin points to specific destination points, and we assume there are regional and local government authorities that want to regulate the hazmat transportations by imposing restrictions on the amount of hazmat traffic over the network links. In particular, the regional authority aims to minimize the total transport risk induced over the entire region in which the transportation network is embedded, while local authorities want the risk over their local jurisdictions to be the lowest possible, forcing the regional authority to assure also risk equity. We provide a linear bilevel programming formulation for this hazmat transportation network design problem that takes into account both total risk minimization and risk equity. We transform the bilevel model into a single-level mixed integer linear program by replacing the second level (follower) problem by its KKT conditions and by linearizing the complementary constraints, and then we solve the MIP problem with a commercial optimization solver. The optimal solution may not be stable, and we provide an approach for testing its stability and for evaluating the range of the its solution values when it is not stable. Moreover, since the bilevel model is difficult to be solved optimally and its optimal solution may not be stable, we provide a heuristic algorithm for the bilevel model able to always find a stable solution. The proposed bilevel model and heuristic algorithm are experimented on real scenarios of an Italian regional network.

Keywords: Hazardous materials; Hazmat transportation network design; Bilevel optimization; Heuristic algorithm.

1. Introduction

The transportation of hazardous materials (hazmats), though may be classified among the more general freight transport issues, is an activity that presents extremely typical characteristics. What differentiates hazmat shipments from the transportation of other materials is the risk associated with an accidental release of hazardous materials during transportation. To reduce the occurrence of dangerous events it is necessary to provide appropriate answers to safety management associated with dangerous goods shipments.

[♣] Corresponding author: Stefano Giordani, Dip. di Ingegneria dell'Impresa, Università di Roma "Tor Vergata", Via del Politecnico 1, I-00133 Roma, Italy. e-mail: giordani@disp.uniroma2.it; tel. +39 06 7259 7358; fax. +39 06 7259 7358

It was estimated that more than 4 billions of hazmat tons were being transported annually at world wide level in the first half of the current decade (Zografos and Androutsopoulos, 2004): in the USA, there are at least 300 million hazmat shipments each year, and totally approximately 3.2 billion tons (Kara and Verter, 2004); in Italy, 74 millions of hazmat tons were transported on trucks in 2001 (Infodatamix, 2002). In USA, in 1998, there were roughly 15,000 incidents related to hazmat transportation, and only 429 of them were classified as serious incidents (Kara and Verter, 2004). Despite the number of such incidents is very small compared to the number of hazmat shipments, due to the potential magnitude of accidents to the population and the environment, the public is very sensitive to the dangers of hazmat transportation activity; therefore, the risk associated with incidents involving hazmat shipments have found considerable attention from the government, encouraging research on hazmat transportation.

Risk assessment and hazmat shipments planning are two of the main research fields in hazmat transportation. In the literature, a lot of work has already been done in risk assessment, by modeling risk probability distribution over given areas, for example, taking into account the risk related to the transported material and the transport modality (Abkovitz et al., 1984) and the environmental conditions (Patel and Horowits, 1994). For a survey the reader is referred to, e.g., Erkut et al. (2007).

One of the main issues of hazmat shipments planning is routing hazmat shipments, that involves a selection among the alternative paths between origin-destination pairs. From a carrier's perspective, shipment contracts can be considered independently and a routing decision needs to be made for each shipment, which we call the local route planning problem. At the macro level, hazmat routing is a "many to many" routing problem with multiple origins and destinations. In the sequel, we refer to this problem as global route planning.

The local route planning problem is to select routes between a given origin-destination pair for a given hazmat, transport mode, and vehicle type. Thus, for each shipment order, this problem focuses on a single-commodity and a single origin-destination route plan. Since these plans are often made without taking into consideration the general context, certain links of the transport network tend to be overloaded with hazmat traffic. This could result in a considerable increase of accident probabilities on some road links as well as leading to inequity in the spatial distribution of risk.

Transport costs are the carriers' main focus. In contrast, a government authority, charged with the management of hazmat shipments within and through its jurisdiction, has to consider the global problem by taking into account all the shipments in its jurisdiction. Although the transportation industry has been deregulated in many countries, hazmat transportation usually remains as part of the governments' mandate mainly due to the associated public and environmental risks. This leads to a harder class of problems that involve multi-commodity and multiple origin-destination routing decisions, i.e., the global route planning problem.

The main concern for a government authority is controlling the risk induced by hazmat transportations over the population and the environment. Besides the minimization of the total risk, a government authority should also promote equity in the spatial distribution of risk. This becomes crucial in the case in which certain populated zones are exposed to intolerable levels of risk as a result of the carriers' routing decisions.

Therefore, in the global route planning for hazmat shipments, the main problem is that of finding minimum risk routes, while limiting and equitably spreading the risk in any zone in which the transportation network is embedded. As a matter of fact, risk equity has to be taken into account also whenever it is necessary to carry out several hazmat shipments from a given origin to a given destination. In this situation, the planning effort has to be devoted to distribute risk uniformly among all the zones of the geographical crossed region. This concept is well defined in (Keeney, 1980), where a measure of the collective risk is determined with explicit reference to the equity.

In the literature, some models have been proposed for determining paths with minimum total risk while guaranteeing equitable risk spreading; see, for example, the models in (Gopalan et al., 1990a; Gopalan et al., 1990b; Current and Radik, 1995). The concept of dissimilar paths has also been considered in order to guarantee the spreading of risk, and different methods have been proposed in the past, also in contexts different from that of hazmat shipments, to generate a number of spatially dissimilar paths (Akgün et al., 2000). Other approaches that address the risk equity find minimum total risk paths with bounded maximum link risk (e.g. see Carotenuto et al., 2007) or consider min-max models which minimize the maximum link risk (e.g., see Bell, 2006).

Hazmat local route planning has attracted the attention of many OR researchers, while the global route planning problem has attained relatively little attention in the literature. The results in this latter area include the works of Gopalan et al. (1990b), Lindner-Dutton et al. (1991) and Marianov and ReVelle (1998). The works of Akgün et al. (2000), Dell'Olmo et al. (2005) and Carotenuto et al. (2007) on the problem of finding a number of spatially dissimilar paths between an origin and a destination can also be considered in this area. For a complete survey of local and global route planning the reader is referred to Erkut et al. (2007).

Typically, a government authority does not have the right to impose specific routes to individual carriers, but it has only the authority to close certain road segments to hazmat vehicles or to limit the amount of hazmat traffic flow on that links. In the context of global route planning, the problem of selecting which road segments to be closed by the government falls in the field of hazmat transportation network design that started to be studied in the academic literature by Kara and Verter (2004), and equity concerns can be incorporated into the design objectives.

Network design has been widely studied in the past, and there are many papers on this field in the open literature (e.g., see the surveys of Magnanti and Wong (1984) and Yang and Bell (1998) for reviews of network design problems for road transportation). Differently from the classical network design problem where one is asked to find the most appropriate ways to expand a given infrastructure, in hazmat network design the question becomes which are the road segments to (partially or entirely) close to hazmat transport in an existing network, for example in order to minimize the total risk induced by the execution of a given set of hazmat shipment requests. Hazmat network design has received little attention from researches and only recently. We review four papers that fall in this research field.

Kara and Verter (2004) consider a hazmat transportation network design problem and propose for it a bilevel integer programming model by considering the roles of carriers and of a government authority. They assume that the carriers, represented by the follower (second level) decision maker in the bilevel model, will always use the cheapest routes on the hazmat transportation network designed by the government authority, which assumes the role of the leader (first level) decision maker in the bilevel model, and has the objective to select the minimum total risk network, taking

into account the cost-minimizing behavior of the carries. In their model, hazmats are grouped into categories based on risk impact, and a network is designed for each group, without considering the interactions among shipments of hazmats of different categories. The bilevel integer programming problem is transformed into a single-level mixed integer linear problem by replacing the follower problem by its KKT conditions and by linearizing the complementary slackness constraints. Then, the latter mixed integer problem is solved using a commercial optimization software. It has to be remarked that the single-level mixed integer linear model may fail to find an optimal stable solution for the bilevel model. In fact, in general, there are multiple minimum-cost routing solutions for the follower over the designed network established by the leader, which may induce different total risk values over the network. We note that Kara and Verter (2004) do not take into account such an issue.

Erkut and Alp (2007) consider a single-level hazmat transportation network design problem, restricting the network to a tree, so that there is a single path between each couple of origin-destination pair; with this restriction, the carriers have no alternative paths on the tree, hence the carriers have no freedom in route selection, with the result that the structure of the proposed model has a single level. They formulate the tree design problem as an integer programming problem with the objective of minimizing the total risk, which is solved using a commercial optimization software. They also propose a greedy heuristic that adds shortest paths to the tree so as to keep the risk increase to a minimum and allow carriers to select cheaper paths.

Erkut and Gzara (2008) consider a similar problem of Kara and Verter (2004), generalizing their model considering the undirected network case and designing the same network for all the shipments. They consider the possible lack of stability of the solution of the bilevel model obtained by solving the single-level mixed integer linear model, and propose a heuristic solution method that always find a stable solution. Moreover, they extend the bilevel model to account for the cost/risk trade-off by including cost in the objective function of the leader (first level) problem.

All the previous papers adopt a link-based formulation for the carriers' problem, while Verter and Kara (2008) provide a new path-based formulation for the hazmat transport network design problem they studied in (Kara and Verter, 2004), where the open links in the given road network chosen by the regulator determine the set of paths that are available to the carriers. This facilitates the incorporation of carriers' cost concerns in regulator's risk reduction decision, and allow to formulate the problem with a single-level integer programming formulation, that assure that the cheapest path among the available ones is used by each carrier.

All the models studied in the literature in the field of hazmat network design consider the government and the carriers points of view trying to mitigate the risk only from a macroscopic point of view but without considering the need to distribute the risk in an equitable way over the region in which the transportation network is embedded; in fact, the choices of the carries, that is related to the cost, could overload, in terms of risk, some links of the network, implying a lack of risk equity. This could be inadequate when there are multiple layers of government authorities being involved in the regulation of dangerous goods shipments (as, for example, is common in Europe and North America), that are responsible at different geographical levels, e.g., regional area authorities and local area authorities. In such a scenario, a regional area authority aims to minimize the total risk over its controlled area, while a local area authority wants the risk over the local populated links of its jurisdiction to be the lowest possible.

In this paper, we study a new model for hazmat network design addressing also the concept of risk equity. The problem we consider is the following: a set of hazmat shipments has to be shipped over a road transportation network in order to transport a given amount of hazardous materials from specific origin points to specific destination points, and we assume that there are government authorities (at different levels, e.g., regional and local) that want to regulate the hazmat traffic by restricting the use of network links to the hazmat shipments with the aim of minimizing the total risk of the shipments, on the one hand, and spreading the risk equitably over the geographical region in which the transportation network is embedded, on the other hand. The former aim concerns that of a regional area authority, while the latter one goes in the direction of the aims of local area authorities (that are responsible to regulate the hazmat traffic inside their local area contained in the regional area) that would like avoiding local populated links in their jurisdictions to be overloaded in terms of induced risk by hazmat transportation. To the best of our knowledge, this is the first work that pays particular attention to the jurisdictional differences within different authorities.

We formulate the hazmat network design problem with a linear bilevel model where at the higher (leader) level there is a meta-local authority (acting on behalf of all the involved local area authorities) that aims to minimize the maximum link risk over populated links of the whole network, that is, risk equity, and at the lower (follower) level there is the regional area authority that aims to minimize the total risk over the network. This corresponds to the existence of two decision makers, one (the regional authority) willing to define a feasible hazmat flow assignment on the network that induces the minimum total risk over the population, and the other (representing the local authorities) that, interpreting the optimal flow assignment of the previous (lower level) decision maker as a flow vector, minimizes the maximum link risk on the network, i.e., aiming at risk equity, by defining capacities over the network links that restrict the possible choices of the regional authority.

The reason for using such a modeling approach follows from the fact that the local and the regional authorities act as multiple decision makers and in many cases they do not cooperate (e.g., because they are controlled by different parties, as often happens in Italy); hence, a simple multi-objective single-level model does not adequately represent such a case, while the bilevel model better represents the scenario where there is a hierarchy of decision makers where the leader (i.e., the meta-local authority) tries to minimize the maximum link risk imposing some restrictions on the amount of hazmat traffic over the links of the network in terms of link capacities, leaving to the follower decision maker (i.e., the regional authority) the freedom to choose the specific amounts of hazmat traffic to be routed over the capacitated network in order to minimize the network total risk.

In Figure 1, we report the conceptual comparison between our model and that of Kara and Verter (2004). In particular, our problem is a network design one, where the goal is not that of determining a subgraph of the whole transportation road network, but it is to determine (link) capacities leading to a balanced risk over the population as evenly as possible. Note that, differently from what happens in the model of Kara and Verter (2004), in our model two types of authority actors are represented, without considering the role of the carriers. Even if we do not take into account the point of view of carriers, the model formulation helps in defining restrictions to the amount of hazmat flowing through the network links without imposing specific routes to carries. In fact, in general, there are multiple-flow assignments for which we get the same minimum total risk

and minimum maximum link risk equal to the values achieved by the bilevel model, and, hence, a certain freedom can be left to the carriers in selecting the cheapest one among these flow assignments.

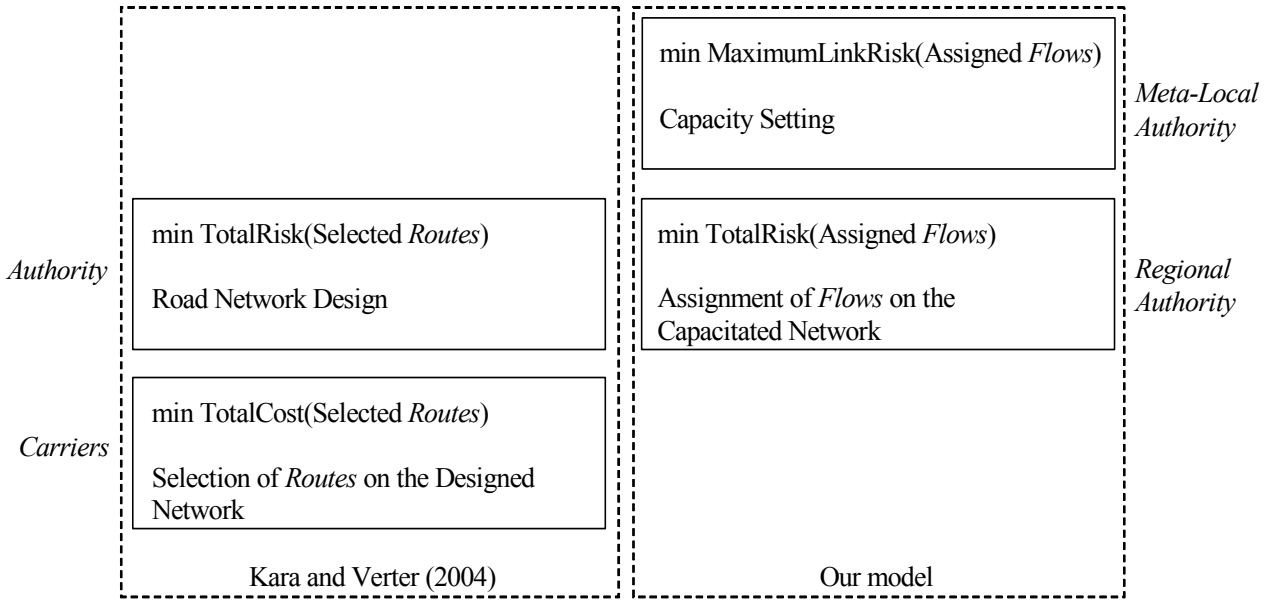

Figure 1: Comparison between Kara and Verter (2004) and the proposed model.

