A Block Cipher Based Pseudo Random Number Generator Secure against Side-Channel Key Recovery

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    - Issue : partial information on the SECRET is leaked by physical media
    - By recovering many pieces of partial info, one can recover the whole secret key

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  - Case Study : Pseudo-Random Number Generator (PRNG)

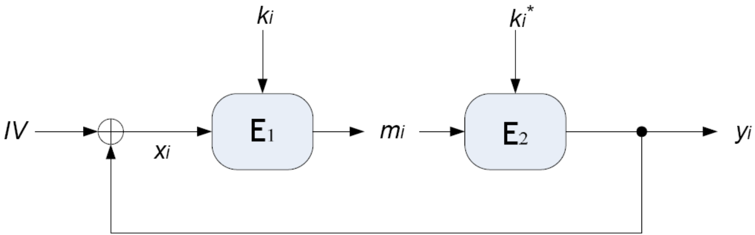
Case Study: PRNG

- Black-Box security (BB) : PRNG
- Grey-Box security (GB): prevent traditional SC cryptanalysis

Talk Overview

- Introduction
- PRNG
  - Construction
  - BB model & security
  - GB model & security
  - PRNG summary
- Conclusion and further work

Construction



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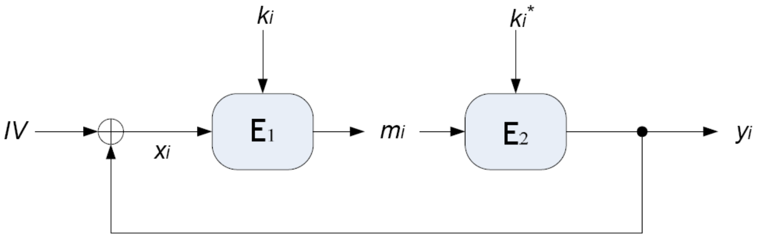

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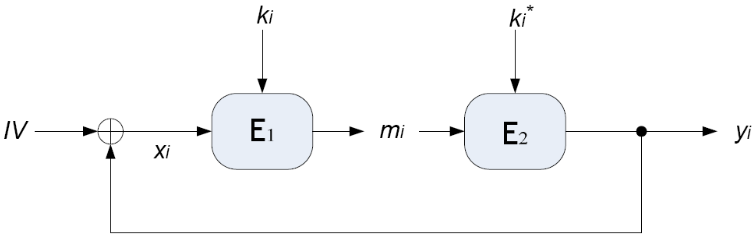

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- First idea (in BB): if E_1 and E_2 are "good", then the y_i's should be PRNs.
- But (in GB) successive leakages allow recovering the whole secret.

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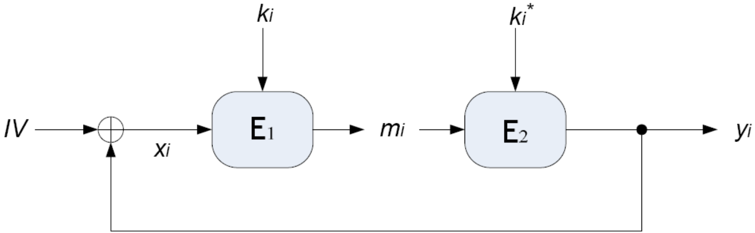

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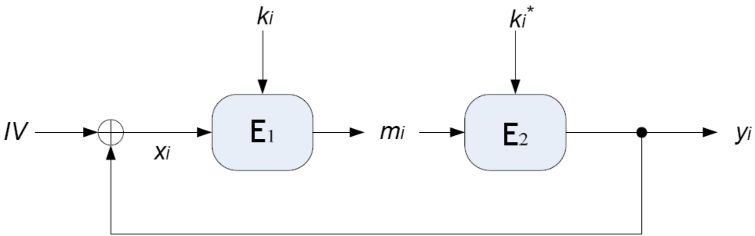

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- Each running key k_i, k^*_i is used to encrypt *only* one message.

Black-Box Model

- Ideal cipher model $\mathsf{E} : \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{M}$
  - (Here $\mathcal{K} = \mathcal{M}$)
  - for each key $k \in \mathcal{K}$, the function $\mathsf{E}_k(\cdot) = \mathsf{E}(k, \cdot)$ is a random permutation on $\mathcal{M}$

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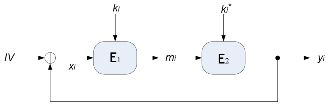
  - G is a PRNG if for any A, $\mathbf{Adv}^{\mathrm{prng}}_{\mathrm{G,A}} \approx 0$.

