A Case for Variable-Range Transmission Power Control in Wireless Multihop Networks

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***Abstract*— In this paper, we study the impact of individual variable-range transmission power control on the physical and network connectivity, network capacity and power savings of wireless multihop networks such as ad hoc and sensor networks. First, using previous work by Steele [16] and Gupta [7] we derive an asymptotic expression for the average traffic carrying capacity of nodes in a multihop network where nodes can individually control the transmission range they use. For the case of a path attenuation factor $\alpha = 2$ we show that this capacity remains constant even when more nodes are added to the network. Second, we show that the ratio between the minimum transmission range levels obtained using common-range and variable-range based routing protocols is approximately 2. This is an important result because it suggests that traditional routing protocols based on common-range transmission can only achieve about half the traffic carrying capacity of variable-range power control approaches. In addition, common-range approaches consume $\sim (1 - \frac{2}{2^\alpha})$ % more transmission power. Second, we derive a model that approximates the signaling overhead of a routing protocol as a function of the transmission range and node mobility for both route discovery and route maintenance. We show how routing protocols based on common-range transmission power limit the capacity available to mobile nodes. The results presented in the paper highlight the need to design future wireless network protocols (e.g., routing protocols) for wireless ad hoc and sensor networks based, not on common-range which is prevalent today, but on variable-range power control.**

I. Introduction

Effective transmission power control is a critical issue in the design and performance of wireless ad hoc networks. Today, the design of packet radios and protocols for wireless ad hoc networks are primarily based on common-range transmission control. We take an alternative approach and make a case for variable-range transmission control. We argue that variable-range transmission control should underpin the design of future wireless ad hoc networks, and not, common-range transmission control. In this paper, we investigate the tradeoffs and limits of using a common-range transmission approach and show how variable-range transmission control can improve the overall network performance. We analyze the impact of power control on the connectivity at both the physical and network layers. We compare how routing protocols based on common-range and variable-range transmission control techniques impact a number of system performance metrics such as the connectivity, traffic carrying capacity, and power conserving properties of wireless ad hoc networks.

Power control affects the performance of the physical layer in two ways. First, power control impacts the traffic carrying capacity of the network. On the one hand, choosing too high a transmission power reduces the number of forwarding nodes needed to reach the intended destination, but as mentioned above this creates excessive interference in a medium that is commonly shared. In contrast, choosing a lower transmission power reduces the interference seen by potential transmitters but packets require more forwarding nodes to reach their intended destination. Second, power control affects how connected the resulting network is. A high transmission power increases the connectivity of the network by increasing the number of direct links seen by each node but this is at the expense of reducing network capacity. In this paper, we consider the use of variable-range transmission control to allow nodes to construct a minimum spanning tree (MST). We show that the use of a minimum spanning tree can lead toward lower total weight than a tree based on common-range transmission links that minimally avoid network partitions.

The type of power control used can also impact the connectivity and performance of the network layer. Choosing a higher transmission power increases the connectivity of the network. In addition, power control impacts the signaling overhead of routing protocols used in mobile wireless ad hoc networks. Higher transmission power decreases the number of forwarding hops between source-destination pairs, therefore reducing the signaling load necessary to maintain routes when nodes are mobile. The signaling overhead of routing protocols can consume a significant percentage of the available resources at the network layer, reducing the end user's bandwidth and power availability.

Existing routing protocols discussed in the mobile ad hoc networks (MANET) working group of the IETF [9] are designed to discover routes using flooding techniques at common-range maximum transmission power. These protocols are optimized to minimize the number of hops between source-destination pairs. Such a design philosophy favors connectivity to the detriment of potential power-savings and available capacity. Modifying existing MANET routing protocols to promote lower transmission power levels in order to increase network capacity and potentially higher throughput seen by applications, is not a trivial nor viable solution [4]. For example, lowering the common transmission power forces MANET routing protocols to generate a prohibitive amount of signaling overhead to maintain routes in the presence of node mobility. Similarly, there is a minimum transmission power beyond which nodes may become disconnected from other nodes in the network. Because of these characteristics MANET routing protocols do not provide a suitable foundation for capacity-aware and power-aware routing in emerging wireless ad-hoc networks.

The main contribution of this paper is that it confirms the need to study, design, implement and analyze new routing protocols based on variable-range transmission approaches that can exploit the theoretical power savings and improved capacity indicated by the results presented in this paper. The structure of this paper is as follows. Section II studies the impact of power control on the physical layer. In Section III, we extend our analysis to the network layer and consider mobility. In particular, we investigate and model the signaling overhead of a common-range transmission based routing protocol considering both route discovery and route maintenance. In Section IV, we present numerical examples to further analyze the models derived in Sections II and III. Section V discuses our results and their implication on the design of future protocols for wireless ad hoc networks. Finally, we present related work in Section VI and some concluding remarks in Section VII

II. PHYSICAL CONNECTIVITY

We represent a wireless ad hoc network as a graph as a means to discuss several results of interest. Consider a graph M with a vertex (e.g., node) set $V = \{x_1, x_2, ..., x_n\}$ and edge (e.g., link) set $E = \{(x_i, x_j)\} : 1 \leq j \leq n$ for $x_i \ \epsilon\ \Re^d, 1 \leq i \leq n$[1]. Here the length of an edge $e = (x_i, x_j) \ \epsilon\ E$ is denoted by $|e|$, where $|e| = |x_i - x_j|$ equals the Euclidean distance from x_i to x_j.

Vertex or nodes in M are allowed to use different transmission power levels P to communicate with other nodes in their neighborhood, $P_{min} \leq P \leq P_{max}$. Connectivity from node x_i transmitting at power P_i to node x_j exists, if and only if $S_j > S_0$, where S_j is the received power at node j and S_0 is the minimum receive power necessary to receive a packet correctly. In this paper we model the received signal using a traditional decay function of the transmitted power, e.g., $S_j \sim \frac{P_i}{|x_i - x_j|^\alpha}$, where $2 \leq \alpha \leq 4$. It is important to note that any propagation model can also be incorporated without modifying the applicability and accuracy of the analysis and results that follow. In the rest of the paper we will use transmission range rather than transmission power for convenience.

Definition: A *route or walk* from node u to node v is an alternating sequence of nodes and links, representing a continuous traversal from node u to node v.

Definition: A graph M is connected if for every pair of nodes u and v there is a walk from u to v.

[1]In this paper, we use the terms edge and link, and vertex and node interchangeably.

Definition: The transmission range of node i transmitting with power P_i, denoted R_i, is the maximum distance from node i where connectivity with another node exists.

Definition: The common transmission range of nodes transmitting with a common transmission power P_{com}, denoted R_{com}, is the maximum distance where two nodes can communicate with each other.

A. Common-range Transmission Control

We analyze the case where all nodes use a *common* transmission range (R_{com}) to communicate with peer nodes in the network. This case is of particular importance because a common transmission range approach is the foundation of most routing protocols in ad hoc networks [8]. Figure 1 (a)(b)(c) illustrates an example of the resulting graph for different common transmission power values. The dotted circles in Figure 1(a)(b)(c) correspond to the transmission range of the transmission by each node.

