

A brief review of metamaterials for opening low-frequency band gaps*

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Abstract Metamaterials are an emerging type of man-made material capable of obtaining some extraordinary properties that cannot be realized by naturally occurring materials. Due to tremendous application foregrounds in wave manipulations, metamaterials have gained more and more attraction. Especially, developing research interest of low-frequency vibration attenuation using metamaterials has emerged in the past decades. To better understand the fundamental principle of opening low-frequency (below 100 Hz) band gaps, a general view on the existing literature related to low-frequency band gaps is presented. In this review, some methods for fulfilling low-frequency band gaps are firstly categorized and detailed, and then several strategies for tuning the low-frequency band gaps are summarized. Finally, the potential applications of this type of metamaterial are briefly listed. This review is expected to provide some inspirations for realizing and tuning the low-frequency band gaps by means of summarizing the related literature.

Key words metamaterial, low-frequency, band gap, vibration suppression

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1 Introduction

The term metamaterial is a composite word of meta and materia. The word meta stems from Greek, carrying a meaning of beyond, and the word materia stems from the Latin word material, carrying a meaning of matter or material^[1]. From the wording components of this term metamaterial, it is clear that metamaterials denote a type of man-made materials capable of achieving properties that are not found in natural materials. In general, the metamaterial is a composite material consisting of substrate materials and embedding materials^[2]. The types of the embedding materials are quite diverse, for instance, metals, plastics, and even air. The embedding materials distribute/encapsulate in the substrate material repeatedly, forming

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a periodic or quasi-periodic structure. Intriguingly, the extraordinary properties of the metamaterials depend neither on the substrate material nor the embedding materials, but on the property of the newly formed structure by the substrate material and embedding materials.

Metamaterials can be categorized broadly as electromagnetic, elastic, acoustic, and structural metamaterials^[3]. With the assistance of the functioning manners such as blocking^[4], absorbing^[5], and enhancing^[6], metamaterials exhibit extraordinary potential for manipulating the electromagnetic waves^[7], acoustic waves^[8], elastic waves^[9], and seismic waves^[10]. Frequency band structure, an analogue of the energy band that stems from quantum mechanics, is the critical property for the achievement of blocking or suppressing the wave propagation along the metamaterials^[11]. In detail, the interaction between the electronic waves and the periodically arranged atomic lattices gives rise to the energy band, which inspires researchers to develop well-designed mechanical structures to open a band structure based on the interactions between the unit cell and acoustic/elastic waves^[12].

For conventional photonic/phononic metamaterials, their band gaps are related to the lattice constant of the designed periodic/quasi-periodic structure. The interferences between travelling waves and reflected waves play a key role in the formation mechanism of band gaps. Based on these interferences, the metamaterials are capable of opening a band gap that is commonly known as the Bragg scattering (BS) band gap^[13]. As the lattice constant should match half of the wavelength, it is difficult for small-size metamaterials to manipulate waves at low frequencies based on the BS mechanism^[14]. An impact of the lattice constant on band structures exists in the mechanical metamaterials as well. For example, a rather bulky configuration is requisite to manipulate low-frequency elastic waves^[15].

Locally resonant (LR) metamaterials are a type of emerging metamaterials capable of attaining band structures owing to energy transfer from the main structure to the resonators^[16]. The resonance of the resonator is the most critical and even only factor for the appearance of an LR band gap. The wavelength is much longer than the lattice constant of the metamaterial when the LR band gap occurs. Hence, the formation mechanism of the LR band gap overcomes the limitation of the BS mechanism. More importantly, the resonance is an essential attribute of the resonator, which is only dependent on its inertia (for instance, the mass^[17–18] and the moment of inertia^[19]) and stiffness (for instance, the vertical stiffness^[20–21] and the torsional stiffness^[22]). Therefore, the LR mechanism provides an appealing avenue to open low-frequency band gaps using small-scale metamaterials with well-designed resonators.

Low-frequency vibration widely exists in engineering structures^[23]. Currently, utilizing metamaterials to suppress low-frequency vibration propagation along the engineering structures has aroused great concerns. The application in the low-frequency vibration attenuation, in turn, drives the development of metamaterials. Researchers tried to develop lots of new configurations of both the BS and LR metamaterials, to obtain band gaps in the low- and even ultralow-frequency ranges^[24]. To date, both passive mechanisms (for example, simple/improved mass-spring structures^[25–26] and the soft materials/structures^[16,27]) and active mechanisms (for example, the piezoelectric materials^[28], the electromagnets^[29], and the pumps^[30]) have been utilized to improve the low frequency band gaps, such as decreasing the frequency^[31], broadening the bandwidth^[32], and improving the wave attenuation within the band region^[33].

It should be admitted that low-frequency band gaps with favorable performance of wave manipulation are very useful but are difficult to be realized. To open wide and deep band gaps in the low-frequency range through metamaterials, lots of efforts and attempts have been carried out, and many related articles on this topic have been published. However, to date, there has been no systematic review summarizing the basic principle of the low-frequency band gaps, fundamental configurations of the metamaterials capable of opening low-frequency band gaps, and potential applications of the low-frequency band gaps. To address this issue, this review makes an overview of existing configurations of metamaterials which are capable of opening the band gaps in the frequency range below 100Hz based on a ground rule of the complexity

and the supporting capacity. The advantages and disadvantages of these configurations and some strategies for tuning the low-frequency band gaps and improving the wave manipulating performance in the band region are detailed. Finally, some potential engineering applications of the metamaterials are summarized briefly.

This review is organized as follows. In Section 2, the basic principle of opening low-frequency band gaps is summarized first. Some typical configurations capable of obtaining low-frequency band gaps are presented and classified according to the carrying capacity of the metamaterial in this section as well. In Section 3, some strategies for tuning the low-frequency band gaps and optimizing the wave manipulation performance in the band gap region are elaborated. Some potential applications of low-frequency metamaterials are briefly summarized in Section 4. Finally, some conclusions about this review are drawn in Section 5.

2 Structural configurations for low-frequency band gaps

One of the key issues of the metamaterials is how to devise suitable configurations to obtain low-frequency band gaps. This section aims to summarize the fundamental configurations that are capable of obtaining band gaps in the frequency range below 100 Hz. The structural configurations of metamaterials can be classified into 5 categories according to their structural complexity and supporting capacity. Moreover, the underlying principles for the achievement of the low-frequency band gaps are reviewed as well in this section.