In order to solve the linear bilevel model proposed, we transform the latter into a single-level mixed integer linear program by replacing the follower problem by its Karush-Kuhn-Tucker (KKT) conditions and by linearizing the complementary constraints, and then by solving the MIP problem with a commercial optimization solver. The optimal solution of the bilevel model may not be stable, and we provide an approach for testing its stability and for evaluating the range of maximum link risk values when the solution is not stable. Moreover, since the bilevel model is difficult to solve optimally and the solution may not be stable, we provide a heuristic algorithm for the bilevel model able to always find a stable solution. The proposed model and heuristic algorithm are experimentally evaluated on an Italian geographical region.

The paper is organized as follows. In Section 2, we recall bilevel optimization and review the existing methods for solving bilevel problems. In Section 3, we give the linear bilevel problem formulation and its reduction to a single-level mixed integer linear program. In Section 4, we show why the solution of the bilevel model could not be stable, and give an approach for checking its stability. In Section 5, we provide the heuristic algorithm. Section 6 presents the model and heuristic algorithm experimentations on real cases of an Italian region. Finally, in Section 7 we provide some conclusions and remarks, and describe how the carriers can identify the cheapest flow assignment among the multiple-flow assignments for which we get the same minimum total risk and minimum maximum link risk equal to the values achieved by the bilevel model.

2. Bilevel optimization review

A bilevel mathematical program (see, e.g., Bialas and Karwan, 1984) concerns two optimization problems hierarchically related. We can interpret such problems as that of two decision makers in which the optimal decision of one of them (called the *leader*) is constrained by the decision of the other decision maker (called the *follower*). The follower decision maker optimizes his/her objective function under a feasible region that is defined by the leader decision maker. The latter, with this setting, is in charge to define all the possible reactions of the second level decision maker and selects those values for the variable controlled by the follower that produce the best outcome for his/her objective function. A general formulation of such problem is the following:

$$\begin{aligned} &\min_{(\xi_1,\xi_2)\in S_1} f_1(\xi_1,\xi'_2) \\ &s.t. \\ &\xi'_2 \in \arg\min_{\xi_2\in S_2(\xi_1)} f_2(\xi_1,\xi_2) \end{aligned}$$

where ξ_1 is a vector of m_1 variables controlled by the leader, and ξ_2 is a vector of m_2 variables controlled by the follower. $f_1(\xi_1, \xi'_2)$ and $f_2(\xi_1, \xi_2)$ are the objective functions of the leader and of the follower, respectively. S_1 is the feasible region of the higher level (leader) problem, and $S_2(\xi_1) = \{\xi_2 \in R^{m_2} : (\xi_1,\xi_2) \in S_2\}$ is the feasible region of the lower level (follower) problem which depends on the values of the variables controlled by the leader. This formulation corresponds to the optimistic one, and implies that whenever the optimal solution set $S'_2(\xi_1) = \arg\min_{\xi_2\in S_2(\xi_1)} f_2(\xi_1,\xi_2)$, i.e., the follower reaction set, does not reduce to a singleton for some ξ_1, the leader may select any solution $\xi'_2 \in S'(\xi_1)$ that suits him best; let ξ^*_2 be such a solution. Nonetheless, there is no guarantee that the follower will choose the best solution ξ^*_2 for the leader, and the value of the leader objective function $f_1(\xi_1, \xi'_2)$ may be grater than $f_1(\xi_1, \xi^*_2)$ for a certain $\xi'_2 \in S'(\xi_1)$. In this case the solution ξ_1 chosen by the leader is not stable.

Bilevel programs are very closely related to the van Stackelberg equilibrium problem (van Stackelberg, 1952), and the mathematical programs with equilibrium constraints (see, e.g., Luo et al. 1996).

The most studied instances of bilevel programming problems have been for a long time the linear bilevel programs, and therefore this subclass is the subject of several dedicated surveys, such as that one by Wen and Hsu (1991). Over the years, more complex bilevel programs were studied and even those including discrete variables received some attention, see, e.g., Vicente et al. (1996). Hence, more general surveys appeared, such as those by Vicente and Calamai (1994) and Falk and Liu (1995) on non-linear bilevel programming. The combinatorial nature of bilevel programming has been reviewed in Marcotte and Savard (2005).

Bilevel programs are hard to solve. In particular, linear bilevel programming has been proved to be strongly NP-hard (see, Hansen et al., 1992); Vicente et al. (1996) strengthened this result by showing that finding a certificate of local optimality is also strongly NP-hard.

Existing methods for bilevel programs can be distinguished into two classes. On the one hand, we have convergent algorithms for general bilevel programs with theoretical properties

guaranteeing suitable stationary conditions; see, e.g., the implicit function approach of Outrata et al. (1998), the quadratic one-level reformulation of Scholtes and Stohr (1999), and the smoothing approaches of Fukushima and Pang (1999) and Dussault et al. (2004). With respect to the optimization problems with complementarity constraints that represent a special way of solving bilevel programs, we can mention the papers of Kocvara and Outrata (2004), Bouza and Still (2007), and Lin and Fukushima (2003, 2005). The first work presents a new theoretical framework with the implicit programming approach. The second one studies convergence properties of a smoothing method that allows the characterization of local minimizers where all the functions defining the model are twice differentiable. Finally, Lin and Fukushima (2003, 2005) present two relaxation methods.

Exact algorithms have been proposed for special classes of bilevel programs, e.g., see the vertex enumeration methods by Candler and Townsley (1982), Bialas and Karwan (1984), and Tuy et al., (1993) applied when the property of extremal solution in bilevel linear program holds.

Complementary pivoting approaches (see, e.g., Bialas et al., 1980, and Júdice and Faustino, 1992) have been proposed on the single-level optimization problem obtained by replacing the second level optimization problem by its optimality conditions. Exploiting the complementarity structure of this single-level reformulation, Bard and Moore (1990) and Hansen et al. (1992), have proposed branch-and-bound algorithms which appear to be among the most efficient. Typically branch and bound is used when the lower level problem is convex and regular, since the latter can be replaced by its Karush-Kuhn-Tucker (KKT) conditions, yielding a single-level reformulation. When one deals with linear bilevel programs, the complementarity conditions are intrinsically combinatorial and in such cases branch-and-bound is the best approach to solve this problem (see, e.g., Colson et al., 2005).

Cutting-plane approach is not frequently used to solve bilevel linear programs. Cutting-plane methods found in literature are essentially based on Tuy's concavity cuts (Tuy, 1964). White and Anandalingam (1993) use these cuts in a penalty function approach for solving bilevel linear programs. Marcotte et al. (1993) propose a cutting-plane algorithm for solving bilevel linear programs with a guarantee of finite termination. Recently, Audet et al. (2007), exploiting the equivalence of the latter problem with a mixed integer linear programming one, propose a new branch and bound algorithm embedding Gomory cuts for bilevel linear programming. For the above reasons, since, as we will show next, our bilevel model is a linear one, in the following we will adopt the single-level mixed integer linear programming formulation for the bilevel model, and a resolution strategy based on branch and bound rules.

3. The bilevel model formulation

Let the road transportation network be represented by an undirected network $G = (N, E)$, with N and E being the set of n nodes (crossing points) and the set of m (undirected) links (road segments that connect the crossing points) of the network, respectively. Let C be a set of hazmat shipments (in the following called also commodities), and, for each hazmat shipment $c \in C$, let s^c and t^c be respectively the source node (origin point) and the sink node (destination point) in the network, and let d^c be the amount of hazmat to be shipped from s^c to t^c in a given time horizon (e.g., a month).

W.l.o.g., we assume that in G there is at least a path connecting the source node s^c to the sink node t^c, for each $c \in C$.

Moreover, w.l.o.g., we assume that each link $<i, j> \in E$ can be traversed in both directions; let $A = \{(i, j), (j, i)\} : <i, j> \in E \}$ be the set of (directed) arcs induced by set E of links, where arc $(i, j) \in A$ represents link $<i, j>$ when traversed from node i to node j.

Let ρ_{ij}^c be the *unitary* risk of arc $(i, j) \in A$ related to commodity $c \in C$, that is, the risk induced over link $<i, j> \in E$ by a unit of flow of commodity $c \in C$ that traverses that link from node i to node j, and let ρ_{ji}^c be the risk induced over the same link $<i, j>$ by a unit of flow of commodity $c \in C$ that traverses that link in the opposite direction. When being not explicitly stated, we assume that $\rho_{ij}^c = \rho_{ji}^c$.

Let x_{ij}^c be the amount of hazmat of commodity c that traverses link $<i, j>$ from node i to node j, that is the amount of flow of commodity c that flown on arc (i, j).

Let y_{ij} be the (bundle) capacity of arc $(i, j) \in A$ that limits the total amount $\sum_{c \in C} x_{ij}^c$ of flow traversing link $<i, j> \in E$ from node i to node j.

Let $\lambda_{<i,j>} = \sum_{c \in C} (\rho_{ij}^c x_{ij}^c + \rho_{ji}^c x_{ji}^c)$ be the link total risk over link $<i, j> \in E$, let λ be the maximum link total risk among the $\lambda_{<i,j>}$ values of each link $<i, j> \in E$, and let $R_{tot} = \sum_{c \in C} \sum_{(i,j) \in A} \rho_{ij}^c x_{ij}^c$ be the network total risk over G.

According to the conceptual scheme of our model the bundle capacities $\{y_{ij}\}$ are the variable controlled by the leader decision maker, who wants to minimize the value of λ by imposing specific limits on the amount of flows $\{x_{ij}^c\}$ on the links of the network, which are the variables controlled by the follower decision maker who aims to minimize the value of R_{tot}.

Referring to the general bilevel formulation given in Section 2, in our model the set S_2 represents all the multi-commodity feasible flow assignments along with bundle capacities in a capacitated transportation network, where the bundle capacity vector $\mathbf{y} = \{y_{ij}\}$ is the vector $\boldsymbol{\xi}_1$ of the variables controlled by the leader, the feasible multi-commodity flow assignment $\mathbf{x} = \{x_{ij}^c\}$ is the vector $\boldsymbol{\xi}_2$, and the feasible region $S_2(\boldsymbol{\xi}_1 \equiv \mathbf{y})$ of the follower problem contains all the multi-commodity feasible flow assignments $\mathbf{x} = \{x_{ij}^c\}$ on the network with bundle capacity vector $\mathbf{y}$. Therefore, once bundle capacities are fixed by the leader decision maker, the lower level (follower) problem that the follower decision maker wants to solve becomes a minimum cost multi-commodity network flow problem, where the arc cost models the unit risk of traversing the arc, with a specific hazmat shipment (commodity) $c \in C$ of d^c units being associated with a couple $(s^c; t^c)$ of source-sink nodes. The set of optimal solutions of this latter problem is the follower reaction set $S'_2(\boldsymbol{\xi}_1)$.

In the following, we give the bilevel formulation.

$$(P1)\quad \begin{cases} \lambda^* = \min_{\mathbf{y}} \lambda \\ \sum_{c\in C} (\rho_{ij}^c x_{ij}'^c + \rho_{ji}^c x_{ji}'^c) \le \lambda, \quad \forall <i,j> \in E & (1) \\ y_{ij} \ge 0, \quad \forall (i,j) \in A & (2) \\ \text{where } \mathbf{x}' = \{x_{ij}^c\} \text{ solves the lower level problem} \end{cases}$$

$$(P2)\quad \begin{cases} R_{tot}^*(\mathbf{y}) = \min_{\mathbf{x}} \sum_{c\in C} \sum_{(i,j)\in A} \rho_{ij}^c x_{ij}^c \\ \sum_{j\in FS(i)} x_{ij}^c - \sum_{j\in BS(i)} x_{ji}^c = \begin{cases} d^c, & i = s^c, \forall c \in C \\ 0, & \forall i \in N \setminus \{s^c, t^c\}, \forall c \in C \\ -d^c, & i = t^c, \forall c \in C \end{cases} & (3) \\ \sum_{c\in C} x_{ij}^c \le y_{ij}, \quad \forall (i,j) \in A & (4) \\ x_{ij}^c \ge 0, \quad \forall (i,j) \in A, \forall c \in C & (5) \end{cases}$$

The first formulation is the *higher level formulation* that models the *leader problem* $P1$ of assuring an equitable distribution of the risk over the network given a minimum network total risk multi-commodity hazmat flow by the follower decision maker. The model minimizes the value of the maximum link total risk λ among the link total risk $\lambda_{<i,j>}$ values of the links $<i, j>$ of the network G, finding appropriate arc capacities y_{ij} for each arc $(i, j) \in A$. Let λ^* be the optimal solution value of the leader problem, that is the minimum possible value for λ. Constraints (1) say that the link total risk over each link $<i, j> \in E$ cannot be greater than λ, while (2) are nonnegative constraints on the arc bundle capacities.

The second formulation is the *lower level formulation* that models the *follower problem* $P2$ of minimizing the network total risk R_{tot} induced by a (feasible) multi-commodity flow assignment $\mathbf{x} = \{x_{ij}^c\}$, given the capacity vector $\mathbf{y} = \{y_{ij}\}$ imposed by the leader decision maker. Let $R^*_{tot}(\mathbf{y})$ be the optimal solution value of the follower problem, that is the minimum network total risk value given the bundle capacity vector $\mathbf{y}$.

Being $FS(i) = \{j \in N : (i, j) \in A\}$ and $BS(i) = \{j \in N : (j, i) \in A\}$ respectively the forward and backward stars of each node $i \in N$, constraints (3) impose the conservation of flow at nodes for each commodity. Constraints (4) say that the total flow on arc $(i, j) \in A$ should not exceed the arc capacity value y_{ij}. Noting that flows x_{ij}^c are variables of the follower problem and capacities y_{ij} the variables of the leader problem, the minimization of the network total risk R_{tot} is assumed over the x_{ij}^c variables, and the minimization of λ over the y_{ij} variables.

The model is a bilevel linear program, i.e., the objective functions and the constraints are linear. Following the approach proposed by Fortuny-Amat and McCarl (1981), we reformulate the linear bilevel model, as made also by Kara and Verter (2004), by replacing the follower problem $P2$ with

the Karush-Kuhn-Tucker optimality conditions into the leader problem $P1$, thus obtaining a single-level optimization problem. Therefore, in the following, we will transform our bilevel program into a new single-level problem called $P3$, assuming that $P2$ is the primal formulation from which we want to define its optimality conditions, i.e., complementary slackness and primal and dual feasibility. To this aim let us define:

γ_i^c = dual variables associated with primal constraints (3), where $i \in N$, $c \in C$;

η_{ij} = dual variables associated with primal constraints (4) where $(i,j) \in A$;

w_{ij} = slack variables of the primal constraints (4), where $(i,j) \in A$;

z_{ij}^c = slack variables of the dual constraints, where $(i,j) \in A$, $c \in C$.

