ARTICLES

A Car Crash Solved—with a Swiss Army Knife

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Look at the accident photo in FIGURE 1. How fast was the white car going? The question has more than academic interest to the author, who once had the experience of being "T-boned" in a car crash. The focus of this article is on the key to unlocking this mystery—a little known gem called Eves's theorem, which is a kind of Swiss Army knife of projective geometry. We'll not only use it to find the speed of the car, we'll use it to revisit classic theorems, illustrate the concept of the geometric mean, and look at windows and other everyday objects in new ways.

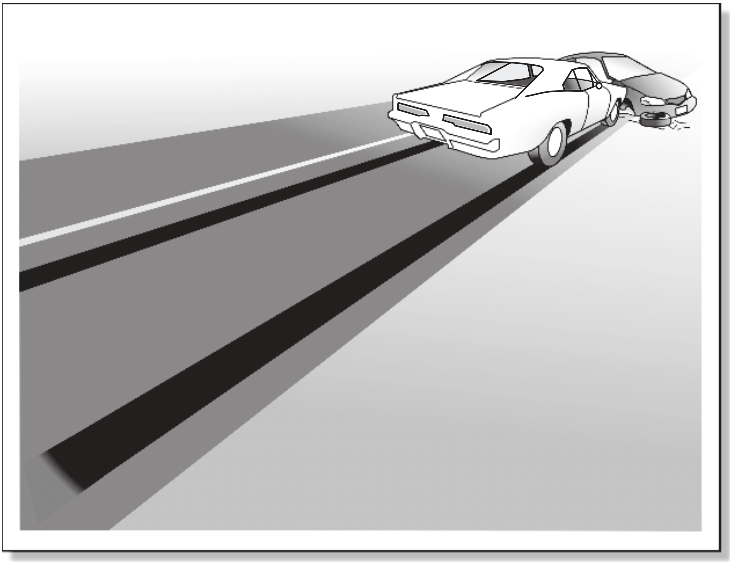

Figure 1 Snapshot of an accident scene. How fast was the white car going?

But first, we refine the car crash question by providing a story to go with the picture, and a little basic physics.

Speed from skid marks

The story goes as follows. The white car and the gray car were headed toward each other in opposite lanes, when the gray car made a left turn in front of the white car

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to enter a parking lot. Upon seeing the gray car making the turn, the driver of the white car slammed on its brakes, locking up its wheels, while managing to hold the car straight in a skid. Unfortunately, the driver of the white car was not able to come to a full stop before striking the gray car in the side as shown. Shortly afterward, a witness snapped the photo in FIGURE 1, just far back enough to show an entire skid mark. The road was repaved a few days later, leaving the photo as the only evidence of the skid marks.

This became important when a dispute arose between the drivers. The driver of the gray car claimed that the white car, a 1969 Dodge Charger, was exceeding the posted speed limit of 35 mi/hr, a claim which the driver of the Charger denied. In addition to the evidence of the photograph, an accident investigator inspected the damaged cars and estimated the speed of the white car at 25 mi/hr at the moment of impact.

There is good news for the driver of the white car: we will give a reasonable analysis that puts an upper bound of 33 mi/hr on the speed of the car at the moment the skid began. We will describe one method of determining the speed—although methods vary in practice—but our greatest emphasis is on showing how Eves's theorem can be used to determine the length of the skid, which is of prime importance in any such analysis.

A little Web searching shows that there are many engineering firms that specialize in accident reconstruction, including skid mark analysis (at least one company provides a "skid speed calculator" [**5**]). We begin by reviewing the problem-solving principles most commonly used when the length of at least one skid mark is known; then we use a rather uncommon method to determine the length of a skid mark in the photograph.

The top part of FIGURE 2 shows a side view of the white car, a 1969 Dodge Charger, along with its skid marks and the specification of its wheelbase (axle-to-axle distance) of 117 inches, or 9.75 feet. The bottom part of FIGURE 2 shows a bird's-eye view of the skid marks, along with a dashed triangle $\triangle ACE$, whose purpose we explain later. The skid mark of the right front tire of the white car ends at point C, but its starting point is obscured by the skid mark of the right rear tire. The skid mark of the right rear tire begins at point A and ends at point B. The distance $|BC|$ is therefore equal to the wheelbase of 9.75 feet. The skid mark of the right rear tire has length $|AB|$. This is the only unobscured skid mark in the witness's photograph.

