A Brief Introduction to the Analysis and Design of Networked Control Systems

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Abstract: Networked Control has emerged in recent years as a new and exciting area in systems science. The topic has many potential applications in diverse areas ranging from control of microrobots to biological and economic systems. The supporting theory is very rich and combines aspects of control, signal processing, telecommunications and information theory. In this paper, we will give a brief overview of recent developments in Networked Control with an emphasis on our contributions. We also point to several open problems in this emerging area.

Key Words: Networked Control, Performance, Data-Rate Constraints, Delays, Data Dropouts

1 Introduction

Traditionally, control theory has dealt with situations where the communication links between plant and controller can be regarded as transparent (see, e.g., [1–3]). There exist, however, cases where the links in a control system are far from being transparent and may become bottlenecks in the achievable performance. Control systems where this happens are collectively referred to as Networked Control Systems (NCS's).

The study of NCS's has emerged as an active research field during the past years (see, e.g., the special issues [4, 5]). Key questions within this framework are related to the way in which network artifacts affect the stability and performance of control loops that employ non-transparent communication channels. Typical channel artifacts include data-rate limits, random delays and data dropouts. A unifying framework for the treatment of general NCS analysis and design problems does not yet exist. Nevertheless, there has been significant progress in the study of several subproblems. For example, data-rate constraints have been studied in, e.g., [6–9]. The issue of data dropouts has been studied in, e.g., [10–12] and random time delays have been considered in, e.g., [13, 14].

A pivotal issue in any closed loop system is that of stability. When focusing on quantization issues, a key result establishes necessary and sufficient conditions on the channel data-rate that allows one to achieve closed loop stability (in an appropriate sense; see, e.g., [7] and the many references therein). These results are given in terms of a lower bound on the channel data-rate (that depends on the unstable plant poles only) over which control and coding schemes can be constructed so as to achieve stability. These coding schemes are quite involved and may not be attractive from a practical point of view. This fact has motivated us to study simple coding schemes that achieve rates *close* to the bounds identified above (see [15]). Recent research has also established an important link between the average data-rate necessary to achieve stability and signal-to-noise ratio requirements (see [15]). This observation reinforces the use of signal-to-noise ratio models in the context of NCS performance analysis (see, e.g., [16]).

For SISO LTI plant models, one can design coding schemes that, for a given initial non-networked controller design, allow one to minimize the impact of quantization effects on overall closed loop performance. These results emphasize the essential role that the available *degrees of freedom* play in NCS's (see [16–18] and also [19–23]).

The issue of performance is particularly interesting in the context of networked control architectures for MIMO plant models. Practical control systems for MIMO plant models often use structurally constrained controllers such as diagonal or triangular ones (see, e.g., [1, 24]). The reasons for this choice are manifold and include ease of design, simplified tuning, and implementation related issues such as cabling or geographic plant distribution. It is well known that restricting the controller architecture generally limits the achievable performance (see, e.g., [25–27]). Within this context, networks can play significative roles. Indeed, it is easy to envisage decentralized control architectures that, when enriched with additional (non-transparent) communication links, may provide enhanced performance. This may (partially) overcome the limitations that arise as a consequence of the controller structure constraint (see, e.g., [28–32]). To illustrate these ideas, in the present paper we will examine the control of MIMO LTI systems for which a decentralized controller has been successfully designed. Unsurprisingly, for high-quality channels it turns out that a networked MIMO architecture outperforms decentralized ones. However, an interesting finding is that, in some situations, the networked architecture will perform better than the decentralized one only if the channels are *extremely* reliable (see [33]).

We finally address the problem of dealing with arbitrary data dropouts and delays. We show how control laws de-

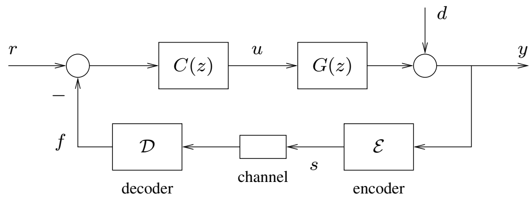

Figure 1: Basic Networked control system (including coding scheme).

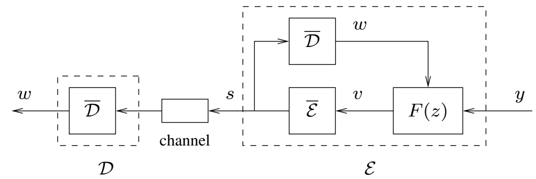

Figure 2: Proposed coding scheme for stabilization.

signed for non-networked control systems can be embellished so as to achieve good performance when used in an NCS that employs unreliable communication channels (see [34] and related work [35–40]).

The remainder of this paper is organized as follows: Section 2 discusses the problem of stabilization of NCS's subject to data rate constraints. Section 3 discusses the design of coding systems that optimize NCS performance. Section 4 studies some aspects in MIMO networked control. Section 5 studies control over unreliable networks. Section 6 draws conclusions.

2 Stability

An overarching issue in any closed loop system is the potential for instability. For this reason, obtaining necessary and sufficient conditions which guarantee closed loop stability has become a central issue in control theory. NCS's give rise to new challenges with respect to stability. This section will give some insights into the stability question.

Consider the networked control situation shown in Figure 1. In that figure, $G(z)$ is a SISO LTI plant, $C(z)$ is a LTI controller, r is a reference signal[1] and d is a disturbance. Unlike standard non-networked control systems (see, e.g., [1–3]) the feedback path in the control system in Figure 1 comprises a non-transparent channel and a coding scheme. The coding scheme is a novel aspect of networked control that has no equivalent in traditional control theory. The coding scheme has two parts: the encoder and the decoder. The encoder is in charge of appropriately processing the signal to be sent over the channel so as to compensate, if possible, channel characteristics and/or to translate the measurements into symbols that the channel can understand. (This is the case of, e.g., digital channels where the symbols are binary words.) On the other hand, the decoder is in charge of translating back the channel symbols into the standard signal domain.

There exist many channel models (see, e.g., [41, 42]). To highlight the ideas, in the present section we will consider error- and delay-free digital channels, i.e., channels that can transmit a countable set of symbols (countable alphabet) without errors or delays. Of course, in practice, channels always have restricted bandwidth and the channel alphabet is, thus, finite (not just countable; see also [7, 43, 44]).

[1] All signals in this paper are assumed wise sense stationary (wss) stationary processes.

2.1 Key results

For the setting described above, a key result states that it is possible to find coders, decoder and controllers such that the resulting system is mean square stable (MSS) if and only if the *average* data-rate in bits per sample, say $\bar{R}$, satisfies (see [6])

$$\bar{R} > \sum_{i=1}^{n_p} \log_2 |p_i| , \tag{1}$$

where $\{p_i\}_{i=1,\cdots,n_p}$ denotes the set of unstable plant poles. The above result is valid for every coder, decoder and controller in the class of (time varying and non-linear) causal systems. Therefore, (1) establishes a fundamental separation between what is achievable in NCS's over digital channels and what is not (when the problem of interest is MSS). We note that bounds similar to (1) arise as solutions to quite a few different problems (e.g., observability, deterministic stability, etc.) and under different assumptions on the channels and coding schemes (see, e.g., [6,9,41,45–47]). Indeed the quantity on the right hand side of (1) is a fundamental measure of the difficulty of manipulating a system, as explored in [8, 48].