Definition: The connectivity measure $k_v(M)$ of graph M indicates the ability of the network to retain connections among its nodes after some links or nodes are removed.

Definition: A graph M is $k - edge$ connected if M is connected and every node has at least k links (i.e., $k_v(M) > k$).

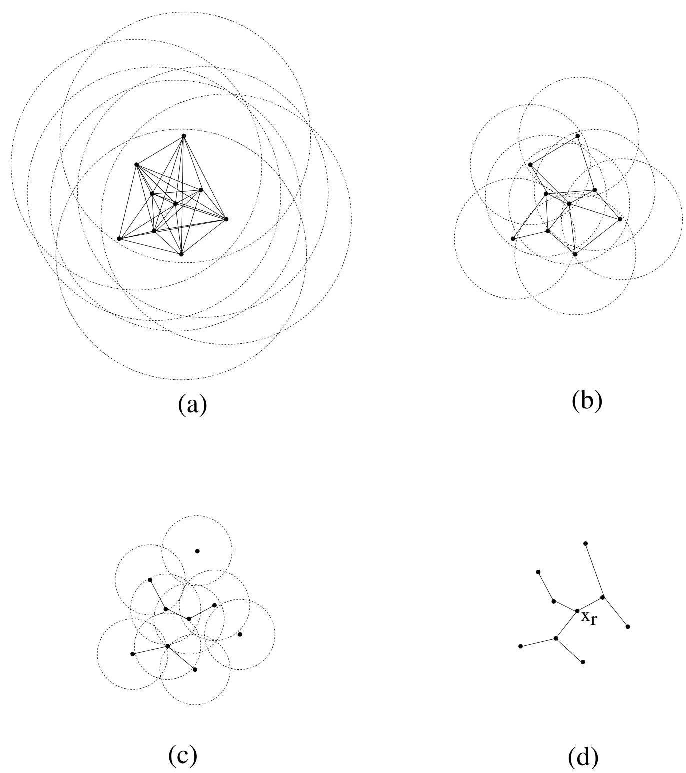


Fig. 1. Transmission Range and Graph Connectivity: (a) illustrates a highly connected network where all nodes are reachable in one hop (e.g., $k_v >> 1$); (b) illustrates a connected network; (c) illustrates the case where at least one node is disconnected forming network partitions; and (d) illustrates a minimum spanning tree that uses variable-range transmission with node x_r as root of the tree.

A high $k_v(M)$ value may be desired from a certain point of view because it provides the graph with several alternative routes in case some edges come down due to nodes powering down, node's movement or links facing severe fading conditions. However, a high level of connectivity may create too much interference for simultaneous transmissions with the resulting channel contention and delays associated with it. Thus, it seems reasonable to reduce the common transmission range to allow for space/frequency-reuse in the network, hence reducing the number of contending/interfering nodes per attempted transmission. Reducing the common transmission range, however, needs careful examination. It is not possible to arbitrarily reduce R_{com} to any value in order to maintain a connected graph. Rather, there is a lower bound of R_{com}, R_{com}^{min}, that is needed to maintain the connected graph.

Definition: The minimum common transmissions range, denoted R_{com}^{min} is the minimum value of R_{com} that maintains a connected graph.

This bound depends on the density and distribution of nodes in the network. Packets transmitted using less power than required to maintain R_{com}^{min} are likely to get lost rather than reaching the final destination node. This may lead to network partitions. In [6], Gupta and Kumar (1998) found an asymptotic expression to characterize the dependence of the common transmission range for asymptotic connectivity (R_{com}) in wireless networks. They found that when the range of R_{com} is such that it covers a disk of area $\frac{\log n + k_n}{n}$ [6], then the probability that the resulting network is connected converges to one as the number of nodes n goes to infinity if and only if $k_n \rightarrow +\infty$. Then the critical transmission range for connectivity of n *randomly* placed nodes in A square meters is shown to be [6],

$$R_{com}^{min} > (1+\epsilon)\sqrt{\frac{A \ln n}{\pi n}}; \epsilon > 0 \tag{1}$$

Definition: A tree T with node set V is called a *spanning tree* of V if each node of V is incident to at least one edge of T.

Definition: A minimum spanning tree for V, denoted MST, is a tree such that the sum of the edge lengths is minimal among all the spanning trees.

In this case the *minimum common transmission range* is the minimum value of the transmission range that permits the construction of a spanning tree. In [7] Gupta and Kumar found the average traffic carrying capacity λ that can be supported by the network to be given by,

$$\lambda(R) \leq \frac{16AW}{\pi \Delta^2 nLR} \tag{2}$$

where A is the total area of the network, L is the average distance between source-destination pairs, each transmission can be up to a maximum of W bits/second. There can be no other transmission within a distance $(1+\Delta)R$ from a transmitting node. The quantity $\Delta > 0$ models the notion of allowing only weak interference. Due to the inverse dependence of the right hand side on R, one wishes to decrease R. As discussed earlier, too low a value of R results in network partitions. This justifies our goal of reducing the common power level to the lowest value at which the network is connected. Combining Equations 1 and 2 it is clear that the average maximum traffic carrying capacity of the network that uses a common transmission power is limited by,

$$\lambda(R_{com}^{min}) \leq \frac{16\sqrt{A}}{\sqrt{\pi}\Delta^2 L}\frac{W}{\sqrt{n \ln n}} \tag{3}$$

If the maximum traffic carrying capacity of the network is bounded by the lowest value of R that keeps the network connected, then one can easily ask the question if the use of variable-range transmission can reduce the value of R beyond the bound given by Equation 1, thus increasing the average traffic carrying capacity and power savings of the network. This intuition motivates the study of variable-range transmission policies that follows.

B. *Variable-Range Transmission*

Now let us assume that each node can dynamically control the transmission power it uses independently of other nodes.

Definition: The weight (or cost) of each individual link e in graph M, denoted $\psi(|e|)$, is the minimum transmission range between two nodes connected by link e.

Definition: The end-to-end weight of a route from node u to node v, is the summation of the weight of the individual links representing a continuous traversal from node u to node v.

Let us also assume there is a unique route between any source-destination pair in the network that minimizes the end-to-end weight and that the average range of each transmission using these unique routes is $\overline{R}$. It is interesting to compare the ratio between R_{com}^{min} and $\overline{R}$ because such a ratio accounts for how much lower a capacity is obtained and extra power is used in the network for holding to a common transmission power approach.