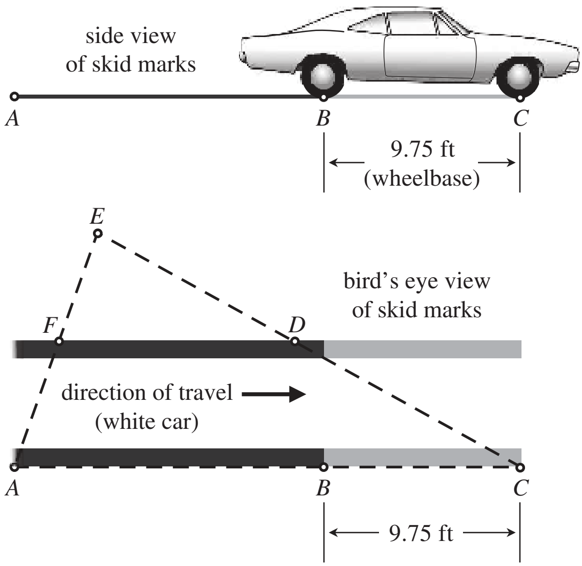

Figure 2 A side view and a bird's-eye view of the skid marks.

Although we don't know the lengths of the other skid marks, it is reasonable to assume they all have length $|AB|$. Let the car have mass m, let v_A denote the car's speed when the right rear tire was at point A, and let v_B denote the car's speed when the right rear tire was at point B. From our earlier information we have an estimate of the impact speed, $v_B \approx 25$ mi/hr, but for the time being we will work with length and time units of feet and seconds. We assume that the road is level, and that during the skid the only external horizontal force acting on the car is the constant deceleration force $\mu m g$, where $\mu \geq 0$ is the dimensionless coefficient of sliding friction between tires and road, and $g (\approx 32.174 \text{ ft/s}^2)$ is the acceleration of gravity.

We take the common approach of idealizing the car as a point mass m in rectilinear motion with constant acceleration (see [**4**, pp. 101–102] for example). We assume readers are familiar with two equations from that theory, namely,

$$v - v_0 = at \quad \text{and} \quad x - x_0 = v_0 t + \frac{1}{2} a t^2,$$

where x and v are the position and velocity at time t of a particle moving on the x-axis with constant acceleration a, and x_0 and v_0 are the position and velocity at time $t = 0$. By eliminating t between these two equations, we get

$$v_0^2 = v^2 - 2a(x - x_0). \tag{1}$$

This is also a basic equation in the theory of rectilinear motion (see [**4**, Eq. (3-16)] or [**6**, Eq. (3-17)].)

To express (1) in terms of our variables, let the x-axis coincide with the line AB in FIGURE 2, with the origin fixed anywhere, and the positive direction to the right. Denote the x-coordinates of A and B by x_A and x_B, respectively. We model the car as a point mass m that moves from x_A at time $t = 0$ to x_B at time t, under a constant acceleration $-\mu g$. Referring to (1), let $x_0 = x_A$, $x = x_B$, $v_0 = v_A$, $v = v_B$, and $a = -\mu g$. Equation (1) then becomes

$$v_A^2 = v_B^2 + 2\mu g(x_B - x_A),$$

or equivalently,

$$v_A^2 = v_B^2 + 2\mu g|AB|. \tag{2}$$

For computational convenience, it is common to express equation (2) in a hybrid form, with v_A and v_B expressed as respective miles-per-hour speeds $\hat{v}_A$ and $\hat{v}_B$, and $|AB|$ expressed in feet. The conversion factor is $k = (3600 \text{ s/hr})/(5280 \text{ ft/mi})$, so we multiply equation (2) by k^2 to obtain

$$\hat{v}_A^2 = \hat{v}_B^2 + 2k^2\mu g|AB|. \tag{3}$$

At this point we need a value for $2k^2\mu g$. Since we are interested in an upper-limit value of $\hat{v}_A$, we use $\mu = 1$, a widely accepted upper bound for this application. We compute

$$2k^2\mu g \leq 2\left(\frac{3600}{5280}\right)^2 (1)(32.174) \approx 29.91,$$

which we round up to 30, again in the interest of obtaining an upper limit. (Readers will find that this constant 30, whose units are $\text{mi}^2\text{ft}^{-1}\text{hr}^{-2}$, appears in many of the basic skid mark analyses on the Internet.) Substituting 30 for $2k^2\mu g$ in (3) and then taking the square root of both sides, we obtain

$$\hat{v}_A < \sqrt{\hat{v}_B^2 + 30|AB|}.$$

Using the investigator's estimate of $\hat{v}_B \approx 25$ mi/hr, this becomes

$$\hat{v}_A < \sqrt{625 + 30|AB|}, \tag{4}$$

where again, $\hat{v}_A$ is in units of miles per hour, and $|AB|$ is in feet. The inequality (4) gives an upper bound $\hat{v}_A$ on the speed of the white car when the skid began, based on the length $|AB|$ of the skid mark visible in the photo.