Proving that the rate at which a stable control system is transmitting data must satisfy (1) (i.e., *necessity*) is fairly simple and employs standard tools (see also [49–51]), whereas constructing an actual coding scheme that achieves stability at any rate above the absolute limit (i.e., the proof of *sufficiency*) is much more involved (see [6, 9]). These observations motivate the remainder of this section.

2.2 Some insights into the problem of stabilization with data-rate constraints

In this section we shed some light into the limitations that finite data-rates impose. To fix ideas, we will consider the coding scheme depicted in Figure 2, where $F(z)$ is a LTI filter, and $\overline{\mathcal{E}}$ and $\overline{\mathcal{D}}$ are abstract systems that translate the discrete time signal v into channel symbols s and channel symbols into the discrete time signal w, respectively. Our presentation will start at an heuristic level and will then move towards a formal approach based on information theoretic considerations.

2.2.1 An additive noise model for quantization

Since the channel is digital, it becomes clear that $\overline{\mathcal{E}}$ must quantize v prior to transmission. We will focus on the most simple situation where $\overline{\mathcal{D}}$ is the identity and $\overline{\mathcal{E}}$ is a finite uniform quantizer (see, e.g., [52,53]), i.e.,[2]

$$s(k) = \mathcal{Q}(v(k)) \triangleq \mathrm{sat}_V \left(\Delta \left\lfloor \frac{v(k)}{\Delta} \right\rfloor \right). \tag{2}$$

In (2), $V > 0$ is the quantizer dynamic range, $\Delta \triangleq 2V(L-1)^{-1}$ is the quantization step and L is the number of quantization levels. We say the the quantizer is *overloaded* if and only if $|v(k)| > V$ for some k. (If in (2) sat_V is removed and Δ is fixed at any arbitrary positive value, then $\mathcal{Q}$ becomes an infinite uniform quantizer.)

Quantization is a deterministic non-linear operation and hence, the exact analysis of quantized systems is difficult (see, e.g., [43, 44, 54]). It has thus become standard, particularly in the signal processing literature (see, e.g., [52, 55, 56]), to approximate quantization noise, i.e., the process

$$q \triangleq w - v, \tag{3}$$

by an additive i.i.d. noise source, uniformly distributed on rthe interval $(-\frac{\Delta}{2}, \frac{\Delta}{2})$ and independent of the input to the quantizer v.[3] This model is valid only if Δ is small enough, the quantizer does not overload and v has a smooth probability density function (see, e.g., [57]). These conditions usually do not hold in the case of quantizers that are embedded in feedback loops, as is the case of NCS's (see, e.g., [58]). We also note that assuming that q is either independent of v or uncorrelated with v is, certainly, not a valid assumption in networked situations. In these cases, it makes more sense to assume that q is independent of (or uncorrelated with) the external signals r and d.

In order to avoid quantizer overload, it is usual in practice to choose a quantizer dynamic range such that the probability of overload is negligible (see, e.g, [52]). Indeed, if v is wss and β is any positive real value, then one can always find a finite α such that[4] $V = \alpha\sigma_v$ guarantees that the probability of overload is less than β; α is called the quantizer *loading factor*.[5] With such a choice for the loading factor, it is immediate to see that, provided q is as in the classical additive model described above,

$$\gamma \triangleq \frac{\sigma_v^2}{\sigma_q^2} = \frac{3}{\alpha^2}(L-1)^2, \tag{4}$$

where γ is the quantizer *signal-to-noise ratio*. Thus, bit-rate limits translate into signal-to-noise ratio constraints.

It is important to note that the previous model for quantization can actually be rendered exact by a simple randomization procedure. Indeed, it suffices to consider a *dithered* uniform quantizer. In this case

$$s(k) = \mathrm{sat}_V \left(\Delta \left\lfloor \frac{v(k) + d_h(k)}{\Delta} \right\rfloor \right),$$
$$w(k) = s(k) - d_h(k),$$

[2] $\mathrm{sat}_V(x) \triangleq x$ if $|x| \leq V$ and $\mathrm{sat}_V(x) \triangleq \frac{x}{|x|}V$ if $|x| > V$; $\lfloor x \rfloor$ denotes the integer part of x.

[3] Sometimes q is assumed only to be uncorrelated with v.

[4] σ_v denotes the standard deviation of v.

[5] For example, if $v(k)$ is Gaussian, then $\alpha = 4$ gives an overload probability of $6.33 \cdot 10^{-5}$.

where d_h is an i.i.d. random process, uniformly distributed on $(-\frac{\Delta}{2}, \frac{\Delta}{2})$ and such that $d_h(k)$ is independent of $v(k)$. In this situation, and provided no overload occurs, q in (3) becomes distributed as $-d_h$ (see, e.g., [53,59,60]). In other words, q becomes just as in the simplified model described above. Again, in order to avoid quantizer overload, one needs to consider an appropriate quantizer loading factor and a signal-to-noise ratio constraint arises.

In practice, implementation of dithered quantizers is not trivial since it requires the availability of the dither signal d_h at both the sending and receiving ends. Usually the dither is generated using pseudo-random number generators that are initialized with the same seeds. Nevertheless, even if one employs a non-dithered uniform quantizer with as small as 4 levels, it turns out that the predictions made using the simple additive quantization model described above are surprisingly accurate (see simulation studies in [16, 17, 55, 61]).

2.2.2 Limits imposed by signal-to-noise ratio constraints

Consider the setting in Figure 2 and assume that $\overline{\mathcal{E}}$ and $\overline{\mathcal{D}}$ are such that q obeys the additive model for quantization described in Section 2.2.1. We note that, since q depends on the way in which $\overline{\mathcal{E}}$ and $\overline{\mathcal{D}}$ are designed, the variance of q, say σ_q^2, becomes a decision variable.[6] Within this setting a basic question arises, namely finding the conditions on the signal-to-noise ratio γ that guarantee MSS.

It is possible to show via standard control theoretic arguments (see [15]) that, for any strictly proper plant model $G(z)$ and any wss r and d, there exists $C(z), F(z)$ and a finite σ_q^2 such that the resulting NCS is MSS if and only if

$$\gamma > \gamma_{\inf} \triangleq \left(\prod_{i=1}^{n_p} |p_i|^2 \right) - 1, \tag{5}$$

where $\{p_1, \cdots, p_{n_p}\}$ is the set of unstable plant poles. From this result we conclude that, as was the case for (1), the degree of instability of the plant plays a key role in the interplay between stability and communications constraints (as expressed by signal-to-noise ratio constraints in our current framework). It is interesting to note that if one adds one and takes logarithms in (5), then one recovers the right hand side in (1). (In [15], we have shown that this connection is not a mere coincidence.)

It is easy to see from (5) and (4) that, if the additive model for quantization holds, then MSS is equivalent to

$$b > b_{\inf} \triangleq \log_2 \left(1 + \sqrt{\frac{\alpha^2}{3} \left(\left(\prod_{i=1}^{n_p} |p_i|^2 \right) - 1 \right)} \right), \tag{6}$$

where $b \triangleq \log_2 L$ is the number of bits in the quantizer. In other words, provided no overload occurs and the noise model for quantization holds, (6) gives a bound on the *instantaneous* data-rate over the channel that guarantees

[6] For the additive model described previously $\sigma_q^2 = \frac{\Delta^2}{12}$.