Now, we give the new single-level optimization problem $P3$, where we considered the optimality conditions of the follower problem $P2$ into the leader problem $P1$.

$$
(P3)\quad
\begin{cases}
\lambda^* = \min\limits_{(\mathbf{x},\mathbf{y})} \lambda \\
\sum\limits_{c\in C}(\rho_{ij}^c x_{ij}^c + \rho_{ji}^c x_{ji}^c) \le \lambda, \quad \forall <i,j> \in E \\
\sum\limits_{j\in FS(i)} x_{ij}^c - \sum\limits_{j\in BS(i)} x_{ji}^c = \begin{cases} d^c, & i = s^c, \forall c \in C \\ 0, & \forall i \in N \setminus \{s^c, t^c\}, \forall c \in C \\ -d^c, & i = t^c, \forall c \in C \end{cases} \\
\sum\limits_{c\in C} x_{ij}^c + w_{ij} = y_{ij}, \quad \forall (i,j) \in A \\
\rho_{ij}^c - \gamma_i^c + \gamma_j^c + \eta_{ij} - z_{ij}^c = 0, \quad \forall (i,j) \in A, c \in C \qquad (5) \\
\eta_{ij} w_{ij} = 0, \quad \forall (i,j) \in A \qquad (6) \\
x_{ij}^c z_{ij}^c = 0, \quad \forall (i,j) \in A, c \in C \qquad (7) \\
z_{ij}^c, x_{ij}^c \ge 0, \quad \forall (i,j) \in A, \forall c \in C \\
w_{ij}, y_{ij}, \eta_{ij} \ge 0, \quad \forall (i,j) \in A \\
\gamma_i^c \text{ free}, \quad \forall i \in N, \forall c \in C \qquad (8)
\end{cases}
$$

Note that the complementary slackness conditions (6) and (7) are quadratic constraints. Moreover (see constraints (8)), dual variables associated with primal equality constraints are free in sign. Constraints (6) and (7) can be linearized by introducing binary variables $\delta_{ij}^1, \delta_{ij}^2, \delta_{ijc}^3$, and δ_{ijc}^4, and large numbers M^1, M^2, M_c^3, M^4, as reported in the next mixed integer linear program $P4$.

$$
(P4)\quad
\begin{cases}
\lambda^* = \min\limits_{(\mathbf{x},\mathbf{y})} \lambda \\
\sum\limits_{c\in C}(\rho_{ij}^c x_{ij}^c + \rho_{ji}^c x_{ji}^c) \le \lambda, \quad \forall < i,j > \in E \\
\sum\limits_{j\in FS(i)} x_{ij}^c - \sum\limits_{j\in BS(i)} x_{ji}^c = \begin{cases} d^c, & i = s^c, \forall c \in C \\ 0, & \forall i \in N \setminus \{s^c, t^c\}, \forall c \in C \\ -d^c, & i = t^c, \forall c \in C \end{cases} \\
\sum\limits_{c\in C} x_{ij}^c + w_{ij} = y_{ij}, \quad \forall (i,j) \in A \\
\rho_{ij}^c - \gamma_i^c + \gamma_j^c + \eta_{ij} - z_{ij}^c = 0, \quad \forall (i,j) \in A, c \in C \\
\eta_{ij} \le M^1 \delta_{ij}^1, \qquad \forall (i,j) \in A \qquad (9) \\
w_{ij} \le M^2 \delta_{ij}^2, \qquad \forall (i,j) \in A \qquad (10) \\
x_{ij}^c \le M_c^3 \delta_{ijc}^3, \qquad \forall (i,j) \in A, c \in C \qquad (11) \\
z_{ij}^c \le M^4 \delta_{ijc}^4, \qquad \forall (i,j) \in A, c \in C \qquad (12) \\
\delta_{ij}^1 + \delta_{ij}^2 \le 1, \qquad \forall (i,j) \in A \qquad (13) \\
\delta_{ijc}^3 + \delta_{ijc}^4 \le 1, \qquad \forall (i,j) \in A, c \in C \qquad (14) \\
z_{ij}^c, x_{ij}^c \ge 0, \qquad \forall (i,j) \in A, c \in C \\
w_{ij}, y_{ij}, \eta_{ij} \ge 0, \qquad \forall (i,j) \in A \\
\gamma_{ij}^c \text{ free}, \qquad \forall (i,j) \in A, c \in C \\
\delta_{ij}^1, \delta_{ij}^2 \in \{0,1\}, \qquad \forall (i,j) \in A \\
\delta_{ijc}^3, \delta_{ijc}^4 \in \{0,1\}, \qquad \forall (i,j) \in A, c \in C
\end{cases}
$$

Note that, due to constraints (13) and (14), pairs of constraints (9)-(10) and (11)-(12) are such that complementary slackness relations in $P3$ are obeyed.

Values M^1 and M^4 are sufficiently large numbers, while we can set $M^2 = \sum_{c \in C} d^c$, since $w_{ij} \le y_{ij} \le \sum_{c \in C} d^c$, and $M_c^3 = d^c$, since $x_{ij}^c \le d^c$.

4. Solving the bilevel model

When solving problem $P4$, besides the optimal solution $\mathbf{y}^* = \{y^*_{ij}\}$ of the bilevel model which is the best leader choice, we also get the best (from the leader point of view) multi-commodity flow assignment $\mathbf{x}_B = \{x^{*c}_{ij}\}$ that the follower may choose among the minimum network total risk flow assignments given bundle link capacities $\{y^*_{ij}\}$, that is, among all his/her indifferent choices over the capacitated network established by the leader. The optimal solution value λ^* of the bilevel model is equal to the maximum link total risk induced by flow assignment $\mathbf{x}_B$. Nevertheless, there is no guarantee that the follower will adopt flow assignment $\mathbf{x}_B$ if there are multiple minimum network total risk flow assignments on the capacitated network, and in this case the optimal solution $\mathbf{y}^*$ of the bilevel model might be unstable, that is, the follower may adopt a flow assignment $\mathbf{x}'$ different from $\mathbf{x}_B$, and the maximum link total risk λ' induced by $\mathbf{x}'$ may be greater than λ^*.

For example, let us consider the network of Figure 2(a) where we have exactly two disjoint paths from node 1 to node 4, i.e., path $P_1 = (1, 2, 4)$ and path $P_2 = (1, 3, 4)$. Let us consider two hazmat commodities $c = 1, 2$, and assume that we have to move 100 hazmat units from node 1 to node 4 for both commodity 1 and 2 (i.e., $d^1 = d^2 = 100$) over the network, where the risk per unit flow flowing on each link $<i, j>$ in both the directions is equal to 1 and 2, respectively for commodity 1 and 2 (i.e., $\rho_{ij}^1 = \rho_{ji}^1 = 1$ and $\rho_{ij}^2 = \rho_{ji}^2 = 2$, for each $<i, j> \in E$). Paths P_1 and P_2 are the only available paths for sending hazmats for both the two commodities. Since, for both commodities, the risk per unit of flow over the two paths P_1 and P_2 is the same (i.e., equal to 2 and 4, for commodity 1 and 2, respectively), every feasible flow assignment on the network links gives the same network total risk R_{tot} of value $R^*_{tot} = 600$, which is the minimum value for R_{tot}. However, one of the best case for the leader would be if the follower divided the total amount of each commodity in two equal amounts of 50 units each, and ship each one of these latter amounts on different paths. That is, for each commodity, 50 units of flow will be assigned to arcs (1, 2) and (2, 4) of path P_1, and 50 units of flow will be assigned to arcs (1, 3) and (3, 4) of path P_2; in this case the value of λ, being the maximum among the link total risk over each link of the network, is equal to $\lambda^* = 150$, which is the lowest possible value for λ; this flow assignment corresponds to the best flow assignment $\mathbf{x}_B$ from the leader point of view (see Figure 2(b)). Therefore, in order to try to force the follower to choose flow assignment $\mathbf{x}_B$, the leader will fix bundle arc capacities y_{ij} to $y^*_{ij} = 100$ for arcs (1, 2), (2, 4), (1, 3) and (3, 4), and to $y^*_{ij} = 0$ for all the other arcs of A; let therefore the capacity vector $\mathbf{y}^*$ be the optimal solution of the bilevel problem. But with these capacity values, the follower has multiple minimum network total risk flow assignments for shipping the two hazmat commodities from node 1 to node 4, which have the same network total risk R_{tot} of value $R^*_{tot} = 600$

but different λ values. For example, the minimum network total risk flow assignment, denoted with $\mathbf{x}_W$ and shown in Figure 2(c), where all the amount of commodity 1 is shipped along path P_1 and all the amount of commodity 2 is shipped along path P_2 has network total risk R_{tot} equal to $R^*_{tot} = 600$ and λ equal to $\lambda_W = 200$. Note that, from the leader point of view, $\mathbf{x}_W$ is the worst flow assignment that the follower may choose among the minimum network total risk flow assignments given bundle link capacities y^*_{ij}, since it produces the maximum value for λ. Therefore, since the leader cannot impose to the follower to choose flow assignment $\mathbf{x}_B$ in order to achieve the minimum value λ^* for λ, the optimal solution $\mathbf{y}^* = \{y^*_{ij}\}$ of the bilevel problem is not stable.

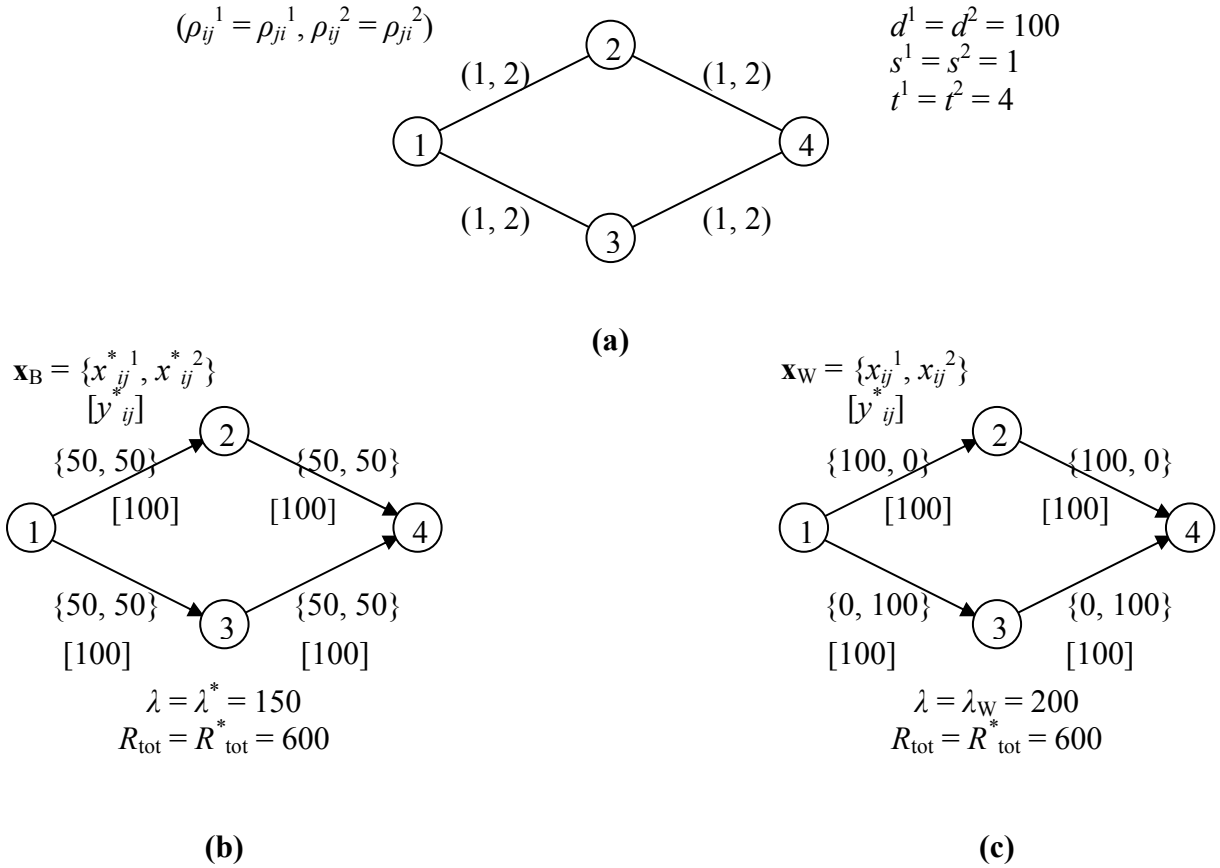

Figure 2: An example for a non-stable solution.

When the optimal solution $\mathbf{y}^* = \{y^*_{ij}\}$ of the bilevel model is not stable, the leader might be interested in evaluating the gap between λ^*, that is the optimal (minimum) value of λ (achieved if the follower adopts the best flow assignment $\mathbf{x}_B = \{x^*_{ij}{}^c\}$ from the leader point of view) and its worst (maximum) value λ_W (achieved if the follower chooses the worst flow assignment $\mathbf{x}_W$, among the minimum-total risk flow assignments given bundle arc capacities y^*_{ij}). If this gap is too large, the leader may prefer to (heuristically) find a stable feasible bundle arc capacity assignment $\{\hat{y}_{ij}\}$.

Before testing the stability of the optimal solution $\mathbf{y}^* = \{y^*_{ij}\}$ of the bilevel model, we reinforce this solution by setting $y^*_{ij} = \min\{y^*_{ij}; \sum_{c \in C} x^*_{ij}{}^c\}$, where $\{x^*_{ij}{}^c\}$ is the solution of problem $P4$ and, hence, corresponds to the best multi-commodity flow assignment $\mathbf{x}_B$ from the leader point of view. With this reinforcement we consider (possibly) another optimal solution of the bilevel model that might have more chance to be stable than the previous one.

To test the stability of solution $\mathbf{y}^* = \{y^*_{ij}\}$, it is sufficient to check if the minimum network total risk multi-commodity network flow problem on the capacitated network with bundle arc capacities y^*_{ij} has multiple solutions with values of λ greater than λ^*. This can be done as follows.

Let us consider the following multi-commodity network flow problem $P5$ with bundle arc capacity vector $\mathbf{y} = \{y_{ij}\}$, an upper bound $\hat{R}_{tot}$ on the network total risk, and where the objective is to maximize the link total risk $\lambda_{<i,j>} = \sum_{c \in C} (\rho^c_{ij} x^c_{ij} + \rho^c_{ji} x^c_{ji})$ over link $<i, j>$ of the network.

$$
(P5)\begin{cases}
\lambda_{<i,j>}(\mathbf{y}, \hat{R}_{tot}) = \max\limits_{\mathbf{x}} \sum\limits_{c \in C} (\rho^c_{ij} x^c_{ij} + \rho^c_{ji} x^c_{ji}) \\
\sum\limits_{c \in C} \sum\limits_{(i,j) \in A} \rho^c_{ij} x^c_{ij} \le \hat{R}_{tot} \\
\sum\limits_{j \in FS(i)} x^c_{ij} - \sum\limits_{j \in BS(i)} x^c_{ji} = \begin{cases} d^c, & i = s^c, \forall c \in C \\ 0, & \forall i \in N \setminus \{s^c, t^c\}, \forall c \in C \\ -d^c, & i = t^c, \forall c \in C \end{cases} \\
\sum\limits_{c \in C} x^c_{ij} \le y_{ij}, \quad \forall (i,j) \in A \\
x^c_{ij} \ge 0, \quad \forall (i,j) \in A, \forall c \in C
\end{cases}
$$

For each link $<i, j>$ of the network, we solve problem $P5$ with $\mathbf{y} = \mathbf{y}^*$ and $\hat{R}_{tot} = R^*_{tot}(\mathbf{y}^*)$, with $R^*_{tot}(\mathbf{y}^*)$ being the total risk value of the minimum-total risk multi-commodity network flow on the capacitated network G with bundle arc capacities vector $\mathbf{y}^*$, that is, $R^*_{tot}(\mathbf{y}^*)$ is the optimal solution value of the follower problem $P2$ given in Section 3; let $\lambda^*_{<i,j>}(\mathbf{y}^*, R^*_{tot}(\mathbf{y}^*))$ be the optimal solution value of problem $P5$, for link $<i, j>$.