Everything depends on the length of that skid mark.

Skid marks from Eves's theorem

In the next section we discuss Eves's theorem in detail. In this section we emphasize how easy it makes finding the skid mark length $|AB|$, and hence the speed of the car.

In FIGURE 2 we have drawn a dashed triangle $\triangle ACE$, where A and C are as described earlier, and E is an arbitrary point on the far side of the skid marks from AC. Side AC contains the point B mentioned earlier, and D and F are the points where the respective sides CE and EA meet the outside edge of the car's left skid mark. Since the skid marks are parallel, we have $|CD|/|DE| = |FA|/|EF|$, hence

$$\frac{|AB|}{|BC|}\frac{|CD|}{|DE|}\frac{|EF|}{|FA|} = \frac{|AB|}{9.75\text{ ft}} \cdot 1 = \frac{|AB|}{9.75\text{ ft}}. \tag{5}$$

The expression on the left hand side of (5) is an example of a *circular product* [**7**]. Given a closed polygon with a point in the interior of each side, one forms a circular product by alternately dividing and multiplying consecutive segment lengths, proceeding clockwise or anticlockwise around the polygon. As discussed in the next section, Eves's theorem implies that circular products are *projectively invariant*, which means that if the points $A, B, C \ldots$ are mapped projectively (as in say, a photograph) to distinct, respective points $A', B', C', \ldots,$ then the corresponding circular product will have the same value. Specifically, we locate in FIGURE 3 the corresponding points $A', B', C', \ldots$ as they would appear in the photograph. We need not worry about locating E' perfectly, because it must correspond to *some* preimage point E as in FIGURE 2.

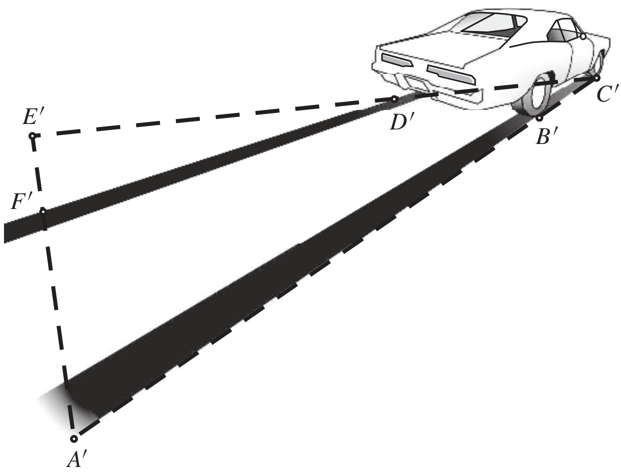

Figure 3 The corresponding image of the triangle in FIGURE 2, as it would appear in the accident photo. For clarity we show the entire side $C'E'$, rather than having part of it disappear under the car.

Having done this, we now use a ruler to measure directly on the photograph in FIGURE 3, and estimate the numerical value of the corresponding circular product. The author did this on a larger image and computed value of just under 1.5 (readers' results will of course vary). Using (5) and the invariance of circular products, we then have

$$\frac{|AB|}{9.75\text{ ft}} = \frac{|AB|}{|BC|}\frac{|CD|}{|DE|}\frac{|EF|}{|FA|} = \frac{|A'B'|}{|B'C'|}\frac{|C'D'|}{|D'E'|}\frac{|E'F'|}{|F'A'|} < 1.5.$$

Hence, rounding up again, we obtain

$$|AB| < (9.75\text{ ft})(1.5) < 15\text{ ft}.$$

Finally, we substitute this result into (4) and round up once more to estimate the white car's speed $\hat{v}_A$ at the beginning of the skid:

$$\hat{v}_A < \sqrt{625 + 30(15)} < 33\text{ mi/hr}.$$

We therefore conclude that the white car was probably *not* exceeding the posted speed limit of 35 mi/hr when the skid began.

If, on the other hand, we were to work on behalf of the driver of the gray car, we would estimate a *lower* bound for v_A, hoping that it would be significantly greater than 35 mi/hr. This is a bit trickier; for example, we would want to estimate a lower bound on the coefficient of friction. Choosing a value of 0 would be unconvincing, so we would need to be more realistic in this case. A realistic estimate of the coefficient of friction depends on the condition of the road surface, the weather, the brand of tires, the condition of the tires, and other factors. Because our emphasis was on the determination of skid mark lengths from a photograph—a key factor in either case—we chose the simpler problem of estimating an upper bound on v_A.