MSS.[7] Clearly, this bound on b is optimistic. Indeed, it is only exact if one employs a dithered uniform quantizer and there is *no overload*. But, if r, d or the initial plant state have unbounded support, then it is impossible to guarantee that the input to the quantizer is deterministically bounded. Even if r, d and the plant initial state have bounded support, and thus v is bounded, the previous analysis usually requires a very large value for the loading factor α. Equation (6) then provides only a conservative bound on $b_{\inf}$.
In practice it is often sufficient to assume that v is such that a sensibly small value for α gives negligible overload probability (a typical value is $\alpha = 4$; see [52] and also Sections 3 and 4).

2.2.3 Average data-rates

In the previous section we dealt with instantaneous data-rates. There are at least two reasons that can be advanced for abandoning that setting. First, if the external signals have unbounded support, then it is impossible to give any guarantee using the above reasoning, unless one employs quantizers with infinitely many levels so as to avoid overload (thus incurring infinite instantaneous data-rates). Second, if the external signals are bounded, then the quantizer loading factor α may need to be quite large in order to accommodate v without overload. This implies that, even in those cases, one needs to use a quantizer with either a large number of bits (which increases the instantaneous rate), or a large quantization step (which implies poor performance). We thus conclude that, at least from a theoretical point of view, it is interesting to study average data-rates.[8] (Of course, guaranteeing that average data-rates are bounded does not ensure that instantaneous rates will be bounded.)
A popular way to deal with average data rate constraints is to use an entropy coded dithered quantizer (ECDQ; see, e.g., [59]). Figure 3 shows the architecture of an ECDQ and its relationship to $\overline{\mathcal{E}}$ and $\overline{\mathcal{D}}$ in Figure 2. An ECDQ uses a dithered infinite uniform quantizer, as described earlier, but instead of sending the quantizer output directly over the channel, it entropy-codes it prior to transmission. The basic idea behind the entropy-coder EC in Figure 3 is to exploit the fact that some of the quantizer output values are more likely than others. Thus, one can assign short channel symbols (i.e., few bits) to very frequent quantizer output values and long symbols (i.e., many bits) to infrequent ones (see [42]). Correspondingly, the task of the entropy-decoder ED is to convert the channel symbols back into the exact quantizer output at the receiving end (see [42]).
A key property of ECDQs is that the average data-rate at which data is transmitted can be related to the quantizer signal-to-noise ratio γ. Indeed, it is possible to show that, if the dither is as in Section 2.2 and is also independent of r, d and the filters initial states, then the coding scheme proposed in Figure 2, when $\overline{\mathcal{E}}$ and $\overline{\mathcal{D}}$ form an ECDQ, is

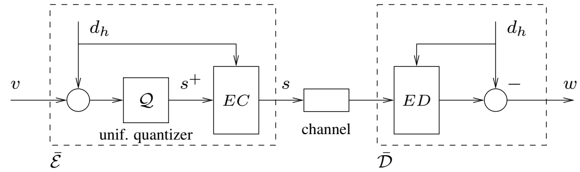

Figure 3: Entropy coded dithered quantizer.

such that the average data-rate is *upper* bounded by (see [15]).

$$\bar{R} \leq \frac{1}{2}\ln(1+\gamma) + \frac{1}{2}\log_2\left(\frac{2\pi e}{12}\right) + 1. \qquad (7)$$

Thus, we see that signal-to-noise ratio considerations give an upper bound on the average data-rate. In other words, there exists a *strong link* between signal-to-noise ratio constraints and average data-rate constraints. This goes beyond the heuristic discussion in Section 2.2.1, and serves as a basis for a simple but rigorous treatment of average data-rate constraints in networked control.
In particular, one can use (7) to show that it is possible to achieve MSS at average data-rates that satisfy (see [15])

$$\bar{R} \leq \sum_{i=1}^{n_p} \log_2 |p_i| + \frac{1}{2}\log_2\left(\frac{2\pi e}{12}\right) + 1 + F(\Delta),$$

where F is a positive function that goes to zero if $\Delta \to \infty$. Therefore, the use of an ECDQ allows one to achieve an average rate that can be made arbitrarily close to the absolute minimum rate in (1) plus $\frac{1}{2}\log_2\left(\frac{2\pi e}{12}\right) + 1$ bits per sample (i.e., 1.25 bits per sample). This additional rate is composed by two terms: the first one is due to the divergence of the distribution of quantization noise from Gaussianity, and the second one originates in the inefficiency of the loss-less coding scheme employed to generate the channel symbols (i.e., the inefficiency of EC).

3 Performance

Next we turn to the question of performance. Motivated by the analysis presented in the previous section, we will utilize an independent additive noise channel model with a signal-to-noise ratio constraint. This hypothesis covers many situations:

- Situations where the actual physical channel is additive and with a signal-to-noise ratio constraint.
- Situations where the channel is digital, has a finite alphabet and one assumes that quantization noise is white, independent, and the quantizer is properly scaled (recall Section 2.2.1).
- Situations where the channel is digital, has a countable alphabet, and one employs an ECDQ (see Section 2.2.3).
- Situations where the channel is analog, but prone to i.i.d. data dropouts (i.e., analog erasure channel; see also [10, 11, 62]). Indeed, it has been recently shown

[7]Clearly, $b_{\inf}$ is in general greater than the right hand side in (1).

[8]Alternatively, one could also examine time varying (i.e., adaptive) quantizers that are scaled on-line according to the statistics of the input signals. This topic is interesting, but is beyond the scope of this paper (see references in [7]).

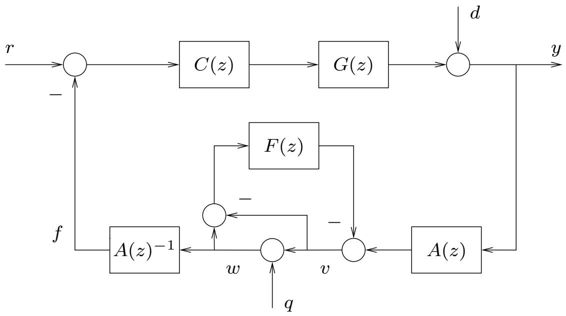

Figure 4: Linear model for the NCS in Section 3.

(see [63]) that, for such data dropout profiles, analog erasure channels are equivalent, up to second order statistics, to an additive white noise channel with a fixed signal-to-noise ratio. (This result is actually inspired by the work [12], where the authors implicitly use a fixed signal-to-noise ratio constraint to design dropout compensators in a specific networked situation.)

3.1 Setup and Analysis

We will focus on the NCS depicted in Figure 4, where the controller $C(z)$ is a *given* admissible controller for $G(z)$.[9] We will show how to design the coding system filters $F(z)$ and $A(z)$ so as optimize overall closed loop performance. (We refer the reader to [64] for controller design procedures.)

It is easy to see from Figure 4 that the variance of the tracking error, i.e.,

$$e \triangleq r - y, \tag{8}$$

is given by

$$\sigma_e^2 = ||T_{dre}(z)\Omega_{dr}(z)||_2^2 + J(A(z), F(z)), \tag{9}$$

where

$$J(A(z), F(z)) \triangleq \frac{||A(z)T_{dry}(z)\Omega_{dr}(z)||_2^2 \left|\left|T(z)A(z)^{-1}(1 - F(z))\right|\right|_2^2}{\gamma - ||T(z) + S(z)F(z)||_2^2},$$

and where $T_{dre}(z)$ and T_{dry} denote the transfer functions from $[d\ r]^T$ to e and y, respectively, $\Omega_{dr}(z)$ denotes a spectral factor of $[d\ r]^T$, γ is the channel signal-to-noise ratio and

$$S(z) = \frac{1}{1 + G(z)C(z)}, \quad T(z) \triangleq 1 - S(z).$$

A straightforward calculation shows that $T_{dre}(z)$ depends on $C(z)$ only, which is fixed in our setting. Thus, to optimize performance we would like to minimize $J(A(z), F(z))$.