Now let us again randomly pick a node in M, say x_r, where $1 \leq r \leq n$, and compute a minimum spanning tree (MST) to all the other $n-1$ nodes in V using node x_r as the root of the MST. Figure 1(d) illustrates an example of a MST with node x_r as the root of the tree[2]. If E is such that the distances $|x_i - x_j|$ are all different then there is a unique MST for V. Dividing the length of the MST (denoted by $M(x_1, x_2, ..., x_n)$) by the number of edges in the tree we get the average range of each transmission for a MST ($\overline{R}_{MST}$). Therefore,

$$\overline{R}_{MST} = \frac{M(x_1, x_2, ..., x_n)}{n-1} \tag{4}$$

To generalize, let $M(x_1, x_2, ..., x_n)$ be the weight of the MST, denoted as

$$M(x_1, x_2, ..., x_n) = min \sum_{e \epsilon E} \psi(|e|) \tag{5}$$

[2]It is outside the scope of this paper to describe how to built a MST. Interested readers may refer to the Prim and Kruskal algorithms for details.

where the minimum is over all connected graphs *T* with node set *V*. The weighting function which is of the most interest is $\psi(|e|) \sim |e|^\alpha$, where $2 \leq \alpha \leq 4$. In [16], Steele (1988) showed that if x_i, $1 \leq i < \infty$ are uniformly distributed nodes and M is the length of the MST of $(x_1, x_2, ..., x_n)$ using the edge weight function $\psi(|e|) = |e|^\alpha$, where $0 < \alpha < d$, then there is a constant $c(\alpha, d)$ such that with probability 1,

$$M(x_1, x_2, ..., x_n) \sim c(\alpha, d) n^{(d-\alpha)/d} \quad as \quad n \to \infty \tag{6}$$

where $c(\alpha, d)$ is a strictly positive constant that depends only on the power attenuation factor α and the dimension d of the Euclidean space being analyzed. Thus the average length of the edges of a minimum spanning tree using Equation 4 is,

$$\overline{R}_{MST} \sim c(\alpha, d) \frac{n^{(d-\alpha)/d}}{n-1}; \quad 0 < \alpha < d \tag{7}$$

1) The Special Case of $\Re^2$*:* In order to compare $\overline{R}_{MST}$ with R_{com}^{min} we need to derive an expression for $\overline{R}_{MST}$ for $\Re^2$ and $\psi(|e|) = |e|^\alpha$, for the particular case where $2 \leq \alpha \leq 4$. Because of the condition $0 < \alpha < d$ in Equation 7, setting $d = 2$ limits the value of α to $\alpha < 2$. Since $\lim_{\alpha \to 2} n^{(d-\alpha)/d} = 1$ for $d = 2$, the following simplification still holds

$$\lim_{\alpha \to 2} \overline{R}_{MST} \sim c(\alpha \to 2, d = 2) \frac{1}{n-1}; \quad n \to \infty \tag{8}$$

Equation 8 assumes the area of the network to be a normalized $1\ m^2$. For a network of area $A\ m^2$ we must scale the previous result by $\sqrt{A}$. Thus the average minimum transmission range of n randomly placed nodes in $A\ m^2$ is,

$$\lim_{\alpha \to 2} \overline{R}_{MST} \sim c(\alpha \to 2, d = 2) \frac{\sqrt{A}}{n-1} \tag{9}$$

Despite its simplicity this expression for $\overline{R}_{MST}$ and $\alpha \to 2$ holds fairly well for large n as we will show later in Section IV when we present numerical examples. However, we can not extend the validity of this expression for the case where $\alpha > 2$ because of the $0 < \alpha < d$ limitation of the model [16]. How much the results change for the general case of $2 \leq \alpha \leq 4$ require further analysis. Comparing the common-range and variable-range transmission expressions we end up comparing expressions $\sqrt{\frac{\log n}{\pi n}}$ for common range with expression $\frac{1}{n-1}$ for variable-range transmissions. These expressions decrease their values asymptotically as n increases. Therefore, the absolute difference between common-range and variable-range transmission values is determined by the respective proportionality constants (e.g., $(1+\epsilon)$ for common and $c(\alpha = 2, d = 2)$ for variable-range transmission). In Section IV, we show results of numerical examples to compute the proportionality constants for both R_{com}^{min} and $\overline{R}_{MST}$. As we show later, a variable-range transmission policy can significantly reduce the average transmission range used compared with the minimum common-range transmission bound. This result has a significant impact on the performance of wireless ad hoc networks since it suggests that a variable-range transmission policy may increase the capacity and power savings of the network.

Capacity Analysis: Now we compute the traffic carrying capacity for variable-range based ad hoc networks. Using the same example by Gupta and Kumar in [7] consider two simultaneous transmissions, one from T to R and another from T' to R', as shown in Figure 2(a). In contrast to the example described in [7] where both transmissions use the same transmission range of r meters, the range of transmission shown in Figure 2(a) from T to R is with a meters while from T' to R' is with b meters, respectively. This illustrates the different transmission ranges that will appear in a variable-range based ad hoc network. Similar to the analysis in [7], for R to hear T and for R' to hear T' we need $|T - R| \leq a$ and $|T - R| \leq b$, respectively. Considering the triangle with vertex points (T,R,R') in Figure 2(a) we have, from the triangle inequality, that $|T - R| + |R - R'| \geq |T - R'| \geq (1 + \Delta)a$, or $|R - R'| \geq (1 + \Delta)a - a \geq \Delta a$. Similarly, for the triangle with vertex points (T',R',R) we get $|R - R'| \geq \Delta b$.

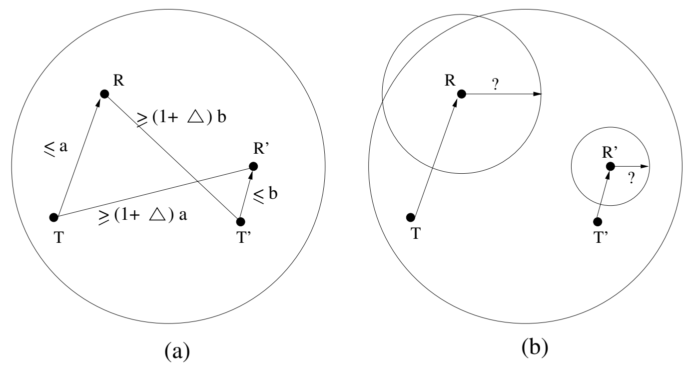


Fig. 2. (a) Protocol model of interference, (b) disks of unknown radii around the receivers are disjoint.

The reader may wonder why we obtain two different values for $|R - R'|$ in Figure 2(a). The answer to this question is that the value of $|R - R'|$ depends on the range used by the transmitting node in each triangle. Lets again go back to the case in [7] where a common transmission range of r meters is used. In this case the minimum distance between two receivers R and R' is always $|R - R'| \geq \Delta r$. As a result disks of radius $\Delta r/2$ around R and R' are disjoint of each other. Dividing the total area of the network by the area of one of these disks we obtain the maximum number of simultaneous receptions in the network, from which Equation 3 follows. In our case illustrated in Figure 2(a) we have two different transmission ranges and that explains why we obtain $|R - R'| \geq \Delta a$ when node T transmits with a range of a meters and $|R - R'| \geq \Delta b$ when node T' transmits with a range of b meters. Due to this variations in the value of $|R - R'|$ it is difficult to find the equivalent of disjoint disks found in [7] (see Figure2(b)). In order to find the area and location of disjoint disks in a variable-range setting it will be necessary to know the range and location of all transmissions in the network, which is difficult to express analytically.