The science of determining 3-D information from the 2-D images in photographs is called *photogrammetry*. Many firms that do accident reconstruction use specially made photogrammetry software to determine skid mark lengths from photographs. In the preceding problem, Eves's theorem allowed us to determine the skid mark length, and thus the car's speed, by simply drawing a triangle on the image of the skid marks and measuring between certain marker points. Unlike other methods for solving such problems (for example, see [**3**]), we did not need to determine the horizon line, vanishing points, or the viewpoint of the photograph.

Eves's theorem—a Swiss Army knife

Having discussed one application of Eves's theorem, it is time to back up a bit and properly discuss the theorem itself. It is a rare occurrence when an important theorem in a field of mathematics goes largely unnoticed (even by many experts in the field) and more amazingly, makes its debut in the pages of a geometry textbook for students with a background in high school mathematics. Nevertheless, that is the case with Eves's theorem. Howard W. Eves (1911–2004) was for many years a professor at the University of Maine and editor of the Elementary Problems section of the *American Mathematical Monthly*. In section 6.1 of his book *A Survey of Geometry* [**1**], Eves presented this simple but powerful theorem, which can be understood and appreciated by anyone. As the distinguished geometer G. C. Shephard wrote [**7**, p. 1280],

> *We feel that Eves' theorem has never been given the recognition it deserves and should be regarded as one of the fundamental results of projective geometry.*

Before stating the theorem, we discuss some terminology. Given two planes π, π' in space and a point O not on either of them, the map that assigns to each point $P \in \pi$ the point $P' \in \pi'$ such that P, P', and O are collinear is called a *perspectivity with center* O. (When OP is parallel to π' it is customary to map P to a *point at infinity*, but we will not need to worry about this in our examples.) The point P' is called the *perspective image* of P. Similarly we can talk about the perspective image in π' of an entire set in π. The two special cases we have in mind are shown in FIGURE 4.

In FIGURE 4(a) the gray arrows in planes π and π' are related by a perspectivity with center O that lies on the opposite side of π' from the arrow in π. We think of a light ray emanating from each point P of the arrow in π, traveling to a viewer's eye at O in a straight line, and on its way piercing the plane π' at the corresponding point P', like passing through a window and leaving an appropriately colored dot on the glass. The arrow on plane π' is the perspective image of that on π. This is the model for perspective drawing and painting developed in the Renaissance. The idea is that if the arrow on π suddenly disappeared, the viewer at O would be unaware of it, because the light rays from the colored dots on π' would still be coming from the same directions as before. Hence (theoretically at least) the painted image on π' is perfectly realistic, as long as the viewer's eye stays at the viewpoint O.

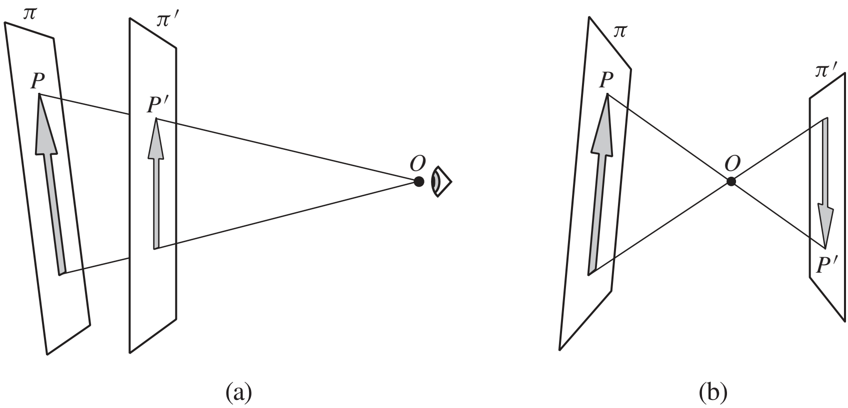

Figure 4 Perspectivities model perspective drawing (a) and photography (b).

FIGURE 4(b) is a simplified model of the photographic process. Here O lies between each pair of corresponding points P, P', causing the perspective image on π' to be inverted. In this model, we think of O as the hole of a pinhole camera, and the line PP' as a light ray passing through it. The arrow on π is an object in the real world, and the inverted image on π' (the screen of the pinhole camera) is its photographic image. Although the structure and function of a lens camera is more complex, to a good approximation the end result is the same—an upside-down perspective image of the given object. Thus for our purposes, we can model both perspective drawing and photography of plane figures with appropriate perspectivities. A composition of perspectivities—for example, a photograph of a photograph—is called a *projectivity*.