[9] i.e., an internally stabilizing controller that defines a well possed control loop (see,e.g., [3]).

It is possible to show that, provided $C(z)$ is an admissible controller for $G(z)$ and the model for quantization holds, the model in Figure 4 is MSS if and only if $F(z)$ is stable, strictly proper, $A(z)$ is stable, minimum phase (MP) and biproper, and the signal to noise ratio satisfies

$$\gamma > ||T(z) + S(z)F(z)||_2^2. \tag{10}$$

It is illustrative to note that, if $\gamma \to ||T(z) + S(z)F(z)||_2^2$, then $J(A(z), F(z)) \to \infty$ (unless all exogenous signals have zero spectral density, in which case $J(A(z)F(z)) \equiv 0$). Also, if $\gamma \to \infty$, then $J(A(z), F(z)) \to 0$ and we recover the "non-networked performance" that is achieved with $C(z)$ and a transparent communication link.

3.2 Coding system design

Based on (9) and noting that $T_{drw}(z)\Omega_{dr}(z)$ is fixed, we next study the problem of finding[10]

$$J_{opt} \triangleq \inf_{\substack{A(z) \in \mathcal{U}_\infty \\ F(z) \in \mathcal{RH}_2 \\ ||T(z)+S(z)F(z)||_2^2 < \gamma}} J(A(z), F(z))$$

and filters $A(z)$ and $F(z)$ that achieve J_{opt} (or approximate J_{opt} arbitrarily well). The exact solution of this problem is not straightforward, so we describe a simple iterative solution.

If $F(z)$ is a given strictly proper and stable filter (as required for MSS), then the Cauchy-Schwarz inequality implies that the optimal filter $A(z)$, say $A_{opt}^{F(z)}(z)$, satisfies (see [16], [18])

$$\left|A_{opt}^{F(z)}(e^{j\omega})\right|^4 = \beta \frac{\left|T(e^{j\omega})(1 - F(e^{j\omega}))\right|^2}{T_{dry}(e^{j\omega})\Omega_d r(e^{j\omega})\Omega_d r(e^{j\omega})^H T_{dry}(e^{j\omega})^H}, \tag{11}$$

where β is any arbitrary positive real. We note that the characterization of $A_{opt}^{F(z)}(z)$ given above is explicit but is in general not satisfied by any stable, biproper and MP rational transfer function (indeed, the $4th$ root of the right hand side in (11) is usually irrational). Nevertheless, since the condition (11) holds on the magnitude of $A(z)$ only, it is always possible to find a filter with the desired characteristics that achieves a performance that is as close as desired to the optimal performance. In practice, it is usually enough to consider reasonably low order filters to approximate $A_{opt}^{F(z)}(z)$ (see also [16]).

Now consider that $A(z)$ is any given stable, MP and biproper transfer function. For any such $A(z)$, it is possible to show that (see [18])

$$F_{opt}^{A(z)}(z) = F_{\epsilon^*}(z), \tag{12}$$

where $F_\epsilon(z)$, $\epsilon \in [0, 1]$, is defined via

$$F_\epsilon(z) \triangleq \arg \inf_{F(z) \in \mathcal{RH}_2} \epsilon J_1(F(z)) + (1 - \epsilon) J_2(F(z)),$$

[10] $\mathcal{RH}_2$ denotes the set of all stable and strictly proper real rational transfer functions, and $\mathcal{U}_\infty$ denotes the set of all stable, MP and biproper transfer functions.

$$J_1(F(z)) \triangleq \left|\left|T(z)A(z)^{-1}(1-F(z))\right|\right|_2^2,$$
$$J_2(F(z)) \triangleq ||T(z)+S(z)F(z)||_2^2,$$

and

$$\epsilon^* \triangleq \arg \min_{\epsilon \in (0,\hat{\epsilon})} \frac{J_1(F_\epsilon(z))}{\gamma - J_2(F_\epsilon(z))}. \qquad (13)$$

In (13), $\hat{\epsilon}$ is defined as follows: If there does not exist an $\epsilon \in (0,1]$ such that $J_2(F_{\epsilon_\gamma}(z)) = \gamma$, then $\hat{\epsilon} = 1$. Otherwise, $\hat{\epsilon} = \epsilon_\gamma$, where ϵ_γ is the unique real in $(0,1]$ such that $J_2(F_{\epsilon_\gamma}(z)) = \gamma$.

We note that the problem of finding $F_\epsilon(z)$ is a standard convex problem that can be tackled using well-known tools (see, e.g., [65, 66]). On the other hand, calculating ϵ^* amounts to solving a (scalar) line search problem. This is also a simple problem to solve.

By utilizing the previous results, it is immediate to envisage an iterative procedure that allows one to construct approximations to the optimal $A(z), F(z)$ pair: In a first step one fixes $A(z)$ (or $F(z)$; trivial choices are $F(z) = 0$ and $A(z) = 1$). Then, one uses (12) to calculate the optimal $F(z)$ for the initial choice of $A(z)$ (or (11) to calculate the optimal $A(z)$ for the initial choice of $F(z)$). This is repeated fixing $A(z)$ or $F(z)$ intermittently. This algorithm is guaranteed to converge, at least, to a local minimum.

In general, $A_{opt}^{F(z)}(z) \neq 1$ and $F_{opt}^{A(z)}(z) \neq 0$. Thus, fixing $A(z)$ or $F(z)$ and optimally choosing the other filter, will obviously provide a coding system that enhances closed loop performance when compared with a non-coded networked situation.[11] From the above, it is also clear that the use of the proposed iterative procedure allows one to design coding systems that will always outperform simple coding systems that do not consider feedback in the quantizer (i.e., where $F(z) = 0$; see [16]). This again highlights the role of architectures (equivalently, available *degrees of freedom*) in NCS's. Further discussions regarding the interplay between coding system architecture and networked performance can be found in [16].

3.3 Example

Consider a nominal loop with plant and controller given by

$$G(z) = \frac{1}{z-0.8}, \quad C(z) = \frac{z-0.8}{z-1}.$$

The disturbance d is assumed zero, whilst the reference is considered to have a power spectral density with spectral factor

$$\Omega_r(z) = \frac{0.02z}{z-0.9}.$$

We consider a finite uniform quantizer with overload factor $\alpha = 4$ and b bits. We send the output of the quantizer directly through the channel (i.e., without entropy coding). Thus, b corresponds to the instantaneous channel data-rate in bits per sample.

Figure 5 shows the steady state tracking error variance σ_e^2 (see (9)) as a function of the number of iterations in the proposed design algorithm for $\gamma = 9.1875$, which corresponds to $b = 3$. Cases 1 and 2 refer to iterations that start with $A(z) = 1$ and $F(z) = 0$. In Case 1 we initially fixed $A(z)$, whereas in Case 2 we start fixing $F(z)$. Case 3 refers to iterations that start with the choices suggested in [17] (where it is assumed that γ is "high enough").

