

# A brief review of basic GPS orbit interpolation strategies

Mark Schenewerk

## Introduction

Almost any GPS data processing involves interpolating a GPS ephemeris. A variety of time-tested software tools are available for this task, but each has advantages and disadvantages. This discussion will consider some of the foibles of interpolating a GPS ephemeris using two common interpolation strategies.

## Discussion

The basic test cycle consisted of interpolating a standard source ephemeris with a 15-min data interval to produce satellite coordinates at 5-min intervals for the entire 24 h of the day. The interpolator output was then compared to a control ephemeris also with a 5-min tabular interval. How closely the interpolator output matched the control ephemeris at all epochs over the full 24 h of the day was the measure of success of the interpolator.

The source ephemeris was structured like a typical International GPS Service (IGS) rapid or final ephemeris which implies an SP3 file format (Spofford and Remondi 1999) with a 15-min table interval spanning 00:00:00 through 23:45:00 GPS Time (GPST) for a single day. The control ephemeris was also in the SP3 format but with a 5-min

interval spanning the entire day plus several hours from both the preceding and subsequent days. The source and control ephemerides were created from the IGS rapid ephemeris for 2002-01-01, igr11472.sp3. The IGS ephemeris was used as input data for an orbit adjustment solution using the PAGES software (Schenewerk et al. 2002). Once the IGS ephemeris was successfully recreated from the orbit adjustment process, a second copy was created from the same adjustment but covering the longer time span with a 5-min interval. These copies became the source and control ephemerides for the interpolator tests. Note that when processing data taken between 23 and 24 h GPST of a day, the source ephemeris must be extrapolated for the last 15 min of the day or a combination with the next day's ephemeris must be created. Both options involve some risk: problems with "ringing" of the interpolated values near the time limits of the file for the former, and omitted satellites and discontinuities between files in the latter for example. For this reason, special emphasis in the evaluation of the interpolators was placed on the last hours of the comparison to the control ephemeris. Also note that the claimed 3D accuracy of the IGS rapid and final products is 5 cm or better (IGS 2001). Thus, 5 cm became a useful benchmark in the evaluation because the interpolator output must exceed this accuracy in representing the ephemeris to avoid significantly degrading the original information.

A common and completely reasonable response in selecting an interpolation strategy is to use

$$C = A_0 + A_1T + A_2T^2 + A_3T^3 + \dots + A_NT^N$$

a simple polynomial as the analytic function approximating the data in the interpolator. Here,  $C$  represents the  $X$ ,  $Y$ , or  $Z$  coordinate value,  $T$  is time and  $A_0$  through  $A_N$  are the coefficients of the polynomial which are adjusted to fit the source ephemeris data. However, when using 32-bit architecture typical of a PC, as was the case in these tests, this form quickly began to fail as the order of the polynomial was increased because of a dynamic range problem. This problem can be mitigated, to some degree, by careful rescaling of the time and coordinate values, but this rescaling made this "simple" function very convoluted to implement. Fortunately, the Neville algorithm (Press et al. 1986) avoids these limitations while providing an equally simple recursive algorithm for computing the function's value. Therefore, the Neville algorithm was selected as one of the functions used in these tests.

The GPS Toolbox is a column dedicated to highlighting algorithms and source code utilized by GPS engineers and scientists. If you have an interesting subroutine or program you would like to share with our readers, please pass it along; e-mail it to us at [gps-toolbox@ngs.noaa.gov](mailto:gps-toolbox@ngs.noaa.gov). To comment on any of the source code discussed here, or to download source code, visit our website at <http://www.ngs.noaa.gov/gps-toolbox>. This column is edited by Stephen Hilla, National Geodetic Survey, NOAA, Silver Spring, Maryland, and Mike Craymer, Geodetic Survey Division, Natural Resources Canada, Ottawa, Ontario, Canada.

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M. Schenewerk  
Give 'em an Inch, 5144 Clark Drive,  
Roeland Park, Kansas 66205-1402, USA  
E-mail: [mschenew@give-em-an-inch.com](mailto:mschenew@give-em-an-inch.com)

A second possible function was suggested by considering the shape of the GPS satellite orbits. Figure 1 shows the coordinates of PRN01 versus time for day 2002-01-01. Although the values for all three coordinates appear cyclic, only the Z coordinate appears to have the anticipated approximately 12-h period. The X and Y coordinates display a more complex curve with an approximately 24-h period. These complex curves result from the expression of the satellite coordinates in a coordinate frame fixed to the surface of the Earth, the IGS2000 in this case, a variant of the International Terrestrial Reference Frame (IERS 2001). Expressing satellite coordinates in this or any Earth-centered, fixed (ECF) frame causes those coordinates to display the effects of both the orbital motion and the rotation of the coordinate frame. Converting the PRN01 coordinates to the Earth's local inertial frame (IN) and plotting, Fig. 2, reveals the expected sinusoidal variation with an approximately 12-h period. The subsequent discussion will apply to interpolating ECF or IN coordinates, but we will revisit this particular point later. Returning to the task at hand, the cyclic character of the coordinates suggested a trigonometric form would be appropriate:

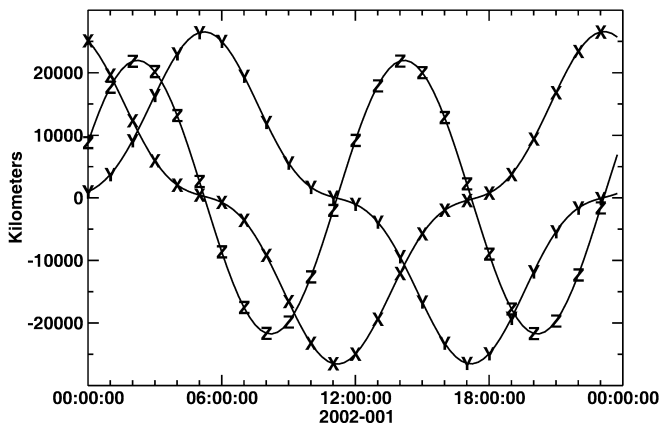


Fig. 1

The Earth-centered, Earth-fixed IGS2000 coordinates for PRN01 from the test ephemeris, igr11472.sp3, plotted versus time

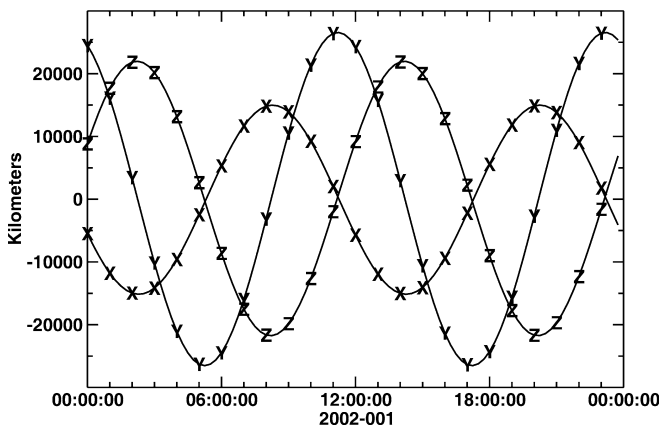


Fig. 2

The inertial coordinates for PRN01 computed from the test ephemeris, igr11472.sp3, plotted versus time

$$C = A_0 + A_1 \sin(WT) + A_2 \cos(WT) + A_3 \sin(2WT) + A_4 \cos(2WT) + \dots + A_N \cos(NWT/2)$$

where  $W$  is a characteristic frequency and the other symbols retain the same meaning as before. When attempting to interpolate ECF coordinates,  $W$  should equal  $2PL$  times the ratio of a solar to sidereal day's length or, approximately,  $2PL \times 1.003 \text{ day}^{-1}$ . A more precise value for this ratio is given in the IERS standards (McCarthy 1996). Having selected two reasonable functions for the interpolator, polynomial and trigonometric, a systematic evaluation was conducted. Tests were performed to determine the optimal number of terms to use in the functions as well as which function performed best. In all cases, the number of source ephemeris epochs used to determine the function coefficients was set equal to the number of coefficients. In this way, the functions always returned the source ephemeris coordinates on the 15-min intervals. The source ephemeris data points used were selected at each interpolation epoch so that the points centered around the epoch where possible, i.e., away from the time limits of the file, and always included the nearest in time source ephemeris epoch. Five to 15 function terms bracketed the best performing variant of the function so, for the sake of brevity, only these results will be reported. Table 1 summarizes the results from the ECF source and control ephemeris tests. For each function used in the interpolator, two columns are given: the population standard deviation of the interpolated minus control differences within the 00:00 to 23:45 time span of the source including the zero differences generated at the source ephemeris epochs, and the maximum difference found within the 00:00 through 24:00 time span of the entire day. This was invariably the 24:00 epoch. With these values, both the capability for interpolation and extrapolation of all function variants could be evaluated.

From the summary in Table 1, it is apparent that either function type is more than adequate for interpolation if 9 to 13 terms are used. Function variants with fewer than 9 terms are unable to match the more subtle variations in the coordinates; forms with more than 13 terms have too much freedom and overreact to those same subtle variations. A distinct success is seen in the 9-term trigonometric interpolator's ability to extrapolate from 23:45 to 24:00.