Black-Box Analysis



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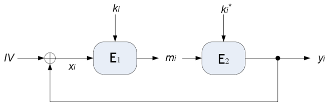

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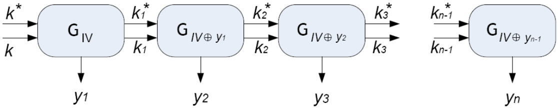

- For each $X \in \mathcal{M} = \mathcal{K}$, let $\mathsf{G}_X : \mathcal{K} \times \mathcal{K} \rightarrow \mathcal{K} \times \mathcal{K} \times \mathcal{K}$

$$\mathsf{G}_X(K, K^*) = (\mathsf{E}_K(X) \oplus K, \mathsf{E}_K(X) \oplus K^*, \mathsf{E}_{K^*}(\mathsf{E}_K(X))).$$

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  - If there is A such that $\mathbf{Adv}^{\text{prng}}_{\mathsf{G}^q,\mathsf{A}} > \epsilon$, then there is A′ such that $\mathbf{Adv}^{\text{prng}}_{\mathsf{G},\mathsf{A}'} > \frac{\epsilon}{q}$ for one of the rounds

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  $\mathsf{P}^q(K, K^*) = (\mathsf{G}^q(K, K^*), \mathsf{L}^q(K, K^*))$
- We show the available information does not permit recovering the secret

Grey-Box Model

- Side-channel key recovery adversary

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- If $\delta(K, K^*) = K_{[0\cdots7]}$

$$\mathbf{Succ}^{\mathrm{sc-kr-}K}_{\mathsf{P}^q(K,K^*),\mathsf{A}} = (\mathbf{Succ}^{\mathrm{sc-kr-}K_{[0\cdots7]}}_{\mathsf{P}^q(K,K^*),\mathsf{A}})^{n/8}$$

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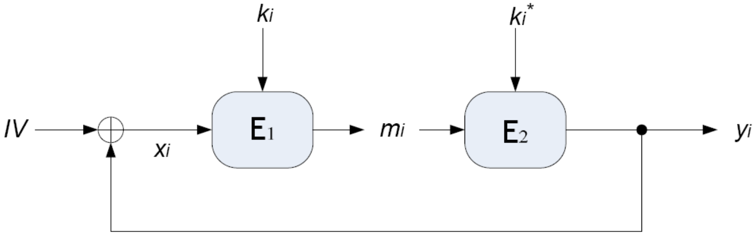

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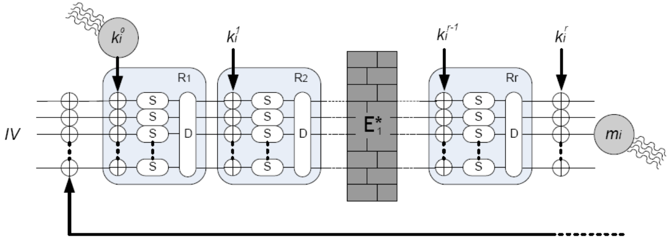

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  - Iterative BC, no key schedule
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  - Form of leakage functions : HW, GHW, NI

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- With observed leakages $\mathbf{l^q} = \{L(k_i), L(m_i)\}$ and relations $k_{i+1} = k_i \oplus m_i$, the best guess is

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- Goal : show that SR remains small as q increases

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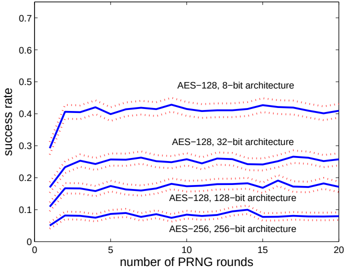
- In this case we compute : $\mathbf{Succ}^{\mathrm{sc-kr-}K_0}_{\mathrm{P}^q(K,K^*),\mathrm{A}} = \frac{n+1}{2^n}$
- High security, independently of q

Noisy Identity Leakages

- Here the above formulae are hard to evaluate analytically
  $\longrightarrow$ Monte-Carlo simulations