We should note that in either case in FIGURE 4, if the planes π and π' are parallel, then the figures in the two planes—the object and its image—are similar. That is, the drawing or photograph is an undistorted likeness of the object, except possibly for resizing. This is the case of the bird's eye view of the skid marks in FIGURE 2.

More generally, however, perspective images are not similar to the real world objects they portray, and it may take some work to recover geometric information about the original object. Eves's theorem shows that there are certain numerical regularities in geometric objects that are *not* changed by the photographic process, such as the circular products associated with the triangles in FIGURES 2 and 3.

There is one other term to explain before stating Eves's theorem. Eves's theorem deals with expressions like the circular product on the left hand side of equation (5), except that each distance such as $|AB|$ is considered to be a *directed distance*, meaning that a positive direction is arbitrarily assigned to the line AB, so that $|AB|$ and $|BA|$ have the same magnitude but opposite signs. This arbitrariness disappears in the expressions under consideration, because every directed distance is divided by, or is divided into, one that is collinear with it. We can therefore think of dealing with signed ratios of collinear pairs of directed distances, a ratio being positive if the two distances are parallel and negative if they are antiparallel. If all collinear pairs have the same direction, then we can simply work with ordinary distances. In [**1**] Eves himself does not explain the origin of the term "h-expression" which follows (but one might think of using it as a mnemonic: "h" for "Howard").

DEFINITION. A product of ratios of directed distances, where all the indicated points lie in one plane, is called an h-expression if it has the following properties:

(1) In each ratio the points that occur are collinear.

(2) Each point appears in the numerator of the product exactly as many times as it does in the denominator.

EVES'S THEOREM. *The value of an h-expression is invariant under any projectivity.*

Observe that the circular product in (5) is an h-expression, which justifies its treatment as a projective invariant in the car problem. Similarly, the well-known cross ratio of four points on a line is an h-expression, hence its projective invariance is a special case of Eves's theorem. Shephard made this observation in [**7**], along with some original applications of the theorem.

This is a significant fact—that the projective invariance of the cross ratio is just a special case of Eves's theorem. We therefore briefly review the cross ratio, and give a simple application of it. The *cross ratio* (AB, CD) of four collinear points A, B, C, D is given by

$$(AB, CD) = \frac{|AC|}{|CB|}\frac{|BD|}{|DA|}, \tag{6}$$

where each quantity such as $|AC|$ is a directed distance. The reader can easily check that (6) is an h-expression. It's important to note that the value of the cross ratio depends not just on the location of the four points, but also on the order of the labels A, B, C, D. Although there are $4!(= 24)$ ways to apply the labels to four given points, it's well known that the cross ratio runs through either 3 or 6 different values as the labels are permuted, depending on the points the labels are being applied to.

For practice, we apply the cross ratio to the perspective drawing of a fence in FIGURE 5. Suppose that in some units of length, the distances between the images of the tops of the fenceposts are as indicated in the figure: $|A'B'| = 10$ and $|B'C'| = 6$. Writing $x = |C'D'|$ as in the figure, we ask: what is the value of x?

To answer the question, we let the positive direction be to the right along the fence rail in FIGURE 5, and we use (6) to compute

$$(A'B', C'D') = \frac{|A'C'|}{|C'B'|}\frac{|B'D'|}{|D'A'|} = \frac{16}{-6}\cdot\frac{x+6}{-(x+16)} = \frac{8x+48}{3x+48}. \tag{7}$$

Next we focus on the side view of the fence in FIGURE 6. We label the tops of the fence posts A, B, C, D, and consider the respective points A', B', C', D' in FIGURE 5

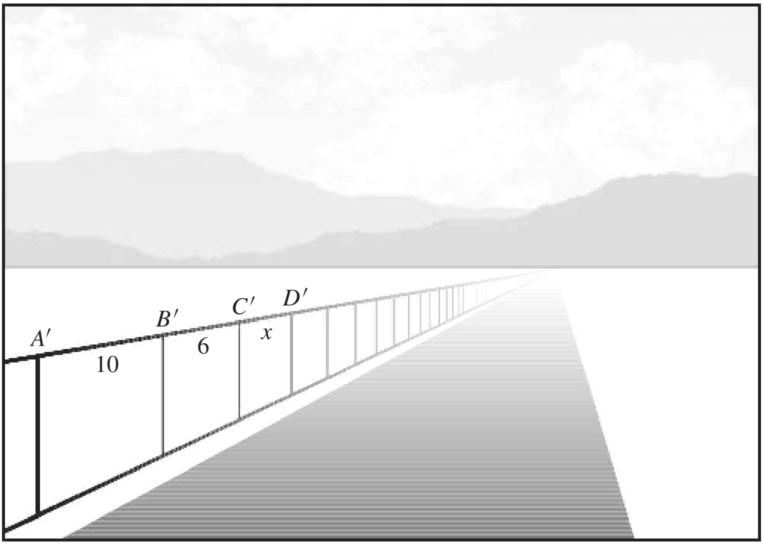

Figure 5 If $|A'B'| = 10$ and $|B'C'| = 6$, then what is $|C'D'|$?