We resolve this problem by taking advantage of the fact that variations around the average length of edges in a MST decrease when the density of nodes increases. We take results from Section IV and present Table I, which shows the mean and standard deviation of the length of edges in a MST when n nodes are randomly positioned in a 200x200 network.

number of nodes 200x200 network	mean value [meters]	standard deviation [meters]
10	54.26	14.10
100	17.81	0.83
1000	5.92	0.02

TABLE I
MEAN VALUE AND STANDARD DEVIATION OF THE LENGTH OF EDGES IN A MST VERSUS NUMBER OF NODES.

The main result from Table I is that for a large n, the length of edges in a MST are roughly constant (or $a \sim b$ in the context of Figure 2(a)). Therefore, the capacity analysis for common-range based ad hoc networks used in [7] can be applied to variable-range as well. For a large n, therefore, it is a good approximation to combine Equations 2 and 9 to obtain the average carrying traffic capacity of the network for a variable-range transmission power policy as

$$\lambda(\overline{R}_{MST}) \leq \frac{16\sqrt{A}W}{\pi\Delta^2 L}; \quad n \rightarrow \infty \tag{10}$$

Equation 10 suggests that using an optimum variable transmission power in the network keeps the per node average traffic carrying capacity constant even if more nodes are added to a fixed area network. This result is quite surprising since intuition says we should expect that per node capacity decreases while adding more nodes in a fixed area as is the case for the common transmission range case. The reason why the capacity remains constant in the variable range case scenario we believe is because the addition of more nodes reduces the average transmission range, and thus increases the capacity in the same proportion as the capacity itself decreases with the addition of more nodes. This result, however, should be taken with caution, first, it will be difficult to achieve the necessary high density of nodes, and second, some minimum transmission power levels may be below P_{min}, the minimum transmission power value allowed in a radio modem. Finally, Equation 10 relies on simulation data and therefore it is an empirical result at this moment. We are currently working on a more formal proof.

The previous analysis for both common-range and variable-range transmissions does not consider node mobility, however. For the case where nodes move in random directions at random speeds the results derived in this section still hold. The reason is that even in the presence of mobility, the distribution of nodes in the network remains homogeneous at any particular time, which is a necessary condition for the analysis shown in this section to be valid. Node mobility, however, does impact the signaling overhead of the routing protocol, and therefore, it affects the available capacity left to mobile nodes (e.g., effective capacity). We quantify the impact of node mobility on the signaling overhead of the routing protocol and its impact on the effective capacity available to mobile nodes given a certain transmission range. The analysis presented in the next section generalizes and extends the results presented in this section for mobile ad hoc networks.

III. NETWORK CONNECTIVITY

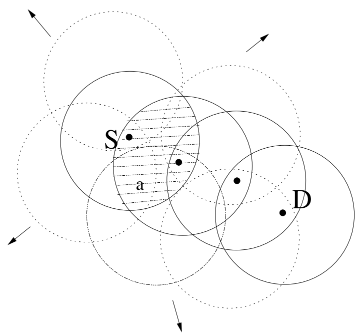
In the previous section we discussed physical connectivity issues, and how they relate to network capacity and power savings in wireless ad hoc networks. Physical connectivity alone, however, does not provide nodes with end-to-end connectivity. A routing protocol is necessary to provide nodes with the means to communicate with each other in a multi hop environment. The transmission range used has a significant impact on the rate of signaling packets required to discover and maintain these "pipes" of connectivity over time in the presence of a node's mobility. The derivations that we present in this section are focused on the behavior of an ideal on-demand common-range transmission based routing protocol. We will discuss the specifics of variable-range transmission based routing protocols at the end of this section.

The choice of the common transmission power used impacts the number of signaling packets required by the routing protocol. The use of a low common transmission power increases the number of intermediate nodes between source-destination pairs. These intermediate nodes move in and out existing routes, requiring the routing protocol to take periodic actions to repair these routes in time. It is expected that the lower the common transmission power used the higher the number of signaling packets required by the routing protocol to discover and maintain routes. Those signaling packets consume capacity and power resources in the network. Choosing a low common transmission power hoping to increase network capacity, as suggested by the analysis in the previous section, may generate too many signaling packets in the presence of node mobility, and therefore, a higher transmission power may be desirable. In what follows, we study this tradeoff.

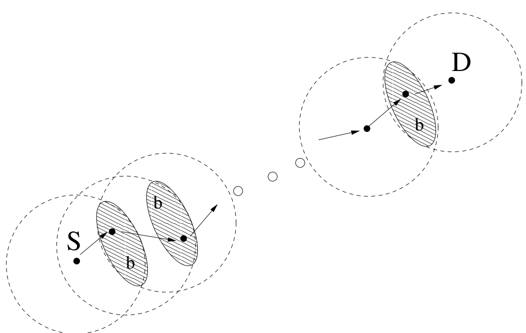
A. Mobility Analysis

In general, there will be none, one, or several intermediate forwarding nodes between source-destination pairs. Figure 3(b) illustrates an example of a route from a source node S to a destination node D involving several forwarding nodes. Each circle in Figure 3(b) represents the transmission range of each forwarding node in this route. The shaded regions illustrated in Figure 3(b) represents the overlapping regions between forwarding nodes.

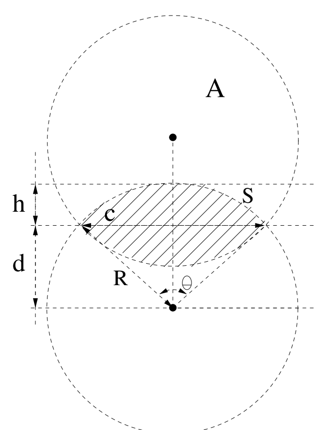
Using the same notation as in Section II, consider a graph M with a node set $V = \{x_1, x_2, ..., x_n\}$ and link set $E = \{(x_i, x_j)\} : 1 \leq j \leq n$ for $x_i \in \Re^2, 1 \leq i \leq n$. Nodes move at a speed of v meters per second in random directions. Figure 3(c) highlights one of these overlapping regions. The length of the arc of the circle subtended by an angle θ, shown as S in Figure 3(c), is $R\theta$. The area of the overlapping region b is



(a) Route Discovery



(b) Route Maintenance



(c) Overlapping Region

Fig. 3. Routing in MANET-type Ad hoc Networks

then given by

$$b = R^2(\theta - \sin\theta) = 2R^2 \arccos(\frac{d}{R}) - 2d\sqrt{R^2 - d^2}$$
$$= 2R^2 \arccos(\tfrac{R-h}{R}) - 2(R-h)\sqrt{2Rh - h^2} \qquad (11)$$

This expression is an approximation only. Forwarding nodes do not always space themselves equally along a path and they may move in random directions with respect to each other. As a result the actual overlapping area for each forwarding node may be smaller or larger in size than b.