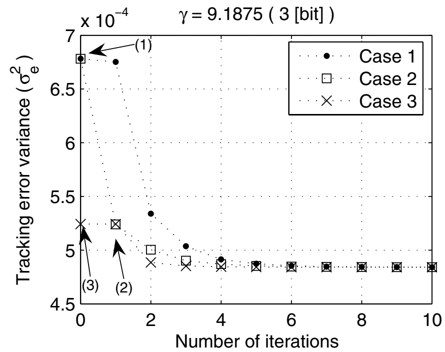

Figure 5: Tracking error as function of the number of iterations in proposed design procedure (see Section 3.3 for details).

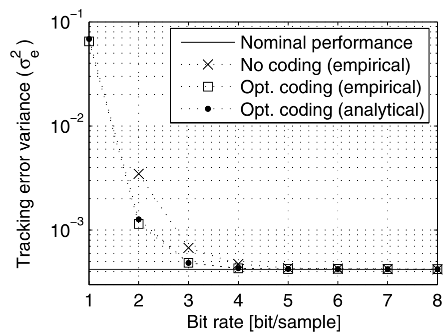

Figure 6: Tracking error as function of the channel bit rate (see Section 3.3).

In Figure 5 we have identified three points. The first of these (point (1)) refers to the performance achieved without coding ($F(z) = 0$ and $A(z) = 1$). The second (point (2)) refers to the performance achieved when employing the optimal coding system proposed in [16]. The third (point (3)) refers to the performance achieved using the approximately optimal filters described in [17].

The results show that coding is, indeed, useful to achieve good closed loop performance. (Compare point (1) with, e.g., the value of σ_e^2 after 10 iterations.) It is also possible to see that use of the proposed procedure yields coding systems that perform better than the simpler proposals in [16,17]; see also the discussion at the end of Section 3.2. (Compare points (2) and (3) with the limiting value for σ_e^2.)

Finally, we examine the behavior of the tracking error variance as a function of the channel bit rate b. The results are presented in Figure 6, where "Nominal performance" refers to the performance achieved by the nominal loop (without quantization), "No coding (empirical)" refers to

[11] Provided both situations use the same channel and the same quantizer.

simulated results[12] when no coding is employed (i.e., when $A(z) = 1$ and $F(z) = 0$), "Opt. coding (empirical)" refers to simulated results obtained with the filters suggested in Section 3.2 (after 10 iterations), and "Opt. coding (analytical)" refers to the corresponding predictions made using the simplified noise model for quantization. One can see that, as expected, the effects of quantization vanish as $b \to \infty$. Interestingly, the predictions made by the additive noise model turn out to be very accurate for every bit rate: indeed, for $b \geq 3$ the relative errors are of less than 1% and, for $b \in \{1, 2\}$, the relative errors are around 8%. (We note that $F(z) = 0$ turns out to be non-admissible for $b = 1$, i.e., (10) is not satisfied. Accordingly, we have omitted the non coded results for $b = 1$.)

4 MIMO systems

Next we turn to MIMO systems. Our development here is largely based on [33]. We focus on control of 2×2 MIMO LTI plants modes and explore the potential benefits of enhancing a traditional diagonal non-networked control architecture with additional non-transparent channels which are subject to signal-to-noise ratio constraints. We will show how to design LTI coding systems which optimize overall performance.

4.1 Setup

As before, $G(z)$ denotes the plant model. We assume that an admissible full MIMO controller, say

$$C(z) = \begin{bmatrix} C_{11}(z) & C_{12}(z) \\ C_{21}(z) & C_{22}(z) \end{bmatrix}$$

has already been designed for $G(z)$. The diagonal terms of this controller are implemented without communication constraints, but the off-diagonal terms communicate using non transparent communication links.

We will focus on a situation where the non transparent communication links are as in Figure 7. In that figure, $F_i(z)$ is the i-th ($i \in \{1, 2\}$) coder transfer function, v_i is the i-th channel input and w_i is the i-th channel output. These signals are related via

$$w_i = v_i + q_i,$$

where q_i is the i-th channel noise. Each noise sequence is considered white, having variance $0 \leq \sigma_i^2 < \infty$ and power spectral density $\Sigma_i(e^{j\omega}) = \sigma_i^2, \forall \omega \in [-\pi, \pi]$. As in previous sections, we assume that σ_i^2 is a design parameter that is proportional to the variance of the channel input (namely, proportional to $\sigma_{v_i}^2$). We define the associated i-th channel signal-to-noise ratio as

$$\gamma_i \triangleq \frac{\sigma_{v_i}^2}{\sigma_i^2}.$$

The NCS which results from employing the links described above to implement the off-diagonal terms of $C(z)$ can be visualized as in Figure 8. In that figure, $u = [u_1 \ u_2]^T$ is the plant input, $y = [y_1 \ y_2]^T$ is the plant output, $r = [r_1 \ r_2]^T$ is the reference sequence, and $e = [e_1 \ e_2]^T$ denotes the tracking error, i.e.,

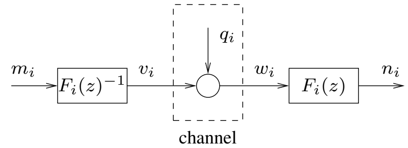

Figure 7: i-th communication link.

$$e \triangleq r - y.$$

4.2 Analysis

It is easy to see from Figure 8 that[13]

$$\sigma_e^2 = ||S(z)\Omega_r(z)||_2^2 + \sum_{i=1}^{2} \sigma_i^2 \, ||S(z)G(z)\varepsilon_i F_i(z)||_2^2 \,, \tag{14}$$

$$\sigma_{v_i}^2 = ||A_i(z)\Omega_r(z)||_2^2 + \sum_{j=1}^{2} \sigma_j^2 \, ||A_i(z)G(z)F(z)\varepsilon_j||_2^2 \,, \tag{15}$$

where $\Omega_r(z)$ is a spectral factor of r and

$$\begin{aligned} F(z) &\triangleq \operatorname{diag}\{F_1(z), F_2(z)\} \,, \\ S(z) &\triangleq (I + G(z)C(z))^{-1}, \\ A_1(z) &\triangleq F_1(z)^{-1} C_{12}(z) \varepsilon_2^T S(z), \\ A_2(z) &\triangleq F_2(z)^{-1} C_{21}(z) \varepsilon_1^T S(z). \end{aligned}$$

Equation (15) allows one to establish that the NCS described above is MSS if and only if both $F_1(z)$ and $F_2(z)$ are stable, MP and biproper,

$$\gamma_1 > B_1 \triangleq \left|\left|C_{12}(z)\varepsilon_2^T S(z) G(z) \varepsilon_1\right|\right|_2^2, \tag{16}$$

$$\gamma_2 > B_2 \triangleq \left|\left|C_{21}(z)\varepsilon_2^T S(z) G(z) \varepsilon_2\right|\right|_2^2, \tag{17}$$

and

$$(\gamma_1 - B_1)(\gamma_2 - B_2) > ||A_1(z)G(z)F(z)\varepsilon_2||_2^2 \, ||A_2(z)G(z)F(z)\varepsilon_1||_2^2 \,. \tag{18}$$

Thus, and as expected from the SISO case examined earlier, we see that MSS imposes limits on the achievable channel signal-to-noise ratio and, hence, on the corresponding channel data-rate.