Let $\lambda_W = \max_{<i,j> \in E} \{\lambda^*_{<i,j>}(\mathbf{y}^*, R^*_{tot}(\mathbf{y}^*))\}$. If $\lambda_W = \lambda^*$ then $\mathbf{y}^*$ is a stable optimal solution of the bilevel problem with maximum link total risk equal to λ^*; otherwise, $\mathbf{y}^*$ is not stable and the maximum link total risk may be as large as λ_W, depending on the follower's choice. In the latter case, a stable solution can be obtained heuristically, as we show in the next section.

5. A heuristic approach

In this section we describe a heuristic approach for finding a stable feasible solution of the bilevel model, inspired by Erkut and Gzara (2008). The heuristic approach we propose is an iterative algorithm that at each iteration constructs a feasible solution $\mathbf{y}_H$ of the bilevel model, and tests its stability using the same approach as the one discussed in the previous section. If $\mathbf{y}_H$ is stable, the algorithm stops; otherwise, the algorithm removes a link and starts a new iteration with the residual network. When the algorithm stops, the stable heuristic solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ on the

original network is obtained by setting $\hat{y}_{ij} = \hat{y}_{ji} = 0$, for each removed link $<i, j>$ from the original network.

In the following, we show the criterion for the selection of the link to be removed, and prove that after a number of iterations less than the number of network links the algorithms stops with a feasible stable solution for the residual network.

At each iteration the algorithm works as follows. It computes a new heuristic solution $\mathbf{y}_H$ of the bilevel model by solving the optimization problem, denoted with *Only_Leader*, of the *over-regulated* scenario where the leader directly imposes the flow assignment on the network that optimizes its own criterion, that is the minimization of the maximum link total risk λ among all the links of the (residual) network G. The problem *Only_Leader* is therefore an uncapacitated multi-commodity network flow problem where the objective is the minimization of the maximum link total risk λ, and is formulated as follows.

$$(Only\_Leader)\begin{cases} \lambda_L = \min_{\mathbf{x}} \lambda \\ \sum_{c\in C}(\rho_{ij}^c x_{ij}^c + \rho_{ji}^c x_{ji}^c) \le \lambda, \quad \forall <i,j> \in E \\ \sum_{j\in FS(i)} x_{ij}^c - \sum_{j\in BS(i)} x_{ji}^c = \begin{cases} d^c, & i = s^c, \forall c \in C \\ 0, & \forall i \in N \setminus \{s^c, t^c\}, \forall c \in C \\ -d^c, & i = t^c, \forall c \in C \end{cases} \\ x_{ij}^c \ge 0, \quad \forall (i,j) \in A, \forall c \in C \end{cases}$$

In order to take also into account the behaviour of the follower, who aims to minimize the network total risk $R_{tot} = \sum_{c\in C} \sum_{(i,j)\in A} \rho_{ij}^c x_{ij}^c$, we modify problem *Only_Leader* by substituting the objective function with the function $f(\lambda, \mathbf{x}) = \lambda + 1/\gamma \cdot R_{tot}$, with $\gamma = \sum_{c\in C} \sum_{(i,j)\in A} \rho_{ij}^c d^c$; let us denote with *Modified_Only_Leader* this latter problem. Note that, with this value of γ, we minimize λ and R_{tot} in lexicographical order (i.e., among the solutions with minimum λ value, we find that one of minimum R_{tot} value).

Let $\hat{\mathbf{x}} = \{\hat{x}_{ij}^c\}$ be the optimal solution of problem *Modified_Only_Leader*, and let $\hat{\lambda} = \max_{<i,j>\in E} \{\sum_{c\in C} (\rho_{ij}^c \hat{x}_{ij}^c + \rho_{ji}^c \hat{x}_{ji}^c)\}$ be the related maximum link total risk. The heuristic solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ is constructed from solution $\hat{\mathbf{x}}$ by setting the bundle capacity value $\hat{y}_{ij}$ of arc (i, j) equal to the total flow on that arc, that is $\hat{y}_{ij} = \sum_{c\in C} \hat{x}_{ij}^c$. Let $\lambda_H = \hat{\lambda}$ be the solution value related to the feasible solution $\mathbf{y}_H$.

Let us assume, therefore, that the leader imposes the bundle capacities $\mathbf{y}_H = \{\hat{y}_{ij}\}$ on the network. The follower finds an optimal flow assignment on the capacitated network that minimizes the network total risk; this corresponds to solve the follower problem $P2$ formulated in Section 3. Let $R^*_{tot}(\mathbf{y}_H)$ be the optimal solution value of this latter problem, that is, the value of the minimum network total risk on the capacitated network G with bundle link capacities vector $\mathbf{y}_H$.

For each link $<i, j>$ of the network, let $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))$ be the maximum link total risk over link $<i, j>$ among all the optimal solutions of the follower problem $P2$ on the capacitated network

with bundle arc capacity vector $\mathbf{y}_H$; recall that $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))$ is the optimal solution value of problem $P5$ with $\mathbf{y} = \mathbf{y}_H$ and $\hat{R}_{tot} = R^*_{tot}(\mathbf{y}_H)$.

If solution $\mathbf{y}_H$ is not stable, there is a link $<i, j>$ of G for which $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))$ is greater than λ_H; let us assume that $<i, j>$ is the link for which $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) = \max_{<i,j>\in E} \{\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))\}$ and that $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) > \lambda_H$, with $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))$ being induced by the follower optimal flow assignment $\{x'^c_{ij}\}$ over the capacitated network with bundle arc capacity vector $\mathbf{y}_H = \{\hat{y}_{ij}\}$.

In order to eliminate the difference between λ_H and $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))$, the algorithm removes link $<i, j>$ from G and starts a new iteration where it searches for a new feasible solution of the bilevel model on the residual network. Note that the following holds in the residual network.

Theorem 1. Given a heuristic solution $\mathbf{y}_H$ of value λ_H, if there is a link $<i, j>$ of G such that $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) > \lambda_H$, in the residual network obtained after the removal of link $<i, j>$ from G, there is at least one path connecting the origin node s^c to the destination node t^c, for each commodity $c \in C$.

Proof: If $\lambda_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) > \lambda_H$ there exists at least a commodity $\hat{c} \in C$ with $x'^{\hat{c}}_{ij} \neq \hat{x}^{\hat{c}}_{ij}$ (or $x'^{\hat{c}}_{ji} \neq \hat{x}^{\hat{c}}_{ji}$). This means that in the residual network there are at least two paths $P_1^{\hat{c}}$ and $P_2^{\hat{c}}$ connecting the source node s^c to the sink node t^c with link $<i, j> \in P_1^{\hat{c}}$ and $<i, j> \notin P_2^{\hat{c}}$, since in the opposite case when all paths from s^c to t^c have in common link $<i, j>$ we should have $x'^{\hat{c}}_{ij} = \hat{x}^{\hat{c}}_{ij} = d^{\hat{c}}$ (or $x'^{\hat{c}}_{ji} = \hat{x}^{\hat{c}}_{ji} = d^{\hat{c}}$ if link $<i, j>$ is traversed from node j to node i when going from s^c to t^c along each path connecting s^c to t^c). Let $P_1^{\hat{c}} = (s^{\hat{c}} \equiv i_1^{\hat{c}}, \ldots, i_h^{\hat{c}}, \ldots, i_q^{\hat{c}} \equiv i, j \equiv j_r^{\hat{c}}, \ldots, j_k^{\hat{c}}, \ldots, j_1^{\hat{c}} \equiv t^{\hat{c}})$ and $P_2^{\hat{c}} = (s^{\hat{c}} \equiv i_1^{\hat{c}}, \ldots, i_h^{\hat{c}}, u, \ldots, v, j_k^{\hat{c}}, \ldots, j_1^{\hat{c}} \equiv t^{\hat{c}})$, with $1 \leq h \leq q$, and $1 \leq k \leq r$. For any other commodity $c \neq \hat{c}$ for which there exists a path P_1^c connecting the origin node s^c to the destination node t^c and with link $<i, j> \in P_1^c$ (e.g., $P_1^c = (s^c \equiv i_1^c, \ldots, i_\varphi^c \equiv i, \ j \equiv j_\omega^c, \ldots, j_1^c \equiv t^c)$, with $\varphi \geq 1$ and $\omega \geq 1$), there exists also path P_2^c (in Figure 3, $P_2^c = (s^c \equiv i_1^c, \ldots, i_\varphi^c \equiv i \equiv i_q^{\hat{c}}, \ldots, i_h^{\hat{c}}, u, \ldots, v, j_k^{\hat{c}}, \ldots, j_r^{\hat{c}} \equiv j \equiv j_\omega^c, \ldots, j_1^c \equiv t^c)$) that does not contain link $<i, j>$. Therefore, in the residual network obtained after the removal of link $<i, j>$ from the network G, there exists at least one path connecting the source node s^c to the sink node t^c, for each commodity $c \in C$. □

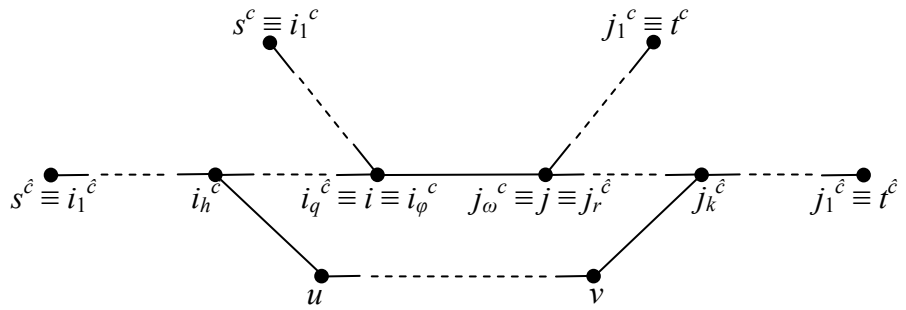

Figure 3: The subgraph of the paths in the proof of Theorem 1.

Theorem 2. The heuristic algorithm always stops with a stable heuristic solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$.

Proof: At each iteration, the heuristic algorithm tests the stability of the heuristic solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ found for the current residual network and stops when this solution is stable; otherwise, it removes a link $<i, j>$ from the network for which $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) > \lambda_H$. In the latter case, according to Theorem 1, in the residual network there exists at least one path connecting the source node s^c to the sink node t^c, for each commodity $c \in C$, and, hence, the set of feasible solutions for the bilevel model is not empty, and the heuristic algorithm finds one of these solutions. The algorithm iterates at most until, for each commodity c of C, there is exactly one path P^c from s^c to t^c in the residual network (which is therefore a forest, that is, a collection of node-disjoint trees), since in this case it always returns a stable solution as shown next. In such case, there is only one feasible solution $\{\hat{x}^c_{ij}\}$ for problem *Modified_Only_Leader* with $\hat{x}^c_{ij} = d^c$ if link $<i, j> \in P^c$ and is traversed from node i to node j when going from s^c to t^c, and $\hat{x}^c_{ij} = 0$ otherwise; therefore, the solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ of the bilevel model, with $\hat{y}_{ij} = \sum_{c \in C} \hat{x}^c_{ij}$, found by the heuristic algorithm is stable, since the existence of exactly one path from s^c to t^c, for each $c \in C$, implies that in the capacitated network with bundle capacities $\hat{y}_{ij}$ the optimal follower solution is unique and equal to solution $\{\hat{x}^c_{ij}\}$. Hence, also in this case, the heuristic algorithm stops with a stable solution. □

For the sake of completeness, in the following we sketch the steps of the proposed heuristic.

Heuristic algorithm

Repeat

Step 1. Solve problem *Modified_Only_Leader*; let $\hat{\mathbf{x}} = \{\hat{x}_{ij}\}$ be the optimal solution and $\hat{\lambda} = \max_{<i,j> \in E} \{\sum_{c \in C} (\rho^c_{ij} \hat{x}^c_{ij} + \rho^c_{ji} \hat{x}^c_{ji})\}$ be the related maximum link total risk.

Step 2. Let $\mathbf{y}_H = \{\hat{y}_{ij}\}$ be a solution of the bilevel model, with $\hat{y}_{ij} = \sum_{c \in C} \hat{x}^c_{ij}$, for each $(i, j) \in A$; let $\lambda_H = \hat{\lambda}$ be the solution value related to solution $\mathbf{y}_H$.

Step 3. Solve the *follower problem* with bundle arc capacity vector $\mathbf{y}_H$; let $R^*_{tot}(\mathbf{y}_H)$ be its optimal solution value.

Step 4. {* Test the stability of solution $\mathbf{y}_H$ *}

Step 4.1. Let $<i^*, j^*>$ be the link of G such that

$$\lambda^*_{<i^*, j^*>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) = \max_{<i, j> \in E} \{\lambda^*_{<i, j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))\},$$

where $\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))$ is the optimal solution value of problem $P5$.

Step 4.2. If $\lambda^*_{<i^*, j^*>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) > \lambda_H$ declare solution $\mathbf{y}_H$ as "*not stable*", otherwise declare solution $\mathbf{y}_H$ as "*stable*".

Step 5. If $\mathbf{y}_H$ is "*not stable*" remove link $<i^*, j^*>$ from the network G.

Until solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ is "*stable*".

Return solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$, with $\hat{y}_{ij} = \hat{y}_{ji} = 0$, for each removed link $<i, j>$ of the original network.