2.1 Spring-mass configuration

The spring-mass configuration is the simplest constitution of the metamaterials but is the most useful model to analyze the formation mechanism of the band gap. Consider the first scenario that all mass elements and spring elements utilized to constitute the spring-mass chain are identical, the spring-mass chain is homogeneous, and its vibration frequency spectrum is composed of only an acoustical branch. For the second case, a unit cell contains two types of mass elements, as illustrated in Fig. 1, and the vibration frequency spectrum of the inhomogeneous metamaterials consists of an acoustical branch and an optical branch. As depicted in the right panel of Fig. 1, a band gap capable of forbidding the wave propagation appears between the acoustical branch and the optical branch^[34]. The beginning frequency and ending frequency of the band gap opened by the diatomic spring-mass chain can be given by^[35]

$$\omega_B = \sqrt{\frac{2k}{M}}, \quad \omega_E = \sqrt{\frac{2k}{m}}, \quad (1)$$

where m and M denote the masses of two types of mass elements. k is the stiffness of the connecting spring. Apparently, the band gap is related to the properties of the spring-mass unit cell, which provides an approach to lower the beginning frequency of the band gap by decreasing the stiffness or increasing the mass.

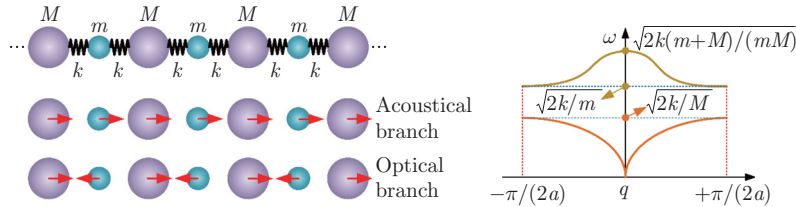


Fig. 1 Schematic diagram of a one-dimensional spring-mass chain and its corresponding vibration frequency spectrum (color online)

Mounting spring-mass local resonators onto the primary structure, for instance, the spring-mass chain^[36], the beam^[25], and the plate^[37], is the second way to open low-frequency band

gaps. The operational principle of opening such low-frequency band gaps is the local resonance. That is, much of the energy transfers from the primary structure to the resonators when the excitation frequency is near the resonant frequency of the spring-mass resonator. Such a phenomenon is referred to as the local resonance, resulting in large-amplitude oscillations of the resonator and a restrained wave propagation along the primary structure^[38]. The band features of the metamaterials, including the frequency and width, are dependent on the mass and stiffness of the spring-mass mechanism. That is, it is possible to obtain a band gap in the frequency range below 100 Hz without taking into account the support capacity^[35].

Generally, the gravity of the resonator is completely supported by the linear spring. However, the limited supporting capacity of the linear spring with a low stiffness may result in the structure destabilization. Therefore, it is just a theoretical possibility for obtaining low-frequency band gaps using spring-mass resonators, leading to difficulty in the engineering applications of the metamaterials. Additionally, the space-consuming configuration of the metamaterials composed by the spring-mass mechanism is the second obstacle for its applications.

2.2 Supportless configurations and quasi-static band gap

Consider a special scenario that the metamaterial is placed on the ground and it does not need to support any weight. Then, it becomes a realistic approach to open low-frequency bands owing to its ultra-low stiffness. The metamaterial is unable to support static load in one or multiple directions on account of the low stiffness of the connecting part in these directions. Such a type of metamaterial is called supportless configuration in this review.

The first way to achieve the supportless feature is replacing the connecting spring between two unit cells by a spin-harnessed spring-mass chain, as presented in Fig. 2(a). The corresponding dispersion relation of such a spin-harnessed spring-mass chain is given by^[39]

$$-\left(M - \frac{p}{\omega^2}\right)\omega^2 = -4\frac{A \cos \alpha^0}{\det \mathbf{W}} \sin^2 \frac{qD}{2}, \quad (2)$$

where M denotes the larger mass of the n th unit cell. q and D stand for the wave vector and the lattice constant, respectively. α^0 represents the angle between the connecting rod and the x -axis at the initial configuration. ω is the angular frequency. \mathbf{W} , p , and A denote three variates that are related to the parameters of the spin-harnessed metamaterial, and their expressions are available in Ref. [39]. The results show that the devised supportless metamaterial possesses the ability to obtain a quasi-static longitudinal wave band gap, and the beginning frequency of the band gap approaches zero, due to the coupling of the spin motion and the longitudinal wave motion.

The second way to obtain the supportless feature is to utilize hinges to connect the unit cells. The quasi-static band gap can be formed by letting the rotational stiffness to be zero in the wave dispersion of a homogeneous continuous beam^[40–42], namely,

$$-\omega^2(\alpha a \rho I_b \kappa^2 + \alpha a \rho A_b - \rho^2 \omega^2 A_b I_b) = 0, \quad (3)$$

where ω and κ denote the angular frequency and the wave vector, respectively, a is the lattice constant, ρ is the density, A_b and I_b stand for the cross-sectional area and the bending momentum of inertia of the beam structure, respectively, and α is the spring coefficient. Two roots of the angular frequency can be obtained by solving Eq. (3). One is zero for all of the wave vector, and the other is related to both the angular frequencies and the material and geometrical parameters. That is, a band gap is achieved within the frequency range between 0 Hz and the second root of the angular frequency. The wave propagation features along the metamaterial with zero rotational stiffness are reported in Fig. 2(e). Clearly, the wave propagations are restrained in the quasi-static frequency range.

A tapered beam achieved by cutting a homogeneous beam periodically is the third approach to acquire a supportless metamaterial and a resultant quasi-static band gap. The schematic