An immediate consequence of the previous result is that, provided (16)-(18) hold,

$$\lim_{(\gamma_1,\gamma_2) \to (\bar{\gamma}_1, \bar{\gamma}_2)} \sigma_e^2 = \infty,$$

where

$$\mathcal{S} \triangleq \left\{(\gamma_1, \gamma_2) \in \mathbb{R}^2 : \ \gamma_1 \text{ and } \gamma_2 \text{ achieve equality in (18)}\right\},$$

[12] All simulations use an actual uniform quantizer with $L = 2^b$ levels. For each b, the results correspond to the average of 200 simulations (each one 10^5 samples long and using a different reference realization).

[13] ε_i is the $i-$th element of the canonical basis in $\mathbb{R}^2$.

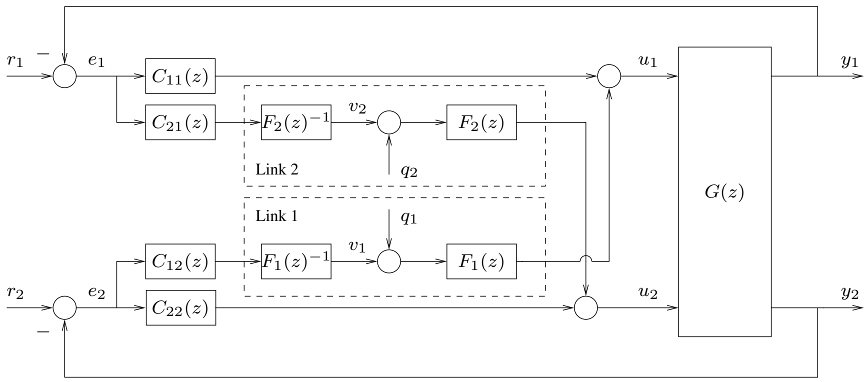

Figure 8: Partly networked MIMO control architecture (see Section 4).

for any $C(z)$, and any choice for $F_1(z)$ and $F_2(z)$. As a consequence, we see that, for any given full MIMO controller, and no matter how the coding system is chosen, there exist sufficiently poor channels which render the performance of the resulting partially networked closed loop arbitrary bad. In these cases, *any* stabilizing decentralized controller (that makes no use of the non-transparent channels) will provide better performance than a NCS. Stated a different way, in the situation under study, there are cases where *poor information is much worse than not having information at all*. Of course, this conclusion is tied to the fact that we are considering a pre-designed controller. If one were to design a centralized controller considering the communication constraints from the very beginning, then one could outperform any decentralized design.

4.3 Coder Design

In this section we show how to design optimal coders $F_1(z)$ and $F_2(z)$ under a mild simplifying assumption.

From (15) one can immediately conclude that, provided (16)-(18) are satisfied and γ_1, γ_2 are large enough, then $\sigma_e^2 \approx J$, where

$$J \triangleq \sum_{i=1}^{2} \frac{||A_i(z)\Omega_r(z)||_2^2 \, ||S_d(z)\varepsilon_i F_i(z)||_2^2}{\gamma_i - B_i}.$$

We will denote the coders that minimize J by $F_1^o(z)$ and $F_2^o(z)$.

Using simple arguments (again based on the Cauchy-Schwarz inequality), it is possible to see that

$$\left|F_i^o(e^{j\omega})\right|^4 = \beta_i \frac{M_i(e^{j\omega})M_i(e^{j\omega})^H}{(S_d(e^{j\omega})\varepsilon_i)^H \, S_d(e^{j\omega})\varepsilon_i},$$

where

$$M_1(z) \triangleq C_{12}(z)\varepsilon_2^T S(z)\Omega_r(z),$$
$$M_2(z) \triangleq C_{21}(z)\varepsilon_1^T S(z)\Omega_r(z),$$

and β_i is an arbitrary positive constant. This results shows how to synthesize coding systems that minimize the impact of the communication links on overall closed loop performance. An interesting feature of the proposed filters is that they do not depend on the channel signal-to-noise ratios. This allows one to conjecture that the optimal filters will perform well for a large class of communication channels.

4.4 Example

We end this section with an example that considers (instantaneous) bit-rate limited channels and, for simplicity, we assume equal bit rates on each channel, i.e., $b_1 = b_2 = b$, and take the sampling interval as $1[s]$. The plant model is given by

$$G(z) = \begin{bmatrix} \frac{0.6}{(z-0.8)} & \frac{0.4}{(z-0.8)} \\ \frac{1}{(z-0.5)} & \frac{1}{(z-0.5)} \end{bmatrix}.$$

For this plant we synthesize the decentralized controller

$$C_d(z) = \begin{bmatrix} \frac{1.3333(z-0.8)}{(z+0.8)(z-1)} & 0 \\ 0 & \frac{0.8(z-0.5)}{(z+0.8)(z-1)} \end{bmatrix},$$

and the full MIMO controller

$$C(z) = \begin{bmatrix} \frac{5(z-0.8)}{z-1} & \frac{-2(z-0.5)}{z-1} \\ \frac{-5(z-0.8)}{z-1} & \frac{3(z-0.5)}{z-1} \end{bmatrix}.$$

We also assume that the reference description is given by

$$\Omega_r(z) = \frac{0.0049627(z + 0.9934)}{(z^2 - 1.97z + 0.9802)} I,$$

and that the quantizers are as in Section 3.3.

Figure 9 shows the tracking error variance in the proposed networked MIMO architecture as a function of the per-channel bit-rate b in several situations: “Analytical no coding” refers to the performance predicted by (14) and (15) when no coding is employed; “Empirical no coding” refers to simulated performance when no coding is considered; “Analytical opt. coding” and “Empirical opt. coding” refer to analytical and simulated performance when the coders suggested by in Section 4.3 are employed. For comparison purposes, Figure 9 also shows the non networked full MIMO performance (“Ideal full MIMO”) and the performance achieved when using $C_{d1}(z)$ (“Decentralized”). The results allow one to conclude that, in this case, the benefits of coding are significative.

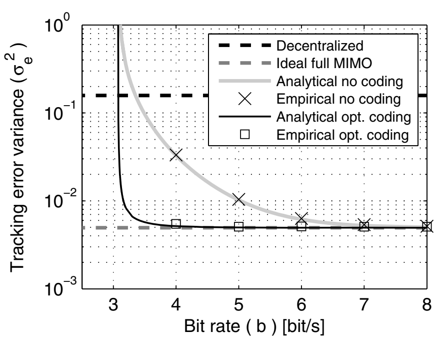

Figure 9: Tracking error variance as a function of the per-channel bit rate (see Section 4.4).

As expected, non-networked full MIMO performance is recovered as $b \to \infty$, irrespective of the coders used. Moreover, it is apparent that the non coded networked full MIMO architecture should be preferred to the decentralized one for $b > 3.36$. On the other hand, the optimally coded networked MIMO architecture provides significant improvement in performance for $b > 3.20$, when compared to the decentralized architecture. It is also interesting to note that, if $b \to 3.07$, then the performance becomes arbitrary poor no matter what the coding is. This is consistent with (16)-(18) as straightforward calculations reveal, and brings back the question of how to actually design MIMO controllers for the considered situation (not just coding systems for a given fixed controller design).

5 Data Loss and Delays

Up to this point in this paper, we have focused on quantization issues. However, in modern network protocols (e.g., Ethernet; see [67]), data is sent in large packets. Accordingly, quantization effects may become negligible in those situations, and the fact that data packets may get corrupted, delayed or lost becomes the dominant issue. In this section (which is based on [34]) we will present an approach that allows one to deal with data loss and random delays. Our proposal exploits the assumption that it is possible to transmit relatively large packets that cover multiple data-dropout and delay scenarios.