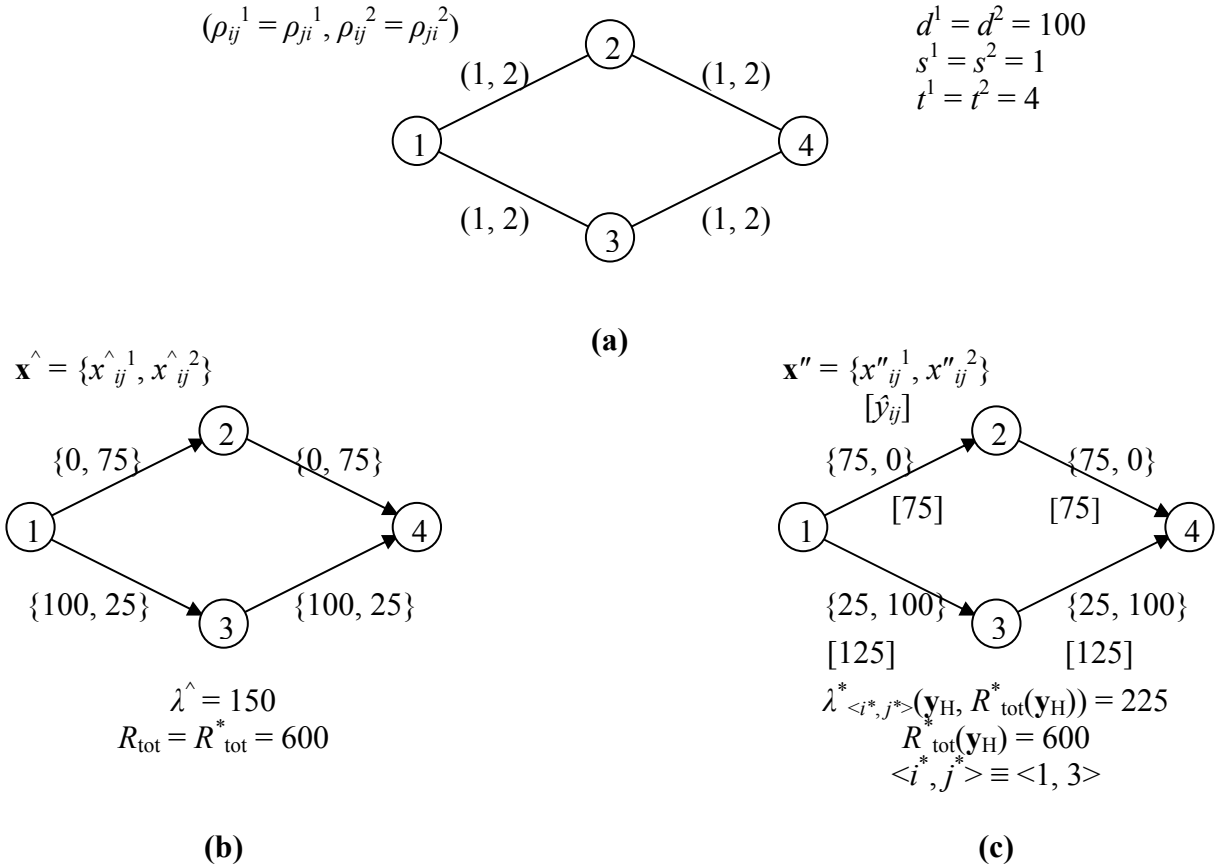

Figure 4: The heuristic solution: iteration 1.

Let us consider again the network example introduced in Section 4, and apply the heuristic algorithm (Figure 4(a) shows the network data). Figure 4 shows the execution of iteration 1. Step 1 solves problem *Modified_Only_Leader*, and the solution $\mathbf{x}^{\wedge} = \{x^{\wedge}_{ij}\}$ is depicted in Figure 4(b) with $\lambda^{\wedge} = 150$. From this solution, Step 2 constructs the heuristic solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ of the bilevel model, with $\hat{y}_{ij} = \sum_{c \in C} x^{\wedge}_{ij}{}^c$, for each $(i, j) \in A$; its solution value is $\lambda_H = \lambda^{\wedge} = 150$. Step 3 solves the follower problem *P*2 with bundle link capacity vector $\mathbf{y}_H$ and finds $R^*_{tot}(\mathbf{y}_H) = 600$. Step 4 tests the stability of $\mathbf{y}_H$, and finds that $\lambda^*_{<1,3>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) = \max_{<i,j> \in E} \{\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))\}$; Figure 4(c) shows the solution $\mathbf{x}''$ of problem *P*5; from this solution $\lambda^*_{<1,3>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) = 225$. Since $\lambda^*_{<1,3>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) > \lambda_H$, the heuristic solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ is not stable. Therefore, the algorithm removes link <1, 3> from the network and executes iteration 2 (see Figure 5). Step 1 solves problem *Modified_Only_Leader* on the residual network depicted in Figure 5(a), and the solution $\mathbf{x}^{\wedge} = \{x^{\wedge}_{ij}\}$, with $\lambda^{\wedge} = 300$, is shown in Figure 5(b). From this solution, Step 2 constructs the new heuristic solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ of the bilevel model, with $\hat{y}_{ij} = \sum_{c \in C} x^{\wedge}_{ij}{}^c$, for each $(i, j) \in A$; its solution value is $\lambda_H = \lambda^{\wedge} = 300$. Step 3 solves the follower problem *P*2 with bundle link capacity vector $\mathbf{y}_H$ and finds $R^*_{tot}(\mathbf{y}_H) = 600$. Step 4 tests the stability of solution $\mathbf{y}_H$, and finds that $\lambda^*_{<i^*j^*>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) = \max_{<i,j> \in E} \{\lambda^*_{<i,j>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H))\} = 300$; Figure 5(c) shows the solution $\mathbf{x}''$ of problem *P*5, with $\lambda^*_{<i^*j^*>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) = 300$ (e.g. with $<i^*, j^*> \equiv <1, 2>$). Since $\lambda^*_{<i^*j^*>}(\mathbf{y}_H, R^*_{tot}(\mathbf{y}_H)) = \lambda_H = 300$ the heuristic solution $\mathbf{y}_H = \{\hat{y}_{ij}\}$ is stable on the residual network. Therefore, the algorithm stops, and the (heuristic) stable solution on the original network is obtained assuming $\hat{y}_{ij} = \hat{y}_{ji} = 0$ for every link $<i, j>$ removed from the original network (i.e., link <1, 3>).

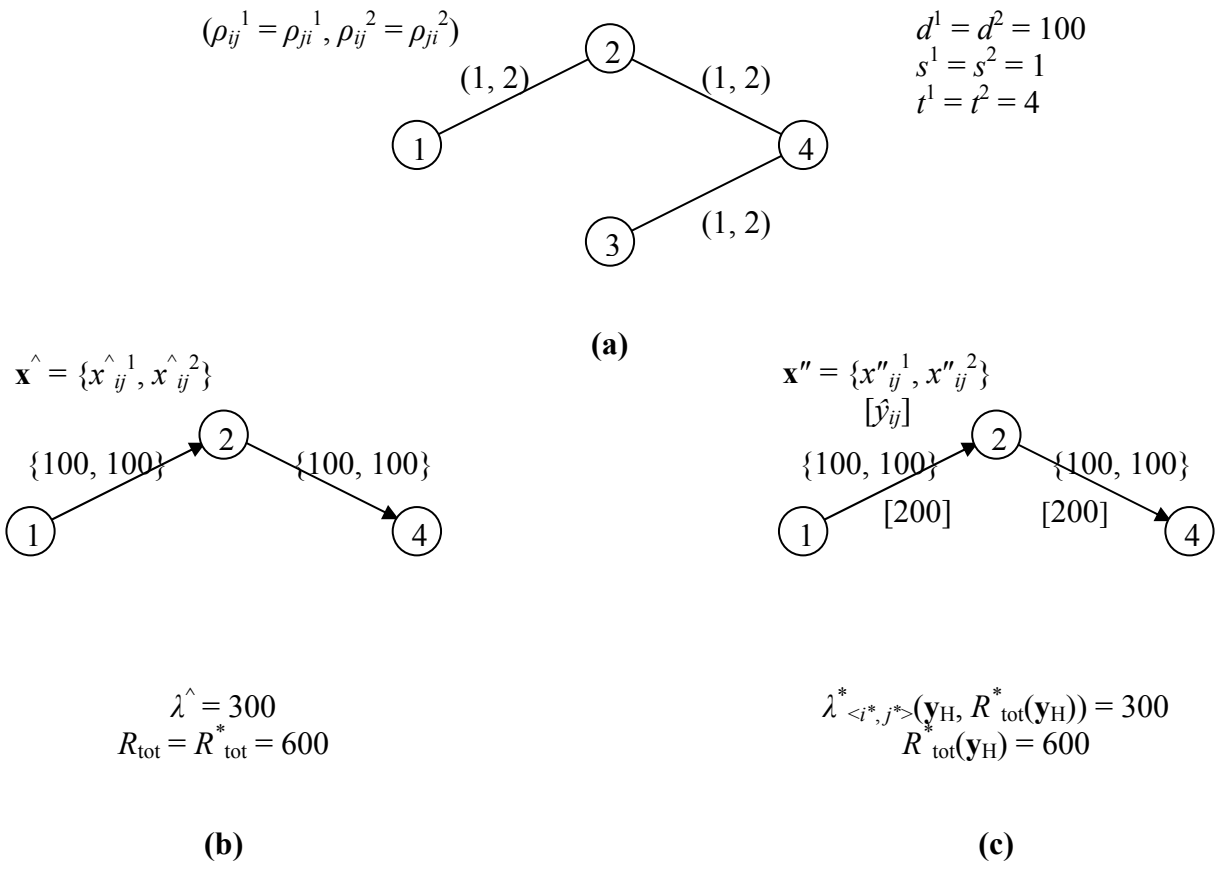

Figure 5: The heuristic solution: iteration 2.

6. Experimental analysis

This section shows the computational results achieved by the proposed bilevel model and the proposed heuristic algorithm. We divide this section into four parts. The first one is devoted to analyze the behavior of the bilevel model, implemented in the AMPL language (www.ampl.com) and optimally solved by means of an on-the-shelf branch and bound algorithm like the one embedded in the CPLEX 8.0.1 solver (www.ilog.com). The second part presents heuristic results. The third and the fourth parts of the section is devoted to the comparison of the results of the optimal and heuristic solutions of the bilevel model with the results coming from two opposite scenarios, called *over-regulated* and *under-regulated* scenarios (see also Erkut and Gzara, 2008), respectively, in order to assess the values of the network total risk R_{tot} and of the maximum link total risk λ obtained by the bilevel model and the heuristic algorithm.

In the over-regulated scenario only the leader decision maker is considered, with the leader that directly imposes the flow assignment on the network G, optimizing his/her own criterion, that is, minimizing the maximum link total risk λ among the links of the network. This consists in solving the linear program *Only_Leader* introduced in Section 5; let $\mathbf{x}_L$ be the optimal solution (flow assignment), and let λ_L be its optimal solution value, that is, the minimum value of the maximum link total risk λ among the links of the network in the over-regulated scenario. Clearly, λ_L is a lower

bound on λ^* and, hence, $\lambda_L \leq \lambda^* \leq \lambda_H$. Moreover, let us denote with R^L_{tot} the network total risk value associated to the flow assignment $\mathbf{x}_L$.

In the under-regulated scenario only the follower decision maker is considered, who therefore aims to find a multi-commodity network flow over the *uncapacitated* network to minimize the network total risk R_{tot}. This corresponds to solve the linear program $P2$ introduced in Section 3 with unlimited bundle capacities (i.e., $y_{ij} = +\infty$, for each $(i, j) \in A$) on the network G; let us denote with *Only_Follower* this latter problem. Let $\mathbf{x}_F$ be the optimal solution (flow assignment) of the *Only_Follower* problem, and let $R^F_{tot} = R^*_{tot}(+\infty)$ be its optimal solution value, that is, the minimum network total risk value in the under-regulated scenario. Clearly, $R^F_{tot} \leq R^*_{tot}(\mathbf{y})$, for any (non-negative) capacity vector $\mathbf{y}$. Moreover, let us denote with λ_F the maximum link total risk value associated to the flow assignment $\mathbf{x}_F$.

Figure 6 shows the optimal solutions of the over-regulated and under-regulated scenarios for the network example introduced in Section 4: Figure 6(a) shows the network data, and Figures 6(b) and 6(c) show the optimal solutions of the over-regulated and under-regulated scenarios, respectively.

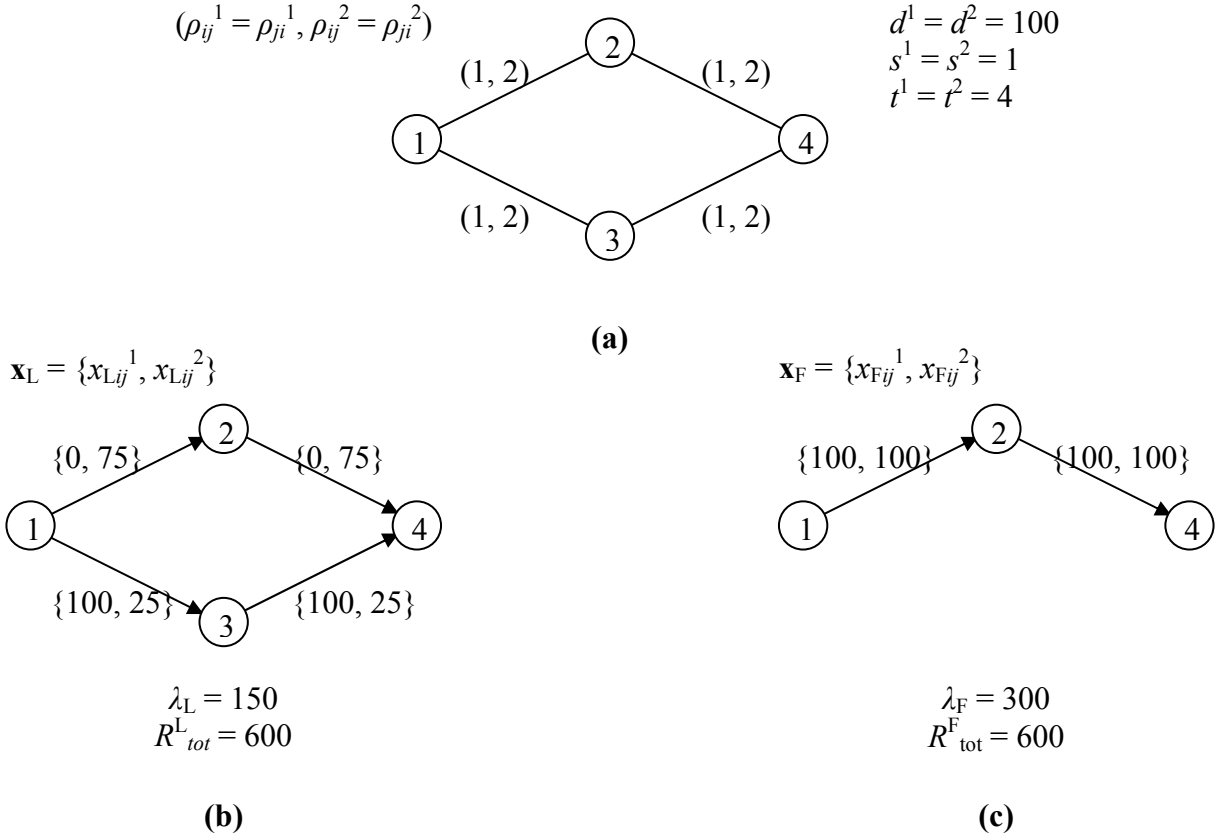

Figure 6: The optimal solutions for the over-regulated and under-regulated scenarios.

The linear programming models for the over-regulated and under-regulated scenarios have been implemented, as for the single-level mixed integer linear programming model reformulation of the bilevel model, by means of the AMPL language and solved with CPLEX 8.0.1. The heuristic algorithm was implemented in the C language. All the algorithms ran on a PC with a Pentium IV processor and 2GB of RAM.