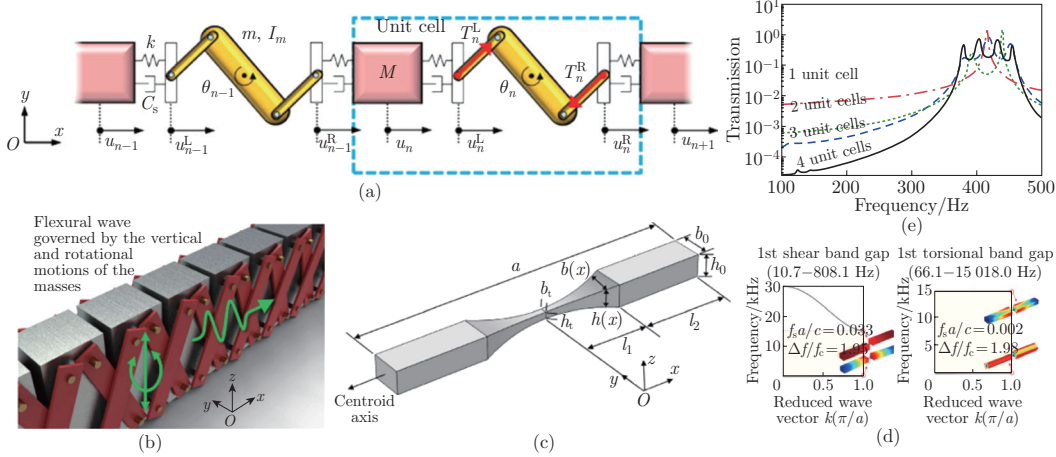


Fig. 2 (a) Spring-mass chain with zero longitudinal stiffness (reprinted with permission from Ref. [39]); (b) metamaterial with zero rotational stiffness (reprinted with permission from Ref. [40]); (c) unit cell with quasi-zero shear and torsional stiffness (reprinted with permission from Ref. [41], 2021, Elsevier); (d) quasi-static band structures (reprinted with permission from Ref. [41], 2021, Elsevier); (e) wave propagations of supportless configurations (reprinted with permission from Ref. [40]) (color online)

diagram of this metamaterial is presented in Fig. 2(c), and its band structures are reported in Fig. 2(d). The change in the regularity of the cross-section parameter of the tapered part was given by Ref. [41],

$$h(x) = (h_0 - h_t)(x/l)^m + h_t, \quad b(x) = (b_0 - b_t)(x/l)^n + b_t, \quad 0 \leq x \leq l, \quad (4)$$

where h_0 and b_0 denote the thickness and the width of the uniform beam, respectively. Apparently, both the thickness $h(x)$ and the width $b(x)$ of the tapered part are related to the minimum thickness h_t and the minimum width b_t . Because the minimum cross-sectional area of the tapered part is quite small, both the bending and torsional stiffness of the connecting part are very low. The extreme stiffness feature of the tapered beam provides the possibility to realize a quasi-static band gap. Note that whether in the spring-mass chain with zero longitudinal stiffness or in the tapered metamaterial, each of the unit cells of the metamaterial is almost free along the direction where the stiffness is quite low or even zero, resulting in difficulties in the practical applications.

2.3 Pentamode metamaterial

The pentamode metamaterial is a kind of artificial periodical structure that is capable of decoupling the compression waves and the shear waves easily^[43]. As its mechanical properties show the greatest similarity with those of the fluids, the pentamode metamaterial is also known as the meta-fluid. The schematic diagram and corresponding band structures of the pentamode metamaterial are delineated in Fig. 3. Compared with the supportless configurations, the pentamode metamaterial possesses the capability of supporting/transmitting static load in one direction. In general, the traditional pentamode metamaterial is able to open an elastic wave band gap whose location is dependent on its material parameters. The relationship between the center frequency of the band gap and the system parameters is given by^[44]

$$f_c = A - B \ln(E/\rho + C), \quad (5)$$

where A , B , and C stand for three different constants. E and ρ denote Young's modulus and the mass density, respectively. Obviously, there are two possible ways to lower the center frequency of the band structure attained by the pentamode metamaterial, namely, decreasing

Young's modulus and increasing the mass density of the material. For instance, the center frequency of the band gap shifts from 3621 Hz to 6.5 Hz by replacing the constituent material from aluminum to rubber only^[44].

Additionally, through imbedding the material element with a large density into the constituent material, for instance, incorporating the lead block into the rubber constituent material, the center frequency of the band gap can be decreased further with the increase in the effective mass density^[44].

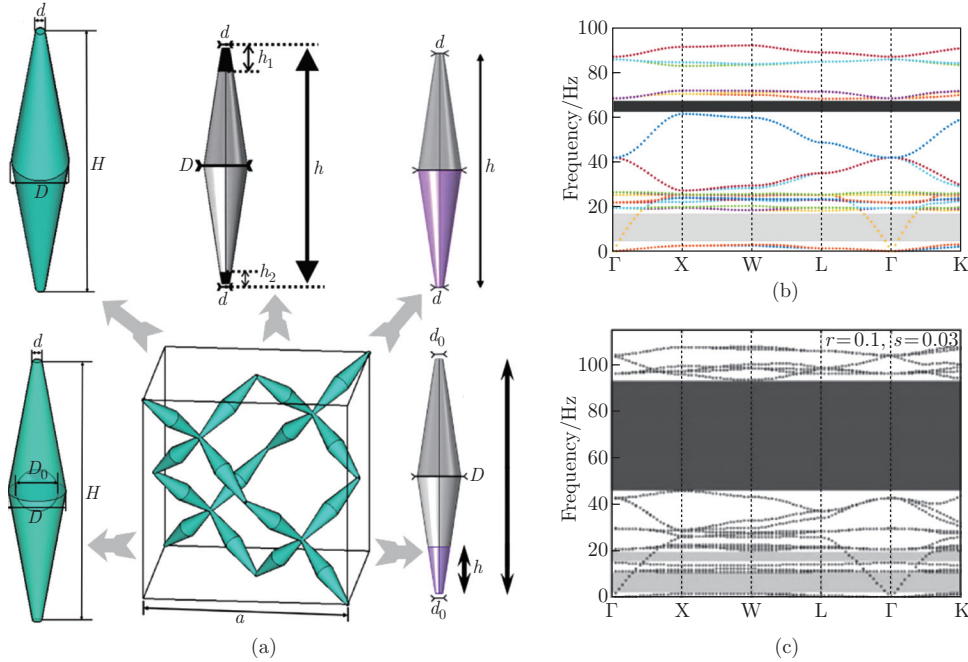


Fig. 3 (a) Schematic diagrams of the pentamode metamaterial and different basic elements (reprinted with permission from Ref. [44] (2021, IOP Publishing), Ref. [45] (2017, AIP Publishing), Ref. [46] (2019, IOP Publishing), and Ref. [47] (2017, IOP Publishing)). (b) and (c) denote band structures achieved by different pentamode metamaterials (reprinted with permission from Ref. [45] (2017, AIP Publishing) and Ref. [46] (2019, IOP Publishing)), respectively (color online)

The second approach for realizing low-frequency band gaps based on the pentamode metamaterial is replacing the traditional pentamode element as an LR one. Figure 3(a) illustrates the schematic diagrams of different types of LR pentamode elements, including the symmetric double-cone element composed of a large percentage of lead solid and a small percentage of silicon rubber^[45], the symmetric double-cone element composed of half of lead solid and half of silicon rubber^[46], and the asymmetrical double-cone element consisted by a large percentage of lead solid and a small percentage of silicon rubber^[47]. The band structures achieved by different LR pentamode metamaterials are reported in Figs. 3(b) and 3(c). As one can see from the figures, the LR pentamode metamaterial not only decreases the position of the band gap, but also broadens the bandwidth by modulating the material ratio of the double-cone pentamode element^[46]. Nevertheless, there is still a drawback of the composite pentamode metamaterial that the added imbedding material/resonating mass could crush the elastic element with a low stiffness, which is consistent with the plight of the spring-mass mechanism.