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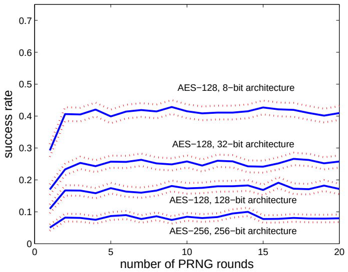

- $\mathbf{Succ}^{\text{sc-kr-K}}_{\text{AES256,A}} \simeq (0.08)^{32} = 2^{-116}$

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  With other countermeasures, leakages on more rounds means better attack

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- Need of theoretical framework for SC
  - unify BB and GB...
  - define physical primitives
  - compose primitives

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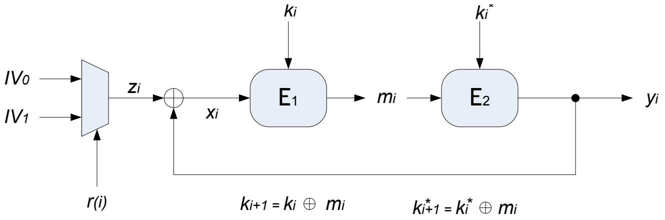

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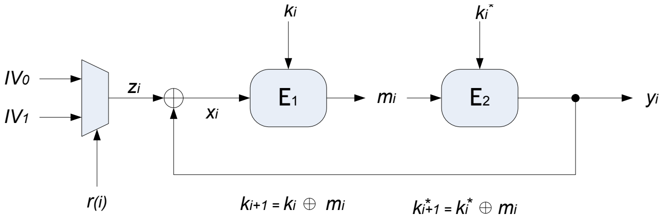

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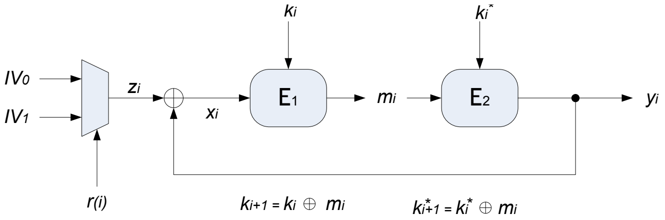

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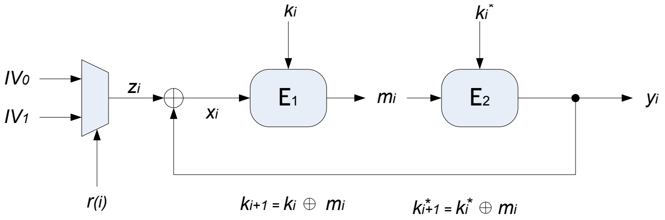

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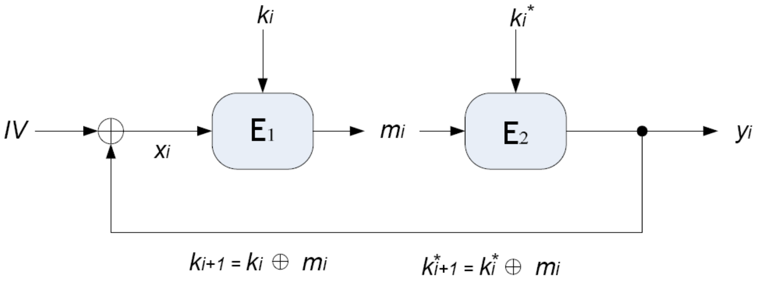

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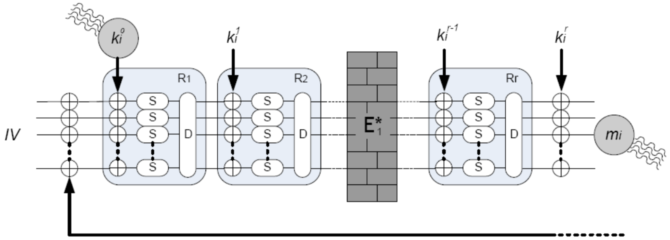

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  - Form of leakage functions : HW, GHW, NI
  - We suppose Bayesian adversary

Discussion about Grey-Box assumptions

- Many assumptions
  - make the proofs cleaner...
  - ...but are not essential.
- Relaxations $\longrightarrow$ same qualitative conclusions
  - key schedule $\longrightarrow$ adapt the leakage model $L(k_i)$
  - targeting not only the first iteration of the PRNG
    $\longrightarrow$ may increase SR, but qualitative results remains