In order to evaluate the effectiveness of the proposed model and algorithm, we concentrated our analysis on a real-world case study. We considered the road network of the Lazio region (located in the middle of Italy), and, in particular, its main transport roads, for an overall size of n = 311 macro-nodes and m = 441 links. Unitary risk ρ_{ij}^{c} values were computed from data provided by a local authority and range from 50 to 250 per ton of hazmat transported. The unitary risk ρ_{ij}^{c} (and ρ_{ji}^{c}) is computed as the ratio between the risk $r_{<i,j>}^{m(c)}$ induced on link <i, j> by a vehicle carrying hazmat of type $m(c)$ and the capacity τ in tons of the vehicle, with $r_{<i,j>}^{m(c)}$ being evaluated as the societal risk computed as the number of people living inside the exposure zone around link <i, j> (whose size depends on the hazmat type $m(c)$) times the accident probability involving the vehicle. We considered from 2 to 10 origin-destination pairs on the network, each one associated with a number of commodities ranging from 1 to 3, for an overall number of shipments (commodities) between 2 and 30 (we recall that in our model each commodity is associated with one origin-destination pair). With each one of the latter scenarios we associated 10 instances; each instance has been generated by assigning uniformly at random a demand from 100 to 1,000 tons to each shipment. Figure 7 shows the road network and the 10 origin-destination points for the hazmat shipments.

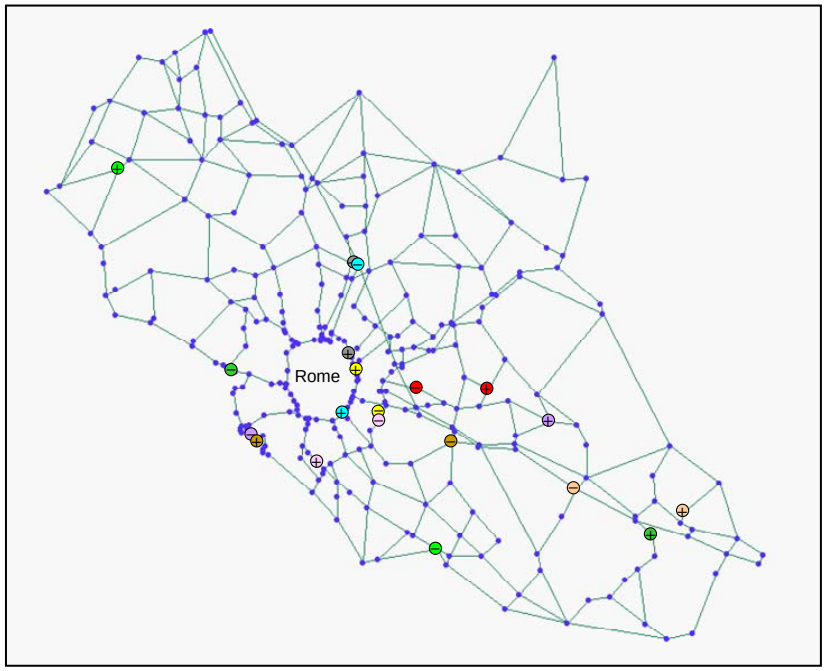

Figure 7: The road transportation network of Lazio with origin-destination pairs.

In Tables 1 and 2, we report the main features of the bilevel model which help in analyzing its performance. In particular, the columns of Table 1 show:

- *ns*, the number of shipments (commodities);
- *Inst,* the instance ID;
- # *reinf.*, the number of capacity values being reinforced after the solution of the bilevel model;
- λ^{*}, the optimal solution value of the bilevel model, i.e., the minimum value of the maximum link total risk λ;
- $R^{*}_{tot}(\mathbf{y}^{*})$, the minimum network total risk over the capacitated network with bundle capacity vector $\mathbf{y}^{*}$ imposed by the leader;

- *Capacity ratio*, computed as the ratio (in percentage) $[\sum_{(i,j)\in A} y_{ij}]/[|A| \sum_{c\in C} d^c]$, the performance indicator that measures the restriction imposed by the leader on the decision of the follower.
- *B&B nodes*, the number of branch and bound nodes generated when solving problem *P*4;
- *Simplex iterations,* the total number of simplex iterations in the bounding process;
- *Cpu time bilevel,* the cpu time (in seconds) spent to solve the bilevel model;
- λ_W, the worst (maximum) value for the maximum link total risk λ among all the optimal flow assignments of the follower on the capacitated network with bundle capacity vector $\mathbf{y}^*$ imposed by the leader;
- *Cpu time stability,* the cpu time (in seconds) needed to check the stability of the solution of the bilevel model.

Table 1 shows results for the case with *ns* = 2 and 3 shipments. Indeed, the B&B algorithm of CPLEX 8.0.1 was able to optimally solve within 3 hours of cpu time limit only these classes of instances. As can be inferred by the table, the solutions found are always stable, i.e. $\lambda^* = \lambda_W$. The cpu time spent to find the optimal solutions ranges from 3 seconds to 4,710 seconds, while the cpu time needed to test to solution stability ranges from 1.2 to 65.5 seconds (note that the stability check is performed by solving at most *m* linear programs). Even if, for each given *ns* value, both λ^* and $R^*_{tot}(\mathbf{y}^*)$ values have a wide range of variability over the 10 instances, we can observe that the λ^* values are quite robust with respect to different *ns* values. In particular, the former ranges from about 4,000 to 19,300, and the latter from about 62,000 to 617,000 when *ns* = 2; when *ns* = 3, λ^* varies from about 6,400 to 19,300 and $R^*_{tot}(\mathbf{y}^*)$ from about 303,000 to 1,330,000. None reinforcement is appreciable for *ns* = 2 and only one instance has such parameter greater than zero (i.e., see instance 4) when *ns* = 3.

ns	*Inst.*	# *reinf.*	λ^*	$R^*_{tot}(\mathbf{y}^*)$	*Capacity ratio* (%)	*B&B nodes*	*Simplex iterations*	*Cpu time bilevel*	λ_W	*Cpu time stability*
2	1	0	12,870.00	263,001.00	0.94%	83	2,983	7.484	12,870.00	4.719
	2	0	4,792.00	93,726.00	0.55%	124	3,372	9.984	4,792.00	10.500
	3	0	4,290.00	62,507.00	0.63%	25	1,735	3.281	4,290.00	7.641
	4	0	10,725.00	177,761.00	0.56%	20	1,472	4.438	10,725.00	9.609
	5	0	17,160.00	290,944.00	0.67%	20	1,757	3.359	17,160.00	1.188
	6	0	4,290.00	87,231.30	0.66%	676	13,390	38.250	4,290.00	9.172
	7	0	6,435.00	115,693.00	0.63%	18	1,689	2.984	6,435.00	5.406
	8	0	6,435.00	233,944.00	1.48%	370	8,773	25.344	6,435.00	9.860
	9	0	4,290.00	106,595.00	0.64%	163	4,276	17.578	4,290.00	16.218
	10	0	19,305.00	416,497.00	1.34%	111	3,091	7.797	19,305.00	1.250
3	1	0	17,160.00	792,980.00	1.80%	14,674	1,112,475	2,359.840	17,160.00	65.454
	2	0	19,305.00	487,101.00	1.18%	700	651,357	147.672	19,305.00	30.109
	3	0	17,160.00	1,332,050.00	2.10%	43,411	2,305,245	4,709.690	17,160.00	40.781
	4	29	10,725.00	306,153.00	0.71%	25,486	769,176	1,665.970	10,725.00	3.546
	5	0	6,435.00	457,294.00	2.49%	155,332	5,884,286	4,637.590	6,435.00	48.594
	6	0	10,725.00	303,822.00	1.17%	612	16,969	17.781	10,725.00	16.438
	7	0	8,580.00	481,618.00	1.29%	3,694	119,789	148.688	8,580.00	25.734
	8	0	10,725.00	439,045.00	1.17%	3,594	104,159	109.750	10,725.00	21.859
	9	0	17,160.00	647,855.00	1.68%	66,695	3,514,594	3,172.220	17,160.00	18.562
	10	0	14,066.20	886,671.00	1.68%	29,276	1,160,354	793.640	14,066.20	52.625

Table 1: Results of the bilevel model for *ns* = 2 and 3.

ns	*Inst.*	λ_{LR}	*Simplex iterations*	*Cpu time bilevel lin. rel.*	*ns*	*Inst.*	λ_{LR}	*Simplex iterations*	*Cpu time bilevel lin. rel.*
5	1	15,629.10	953	0.813	**20**	1	47,190.00	3,440	10.406
	2	15,015.00	902	0.781		2	53,625.00	3,176	6.281
	3	7,814.50	911	0.828		3	53,625.00	3,201	7.750
	4	21,450.00	878	0.750		4	42,900.00	3,323	7.843
	5	15,015.00	937	0.953		5	40,755.00	3,253	7.343
	6	17,160.00	882	0.735		6	53,625.00	3,154	7.765
	7	15,015.00	911	0.860		7	45,045.00	3,289	7.313
	8	17,160.00	927	0.766		8	55,770.00	3,127	6.328
	9	17,160.00	893	0.766		9	53,625.00	2,896	6.125
	10	21,450.00	979	0.797		10	47,190.00	3,304	7.469
10	1	38,610.00	1,736	2.406	**30**	1	70,785.00	4,912	13.392
	2	23,595.00	1,901	2.375		2	55,770.00	4,814	13.527
	3	25,740.00	1,708	2.203		3	79,365.00	4,532	12.593
	4	38,610.00	1,747	2.609		4	98,670.00	5,239	14.113
	5	21,450.00	2,084	2.531		5	68,640.00	4,932	13.183
	6	11,261.90	1,902	2.484		6	98,670.00	5,192	14.082
	7	25,740.00	1,674	2.328		7	71,325.30	5,032	14.822
	8	32,175.00	2,050	2.562		8	96,525.00	4,802	13.926
	9	21,450.00	1,982	2.625		9	70,785.00	4,924	13.845
	10	25,740.00	1,662	2.437		10	83,655.00	5,102	14.634

Table 2: Results of the linear relaxation of the single-level MIP formulation of the bilevel model for *ns* = 5, 10, 20 and 30.

ns	*Inst.*	λ_H	$R^*_{tot}(\mathbf{y}_H)$	*Capacity ratio* (%)	*Cpu time heuristic*	*ns*	*Inst.*	λ_H	$R^*_{tot}(\mathbf{y}_H)$	*Capacity ratio* (%)	*Cpu time heuristic*
2	1	12,870.00	179,535.00	0.63%	3.797	**10**	1	38,610.00	1,633,280.00	1.20%	39.250
	2	4,792.00	86,434.60	0.54%	8.500		2	23,595.00	1,655,610.00	1.58%	155.203
	3	4,290.00	59,845.00	0.63%	3.797		3	25,740.00	1,536,850.00	1.41%	170.469
	4	10,725.00	163,590.00	0.61%	4.594		4	38,610.00	2,099,500.00	1.39%	107.532
	5	17,160.00	234,721.00	0.63%	2.578		5	21,450.00	1,641,710.00	1.52%	167.672
	6	4,290.00	74,010.70	0.58%	3.859		6	11,261.90	1,151,150.00	1.53%	322.282
	7	6,435.00	94,426.60	0.62%	3.797		7	25,740.00	1,095,820.00	1.28%	101.468
	8	6,435.00	89,767.50	0.63%	4.610		8	32,175.00	2,355,110.00	1.50%	156.391
	9	4,290.00	83,594.70	0.55%	6.484		9	21,450.00	1,644,900.00	1.61%	182.797
	10	19,305.00	255,326.00	0.64%	1.797		10	25,740.00	1,191,100.00	1.26%	106.015
3	1	17,160.00	325,055.00	0.76%	2.969	**20**	1	47,190.00	3,116,820.00	1.46%	221.359
	2	19,305.00	410,710.00	0.77%	4.985		2	53,625.00	3,066,620.00	1.32%	159.813
	3	17,160.00	570,626.00	0.93%	11.218		3	53,625.00	3,607,980.00	1.41%	340.047
	4	10,725.00	239,512.00	0.83%	3.031		4	42,900.00	3,318,950.00	1.41%	362.484
	5	6,435.00	349,033.00	1.19%	16.719		5	40,755.00	2,304,570.00	1.45%	101.310
	6	10,725.00	230,194.00	0.89%	2.234		6	53,625.00	2,864,430.00	1.22%	59.047
	7	8,580.00	231,618.00	0.79%	7.234		7	45,045.00	3,545,330.00	1.49%	104.750
	8	10,725.00	317,008.00	0.83%	4.890		8	55,770.00	3,921,850.00	1.49%	133.860
	9	17,160.00	444,601.00	0.82%	6.657		9	53,625.00	3,559,550.00	1.40%	30.844
	10	14,066.20	661,654.00	1.27%	4.953		10	47,190.00	3,036,860.00	1.35%	112.375
5	1	15,629.10	1,159,620.00	1.41%	31.375	**30**	1	70,785.00	5,840,820.00	1.43%	408.935
	2	15,015.00	829,657.00	1.31%	23.734		2	55,770.00	5,654,790.00	1.55%	281.766
	3	7,814.50	476,081.00	1.25%	30.546		3	79,365.00	6,260,840.00	1.50%	211.563
	4	21,450.00	680,472.00	0.92%	22.812		4	98,670.00	5,618,600.00	1.24%	54.125
	5	15,015.00	606,019.00	1.08%	25.047		5	68,640.00	4,142,590.00	1.25%	112.390
	6	17,160.00	599,950.00	0.97%	7.953		6	98,670.00	5,505,330.00	1.43%	55.546
	7	15,015.00	469,690.00	1.24%	19.063		7	71,325.30	6,514,180.00	1.53%	176.547
	8	17,160.00	598,347.00	1.07%	33.312		8	96,525.00	6,107,060.00	1.33%	85.469
	9	17,160.00	703,746.00	0.97%	19.922		9	70,785.00	4,527,220.00	1.23%	141.297
	10	21,450.00	1,190,630.00	1.12%	23.563		10	83,655.00	4,327,560.00	1.27%	102.703

Table 3: Results of the heuristic algorithm.

Clearly, the number of generated branch and bound nodes tends to grow-up substantially (sometimes by more than two orders of magnitude) as soon as we pass from 2 to 3 shipments, and this sheds light on why CPLEX 8.0.1 is unable to optimally solve, within the imposed time limit of 7,200 seconds, instances with more than 3 shipments. For this reason, when *ns* is greater than 3, we optimally solved the linear relaxation of the single-level mixed integer linear programming formulation of the bilevel model in order to have a lower bound on the optimal solution value.

In Table 2, we report such values, denoted with λ_{LR}, for all the instances and reported the number of simplex iterations (see column *Simplex iterations*) and cpu time as well (see column *Cpu time bilevel lin. rel.*). Clearly, cpu times are very limited, i.e., about 1 second for *ns* = 5 to about 10 seconds for *ns* = 30. This lower bounding phase is particularly useful as soon as in the following we discuss heuristic results (reported in Table 3).