The factor h plays a crucial role in the operation and performance of routing protocols for wireless ad hoc networks. As illustrated in Figure 3(c), h accounts for how much area between adjacent forwarding nodes overlaps. Setting the value of h in a real network is rather difficult and, in general, the value of h is constantly changing as forwarding nodes may move in different directions at different speeds. Clearly the value of h ranges from a minimum of 0 meters to a maximum of R meters. When a forwarding node moves outside its forwarding region a new node in that region needs to take its place. We call this process a *route-repair* event. Having $h = 0$ indicates that forwarding nodes are located on a straight line connecting source-destination nodes and there is a minimum number of hops involved. Having $h = 0$, similarly, means that any node's movement results in a route-repair event. As a result having small h is only feasible in static networks (e.g., sensor networks). On the other hand having $h \rightarrow R$ minimizes the number of route-repair events seen by the routing protocol at the expense of significantly increasing the number of forwarding nodes per route.

In most on-demand routing protocols [2] for ad hoc networks there are route-discovery and route-maintenance phases. Route-discovery is responsible for finding new routes between source-destination pairs whereas route-maintenance is responsible for updating existing routes in the presence of node mobility. In what follows, we derive a model to compute the signaling overhead of each component in an on-demand routing protocol as a function of the common transmission range being used.

B. Route Discovery

A source node intending to transmit a packet to a destination node outside its transmission range needs a chain of one or more forwarding nodes in order to successfully reach the intended destination node. We call this process of finding such a chain of nodes *route-discovery*. Figure 3(a) illustrates a route-discovery process where node S searches for a route toward node D. The solid circles in Figure 3(a) illustrate the transmission range of the nodes associated with the final route, whereas the dotted circles illustrate the transmission range of nodes in all other directions that did not become part of the final route. Route-discovery can become very demanding in terms of both the number of signaling packets generated as well as the delay involved in finding the intended receiver. An important part of the complexity found in most routing protocols for on-demand ad hoc networks is how to reduce this overhead. In this analysis, however, we will consider that the process of route-discovery consist of flooding the entire network with a route-discovery request.

A node searching for a route broadcasts a route-discovery message which is heard within a circular region $A = \pi R^2$. Assuming that the intended receiver is not located within this region, then another node in region A will re-broadcast the original message, thus extending the region unreachable by the original broadcast message, and so on [12]. A percentage of the second broadcast is wasted because it overlaps with the area covered by the first broadcast message (see Figure 3(a)), however. This problem is also addressed in [11]. As a result there is an inherent space-waste while flooding the network with broadcast messages. The node transmitting the second broadcast message can be located anywhere between 0 and R meters from the node transmitting the first broadcast message. This is equivalent to varying the parameter h between $\frac{R}{2}$ and R (see Figure 3(c)). The average overlapping area of a re-broadcast message is,

$$\overline{a} = \frac{2}{R}\int_{\frac{R}{2}}^{R}\left[2R^2\arccos\left(\frac{R-h}{R}\right) - 2(R-h)\sqrt{2Rh-h^2}\right]dh \quad (12)$$

integrating by parts and simplifying we obtain $\overline{a} =\sim 0.68A$. Clearly a re-broadcast message may overlap not only with the originating node, but potentially with regions covered by re-broadcast messages by other nodes. Therefore the value of $\overline{a}$ may be even lower than ~ 0.68. If the total area of the network is A_T, then the total number $Q(R)$ of broadcast messages at range R necessary to successfully flood the network entirely is ,

$$Q(R) \sim \frac{A_T}{(1-0.68)A} = \frac{A_T}{(1-0.68)\pi R^2} \quad (13)$$

Due to the reciprocal square dependence of the right hand side on R^2 in Equation 13 reducing the transmission power may generate a prohibitive number of broadcast messages necessary to completely flood the network for low values of R. As a result, the use of a higher transmission range may provide better performance (e.g., higher per node average capacity).

C. Route Maintenance

A property of most MANET-style routing protocols is that they attempt to minimize the number of forwarding nodes per route in the network. The resulting effect of applying this routing policy is that routes seem to fall on a region connecting source and destination nodes (see Figure 3(b)). From the point of view of the routing protocol being used there is a region b where a potential forwarding node may be located as the next hop in the route toward the destination (assuming a high density of nodes allows for several nodes to be located in that region). Figure 3(b) illustrates an example of this region for each forwarding node in the route toward the destination. In what follows, we analyze how much node mobility and transmission range impact the number of route-repair events per second generated by the routing protocol.

The number of nodes per second crossing region b, denoted by M, is given by $\frac{\rho v F}{\pi}$. Here ρ is the density of nodes in the network, v is the velocity of nodes and F is the area boundary length or perimeter of region b. The perimeter F of region b is given by ,

$$F = 2S = 2R\theta = 4R\arccos\left(\frac{R-h}{R}\right) \quad (14)$$

therefore,

$$M = \frac{4\rho v R\arccos\left(\frac{R-h}{R}\right)}{\pi} \quad (15)$$

Equation 15 assumes that nodes move in a random direction at a constant velocity and there is always conservation of flow in the shaded region. Let $N = \rho b$ be the average number of nodes in region b. A node entering region b at speed v remains an average of $T = N/M$ seconds inside the region before leaving. Using equations 14 and 15 we can compute $T(R)$ as,

$$T(R) = \frac{\pi R^2\arccos\left(\frac{R-h}{R}\right) - \pi(R-h)\sqrt{2Rh-h^2}}{2vR\arccos\frac{R-h}{R}} \quad (16)$$

The parameter T directly relates to network connectivity because it accounts for how long a node in a route remains in a forwarding position before it needs to be replaced by a new forwarding node. We can assume, therefore, that the number of route-repair events in the network per second is proportional to $\frac{1}{T}$. If L is the average length in meters separating source-destination pairs in the network over time, then there are L/d forwarding nodes per route on the average. Therefore, the average number of route-repair events per second per route, $J(R)$, is proportional to $\frac{L}{R-h}\frac{1}{T}$ or,

$$J(R)\Theta\frac{L}{(R-h)}\left(\frac{2vRarccos(\frac{R-h}{R})}{\pi R^2 arccos(\frac{R-h}{R}) - \pi(R-h)\sqrt{2Rh-h^2}}\right) \quad (17)$$

The factor R^2 in the denominator of Equation 17 dominates the behavior of $J(R)$, and thus a higher value of transmission range R keeps a forwarding node in the route for a longer interval before there is a need to replace it, thus requiring less signaling messages to maintain existing routes. The actual number of signaling messages necessary to maintain a route after a route-repair event occurs depends on the actual operation of the routing protocol being deployed.