to be their perspective images. Assuming that the fence posts are equally spaced, we indicate in FIGURE 6 that $|AB| = |BC| = |CD| = w$ for some positive number w. Again letting the positive direction be to the right, we have by (6),

$$(AB, CD) = \frac{|AC|}{|CB|}\frac{|BD|}{|DA|} = \frac{2w}{-w} \cdot \frac{2w}{-3w} = \frac{4}{3}. \tag{8}$$

The projective invariance of the cross ratio implies that the results of (7) and (8) are equal, hence

$$\frac{8x + 48}{3x + 48} = \frac{4}{3}.$$

Solving this for x, we get $x = 4$. (Observe that if only the first three fence posts were drawn in FIGURE 5, we could locate and draw the rest of them by recursively applying this method to the last two known distances between their tops.)

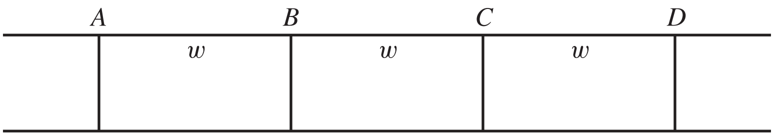

Figure 6 Side view showing equal spacing of the fence posts.

Of course Professor Eves was not unaware of the utility of his theorem. For example, he showed [**1**, p. 290] how to apply the theorem to the proof of the classic theorems of Ceva and Menelaus. Ceva's theorem says that if points D, E, F lie on the respective sides BC, CA, AB of a triangle $\triangle ABC$ as in FIGURE 7(a), the lines AD, BE, CF are concurrent at a point G if and only if the corresponding circular product around the triangle satisfies

$$\frac{|AF|}{|FB|}\frac{|BD|}{|DC|}\frac{|CE|}{|EA|} = 1.$$

Menelaus' theorem says that the previously mentioned points D, E, F are collinear as in FIGURE 7(b) if and only if

$$\frac{|AF|}{|FB|}\frac{|BD|}{|DC|}\frac{|CE|}{|EA|} = -1.$$

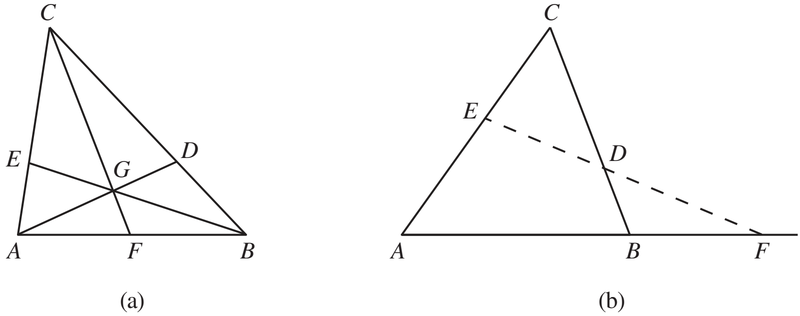

Figure 7 Diagrams for the theorems of Ceva (a) and Menelaus (b).

To give an example of how Eves's theorem applies, Eves proved [**1**, p. 288] that given a configuration like that in FIGURE 7(a), there exists a perspectivity that maps the points $A, B, \ldots$ to respective points $A', B', \ldots,$ such that G' is the centroid of $\triangle A'B'C'$. Since the lines $A'D'$, $B'E'$, $C'F'$ are concurrent at G', the points D', E', F' are midpoints of their respective sides. The "only if" part of Ceva's theorem then follows from an application of Eves's theorem; given the concurrency at G in FIGURE 7(a), we have

$$\frac{|AF|}{|FB|}\frac{|BD|}{|DC|}\frac{|CE|}{|EA|} = \frac{|A'F'|}{|F'B'|}\frac{|B'D'|}{|D'C'|}\frac{|C'E'|}{|E'A'|} = (1)(1)(1) = 1.$$