Note that we have not reported in Table 2 the values of $R^*_{tot}(\mathbf{y}_{\mathrm{LR}})$ associated with the capacity vector $\mathbf{y}_{\mathrm{LR}}$ established by the optimal solution of the linear relaxation of the single-level mixed integer linear programming formulation of the bilevel model: in fact, since the solution can be fractional, the $R^*_{tot}(\mathbf{y}_{\mathrm{LR}})$ values are not interpretable neither as lower bound nor as upper bound on the network total risk.

Similarly for Table 1, in Table 3 we reports the results of the heuristic algorithm listing the λ_{H} values (i.e., the solution values returned by the heuristic algorithm), the $R^*_{tot}(\mathbf{y}_{\mathrm{H}})$ values (i.e., the minimum network total risk over the capacitated network with bundle capacity vector $\mathbf{y}_{\mathrm{H}}$), the capacity ratio, and the cpu time in seconds spent by the algorithm (see column *Cpu time heuristic*).

We start noting that the heuristic algorithm performed only one iteration for every instances (this is why we did not list the number of iterations); this means that the heuristic solutions $\mathbf{y}_{\mathrm{H}} = \{\hat{y}_{ij}\}$ of the bilevel model determined at the first iteration (with $\hat{y}_{ij} = \sum_{c \in C} \hat{x}^c_{ij}$ for each $(i, j) \in A$, where $\hat{\mathbf{x}} = \{\hat{x}^c_{ij}\}$ is the optimal solution of problem *Modified_Only_Leader* on the given network *G*) is stable, and, therefore, the algorithm immediately exits from the repeat-until loop (see the algorithm pseudo-code for details).

Comparing the λ_{H} values obtained by the heuristic to the lower bound values λ_{LR} in Table 2 and the optimal values λ^* in Table 1, we note that $\lambda_{\mathrm{H}} = \lambda^*$, for *ns* = 2, 3, and $\lambda_{\mathrm{H}} = \lambda_{\mathrm{LR}}$, for *ns* = 5, 10, 20, 30; hence, for all the test instances, the solution of the bilevel model determined by the heuristic algorithm is indeed optimal. The capacity ratio increases on average with *ns* from 0.60% (for *ns* = 2) to 1.37% (for *ns* = 30).

Heuristic algorithm running times are quite limited: indeed, they are never greater than 7 minutes over all the instances tested. Recall that, for each iteration, the heuristic algorithm solves $O(m)$ linear programs.

In Table 4, we report the values obtained by solving problem *Only_Leader* for the over-regulated scenario. In particular, its optimal solution values λ_{L} are useful to assess the quality of the λ values for the bilevel model obtained optimally or heuristically. In fact, the λ_{L} values in Table 4 represent the best (lowest) values for the maximum link total risk that can be obtained by the leader, i.e., in the over-regulated scenario in which the leader directly decides the flow assignment. It is interesting to note that for the bilevel model both its optimal solutions (for *ns* = 2, 3) and the solutions provided by the heuristic algorithm (for *ns* = 5, 10, 20, 30) are able to produce maximum link total risk λ

values equal to the optimal solution values λ_L of problem *Only_leader*, meaning that these former values are the best achievable ones for all the test instances under consideration.

Running times for solving (linear programming) problem *Only_Leader* are very limited, i.e., less than 4 seconds over all the instances.

ns	*Inst.*	λ_L	R^L_{tot}	*Simplex iterations*	*Cpu time Only_Leader*	*ns*	*Inst.*	λ_L	R^L_{tot}	*Simplex iterations*	*Cpu time Only_Leader*
2	1	12,870.00	310,542.00	33	0.016	**10**	1	38,610.00	3,690,230.00	485	0.078
	2	4,792.00	145,603.00	33	0.015		2	23,595.00	2,898,750.00	623	0.313
	3	4,290.00	103,514.00	33	0.015		3	25,740.00	2,911,950.00	594	0.297
	4	10,725.00	280,349.00	31	0.016		4	38,610.00	4,260,680.00	583	0.266
	5	17,160.00	406,868.00	33	0.016		5	21,450.00	2,770,830.00	666	0.328
	6	4,290.00	122,434.00	38	0.016		6	11,261.90	1,828,290.00	1,041	0.594
	7	6,435.00	162,459.00	33	0.310		7	25,740.00	2,084,360.00	430	0.188
	8	6,435.00	155,271.00	33	0.031		8	32,175.00	4,153,500.00	545	0.266
	9	4,290.00	133,083.00	45	0.016		9	21,450.00	2,757,680.00	718	0.391
	10	19,305.00	444,249.00	33	0.015		10	25,740.00	2,784,320.00	432	0.203
3	1	17,160.00	954,293.00	99	0.031	**20**	1	47,190.00	5,880,280.00	924	0.610
	2	19,305.00	1,134,640.00	135	0.047		2	53,625.00	5,916,010.00	850	0.578
	3	17,160.00	1,356,290.00	157	0.047		3	53,625.00	6,420,990.00	1,131	0.796
	4	10,725.00	736,880.00	109	0.047		4	42,900.00	5,472,430.00	1,107	0.750
	5	6,435.00	684,595.00	169	0.063		5	40,755.00	4,657,630.00	979	0.578
	6	10,725.00	688,177.00	119	0.047		6	53,625.00	6,039,160.00	995	0.625
	7	8,580.00	619,037.00	116	0.047		7	45,045.00	6,102,030.00	1,206	0.891
	8	10,725.00	843,147.00	135	0.016		8	55,770.00	6,899,530.00	1,177	0.782
	9	17,160.00	1,265,060.00	113	0.047		9	53,625.00	7,351,630.00	967	0.657
	10	14,066.20	1,442,490.00	163	0.047		10	47,190.00	6,174,310.00	1,126	0.750
5	1	15,629.10	2,049,860.00	322	0.047	**30**	1	70,785.00	9,139,430.00	2,401	3.563
	2	15,015.00	1,490,600.00	225	0.031		2	55,770.00	8,446,490.00	2,980	4.031
	3	7,814.50	891,807.00	263	0.031		3	79,365.00	10,329,000.00	1,842	2.094
	4	21,450.00	1,586,260.00	180	0.047		4	98,670.00	11,719,500.00	1,230	1.235
	5	15,015.00	1,235,670.00	252	0.047		5	68,640.00	8,486,060.00	1,371	1.250
	6	17,160.00	1,395,520.00	217	0.047		6	98,670.00	11,304,400.00	1,075	1.016
	7	15,015.00	979,442.00	195	0.031		7	71,325.30	10,174,000.00	2,460	3.000
	8	17,160.00	1,112,880.00	186	0.031		8	96,525.00	12,065,000.00	1,501	1.563
	9	17,160.00	1,452,790.00	213	0.047		9	70,785.00	9,399,350.00	1,728	1.922
	10	21,450.00	2,162,790.00	357	0.063		10	83,655.00	9,153,580.00	1,355	1.281

Table 4: Results for the over-regulated scenario.

For completeness, Table 4 also lists the network total risk values R^L_{tot} induced by the optimal solutions $\mathbf{x}_L$ of problem *Only_Leader*; note that, for all the test instances, the R^L_{tot} values are greater than the $R^*_{tot}(\mathbf{y})$ values (see Tables 1 and 3) obtained for the bilevel model (with $\mathbf{y} = \mathbf{y}^*$ if optimally solved and $\mathbf{y} = \mathbf{y}_H$ if heuristically solved). Moreover, in Table 5, we report the averages values $Avg\_R^*_{tot}(\mathbf{y})$ and $Avg\_R^L_{tot}$ of $R^*_{tot}(\mathbf{y})$ and R^L_{tot}, respectively, over the 10 instances for each *ns* value; moreover, we also list the average values *Avg_gap* (in percentage) of the gaps $(R^*_{tot}(\mathbf{y}) - R^L_{tot})/R^L_{tot}$ between $R^*_{tot}(\mathbf{y})$ and R^L_{tot} values. As it can be inferred by the values listed in the table, both the $Avg\_R^*_{tot}(\mathbf{y}^*)$ values (obtained when the bilevel model is optimally solved) and the $Avg\_R^*_{tot}(\mathbf{y}_H)$ values (obtained when the bilevel model is heuristically solved) are considerable lower than the $Avg\_R^L_{tot}$ values obtained for the over-regulated scenario, with the average gap ranging from -61.5% to -18.9%.

ns	$Avg\_R^*_{tot}(\mathbf{y}^*)$	$Avg\_R^L_{tot}$	*Avg_gap* (%)
2	184,789.93	226,437.20	-18.9%
3	613,458.90	972,460.90	-38.1%
ns	$Avg\_R^*_{tot}(\mathbf{y}_H)$	$Avg\_R^L_{tot}$	*Avg_gap* (%)
2	132,125.11	226,437.20	-41.2%
3	378,001.10	972,460.90	-61.5%
5	731,421.20	1,435,761.90	-49.4%
10	1,600,503.00	3,014,059.00	-46.3%
20	3,234,296.00	6,091,400.00	-46.9%
30	5,449,899.00	10,021,681.00	-45.3%

Table 5: Comparison between the bilevel model and the over-regulated scenario average network total risk values.

ns	*Inst.*	λ_F	R^F_{tot}	*Simplex iterations*	*Cpu time Only_Follower*	*ns*	*Inst.*	λ_F	R^F_{tot}	*Simplex iterations*	*Cpu time Only_Follower*
2	1	25,740.00	130,914.00	27	0.015	**10**	1	42,900.00	1,599,170.00	686	0.093
	2	9,584.00	71,592.00	26	0.001		2	53,584.00	1,385,320.00	687	0.093
	3	8,580.00	43,638.00	27	0.001		3	53,584.00	1,485,750.00	687	0.110
	4	21,450.00	123,072.00	26	0.016		4	50,235.00	2,005,476.00	689	0.156
	5	34,320.00	169,893.00	26	0.015		5	36,465.00	1,491,130.00	686	0.125
	6	8,580.00	57,615.00	26	0.015		6	36,839.00	961,389.00	687	0.110
	7	12,870.00	70,116.00	26	0.015		7	36,839.00	1,023,496.00	687	0.013
	8	12,870.00	65,457.00	27	0.001		8	63,631.00	2,067,980.00	688	0.125
	9	8,580.00	66,933.00	26	0.001		9	30,141.00	1,447,520.00	689	0.110
	10	38,610.00	182,394.00	26	0.015		10	30,141.00	1,172,290.00	687	0.125
3	1	34,320.00	260,227.00	133	0.031	**20**	1	60,060.00	2,855,750.00	1,375	0.250
	2	38,610.00	336,531.00	133	0.031		2	56,933.000	2,857,390.00	1,376	0.266
	3	34,320.00	454,872.00	134	0.031		3	68,640.00	3,398,250.00	1,372	0.234
	4	21,450.00	194,770.00	134	0.015		4	62,205.00	3,049,050.00	1,372	0.234
	5	13,396.00	250,784.00	134	0.016		5	46,886.00	2,243,410.00	1,373	0.022
	6	21,450.00	185,452.00	134	0.031		6	63,631.00	2,697,540.00	1,370	0.218
	7	17,160.00	191,587.00	133	0.031		7	83,725.00	3,208,050.00	1,373	0.218
	8	21,450.00	258,573.00	133	0.015		8	97,121.00	3,592,110.00	1,372	0.234
	9	34,320.00	364,538.00	133	0.016		9	68,640.00	3,306,939.00	1,374	0.235
	10	30,141.00	487,413.00	134	0.031		10	80,376.00	2,710,930.00	1,372	0.218
5	1	33,490.00	817,369.00	335	0.031	**30**	1	137,309.00	5,175,810.00	2,057	0.453
	2	23,443.00	622,304.00	224	0.016		2	127,262.00	4,742,380.00	2,059	0.359
	3	16,745.00	359,378.00	223	0.016		3	160,752.00	5,485,960.00	2,061	0.359
	4	42,900.00	588,807.00	222	0.031		4	120,564.00	5,374,570.00	2,059	0.359
	5	30,030.00	482,787.00	223	0.015		5	80,376.00	3,813,510.00	2,057	0.359
	6	34,320.00	503,096.00	222	0.031		6	122,265.00	5,279,520.00	2,063	0.359
	7	20,022.00	378,396.00	223	0.016		7	148,005.00	5,765,640.00	2,063	0.359
	8	26,696.00	482,278.00	223	0.016		8	117,215.00	5,790,530.00	2,058	0.359
	9	34,320.00	579,274.00	223	0.016		9	103,819.00	4,265,520.00	2,061	0.359
	10	42,900.00	902,614.00	222	0.016		10	113,866.00	4,244,050.00	2,060	0.375

Table 6: Results for the under-regulated scenario.

Following what we have done in Table 4, in Table 6 we report the values obtained by solving problem *Only_Follower* for the under-regulated scenario. In particular, its optimal solution values R^F_{tot} are useful to assess the quality of the network total risk values $R^*_{tot}(\mathbf{y})$ for the bilevel model (with $\mathbf{y} = \mathbf{y}^*$ if optimally solved and $\mathbf{y} = \mathbf{y}_H$ if heuristically solved). Indeed, the R^F_{tot} values in Table 6 are the best (lowest) values for the network total risk that can be obtained by the follower, i.e., in the under-regulated scenario where the follower decides the flow assignment without limitations on flows coming from the leader. For completeness, Table 6 also lists the maximum link total risk values λ_F induced by the optimal solutions $\mathbf{x}_F$ of problem *Only_Follower*; note that, for all the test

instances, the λ_F values are greater than the λ^* values (see Tables 1) and the λ_H values (see Table 3) obtained for the bilevel model when optimally and heuristically solved, respectively. Recall that, as we noted before, for all the test instances, these latter values are the best (minimum) achievable values for the maximum link total risk, since they are equal to the optimal solution values λ_L of problem *Only_Leader*.

Similarly to what happened for solving problem *Only_Leader*, running times for solving the linear programming problem *Only_Follower* are negligible, i.e., less than half a second.

ns	$Avg\_R^*_{tot}(\mathbf{y}^*)$	$Avg\_R^F_{tot}$	*Avg_gap* (%)
2	184,789.93	98,162.40	85.2%
3	613,458.90	298,474.70	102.6%
ns	$Avg\_R^*_{tot}(\mathbf{y}_H)$	$Avg\_R^F_{tot}$	*Avg_gap* (%)
2	132,125.11	98,162.40	33.1%
3	378,001.10	298,474.70	26.0%
5	731,421.20	571,630.30	27.0%
10	1,600,503.00	1,463,952.10	9.6%
20	3,234,296.00	2,991,941.90	8.0%
30	5,449,899.00	4,993,749.00	9.0%

Table 7: Comparison between the bilevel model and the under-regulated scenario average network total risk values.

As for Table 5, in Table 7, we list the averages values $Avg\_R^*_{tot}(\mathbf{y})$ and $Avg\_R^F_{tot}$ of $R^*_{tot}(\mathbf{y})$ and R^F_{tot}, respectively, over the 10 instances for each *ns* value; moreover, we also list the average values *Avg_gap* (in percentage) of the gaps $(R^*_{tot}(\mathbf{y}) - R^F_{tot})/R^F_{tot}$ between $R^*_{tot}(\mathbf{y})$ and R^F_{tot} values. Clearly, differently from what happened between the $R^*_{tot}(\mathbf{y})$ values coming from the bilevel model and the R^L_{tot} values coming from the optimal solutions of problem *Only_Leader* (see Table 5), now the network total risk $R^*_{tot}(\mathbf{y})$ values should be greater than (or at most equal to) the R^F_{tot} values, since the latter ones are the optimal solution values of problem *Only_Follower*. Indeed, we experimentally noted that the $R^*_{tot}(\mathbf{y})$ values are greater than the R^F_{tot} values, and we can infer from Table 7 that the average percentage gap between these values, ranges from 8.0% to 33.1% if the bilevel model is heuristically solved, while much higher gaps are obtained when the bilevel model is optimally solved.