D. Capacity and Signaling Overhead

Clearly the rate of signaling packets generated by the routing protocol has an impact on the capacity available to nodes for data transmission. In Equation 2 we showed an expression for $\overline{\lambda}(R)$, the average traffic carrying capacity per node that can be supported by the network. Now let C be the number of bits exchanged by the routing protocol triggered by a route-repair event. The value of C depends on the number of signaling messages exchanged during a route-repair operation and the average size of each signaling message. Then the total capacity available to nodes using a transmission range R removing the portion of the capacity used by the routing protocol is

$$\overline{\lambda}(R,t) = \lambda(R,t) - CJ(R,t) \tag{18}$$

The route-discovery process occurs once per each route, and thus the corresponding amount $Q(R, t_0)$ is subtracted from the available capacity of the network once, and therefore not taken into account in Equation 18. This is in contrast with the signaling overhead of the route-maintenance process, which continuously uses a portion of the available capacity. We mentioned previously that R must be made as small as possible to maximize the traffic carrying capacity of the network. In the previous section we showed that R is limited by Equation 1 if a common transmission range is used, and by Equation 7 if a variable-range transmission is used. Reducing the transmission range, however, has the effect of increasing the number of signaling packets transmitted to discover and maintain routes in the presence of node mobility. Clearly there is an optimum setting of R for a given node mobility v that maximizes the network capacity available to nodes. Because route-discovery occurs once we do not include $Q(R, t_0)$ in the derivation of R_{opt},

$$\frac{d}{dR}\overline{\lambda}(R) = \frac{d}{dR}\frac{16AW}{\pi\Delta^2 nLR} + \\ -\frac{d}{dR}\frac{4CLvarccos(\frac{R-h}{R})}{3(\pi R^2 arccos(\frac{R-h}{R}) - \pi(R-h)\sqrt{2Rh-h^2})} \tag{19}$$

In order to remove the dependency on h in Equation 19, we first compute the average overlapping region b between two forwarding nodes in a route. Because h can vary between 0 and $\frac{R}{2}$ in this case, the average overlapping region is,

$$\overline{b} = \frac{2}{R}\int_0^{\frac{R}{2}} \left[[2R^2 \arccos\left(\frac{R-h}{R}\right) + \\ -2(R-h)\sqrt{2Rh-h^2} \right] dh = \sim 0.16A \tag{20}$$

which corresponds to a value of $\overline{h} = 0.265R$ or $\overline{h} \sim \frac{1}{4}R$. Substituting this value in Equation 19 and using the chain rule we obtain after some simplifications,

$$\frac{d}{dR}\overline{\lambda}(R) = -\frac{16AW}{\pi\Delta^2 nLR^2} + \\ -\frac{128LCv \arccos\left(\frac{3}{4}\right)}{9\sqrt{7}\pi R^3 - 48\pi R^3 \arccos\left(\frac{3}{4}\right)} \tag{21}$$

making Equation 21 equal to zero we find the value of R that maximizes $\overline{\lambda}(R,t)$ as,

$$R_{opt} = \frac{8C\Delta^2 L^2 nv \arccos\left(\frac{3}{4}\right)}{AW(48\arccos\left(\frac{3}{4}\right) - 9\sqrt{7})} \tag{22}$$

Results from this section show that there is an optimum setting for the transmission range, not necessarily the minimum value we found in Section II-B based on connectivity issues only, which maximizes the capacity available to nodes in the presence of node mobility. This result contrasts the main result of the previous section that pointed toward minimizing the transmission range as a means to increase the capacity of static networks.

The previous analysis is focused on the behavior of an ideal on-demand common-range transmission based routing protocol. Most of the insights obtained from this section, however, apply to variable-range transmission based routing protocols as well. This is because the general trend "the lower the transmission range used the higher the number of signaling packets required by the routing protocol to discover and maintain routes" applies to both common-range and variable-range based routing protocols as well (this trend is supported by extensive simulations in Section IV). There are, however, important differences that is necessary to consider. In the case of an ideal variable-range based routing protocol a node always use the minimum transmission range to communicate with another node. The use of a minimum transmission range implies that the parameter $T(R)$ (the time interval that a moving node remains in a route) is always equal to zero. As a result, even the smallest movement of a node could trigger a route-repair operation by the routing protocol. In the presence of mobility, is then necessary to increase this minimum transmission range in order to increase the factor $T(R)$ to reduce the signaling overhead. This solution, of course, will lessen the advantages of variable-range based routing protocols found for static networks. We are currently investigating this tradeoff at this moment.

IV. Numerical Examples

In what follows, we present numerical examples about physical and network connectivity. We analyze the fundamental relationship (i.e., the ratio) between the R_{com}^{min} and $\overline{R}_{MST}$. In addition, we quantify the signaling overhead of the network layer in the presence of node mobility.

A. Physical Connectivity

The main limitation with the previous derivations of both R_{com}^{min} and $\overline{R}_{MST}$ is that the analytical results presented only hold for large values of n and, similarly, the proportionality constants of both bounds remain unknown. In order to quantitatively compare the two bounds we performed extensive computations to find these constants. Figure 4 shows the transmission range in a 200x200 square network for different numbers of nodes randomly distributed in the network. For each point in Figure 4 we performed 50 experiments, each of them using a different *seed* number to vary the location of nodes in the network. Figure 4 contrasts $\overline{R}_{MST}$ with R_{com}^{min} (the numerical values corresponds to the 99% confidence interval).

There are several interesting observations we can make from Figure 4. As expected from equations 1 and 7, the values of R_{com}^{min} and $\overline{R}_{MST}$ decrease as the density of nodes per unit area increases. This behavior is quite intuitive. The minimum transmission range that keeps the network connected is sensitive to the average number of nodes seen by any node within its current transmission range. The more nodes in the network

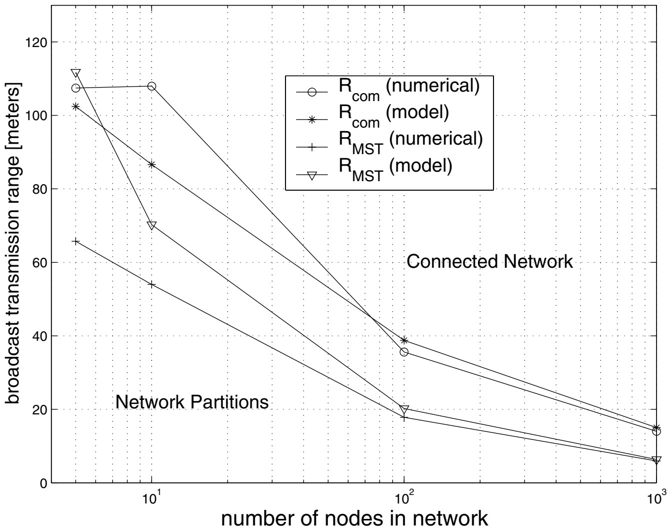


Fig. 4. Transmission Range in Wireless Ad hoc Networks

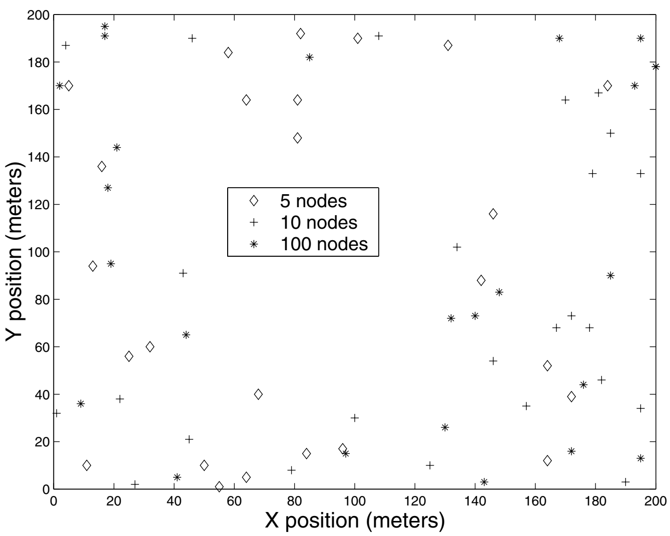


Fig. 5. Edge Effects in Wireless Ad hoc Networks

the more stable the average number of neighbors per coverage area seen by a node, and thus, the lower the transmission range required to keep them connected. A key observation from Figure 4 relates to the ratio $R^{min}_{com}/\overline{R}_{MST}$ which remains roughly constant and is ∼**2**. This results indicates, *that the value of the minimum common-range transmission is approximately twice the average value of the minimum variable-range transmission for similar routes*.