2.4 Elastic materials/structures

All of simple spring-mass mechanisms (only composited by one spring and one mass block), supportless configurations, and pentamode metamaterials possess the capability of realizing an

elastic wave band gap in the low-frequency range. However, the incapability of supporting and transmitting the static load limits the practical application of these kinds of metamaterials with low-stiffness connecting elements. Therefore, both the load supporting capability and compact configurations should be taken into account to design the metamaterials in engineering applications. By analogy with the atomic chain, a one-dimensional metamaterial composed of DNA-inspired connecting components and central plates was put forward to obtain low-frequency band gaps. The schematic diagram of the DNA-inspired metamaterial is reported in Fig. 4(a). Note that the connecting components can be considered as springs and the central and lateral plates as mass blocks. The stiffness of the DNA-inspired connecting component is dependent on the number of the interconnections between the double helices. Consequently, a potential avenue for opening band gaps in the low-frequency range based on the DNA-inspired metamaterial is to decrease the number of the interconnections between the double helices^[48].

Similarly, the local resonator can be devised through elastic materials or structures as well. As reported in Figs. 4(b)–4(e), there are lots of conceptions for the structured resonator. By replacing the linear spring into elastic materials or structures, for instance, silicon rubber^[49,51,55–57], membrane^[50,58], cantilever beam^[52], and polymer concrete^[59], the size of the metamaterial is drastically reduced. An additional advantage of replacing the spring by the

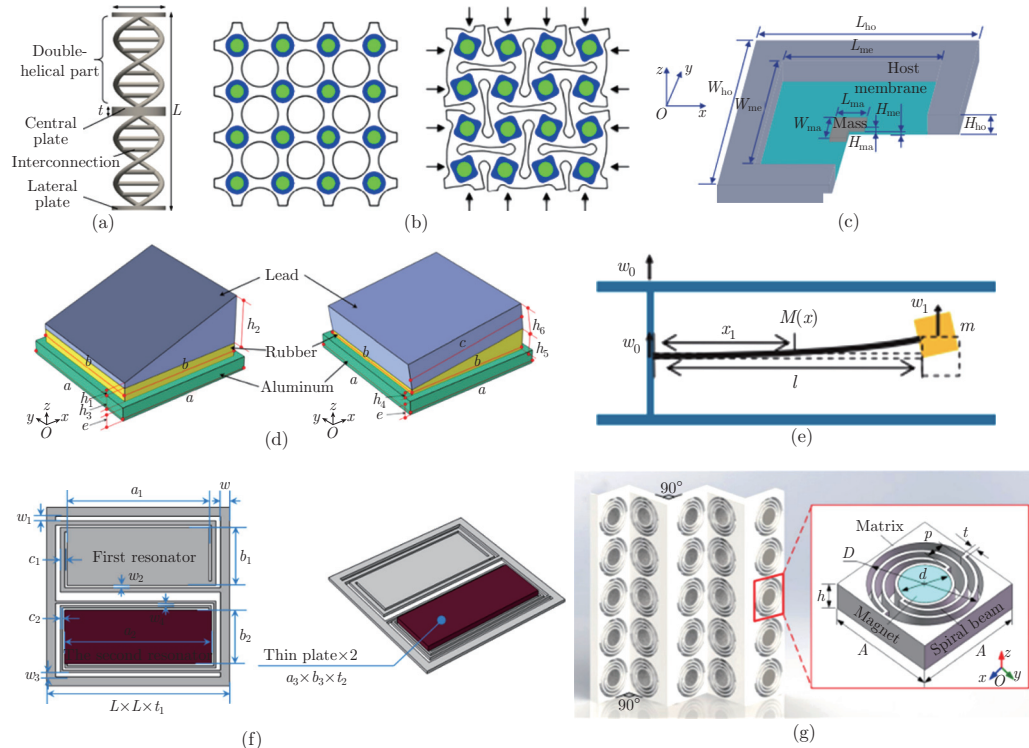


Fig. 4 (a) DNA-inspired structure (reprinted with permission from Ref. [48], 2017, IOP Publishing). (b) Cellular metamaterial with resonators consisted by elastomeric coating and mass block (reprinted with permission from Ref. [49], 2020, Elsevier). (c) Local resonator consisted by mass block and membrane (reprinted with permission from Ref. [50], 2021, IOP Publishing). (d) Unit cell composed by silicon rubber and lead (reprinted with permission from Ref. [51], 2020, Elsevier). (e) Resonator consisted by cantilever beam and mass block (reprinted with permission from Ref. [52], 2020, Elsevier). (f) and (g) Local resonator formed by hollowing out a continuous structure (reprinted with permission from Ref. [53] (2017, AIP Publishing) and Ref. [54] (2019, IOP Publishing)), respectively (color online)

elastic material is that the elastic material is capable of providing the restoring force in multiple directions. That is, the resonators consisted by elastic materials are able to open multiple types of band gaps, such as the longitudinal wave band gap, the bending wave band gap, and the torsional wave band gap^[57]. Hollowing out a continuum, such as beam or plate, to form some special geometries is another avenue to design a resonator with a low resonant frequency^[53–54], as presented in Figs. 4(f) and 4(g). Compared with the continuum, the stiffness of the hollowed-out part is much smaller, which is the crucial reason for the low resonant frequency and the resultant low-frequency band gap. Consistent with the simple spring-mass mechanism, however, the contradiction between the support capacity and the low stiffness feature of the elastic material still exists. That is, the ordinary elastic materials/structures are not the most suitable options to decrease the band frequency further.