For projective geometry enthusiasts, we give the following hint for using Eves's theorem to prove the "only if" part of Menelaus' theorem. Suppose the points D, E, F are collinear in FIGURE 7(b). Then what line can be projectively mapped to infinity to give the result

$$\frac{|A'F'|}{|F'B'|} = \frac{|B'D'|}{|D'C'|} = \frac{|C'E'|}{|E'A'|} = -1?$$

The geometric mean in perspective

It is well known that the geometric mean $\mathrm{GM}(x_1, \ldots, x_n)$ of n nonnegative numbers $x_1, \ldots, x_n$ is defined as

$$\mathrm{GM}(x_1, \ldots, x_n) = \left(\prod_{i=1}^{n} x_i\right)^{1/n}.$$

Unfortunately for visual thinkers, most of the common pedagogical examples of the geometric mean don't admit a visual interpretation for values of n greater than 2 or 3. Eves's theorem makes it easy to construct an example of the geometric mean that includes a picture for each value of n greater than 2. FIGURE 8(a) shows a perspective image of one regular pentagon inscribed in another, with a little thickness added to suggest say, a decorative tile. Suppose we want to use this picture to construct the undistorted bird's-eye view in FIGURE 8(b), in which the vertices of the smaller pentagon divide each side of the larger one into lengths p and q. Given the perspective view in (a), we could make a scale drawing of the bird's-eye view in FIGURE 8(b) if we could just determine the value of the *edge ratio* p/q. By Eves's theorem, the circular product $\prod_{i=1}^{5}(p_i/q_i)$ associated with FIGURE 8(a) must be equal to the circular product $(p/q)^5$ associated with FIGURE 8(b), hence the edge ratio p/q satisfies

$$\frac{p}{q} = \mathrm{GM}\left(\frac{p_1}{q_1}, \ldots, \frac{p_5}{q_5}\right).$$

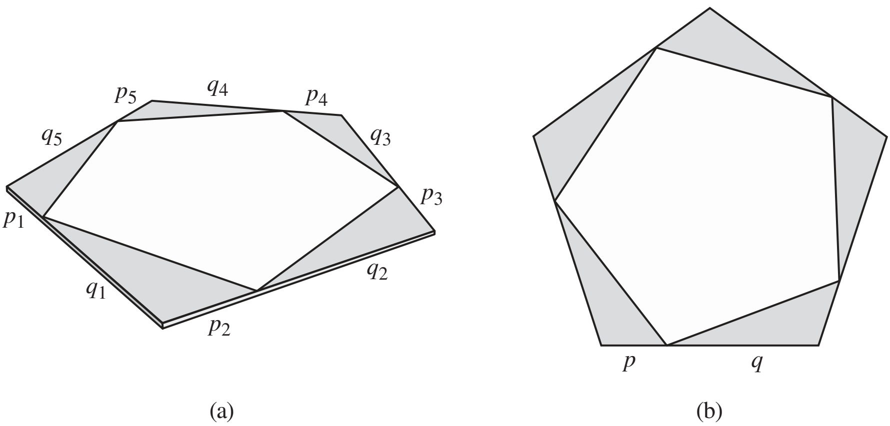

Figure 8 A perspective view (a) and an undistorted bird's-eye view (b) of one regular pentagon inscribed in another. The edge ratio p/q is the geometric mean of the five corresponding edge ratios in the perspective view.

More generally, suppose that for $n \geq 3$ we have a regular n-gon inscribed in a larger one, so that each vertex of the smaller n-gon divides a side of the larger one into lengths p and q. Then, given a perspective drawing or photograph of the configuration in which the image of the ith side of the outer n-gon is divided into corresponding nonzero lengths p_i, q_i for $i = 1, \ldots, n$, the "true" edge ratio p/q is the geometric mean of those in the perspective image:

$$\frac{p}{q} = \mathrm{GM}\left(\frac{p_1}{q_1}, \ldots, \frac{p_n}{q_n}\right). \tag{9}$$

Experimenting with Eves's theorem

A nice feature of Eves's theorem is that it is evident in many of the photographs we see every day in magazines, newspapers, and on the Internet. That's because it is possible to associate h-expressions with many everyday objects such as tile floors, brick walls, windows, parking lot markings, athletic fields, and much more. The exact values of the h-expressions are easy to deduce from the designs of the objects. The goal of such an experiment is to see if the predicted value does indeed result when we make careful measurements on a given photograph. All we need is a photograph of such an object from an interesting angle, a ruler marked in fine graduations such as millimeters, and a little mathematical curiosity.