As a caveat, these are numerical results and therefore the results apply to the network settings only, and cannot be extended to other network topologies without further experimentation. This result has its power consumption counterpart. Using a common transmission power approach to routing results in routes that consume $\sim (1 - \frac{2}{2^\alpha})$ % $(2 < \alpha < 4)$ more transmission power than routes that use a variable-range transmission. Figure 4 also shows the theoretical bound for both $\overline{R}_{MST}$ and R^{min}_{com} using the respective equations introduced earlier. We found that the proportionality constant for $\overline{R}_{MST}$, $C(\alpha, d) \sim 1$ whereas the proportionality constant for R^{min}_{com}, $\epsilon \sim 2$. Figure 4 clearly shows that the model breaks down for a density below 0.0025 $nodes/meter^2$ (e.g., $n < 100$).

Homogeneous distribution of nodes refers to the fact that the number of neighbors seen by each node within its transmission range remains more or less constant at least for a large n. Because of edge effects this property, unfortunately, does not hold even when nodes are uniformly distributed in the network. A node located right at the edge of the network has 1/2 as many neighbors while a node located in one of the corners (e.g., for a square network) has 1/4 as many neighbors on the average compared with a node located in a more central position of the network. In Figure 5 we recorded the position of the node triggering the first partition of the network while finding R^{min}_{com} in each of the 50 experiments of Figure 4. We found that approximately 50-60% of the time the node triggering the partition is located in a position within 10% from the edge of the network. This confirms the fact that edge effects can play a critical role in determining the value of R^{min}_{com}.

B. Network Connectivity

In Figure 6 we plot the average capacity per node, the signaling overhead of route-maintenance and the average capacity left per node after removing the capacity used by the signaling packets. The value of the parameters used for this plot is as follows: L=50 meters; A=10000 square meters; v= 10 meter/second; W = 2000000 bits/second; C = 150 bits; Δ = 10 meters; and n = 1000 nodes. As Figure 6 shows the average available capacity per node increases as the common transmission range decreases up to a certain point P_{opt}. After that point the signaling overhead component dominates the performance and the available average capacity per node decreases sharply.

C. MANET Routing Protocols

In order to complement the previous analysis we performed a series of simulations to observe the behavior of a MANET-type on-demand routing protocol stressing the impact that varying transmission range has on the rate of signaling messages generated. We use the ns2 simulator and the CMU wireless extensions. Our simulation settings are as follows: there are 50 nodes in a 1500x300 meters network, nodes move at a maximum speed of v meters/second and there are 20 CBR connections among the 50 nodes. Each CBR connection transmits 4 packets (512 bytes long) per second for the 900-second simulation scenario. We use the Dynamic Source Routing (DSR) protocol [8]. The mobility model in the simulator works in the following way. A node randomly selects a destination point within the network limits and then moves toward that point at a speed selected uniformly between 0 and a maximum speed. After reaching the destination point a node pauses for a period of time before moving to a new randomly selected

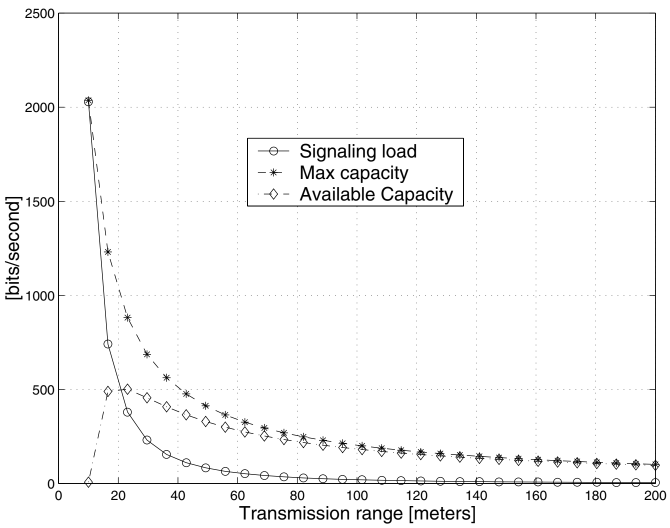


Fig. 6. Route Maintenance and Available Capacity

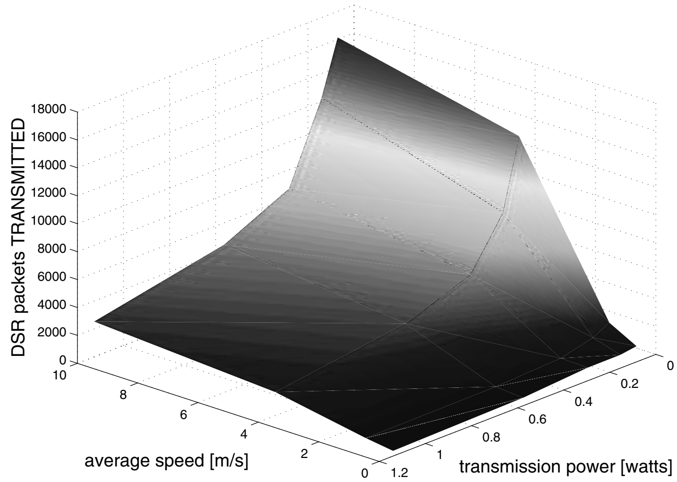


Fig. 7. Signaling Load in MANET Protocols

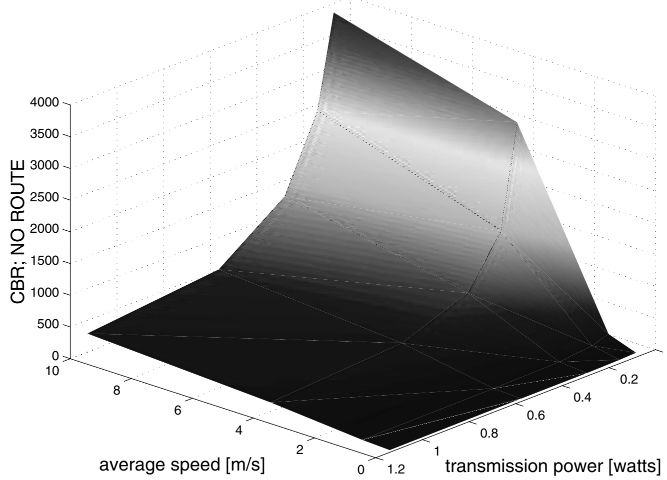


Fig. 8. Network Partitions in MANET Protocols

destination at a new speed. Figure 7 shows the signaling overhead of the routing protocol versus the transmission power and node speed. As shown in Figure 7, the number of signaling packets is low for high transmission power values, and grows in an exponential manner when transmission range approaches the minimum common transmission range. A similar behavior is observed in Figure 8 which shows the number of times a received packet found no routing information to continue its journey toward the destination (e.g., because of the number of network partitions). These results highlight the fact that MANET-style routing protocols do not provide a suitable foundation for the development of routing protocols that are capacity-aware and power-aware. The choice of DSR in these experiments does not limit us from generalizing these results to other MANET routing protocols. This is because all MANET routing protocols to our knowledge use a common broadcast transmission range to discover and maintain routes. It is this particular feature what shapes the results shown in figures 7 and 8.