5.1 Setup

We consider a discrete-time nonlinear plant model described in state-space form via:

$$x(k+1) = f(x(k), u(k)). \tag{19}$$

The plant input and state are constrained according to:

$$u(k) \in \mathbb{U} \subseteq \mathbb{R}^{\nu}, \quad x(k) \in \mathbb{X} \subseteq \mathbb{R}^{\eta}, \quad \forall k \in \mathbb{N}_0 \triangleq \mathbb{N} \cup \{0\},$$

where $\mathbb{N}$ denotes the set of positive integers, ν denotes the dimension of the plant input, whilst η refers to the state dimension.

To control (19), one can employ standard discrete-time controllers operating at the same sampling rate as the plant

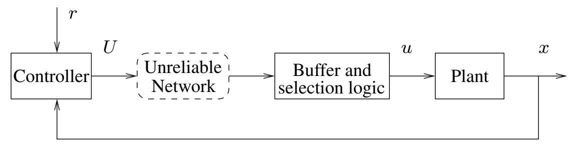

Figure 10: NCS over an unreliable network.

(see, e.g., [2, 3, 68, 69]). Assuming that the state x is available for measurement, in a non-networked case, the plant input can be considered as if were given by a (possibly time-varying and dynamic) mapping of the plant state, say

$$u(k) = \kappa_k(x(k)), \quad k \in \mathbb{N}_0, \tag{20}$$

where $\kappa_k \colon \mathbb{X} \to \mathbb{U}$ is the control policy.

We will now show how one can embed the control law (20) in a more general control strategy where the controller communicates with the plant input via an unreliable network (see Figure 10). This network is assumed to be able to carry large packets of data (say, a few kilobytes). However, each packet, say $U(k)$, may be delayed or even lost (due to buffer overflows and transmission errors).

We assume that the delay experienced by $U(k)$ has both a fixed component, which can be included in the plant model (19), and a time-varying component, say $\tau(k)$. The latter depends on several factors including network load. Since we will concentrate on a configuration where sensors, actuators and controller operate at the same sampling rate, it suffices to consider $\tau(k) \in \mathbb{N}_0$. Packets which are delayed by more than a given value, say $\tau_{\max}$, will be regarded as lost. Thus, we will use the term "lost" to refer to those packets which are effectively dropped by the network as well as to those which do not arrive during a prespecified time frame. Lost packets will not be used further in the NCS strategy described in this section.

The maximum delay $\tau_{\max}$ constitutes a "timeout" value which can be designed based upon network delay and dropout characteristics, see, e.g., [70]. The model adopted above includes pure erasure channels (see, e.g., [10, 39, 40]), as a particular case, by setting $\tau_{\max} = 0$.

5.2 Scenario Based Networked Control

This section briefly describes a control strategy to deal with the situation described above. A detailed description of the control policy can be found in [34].

At the plant side, there exists a buffer, say $\vec{b}(k)$, that contains N control input values to be sequentially passed on to the actuator. The buffer is updated as follows: If the packet $U(k)$ is received (without error) at time instant $k + \tau(k)$ and none of the packets

$$\{U(k+1), U(k+2), \ldots, U(k+\tau(k))\} \tag{21}$$

have been received by time $k + \tau(k)$, then $U(k)$ is used to replace the current content of the buffer. Otherwise, if any of the more recent packets in (21) have been received by time $k + \tau(k)$, then $U(k)$ is discarded. This guarantee that only fresh data is used at the plant side.

At each time instant the controller needs to send a packet $U(k)$. This packet is formed assuming knowledge of the current plant state $x(k)$, the buffer state in the previous sampling instant $\vec{b}(k-1)$,[14] and the previously sent packets. With this information, the controller uses (20) to generate a set of N-length sequences of control inputs, each one taking into account one possible value for the delay[15] $\tau(k) \in \{0, 1, \ldots, \tau_{\max}\}$ and one possible set of packets that may arrive before $U(k)$. Each one of these transmission outcomes constitutes a *scenario*. Upon reception, and if $U(k)$ arrived before $U(k+i)$, $i \geq 1$, the buffer selection logic at the plant uses time-stamping to determine the delay experienced by $U(k)$ and, based upon the knowledge of which packets where received in $[k, k+\tau(k)-1]$, selects the corresponding control inputs sequence.

It is possible to analyze the previous strategy in a precise fashion. Indeed, one can derive deterministic performance guarantees (see [34]). Here, we concentrate on a key result that states that controlling a nonlinear constrained unstable plant model over an unreliable channel by means of SBNC amounts (essentially) to controlling the same plant over an erasure channel with *equivalent data dropout probability* $p_{eq}(k)$ given by[16]

$$
\begin{aligned}
p_{eq}(k) &\triangleq 1 - \mathcal{P}\{\tau(k)=0\} \\
&- \sum_{\substack{j=0 \\ (i,j)\neq(0,0),\, i+j\leq k}}^{N-1} \sum_{i=0}^{\tau_{\max}} \mathcal{P}\Big\{\tau(k-i-j)=i \wedge \tau(k)>0 \\
&\wedge \tau(k-1)>1 \wedge \cdots \wedge \tau(k-j-i+1) > j+i-1\Big\}.
\end{aligned}
$$

This important result, implies that, in order to analyze or design SBNC loops, it suffices to consider a setting wherein the network is modeled via an erasure channel with a given dropout probability (see, e.g., [10–12, 71, 72] and the many references therein).

It should be emphasized here that if $\{\tau(k)\}_{k\in\mathbb{N}_0}$ is a sequence of independent random variables, then

$$p_{eq}(k) = p_{eq},$$

i.e., the equivalent probability, is a constant. However, the sequence of dropouts in SBNC is, in general, not a sequence of independent random variables, even if the underlying network delay and dropout distributions are.

The equivalent dropout probability characterizes closed loop control performance. It can be used as a guideline for choosing the horizon length N and the timeout value $\tau_{\max}$. Indeed, $p_{eq}(k)$ can be made arbitrarily small (and, thus, the performance will become indistinguishable from that achieved in the non-networked case). This is achieved by choosing sufficiently large values for N and $\tau_{\max}$, albeit at an increase in computational complexity (for details see [34]). Fortunately, in practice, choosing moderate values for N and $\tau_{\max}$ is often sufficient to achieve good performance as illustrated below.

[14] This implies that the network protocol must have acknowledgements, as in TCP.

[15] This implies that the actual delay distribution does not play any role.

[16] $\mathcal{P}\{\cdot\}$ stands for probability of $(\cdot)$.

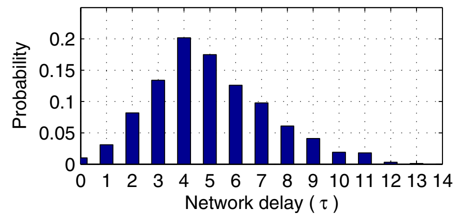

Figure 11: Delay distributions of the network examined in Section 5.3.

5.3 Example

This section presents an example to illustrate the performance of SBNC. We will assume that the network has, for every k, a delay profile as illustrated in Figure 11. Thus, if, for example, one sets $\tau_{\max} \leq 4$, then most packets will be effectively lost.