Table 1

The sample standard deviation and maximum deviation of the function representing the ephemeris' Earth-centered, Earth-fixed coordinates for PRN01 versus the number of terms used in the function. Both the polynomial and trigonometric function results are given

Terms	Polynomial		Trigonometric	
	SD (cm)	Max. (cm)	SD (cm)	Max. (cm)
5	972.7	184738.1	451.8	82512.9
7	12.8	6615.0	0.6	303.5
9	0.3	248.1	0.1	10.3
11	0.2	72.4	0.2	61.7
13	0.3	269.3	0.3	242.4
15	1.0	1017.3	0.9	863.6

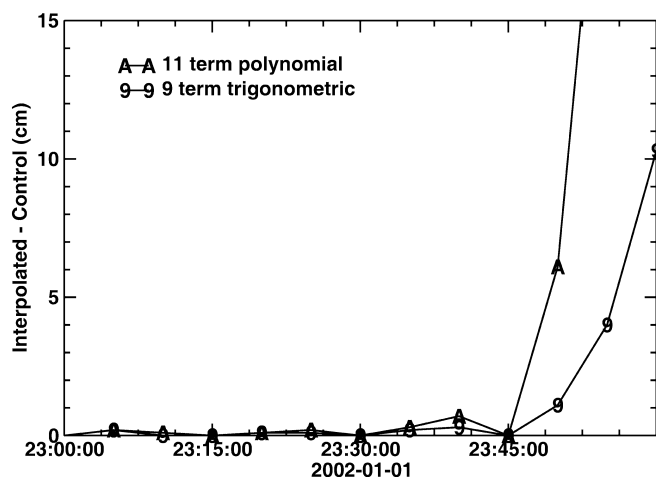


Fig. 3

The interpolated minus control ephemeris position 3D differences for PRN01 versus time. Specifically, the time span from 23 h through 24 h of 2002-01-01 is shown to further emphasize the possible problems of interpolating for coordinates near the end of and extrapolating 15 min beyond the end of the ephemeris. Only the results from the 11-term polynomial and 9-term trigonometric function representations of the ephemeris, the best representations from each function form, are shown for clarity

Figure 3 further emphasizes this performance by plotting differences from the best polynomial and trigonometric functions, 11 and 9 terms, respectively, for the time span 23:00 through 24:00. In retrospect, the success of the trigonometric interpolator should have been expected because, intuitively, it is a more accurate representation of the orbit. Finally, recall that a typical GPS ephemeris provides satellite coordinates in an ECF frame. Therefore, the orbital motion represented by these coordinates is “corrupted” by the rotation of the ECF frame itself. A similar analysis using IN rather than ECF coordinates results in the values shown in Table 2. Here, the fundamental period for the trigonometric function was one-half a sidereal day rather than one sidereal day. Here, the 7-term trigonometric interpolator outperforms all other cases providing greater accuracy with fewer terms.

## Summary

This brief review has demonstrated some of the strengths and weakness of interpolating GPS orbits and interpolating data in general. Two basic functional representations of the ephemeris data were tried, polynomial and trigonometric, and both performed more than adequately for interpolation assuming an appropriate number of terms in the function were used. The trigonometric form, which more closely represents the physical character of the ephemeris data, performs better in extrapolation over short time spans. There is some additional computational burden involved in computing the trigonometric terms but this is minimized, at least in part, by requiring fewer terms. Ultimately, as is always the case, the requirements for speed and accuracy must be balanced when coding this or any software tool.

Table 2

The sample standard deviation and maximum deviation of the function representing the ephemeris’ inertial coordinates for PRN01 versus the number of terms used in the function. Both the polynomial and trigonometric function results are given

Terms	Polynomial		Trigonometric	
	SD (cm)	Max. (cm)	SD (cm)	Max. (cm)
5	556.1	101162.1	0.4	70.1
7	3.3	1780.6	0.1	8.2
9	0.1	41.7	0.1	14.5
11	0.1	38.8	0.1	29.4
13	0.2	155.4	0.2	103.8
15	0.7	831.3	0.5	428.2

## Software

The test cycle was implemented using the program, atest. This program allows one to enter the source and control ephemerides, the PRN number of the satellite to use when creating interpolation – control differences, the number of data points (centered on each epoch) used in computing the interpolation coefficients, the number of terms to use in the interpolation function, and a character specifying the function. For example, atest src\_eph cnt\_eph 1 8 8 N causes atest to use:

- “src\_eph” as the source ephemeris,
- “cnt\_eph” as the control ephemeris,
- PRN01 in the comparison,
- 8 data points around each epoch to compute the function coefficients,
- 8 terms of the function, e.g. 8 implies a seventh-order polynomial,
- “N” implies the Neville algorithm is used.

This program and interpolation routines, all in C, are provided at <http://www.ngs.noaa.gov/gps-toolbox>.

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