V. Discussion

Now we discuss some deployment issues that motivates further study of variable-range transmission support in the design of protocols for wireless ad hoc networks. At the physical layer we show that using a common-range transmission based routing protocol results in routes that, at best, involve transmission range levels that approximately double the average range in variable-range transmission based routing protocols for similar routes. In practice, however, it is relatively difficult to discover R_{com}^{min} from a practical implementation point of view. Similarly, nodes in a real network are not uniformly distributed in the network, but follow terrain and building layouts in complex ways. These facts increase the gap between R_{com}^{min} and $\overline{R}_{MST}$ for real network deployments. A common and safe approach used in most MANET-type routing protocols for ad hoc networks is to set $R_{com} >> R_{com}^{min}$, or simply, $R_{com} = R_{max}$. These solutions, while improving the physical connectivity of the network, achieve that goal at the expense of sacrificing network capacity and wasting transmission power in the network significantly.

Figure 9 illustrates the main drawback of a common transmission range approach to routing. In this example the smaller circle in Figure 9 corresponds the minimum common transmission range where node x_i is not part of the graph. Once node x_i is part of the graph then the new minimum common transmission range becomes the larger circle. For real networks where nodes follow building and street layouts this type of scenario is the common case and not an exception of the rule.

At the network layer we also show that in the presence of node's mobility, reducing the transmission range as a means to increase network capacity could be harmful to the available capacity remaining for nodes. The tradeoff between network connectivity and network capacity presents a very interesting paradigm: is it possible to maintain low overhead for the routing protocol while at the same time provide higher capacity to the nodes in the network? Following the design and performance of common-range transmission MANET-type routing protocols the answer is "no", unless a different method for discovering and maintain routes that departs from common transmission range broadcast technique is used. Recently, there has been some initial work in this area [14] [5] [13] that provides variable-range transmission support for routing protocol operation.

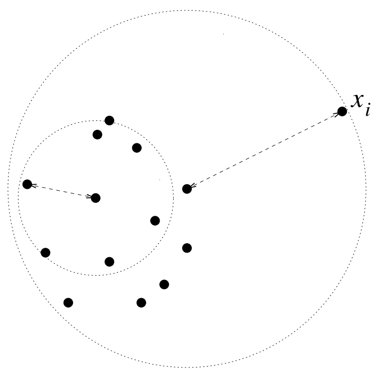


Fig. 9. Disadvantages of Common-range Transmissions

Most ad hoc network designs simply borrowed MAC protocols designed for wireless LAN operation. IEEE 802.11 as well as most CSMA MAC protocols use a common-range transmission and are not flexible enough to exploit the spectral reuse potential of the network. In general, nodes transmitting with lower transmission power levels may not be noticed by nodes transmitting with higher transmission power levels and as a result collisions may be difficult to avoid. Fortunately there are some new proposals in MAC design that overtake this limitation and take full advantage of the spectral reuse potential acquired when using dynamic power control [10].

VI. Related Work

In what follows, we discuss how our contribution discussed in this paper contrasts to the related work in the area. The work by Gupta and Kumar [6] [7] on the mathematical foundations of common-range transmission in wireless ad hoc networks represents the seminal related research in this area. In this paper, we take a similar approach to Gupta and Kumar but consider variable-range transmission in contrast to common-range transmission.

The work presented in this paper on the bounds of variable-range transmissions in wireless ad hoc networks uses traditional graph theory. In particular, we used the theory explaining the behavior of minimum spanning trees (MST) to compute the weight of a minimum spanning tree [16]. In the work described in [1], the authors discuss the impact on TCP throughput on the number forwarding nodes in static wireless ad hoc networks for unreliable links. In [3], the authors study the throughput capacity of wireless multihop networks for UDP traffic.

Systems based on common-range transmission control like MANET protocols [9] usually assume homogeneously distributed nodes. As discussed earlier, such a regime raises a number of concerns and is an impractical assumption in real networks. The authors in [13] and [17] discuss this problem and propose different methods to control the transmission power levels in order to control the network topology. The work in [13] and [17] is concerned with controlling the connectivity of non-homogeneous networks, but it does not provide a mathematical description of the problem space, and ignores the power savings and traffic-carrying capacity aspects of the problem. We address these issues in this paper.

In [15], the authors present several link cost functions that take into account the power reserves of mobile nodes. The work in [14] [5] intuitively suggest that a variable-range transmission approach can outperform a common-range transmission approach in terms of power savings, however, no definite analytical results are provided. In [14], wireless-enabled nodes discover energy-efficient routes to neighboring nodes and then use the shortest path Bellman-Ford algorithm to discover routes to other nodes in the network. The PARO protocol [5], uses redirectors to break longer-range transmissions into a set of smaller-range transmissions.

Mobility management in cellular and mobile networks is concerned with the rate of cellular/mobile nodes crossing cell boundaries. In most MANET routing protocols, mobility analysis relies on simulations [2] due to the lack of a mobility model for this environment. For the specific case of route discovery, the work by [11] shows that the inherited space-waste involved while flooding the network with broadcast messages. However, no comprehensive mobility management analysis is presented. To our knowledge, our analysis of mobility management is a first attempt at modeling the various aspects of mobility in multihop wireless ad hoc networks.

VII. Conclusion

There has been little analysis in the literature that quantifies the pros and cons of common-range and variable-range transmission control on the physical and network layer connectivity. In this paper, we provide new insights beyond the literature that strongly support the development of new variable-range transmission based routing protocols. Our results indicate that a variable-range transmission approach can outperform a common-range transmission approach in terms of power savings and increased capacity. We derive an asymptotic expression for the computation of the average variable-range transmission and traffic capacity in wireless ad hoc networks. We show that the use of a variable-range transmission based routing protocol uses lower transmission power and increases capacity compared with common-range transmission approaches. We also derive expressions for the route-discovery and maintenance phases of an ideal on-demand routing protocol. We show that there is an optimum setting for the transmission range, not necessarily the minimum, which maximizes the capacity available to nodes in the presence of node mobility. These results motivate the need to study, design, implement and analyze new routing protocols based on variable-range transmission approaches that can exploit the theoretical power savings and improve capacity indicated by the results presented in this paper.

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