Finally in Table 8, referring to the maximum link total risk values, we summarize the results for the comparison between the bilevel model and the under-regulated scenario, listing the average values Avg_λ^* (Avg_λ_H) and Avg_λ_F of λ^* (λ_H) and λ_F, respectively, over the 10 instances for each *ns* value; moreover, we list the average values *Avg_gap* (in percentage) of the gaps $(\lambda^* - \lambda_F)/\lambda_F$ between the optimal solutions values λ^* of the bilevel model and the λ_F values, and the average values of the gap $(\lambda_H - \lambda_F)/\lambda_F$ between the solution values λ_H of the bilevel model obtained with the heuristic algorithm and the λ_F values. As it can be inferred by the values listed in the table, both the Avg_λ^* values (obtained when the bilevel model is optimally solved) and the Avg_λ_H values (obtained when the bilevel model is heuristically solved) are considerable lower than the Avg_λ_F values obtained for the under-regulated scenario, with the average gap ranging from -50.5% to -26.1%.

ns	*Avg_* λ^*	*Avg_* λ_F	*Avg_gap* (%)
2	9,059.20	18,118.40	-50.0%
3	13,204.12	26,661.70	-50.5%
ns	*Avg_* λ_H	*Avg_* λ_F	*Avg_gap* (%)
2	9,059.20	18,118.40	-50.0%
3	13,204.12	26,661.70	-50.5%
5	16,286.86	30,486.60	-45.3%
10	26,437.19	43,435.90	-37.5%
20	49,335.00	68,821.70	-26.1%
30	79,419.03	123,143.30	-33.5%

Table 8: Comparison between the bilevel model and the under-regulated scenario average maximum link total risk values.

7. Conclusions and final remarks

In this work, we have proposed a bilevel network flow model for hazmat transportation network design. The proposed model aims to minimize total risk and to guarantee risk equity. The model represents a scenario where there are multiple layers of government authorities being involved in the regulation of dangerous goods shipments (as, for example, is common in Europe and North America), that are responsible at different geographical levels, e.g., regional area authorities and local area authorities. In such a scenario, a regional area authority aims to minimize the total risk over its controlled area, while a local area authority wants the risk over the local populated links of its jurisdiction to be the lowest possible.

We formulate the considered hazmat network design problem with a linear bilevel model, where at the higher (leader) level there is a meta-local authority (acting on behalf of all the involved local area authorities) that aims to minimize the maximum link risk over populated links of the whole network, that is, risk equity, and at the lower (follower) level there is the regional area authority that aims to minimize the total risk over the network. This corresponds to the existence of two decision makers, one (the regional authority) willing to define a feasible hazmat flow assignment on the network that induces the minimum total risk over the population, and the other (representing the local authorities) that, interpreting the optimal flow assignment of the previous (lower level) decision maker as a flow vector, minimizes the maximum link risk on the network, i.e., aiming at risk equity, by defining capacities over the network links that restrict the possible choices of the regional authority.

The proposed linear bilevel model is transformed into a single-level mixed integer linear problem by replacing the follower problem by its KKT conditions and by linearizing the complementary slackness constraints. Then, the latter mixed integer linear problem is solved using CPLEX 8.0.1. The MIP formulation is difficult to be solved exactly, and, more importantly, the optimal solution obtained may be a non-stable solution of the bilevel problem; therefore, we also provide a heuristic algorithm for the bilevel model able to always find a stable solution. The proposed model and heuristic algorithm are experimentally evaluated on an Italian geographical region.

The experimentation shows that CPLEX 8.0.1 were able to solve the MIP formulation within a reasonable time only for instances with at most three shipments on the given network with 311 nodes and 441 links, and that the heuristic algorithm was able to give always an optimal stable solution in all the test cases, that is, with the best (minimum) possible maximum link total risk. Moreover, if we compare the solutions given by the MIP solver and that ones provided by the heuristic algorithm, we can note that the latter ones are able to give a better solution in terms of network total risk values. This is due to the fact that the heuristic constructs a feasible solution for the bilevel model taking into account the objective of the follower, who aims to minimize the network total risk, while the bilevel model only considers the reaction of the follower for a given choice of the leader. This would suggest a possible future work devoted to study a multi-objective version of the proposed bilevel model, that accounts for maximum link risk/network total risk trade-off by including the network total risk in the higher level objective. Several other refinements may be introduced to the basic model presented in this paper. For example, one may change the objective of the lower level problem considering the total cost of the shipments; another possible modification could be considering also the population density of the people living in the neighbour of each link, and in the higher level formulation minimize the maximum exposure of the population of each link.

Even if the proposed model does not consider explicitly the carriers' point of view, it should be pointed out that the solution $\hat{\mathbf{y}}$ of the bilevel model is not entirely prescriptive. Indeed, by imposing the values of $\hat{\lambda}$ and $R^*_{tot}(\hat{\mathbf{y}})$ as upper limitations, the carriers can choose the solution that optimizes their objective function (e.g., the total cost minimization). Such a solution may be found by solving a minimum-cost multi-commodity network flow problem, where, in addition to the mass balance constraints and the bundle capacity constraints (e.g., see constraints (3) and (4) of problem *P*2 of Section 3) with bundle capacity vector of value $\hat{\mathbf{y}}$ established by the solution of the bilevel model and representing the leader choice, there are the constraint

$$\sum_{c \in C} \sum_{(i,j) \in A} \rho^c_{ij} x^c_{ij} \leq R^*_{tot}(\hat{\mathbf{y}}),$$

and the constraints

$$\sum_{c \in C} (\rho^c_{ij} x^c_{ij} + \rho^c_{ji} x^c_{ji}) \leq \hat{\lambda}, \text{ for each link } <i,j> \in E$$

of the given network $G = (N, E)$, where $R^*_{tot}(\hat{\mathbf{y}})$ is the minimum network total risk achievable by the follower with the given leader choice $\hat{\mathbf{y}}$, and $\hat{\lambda}$ is the solution value of the bilevel model (i.e., the maximum link total risk value) related to solution $\hat{\mathbf{y}}$. Therefore, the maximum link total risk and the network total risk associated with the solution that the carriers would adopt are not grater than those offered by the bilevel model solution.

A final remarks is devoted to the way the authorities can implement the flow levels and the desired capacities. Indeed, our model is in charge to provide the bundle arc capacities associated with a given set of transportation demands in a certain time horizon, in order to design the network, and to establish the hazmat flow level, that is, the amount of hazmat shipped along each network arc in the given time horizon, for each shipment; the operational phase, i.e., the phase of assigning a schedule to the carriers' routes, is successive to the tactical one considered in our model, and has to be implemented also by monitoring the fleet of vehicles scheduled over time, that is, by monitoring

that, within the given time horizon, the total amount of hazmat of each shipment, that is shipped along each network arc, does not exceed the limit established by the flow assignments on that arc decided in the tactical phase.

Acknowledgments

This work has been partially supported by grant PRIN-2005 N. 2005018500 from the Italian Ministry of University and Research. The authors are grateful to the anonymous referees for their valuable comments and suggestions which contributed to improve the quality of the paper.

References

Abkovitz, M., Eiger, M., Srinivasan, S., 1984. Estimating the Release Rates and Costs of Transporting Hazardous Waste. Transportation Research Record (977), 22–30.

Akgün, V., Erkut, E., Batta, R., 2000. On finding dissimilar paths. European Journal of Operations Research (121), 232–246.

Audet, C., Savard, G., Zghal, W., 2007. New Branch-and-Cut Algorithm for Bilevel Linear Programming. Journal of Optimization Theory and Applications (134), 353–370.

Bard, J.F., Moore, J., 1990. A branch-and-bound algorithm for the bilevel programming problem. SIAM Journal of Scientific and Statistical Computing (11), 281–292.

Bell, M.G.H., 2006. Mixed Route Strategies for the Risk-Averse Shipment of Hazardous Materials. Networks and Spatial Economics (6), 253–265.

Bialas, W., Karwan, M., 1984. Two-level linear programming. Management Science (30), 1004–1020.

Bialas, W., Karwan, M., Shaw, J., 1980. A parametric complementarity pivot approach for two-level linear programming. Operations Research Program Report 80-2, State University of New York at Buffalo.

Bouza, G., Still, G., 2007. Mathematical Programs with Complementarity Constraints: Convergence Properties of a Smoothing Method. Mathematics of Operations Research (32), 467–483.

Candler, W., Townsley, R., 1982. A linear two-level programming problem. Computers and Operations Research (9), 59–76.

Carotenuto, P., Giordaṇi, S., Ricciardelli, S., 2007. Finding Minimum and Equitable Risk Routes for HazMat Shipments. Computers and Operations Research (34), 1304–1327.

Colson, B., Marcotte, P., Savard, G., 2005. Bilevel programming: A survey, 4OR (3) 87–107.

Current J., Ratick S., 1995. A model to assess risk, equity and efficiency in facility location and transportation of hazardous materials. Location Science (3), 187–201.

Dell'Olmo, P., Gentili, M., Scozzari, A., 2005. On finding dissimilar Pareto-optimal paths. European Journal of Operations Research (162), 70–82.

Dussault, J.-P., Marcotte, P., Roch, S., Savard, G., 2004. A smoothing heuristic for a class of bilinear bilevel programs. Cahier du GERAD, GERAD.

Erkut, E., Alp, O., 2007. Designing a road network for hazardous materials shipments. Computers and Operations Research (34), 1389–1405.

Erkut, E., Gzara, F., 2008. Solving the HazMat Transport Network Design Problem, Computers and Operations Research (35), 2234–2247.

Erkut, E., Tjandra, S.A., Verter, V., 2007. Hazardous Materials Transportation. In: Barnhart, C., Laporte, G. (Eds.), Operations Research Handbook on Transportation, Elsevier, pp. 539–621.

Falk, J.E., Liu, J., 1995. On bilevel programming, Part I : general nonlinear cases. Mathematical Programming (70), 47–72.

Fortuny-Amat, J., McCarl, B., 1981. A Representation and Economic Interpretation of a Two-Level Programming Problem. Journal of the Operational Research Society (32), 783–792.

Fukushima, M., Pang, J.S., 1999. Complementarity constraint qualifications and simplified b-stationarity conditions for mathematical programs with equilibrium constraints. Computational Optimization and Applications (13), 111–136.

Gopalan, R., Batta R., Karwan M.H., 1990a. The equity constrained shortest path problem. Computers and Operations Research (17), 297–307.

Gopalan, R., Kolluri, K.S., Batta, R., Karwan, M.H., 1990b. Modeling Equity of Risk in the Transportation of Hazardous Materials. Operations Research (38), 961–973.

Hansen, P., Jaumard, B., Savard, G., 1992. New branch-and-bound rules for linear bilevel programming. SIAM Journal of Scientific and Statistical Computing (13), 1194–1217.

Infodatamix 2002. Il traffico di merci pericolose. Rep. of Ministero delle Infrastrutture e dei Trasporti—Servizio Sistemi Informativi e Statistica, CNIT 2001, Cap. X, Roma, Italy, 2002 (in Italian).

Júdice, J., Faustino, A., 1992. A sequential LCP method for bilevel linear programming. Annals of Operations Research (34), 89–106.

Kara, B.Y., Verter, V., 2004. Designing a Road Network for Hazardous Materials Transportation. Transportation Science (38), 188–196.

Keeney, R.L., 1980. Equity and Public Risk. Operations Research (28), 527–534.

Kocvara, M., Outrata, J.V., 2004. Optimization problems with equilibrium constraints and their numerical solution. Mathematical Programming (101), 119–149.

Lin, G.H., Fukushima, M., 2003. New Relaxation Method for Mathematical Programs with Complementarity Constraints. Journal of Optimization and Its Applications (118), 81–116.

Lin, G.H., Fukushima, M., 2005. A Modified Relaxation Scheme for Mathematical Programs with Complementarity Constraints. Annals of Operations Research (133), 63–84.

Lindner-Dutton, L., Batta, R., Karwan, M., 1991. Equitable sequencing of a given set of hazardous materials shipments. Transportation Science (25), 124–137.

Luo, Z.Q., Pang, J.S., Ralph, D., 1996. Mathematical Programs with Equilibrium Constraints. Cambridge University Press, Cambridge.

Magnanti, T.L., Wong, R.T., 1984. Network design and transportation planning: models and algorithms. Transportation Science (18), 1–55.

Marcotte, P., Shiquan, W., Chen, Y., 1993. A cutting-plane algorithm for the linear bilevel programming problem. CRT Report 925.

Marcotte, P., Savard, G., 2005. Bilevel programming: A combinatorial perspective. In: D. Avis et al. (Eds.). Graph Theory and Combinatorial Optimization. Kluwer Academic Publishers, Boston.

Marianov, V., ReVelle, C., 1998. Linear, non-approximated models for optimal routing in hazardous environments. Journal of the Operational Research Society (49), 157–164.

Outrata, J.V., Kocvara, M., Zowe, J., 1998. Nonsmooth Approach to Optimization Problems with Equilibrium Constraints. Kluwer Academic, Dordrecht.

Patel, M.H., Horowitz, A.J., 1994. Optimal Routing of Hazardous Materials Considering Risk of Spill. Transportation Research A (28), 119–132.

Scholtes, S., Stohr, M., 1999. Exact penalization of mathematical programs with equilibrium constraints. SIAM Journal on Control and Optimization (37), 617–652.

Tuy, H., 1964. Concave programming under linear constraints. Soviet Mathematics 5, 1437–1440.

Tuy, H., Migdalas, A., Värbrand, P., 1993. A global optimization approach for the linear two-level program. Journal of Global Optimization (3), 1–23.

van Stackelberg, H., 1952. The Theory of Market Economy. Oxford University Press, Oxford.

Verter, V., Kara, B.Y., 2008. A Path-Based Approach for Hazmat Transport Network Design. Management Science (54), 29–40.

Vicente, L.N., Calamai, P.H., 1994. Bilevel and multilevel programming: a bibliography review. Journal of Global Optimization (5) 291–306.

Vicente, L.N., Savard, G., Júdice, J.J., 1996. Discrete linear bilevel programming problem. Journal of Optimization Theory and Applications (89), 597–614.

Wen, U., Hsu, S., 1991. Linear Bi-level Programming Problems–A Review. Journal of the Operational Research Society (42), 125–133.

White, D., Anandalingam, G., 1993. A penalty function approach for solving bilevel linear programs. Journal of Global Optimization (3), 397–419.

Yang, H., Bell, M.G.H., 1988. Models and algorithms for road network design: a review and some new developments. Transportation Review (18), 257–278.

Zografos, K.G., Androutsopoulos, K.N., 2004. A heuristic algorithm for solving hazardous materials distribution problems. European Journal of Operations Research (152), 507–519.