2.5 Quasi-zero-stiffness mechanisms

A low stiffness feature and a sufficient supporting capacity are two requirements to obtain a low-frequency LR band gap. However, in terms of linear spring-mass mechanisms or elastic materials/structures, it is hard to obtain these two features simultaneously. To isolate the low-frequency vibration, researchers put forward a type of nonlinear isolator named the quasi-zero-stiffness isolator^[60–64]. Its supporting capacity is guaranteed by a positive stiffness element, but its dynamic stiffness is neutralized by a negative stiffness mechanism, leading to a win-win situation on both the supporting capacity and the low stiffness feature. Inspired by an analogy with the quasi-zero-stiffness isolator, Zhou et al.^[65] firstly devised a quasi-zero-stiffness local resonator based on a negative stiffness mechanism formed by two oblique springs, as reported in Fig. 5(a). The nonlinear expression between the displacement and the stiffness of the resonator can be given by^[66]

$$k = k_x - (1 - \gamma) \frac{a}{b - a} \left(\frac{a^2}{(a^2 + x^2)^{\frac{3}{2}}} - 1 \right), \quad (6)$$

where k and k_x denote the stiffnesses of the resonator and the original linear spring utilized to form the traditional spring-mass mechanism, respectively, a and b are two geometrical parameters of the resonator, x stands for the displacement the mass block moved, γ denotes the stiffness ratio of the resonator with and without the negative stiffness mechanism, and its value can be varied through adjusting the level of the negative stiffness mechanism engagement.

Here, it should be pointed out that the negative stiffness mechanism is the critical component of the quasi-zero-stiffness resonator. Other than the oblique spring mechanism, there are other configurations which are capable of forming a negative stiffness mechanism as well, for instance, the X-shaped mechanism^[67], the magnetic ring^[68], the cam-roller mechanism^[22], and the hemisphere-roller mechanism^[69]. Connecting the negative stiffness mechanism by the positive stiffness element, such as the linear spring and the rubber ring, a variety of quasi-zero-stiffness resonators have been devised. However, the contact between the positive and negative stiffness mechanisms may yield friction, which would have a significant impact on the band gap. The relatively large self-weight of the quasi-zero-stiffness resonator and the space-consuming configuration of the negative stiffness mechanism could make the inapplicability of the metamaterial in the practical application as well. To address these problems, an integrated and lighter quasi-zero-stiffness resonator is devised based on two pairs of folded beams and two pairs of buckled beams. As reported in Fig. 5(f), folded beams and buckled beams are connected directly, which avoids the contact friction^[70].

Along with the improvement of the structured design of the quasi-zero-stiffness mechanism, the positive stiffness element and the negative stiffness element disappeared, to be replaced by multi-pieces of curved beams. The fundamental configuration of the integrated quasi-zero-stiffness resonator is illustrated in Figs. 5(g) and 5(h). After an optimization of geometrical parameters, the multi-pieces of curved beams are able to produce a quasi-zero-stiffness characteristic. More importantly, the stiffness features of the integrated quasi-zero-stiffness resonator

can be tuned easily by changing the deformation degree of the curved beams only, which provides a promising approach to tuning the low-frequency band gap^[71–72].

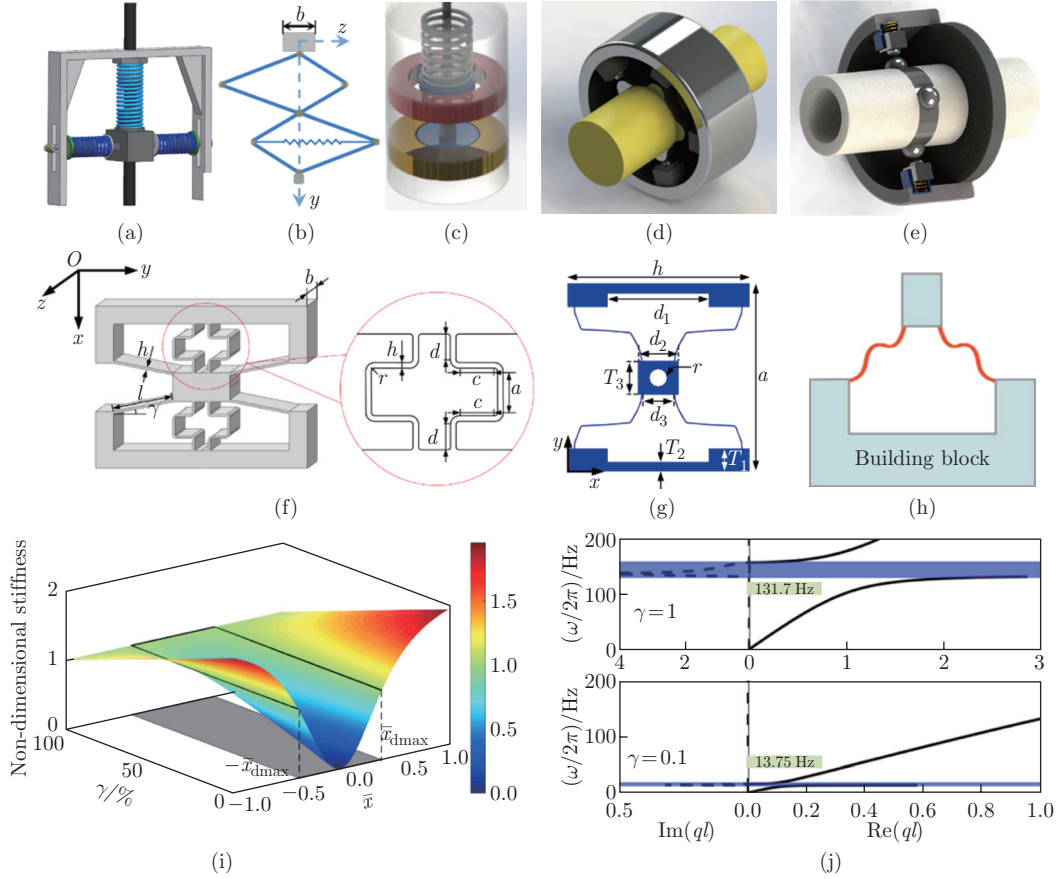


Fig. 5 Quasi-zero-stiffness local resonators composed of (a) oblique spring mechanism (reprinted with permission from Ref. [65], 2017, AIP Publishing), (b) X-shaped mechanism (reprinted with permission from Ref. [67], 2019, Elsevier), (c) permanent magnet ring (reprinted with permission from Ref. [73], 2020, Elsevier), (d) cam-roller mechanism (reprinted with permission from Ref. [22], 2019, IOP Publishing), (e) hemisphere-roller mechanism (reprinted with permission from Ref. [69], 2021, Elsevier), and (f) folded beams and buckled beams (reprinted with permission from Ref. [70], 2020, Elsevier). (g) Structured quasi-zero-stiffness resonator (reprinted with permission from Ref. [72], 2021, Springer Nature). (h) Quasi-zero-stiffness local resonators composed of multi-pieces of curved beams (reprinted with permission from Ref. [74], 2021, John Wiley and Sons). (i) Stiffness characteristics of the resonator (reprinted with permission from Ref. [66], 2019, Elsevier) and (j) band structures of the quasi-zero-stiffness metamaterial (reprinted with permission from Ref. [66], 2019, Elsevier) (color online)