Consider the following stable nonlinear plant having scalar input

$$
\begin{aligned}
x_1(k+1) &= x_1(k) + 0.01\left(x_2(k) + x_2(k)^3\right) \\
x_2(k+1) &= x_2(k) + 0.01\big(-2x_1(k) - 3x_2(k) \\
&\qquad + u(k)\big(1 + 0.1x_1(k)^2\big)\big) \\
y(k) &= x_1(k) + d(k),
\end{aligned}
$$

where $x(k) = [x_1(k)\ x_2(k)]^T$ is the plant state at time k, $\{u(k)\}$ is the plant input, $\{y(k)\}$ is the plant output, and $\{d(k)\}$ is a piecewise constant output disturbance having infrequent steps at random times. Both the plant state and disturbance measurements are affected by Gaussian white noise of variance $\sigma_d^2 = 0.01$ and $\sigma_x^2 = 0.04I_2$, respectively. The control objective is to achieve reference tracking for the plant output. Whilst future reference values are known to the controller, future disturbances are not. For simplicity, the latter are assumed to remain at their current value.

The underlying controller is given by:

$$\kappa_k(x(k)) = \frac{r_y(k) - d(k) - B(x(k))}{A(x(k))},$$

where $\{r_y(k)\}$ is the reference for $y(k)$,

$$
\begin{aligned}
A(x) &= (1+3x_2^2)(1+0.1x_1^2)/9, \\
B(x) &= -(2x_1 + 6x_2^2x_1 + 9x_2^3 - 4x_2 - 4x_2^3 - 9x_1)/9.
\end{aligned}
$$

We illustrate the effect on performance of the SBNC design parameters $\tau_{\max}$ and N in Figure 12,[17] where typical plant output trajectories for step-like reference and disturbances are shown. For each pair $(N, \tau_{\max})$ we also calculated the equivalent dropout probability $p_{eq}(k)$ (which is constant in this case).

As expected, closed loop performance improves when N and $\tau_{\max}$ are increased. However, it is also appreciated that there are cases where increasing N has no obvious effect on SBNC performance. This is easily explained if one notes that, as suggested by our comments in Section 5.2,

[17] As mentioned in Section 5.2, SBNC does not require knowledge of the statistical properties of the delay profiles.

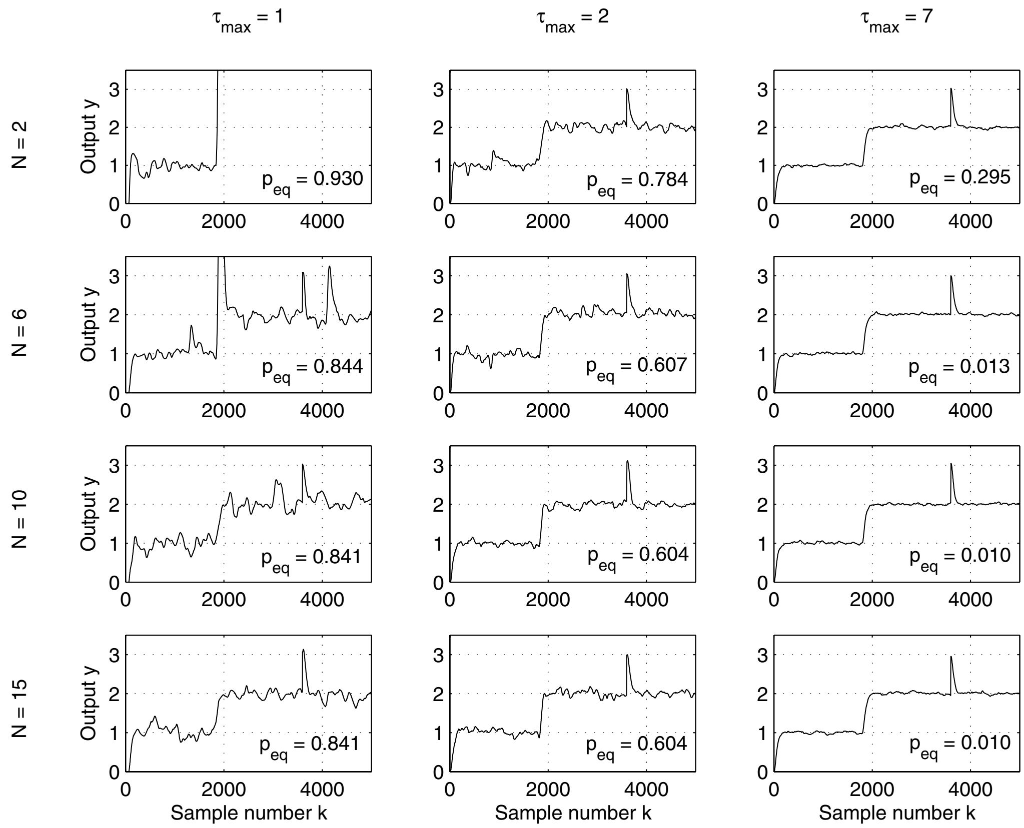

Figure 12: SBNC closed loop performance as a function of N and $\tau_{\max}$ (see Section 5.3).

the equivalent dropout probability p_{eq} is the key parameter determining SBNC performance. Indeed, cases with very disparate values of N or $\tau_{\max}$ can exhibit similar performance if the corresponding equivalent dropout probabilities are similar.[18]

It is also informative to compare the probability of a packet being effectively lost due to the choice of $\tau_{\max}$, and the equivalent dropout probability p_{eq} (which depends on the interplay between N and $\tau_{\max}$). It follows from Figure 11 that if $\tau_{\max} \in \{1, 2, 7\}$, then $\mathcal{P}\{\tau(k) > \tau_{\max}\}$ equals $0.959, 0.877$ and 0.143, respectively. The above results show that by a proper choice of N one can decrease this probability dramatically. It must be noted, however, that if one chooses N too small (as compared with $\tau_{\max}$), then p_{eq} could be larger than $\mathcal{P}\{\tau(k) > \tau_{\max}\}$ (see the case $(N, \tau_{\max}) = (2, 7)$). This is due to the fact that, in those cases, the buffer will frequently run out of data.

It is also worth mentioning that, for this problem, using $\tau_{\max} = 0$, does not stabilize the loop for *any* value of N. In addition, the simple policy that regards all delayed data as lost and only sends the current plant input also gives unstable behaviour. This strongly supports the use of the SBNC scheme advocated here.

[18]Consider, for example, the cases $(N, \tau_{\max}) = (15, 1)$ and $(N, \tau_{\max}) = (2, 2)$ or, more dramatically, $(N, \tau_{\max}) = (6, 7)$ and $(N, \tau_{\max}) = (15, 7)$.

6 Conclusions

Networked Control Systems are control systems in which the communication channels between plant and controller are not transparent. Specifically, NCS's are control systems which include non standard features such as quantization (i.e. data rate limits), data dropouts and random delays. NCS theory brings together fundamental (and contemporary) ideas from control theory, information theory, signal processing and communications theory. It is, thus, an excellent area for the systems scientist to contribute to.

In specific areas, significant results have been found that allow one to analyze and synthesize networked control strategies. Nevertheless, we feel that the big picture is still missing. Many questions remain unanswered. For example, the issues of robustness to plant and channel model errors and adaptation need further work. More ambitious is the search of explicit solutions to certain kinds of problems, which would certainly increase the understanding of the interactions between control objectives and communications constraints.

Beyond technical details, the key open problem in NCS's is the development of a unified framework which takes channel issues into account and gives practice-oriented design methodologies. This makes this research area a challenging and interesting one, with a high development potential.

Acknowledgment

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