As a simple example, FIGURE 9 shows a photograph of a large window at a retail store. The inset of the figure is a qualitative front view of the window; the segments HD and BF, which represent the dividers between the windowpanes, are parallel to the sides AC and CE, respectively. It should be easy for the reader to verify that

$$\frac{|AB|}{|BC|}\frac{|CD|}{|DE|}\frac{|EF|}{|FG|}\frac{|GH|}{|HA|}=1,$$

since $|AB|=|FG|$, and so on. In the photograph, the perspective images of these points are labeled with primed versions of the same letters, and according to Eves's theorem, we must have

$$\frac{|A'B'|}{|B'C'|}\frac{|C'D'|}{|D'E'|}\frac{|E'F'|}{|F'G'|}\frac{|G'H'|}{|H'A'|}=\frac{|AB|}{|BC|}\frac{|CD|}{|DE|}\frac{|EF|}{|FG|}\frac{|GH|}{|HA|}=1.$$

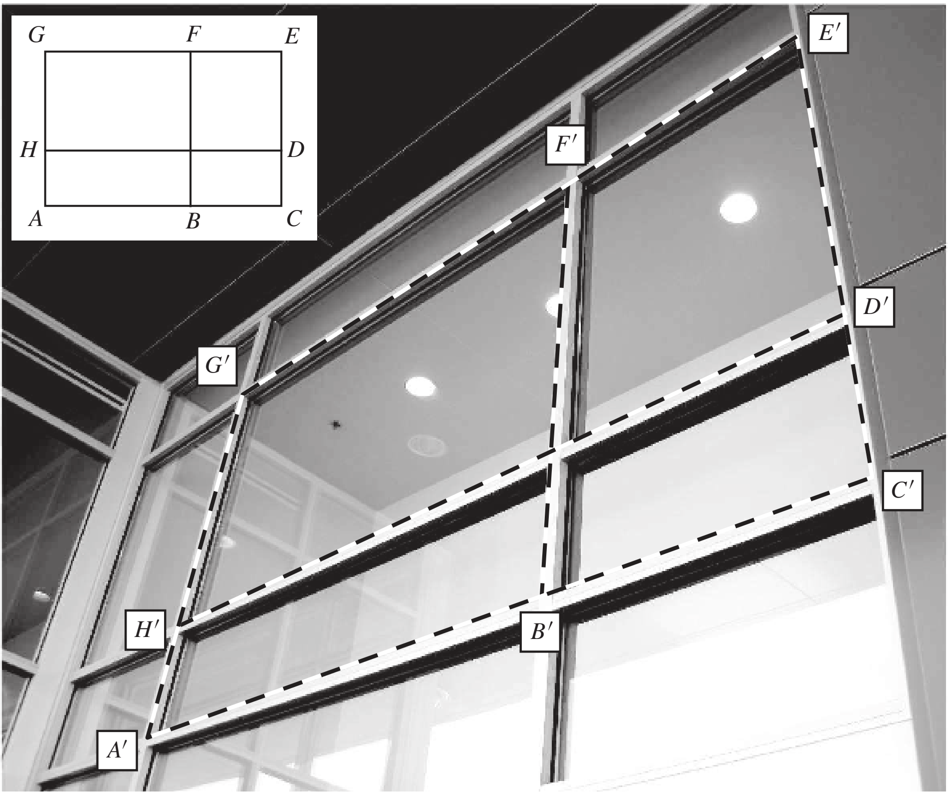

Figure 9 Photograph of a window, and a front view (inset).

We invite the reader to carefully measure the indicated lengths along the dashed lines with, say a ruler marked in millimeters, and then perform the above computation. (Since we must deal with mathematical lines, we drew the dashed lines to represent certain edges of the window frame and the windowpane dividers.) If such measurements are done carefully enough, the result should be reasonably close to 1. Is it?

Exercise Look around you right now. How many objects do you see that you could apply Eves's theorem to?

Conclusion

It is interesting to note that among the pictorial examples of the geometric mean appearing in the literature are some nice ones presented in Professor Eves's last published paper, which appeared in this MAGAZINE [**2**]. We hope the applications presented here, made possible by his theorem, are ones he would have enjoyed.

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Summary Though not well known, Eves's theorem is a fundamental result of projective geometry—a tool as versatile as a Swiss Army knife. Named for the late Howard W. Eves (1911–2004), the theorem establishes a class of numerical projective invariants, of which the famous cross ratio is a special case. We illustrate the versatility of Eves's theorem by applying it to accident scene reconstruction, to the circular products in the theorems of Ceva and Menelaus, and to perspective illustrations of the geometric mean. In addition, we show that the theorem is illustrated by everyday photographs of buildings and other common objects.

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