In terms of the quasi-zero-stiffness resonator formed by connecting the positive stiffness mechanism and the negative stiffness mechanism, its stiffness features are related to the involvement level of the negative stiffness mechanism which is quantified by the stiffness ratio. The dependence of the stiffness features on both the stiffness ratio and the displacement is illustrated in Fig. 5(i). When the stiffness ratio γ is equal to 0, clearly, the local resonator updates to a quasi-zero-stiffness configuration whose stiffness is equal to zero at the equilibrium position and approaches zero nearby the equilibrium position^[60]. With the augment of the stiffness ratio, the magnitude of the resonator stiffness at the equilibrium position increases

obviously. When the stiffness ratio equals one, there is an extreme scenario that the effect of the negative stiffness mechanism on the original linear spring disappears. The resonator degenerates to a traditional linear spring-mass resonator. The corresponding band structures of the metamaterial with quasi-zero-stiffness local resonators are depicted in Fig. 5(j), where the shadows denote the band gap range. It is evident from the figure that with the decrease in the stiffness ratio, the location of the band gap shifts from a high frequency range to a low-frequency one owing to the decline of the resonant frequency^[69]. That is, in theory, the metamaterial with quasi-zero-stiffness resonators is capable of obtaining a quasi-static band gap at a very low-frequency range.

Despite these huge advantages of quasi-zero-stiffness mechanisms in the field of realizing low-frequency band gaps through metamaterials with enough support capacity, some drawbacks still abound. First, the research results indicate that the width of the band gap becomes narrower, and the wave suppression performance in the band gap region degrades as the center frequency of the band gap decreases. Furthermore, as far as the traditional quasi-zero-stiffness mechanisms are concerned, their configurations are complex and the contact between the mechanical parts brings about frictions that could affect the realization of low-frequency band gaps. Therefore, although some attempts have been carried out, a large body of work needs to be implemented to devise compact and lighter quasi-zero-stiffness resonators and then realize wide low-frequency band gaps.

3 Strategy for band gap tuning

Compared with the BS mechanism, the LR mechanism is appropriate to obtain a band gap in the low-frequency range, due to the fact that it eliminates the dependence of the band frequency on the lattice constant of the metamaterial. This section is largely concentrated in approaches that are utilized to realize the regulation of the LR band gap. The fundamental strategies of tuning the low-frequency band gap are grouped into two categories, namely, adjusting the stiffness or mass of the resonator. Some strategies for improving the wave manipulation performance within the band region are presented as well in this section.

3.1 Band gap tuning based on adjustable mass

The essential pathways to tune the resonator mass include changing (decrease/increase) its mass directly and enlarging the effective mass by an inertial amplification. Figure 6 delineates configurations of the metamaterial with tunable resonator mass and its corresponding band structures^[75–78]. In reviewed literature, the approach for directly varying the resonator mass is to transfer a part of mass between the primary structure and the resonator. The fundamental structures utilized to carry out the mass switch include electromagnets^[75] and pumps^[76], as elucidated in Figs. 6(a) and 6(b). In terms of the mass switch achieved by the electromagnet, its basic principle is to manipulate the current to make two electromagnets attach (Att) or detach (Det) and then change the resonator mass. The comparison between these two modes (attach/detach) is reported in Fig. 6(d). Clearly, the band structure can be adjusted obviously through switching the current off/on only.

The second approach for tuning the center frequency of the band gap based on the resonator mass is to introduce an inertial amplification mechanism into the metamaterial. Different from the approach that varying the resonator mass directly by electromagnets or pumps, the inertial amplification mechanism forms a large effective mass based on a small resonator mass^[79–81]. Figure 6(e) delineates the comparison of the wave propagation features of metamaterials with and without the inertial amplified resonator. Clearly, the introduction of the inertial amplification mechanism plays a positive role for tuning (decreasing) the band frequency^[78].

It should be pointed out that the inertial amplification mechanism not only can constitute the metamaterial alone but also can be coupled with other mechanisms to tune the band frequency. Figure 6(c) depicts the schematic diagram of a quasi-zero-stiffness resonator with

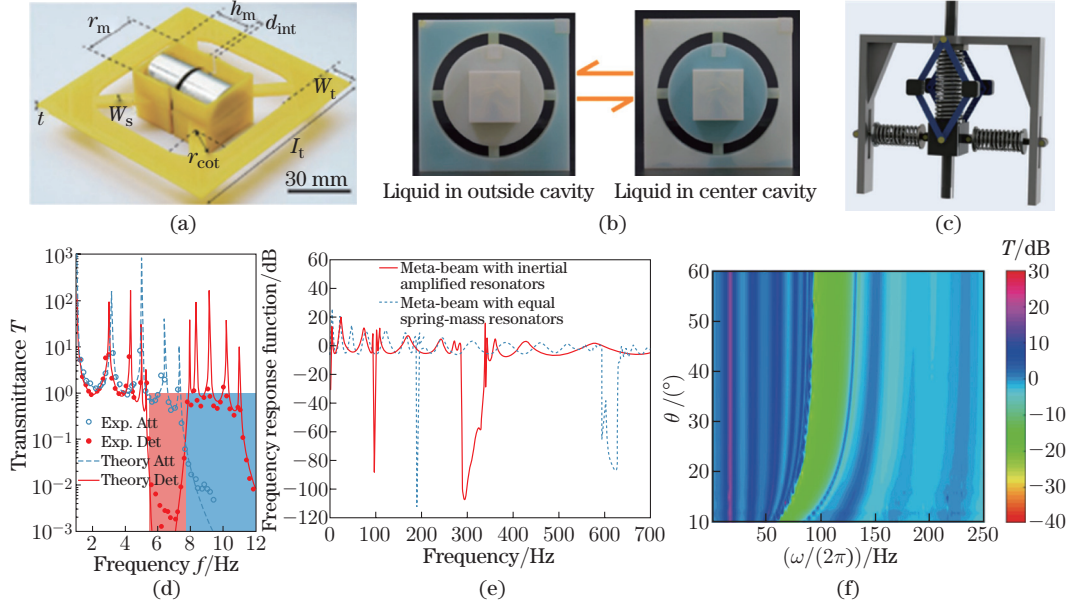


Fig. 6 Adjustable resonator mass realized by (a) electromagnet (reprinted with permission from Ref. [75], 2016, John Wiley and Sons) and (b) pump (reprinted with permission from Ref. [76], 2018, AIP Publishing). (c) A resonator with inertial amplification mechanism (reprinted with permission from Ref. [77], 2019, Springer Nature). (d) Effect of the resonator mass on the wave attenuation feature (reprinted with permission from Ref. [75], 2016, John Wiley and Sons). (e) Influence of the inertial amplification mechanism on the wave transmittance (reprinted with permission from Ref. [78], 2021, Elsevier). (f) Influence of the inertial amplification mechanism on the band structures (reprinted with permission from Ref. [77], 2019, Springer Nature) (color online)

an inertial amplification mechanism. The effective mass of the resonator is given by^[77]

$$M_{\text{eff}} = \left(1 + 2\gamma \frac{l^2}{4l^2 \sin^2 \theta - 4yl \cos \theta - y^2} \right) m_r, \quad (7)$$

where γ denotes the mass ratio of the inertial amplification mechanism (tip mass block) to that of the resonator, l is the length of the light rod in the inertial amplification mechanism, θ is the angle between the axis of the rod and the vertical spring at the initial configuration, and m_r stands for the mass of the local resonator. Obviously, the effective mass of the resonator with the inertial amplification mechanism is larger than that of the original one. The influence of the inertial amplification mechanism on the band structures is depicted in Fig. 6(f), where the frequency of the band gap decreases obviously when the inertial amplification mechanism is introduced into the resonator.

3.2 Band gap tuning based on adjustable stiffness

There are two approaches that can be utilized to modulate the resonator stiffness. One is to devise the mechanical structure to decrease the stiffness, and the other is to adjust the stiffness real-time. Introducing the negative stiffness mechanism^[67,82], decreasing the cross-section of the elastic material^[56–57,83–84], and adjusting the amount of compression^[71–72,74,85] are the mechanical ways to change the resonator stiffness. Although these mechanical structures are in favor of obtaining band gap in the low-frequency range, these configurations are difficult to change once they were designed.

The second way of tuning is to devise the metamaterial with real-time controllable configura-

tions, as illustrated in Fig. 7. To be specific, the electrical elements (current/voltage/inductance/resistance)^[73,86–91], the air pressure (see Fig. 7(a))^[30,92], and the temperature (see Fig. 7(b))^[93–94] are primary controllable variables. The piezoelectric actuator and the electromagnet are two main electronic components for the stiffness tuning based on the electrical elements. Their regulatory effects on the wave propagation along the metamaterial are depicted in Figs. 7(c)–7(d). The piezoelectric actuator is a primary device to achieve smart metamaterials as well. For instance, a thin beam with periodically mounted piezoelectric actuators is capable of opening two types of band gap (i.e., the BS and LR band gaps) simultaneously using a digital synthetic impedance circuit. More importantly, the frequency of the LR band gap can be tuned easily by adjusting the inductance or the resistance.

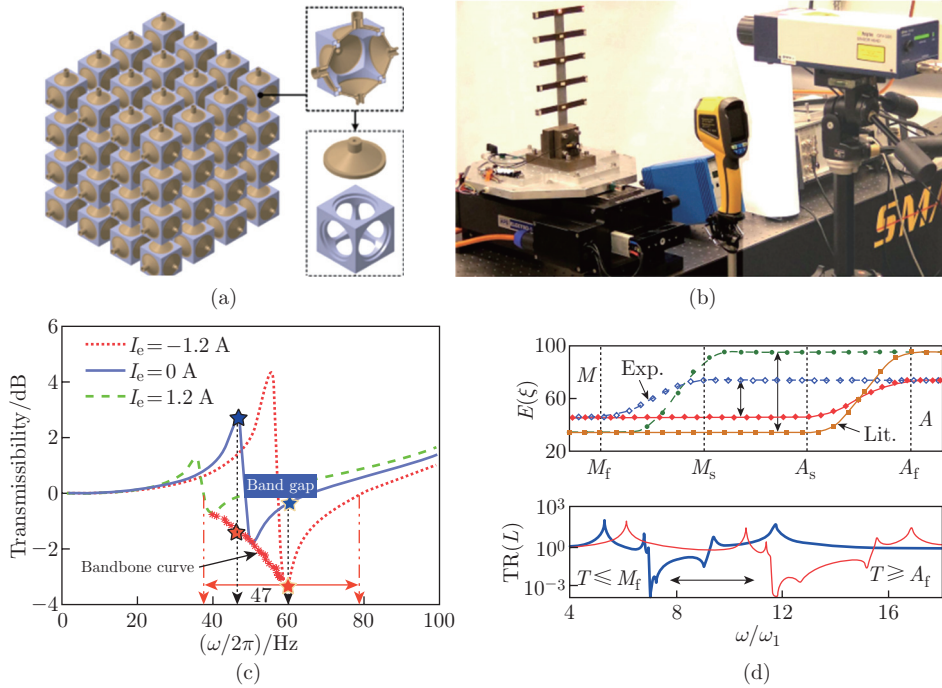


Fig. 7 Fundamental configurations of the metamaterial composed by (a) gasbags (reprinted with permission from Ref. [30], 2020, Elsevier) and (b) shape memory alloy (reprinted with permission from Ref. [93], 2018, AIP Publishing). Wave propagation features of the metamaterial influenced by (c) the current (reprinted with permission from Ref. [73], 2021, Elsevier) and (d) the temperature (reprinted with permission from Ref. [93], 2018, AIP Publishing) (color online)

The schematic diagram of a metamaterial consisted by gasbags is illustrated in Fig. 7(a). Controlling the pressure of the gasbags is the fundamental mechanism to vary the structural stiffness of the resonator^[30,92]. With the increase in the pressure, the structural stiffness of the gasbags augments accordingly. Due to the dependence of the center frequency on the structure stiffness of the resonator, the center frequency of the band gap can be tuned easily by varying the pressure^[95]. Furthermore, the stiffness can also be tuned by changing the temperature for the resonator made of the shape memory alloy, as shown in Fig. 7(b). The temperature can affect the structural stiffness of the resonator and thus tune the band gap. Specifically, the transformation between the martensitic phase and the austenitic phase induced by the temperature variation gives rise to the elastic modulus change of the shape memory alloy, which is the underlying reason for the variation of the resonator stiffness^[93].

3.3 Improvement of wave suppression performance

Generally, the bandwidth gets narrower, and the wave suppression becomes less effective, as the center frequency of the LR band gap decreases. Consequently, the wave manipulation in the low-frequency range is challengeable for the LR metamaterial. In the ordinary way, the major approaches used to broaden the low-frequency bandwidth include devising multi-degree-of-freedom resonator^[96–101], introducing an inerter into the local resonator^[102–103], establishing coupling between different resonators^[104–108], devising the graded local resonator by varying the mass and/or the stiffness of the resonator regularly^[83,109–110], and loading axial forces on the primary structure^[111]. Although these efforts belong to passive approaches, they have achieved an improvement of broadening the bandwidth of the low-frequency band gap.

Aside from the attempts of broadening the bandwidth in the low-frequency range, modulating the wave propagations in the pass band region (permitting propagation of the elastic wave) is another way to improve the wave suppression performance. In general, mounting resonators on a primary structure will introduce more resonant peaks, that is, the wave propagation will be enlarged in the frequency ranges outside the band gaps. Suppressing the resonant peaks, therefore, is a practicable alternative to improve the wave suppression performance in the low-frequency range as well, and is beneficial to the engineering application of the metamaterials. Aimed at this issue, a nonlinear metamaterial is devised by replacing the linear springs in the resonator as nonlinear ones. The results show that resonant peaks in the frequency range between two band gaps are attenuated obviously owing to the bridging coupling effect^[26,112–116]. Such a frequency range is called the chaotic band gap, which provides a promising way to suppress the wave propagation in a much larger frequency range. The chaotic band gap induced by the bridging coupling is dependent on the excitation amplitude, which would limit its engineering applications. To break the dependence of the chaotic band gap on the excitation amplitude, a linear metamaterial with dipolar resonance is devised and provides an avenue to obtain a bridging coupling uncorrelated with the excitation amplitude^[117].

Here, it should be pointed out that the wave suppression performance in the band gap realized by changing the resonator mass is superior to that by changing the resonator stiffness^[70]. Due to the limited supporting capacity of a traditional local resonator with a low stiffness, nevertheless, its stability is likely to be disrupted regardless of changing the resonator mass or the resonator stiffness within a large range. Therefore, it is difficult for the traditional active metamaterial to tune the elastic wave band gap in a large frequency range. To achieve the aim of tuning the band gap in a large frequency range, it is a promising direction to develop the active quasi-zero-stiffness metamaterial, and thus to fulfill low-frequency band gap tuning.

4 Potential applications

The band gap that is capable of suppressing the wave propagation is an important property of the metamaterial. The first application of metamaterials, therefore, is to suppress the vibration propagation along the engineering structures based on the band gaps. It is found that the vibration can be attenuated by more than 20 dB in the frequency range of the band gap^[80,83,105,107,118–119]. Due to the remarkable performance of vibration attenuation, metamaterials have been introduced into the vibration isolation floating raft to improve the vibration attenuation and acoustic stealthy performance of underwater vehicles^[120]. Under the condition of multiple-frequency excitation, the introduced metamaterial displays a prominent performance for attenuating the vibration propagation along the floating raft system. Additionally, metamaterials were utilized to control the vibration of solar arrays as well^[121]. Results indicate that the proposed metamaterial is capable of suppressing the vibration of the solar arrays in multiple frequency domains, which provides a promising approach to improve the safety performance of the solar arrays.

Besides attenuating the vibration by metamaterials, executing logic operations (gate/switch/

cascade) for information processing is another application of the metamaterials^[122]. The operation principle of the mechanical logic gate composed of metamaterials is in line with the electronic one. The realization forms in the mechanical computation are highly varying, including coil springs^[122], bistable series connected by soft coupling elements^[123], and conductive polymers^[124]. Whatever the realization configuration is, the devised metamaterials have been proved effective to execute some simple calculations. Aside from the application in the mechanical computation, devising wave guides to achieve a directed transmission of elastic wave along a designed path is another important application of metamaterials^[75]. Actually, there are many other applications of metamaterials, such as energy harvesting^[118] and acoustic black hole^[125].

5 Conclusions and prospects

Metamaterials are important tools for suppressing the low-frequency vibration propagation along the engineering structures by means of band gaps. This review summarizes some primary structures that are utilized to decrease the band frequency and are classified based on the constituent elements, operating principles, and supporting capacity. The categories of these metamaterials include spring-mass mechanisms, supportless configurations, pentamode metamaterials, elastic materials/structures, and quasi-zero-stiffness mechanisms. The advantages and disadvantages of these configurations were dissected briefly to quest new approaches to realize and optimize (broaden the bandwidth and improve the vibration suppressing performance) band gaps in the frequency range below 100 Hz. Additionally, two kinds of basic strategies for tuning the low/ultralow-frequency band structures, based on stiffness or inertia, were summarized, and followed by reviewed contents about attempts for the improvement of wave suppression in the range of band gap. Some potential applications of metamaterials in the low/ultralow-frequency range are incorporated into this review as well.

Even though various configurations and strategies have been devised and presented to obtain and tune the low-frequency elastic wave, the space-hogging and complex structure limit the engineering application of the metamaterial. As the band gap shifts from the high-frequency range to a low-frequency one, the bandwidth gradually becomes narrower and the wave manipulation performance within the band gap range gets less effective, which is also a large obstacle to the engineering application of the metamaterial. Therefore, the design of a compact and light structure, the formulation mechanism of an ultra-wide band gap with excellent wave manipulation performance and the strategy of fast, flexible, and real-time control should be focused in the future research on elastic wave metamaterials.

It is believed that some novel configurations and even new band formation mechanisms will be explored according to the occurring research works, and then almost perfect band gaps in the frequency range below 100 Hz can be realized. Metamaterials will eventually be successfully used for vibration or wave manipulation in marine, transportation, aviation, and other fields.

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