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A CHAOTIC ECONOMIC GROWTH MODEL AND THE AGRICULTURAL SHARE OF AN OUTPUT

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Abstract: The agricultural share of a total output generally declines in the process of economic growth. The major reason for this is that consumer demand for food increases only slightly with rising incomes. However, a small, open economy can overcome this constraint to the growth of agricultural production by expanding its net exports.
The basic aim of this paper is to set up a chaotic growth model of the gross domestic product that is capable of generating stable equilibria, cycles, or chaos depending on parameter values.
Key words: agricultural share of an output, gross domestic product, rate of growth, chaos.

Introduction

Beginning with a stage in which the country was predominantly agricultural, the extension of more advanced techniques served both to raise productivity in agriculture and to develop new industries which permitted a better utilization of agricultural products and the employment in other pursuits of the growing population and of the labor force released from agriculture. Because the nature of human wants is such that, with the growing supply of goods per capita, an increasingly smaller proportion of consumer needs is satisfied by agricultural products, the growing products per capita meant a decreasing proportion in the national economy of agriculture and extrative industries in general; and a correspondingly higher proportion of non-agricultural activities – manufacturing, construction, and transportation in the earlier phases; service industries in the later phases. Industrialization can be defined formally as a sustained increase in the proportion of a country's economy devoted to pursuits other than agriculture,

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accompanied by a marked increase in total production and associated with the spread of techniques based upon empirical science. All three elements are important in the process; but the spread of modern technology, physical and social , was the base; growth in production its most important result; and the diversion away from agriculture a corollary consequence, even though superficially most conspicuous (K u z n e t s, 1971).

The economic growth can thus be defined on the quantitive side as a sustained increase in total output; and on the structural side, as the extension of scientific technology resulting in diversification away from agriculture, the emergence of a host of new industries of urban character, rapid growth of productivity in all industries including agriculture (K u z n e t s, 1971).

Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. The number of nonlinear business cycle models use chaos theory to explain complex motion of the economy. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in Economics by Benhabib and Day (1981,1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), among many others.

The model

Irregular movement of the gross domestic product (GDP) , Y , can be analyzed in the formal framework of the chaotic growth model .

The autonomous rate of the gross domestic product (GDP) , Y , growth is α , or

$$\frac{Y_{t+1} - Y_t}{Y_t} = \alpha \qquad (1)$$

Further , it is supposed that the increase in the GDP , $(Y_{t+1} - Y_t)/Y_t$, depends on productivity (Y/L) and the share of agriculture , P, in the gross domestic product (GDP) ,Y, (P/Y) ,i.e.

$$\frac{Y_{t+1}-Y_t}{Y_t}=\alpha-\beta\frac{L_t}{Y_t}-\gamma\frac{P_t}{Y_t} \quad (2)$$

Where

$$Y_t=\lambda\, L_t^{1/2} \quad (3)$$

is the production function., L is labour force, γ is the parameter.
On the other hand , agriculture takes part in GDP creation, i.e.

$$P_t = \delta\, Y_t \quad (4)$$

Substitution (3) and (4) in (2) gives

$$\frac{Y_{t+1}-Y_t}{Y_t}=\alpha-\frac{\beta}{\lambda^2}Y\,t-\gamma\,\delta \quad (5)$$

Solving this last equation yields the nonlinear, first-order difference equation for the gross domestic product, Y , or the economic growth model

$$Y_{\ t+1}=(1+\alpha-\gamma\,\delta)\,Y_t-\frac{\beta}{\lambda^2}Y\,t^{\ 2} \quad (6)$$

Further, it is assumed that the current value of the gross domestic product (GDP), Y, is restricted by its maximal value in its time series, Y^m . This premise requires a modification of the growth law. Now, the GDP growth rate depends on the current size of the GDP , Y, relative to its maximal size in its time series Y^m. We introduce y as $y = Y/Y^m$. Thus y range between 0 and 1. Again we index y by t, i.e. write y_t to refer to the size at time steps t=0,1,2,3,... Now growth rate of the gross domestic product is measured

$$y_{\ t+1}=(1+\alpha-\gamma\,\delta)\,y_t-\frac{\beta}{\lambda^2}\,y\,t^{\ 2} \quad (7)$$

This model given by equation (7) is called the logistic model. For most choices of α , β , γ, λ and δ, there is no explicit solution for (7). Namely, knowing α , β , γ, λ and δ, and measuring y_0 would not suffice to predict y_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (7) can lead to very interesting dynamic behavior , such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of y_t. This difference equation (7) will possess a chaotic region. Two properties of the chaotic solution are important:firstly, given a starting point y_0 the solution is highly sensitive to variations of the parameters α , β , γ, λ and δ ; secondly, given the parameters α , β , γ, λ and δ the solution is highly sensitive to variations of the initial point y_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t) \qquad z_t \in [0,1] \tag{8}$$

is equivalent to the iteration of growth model (6) when we use the identification

$$z_t = \frac{\beta}{\lambda^2 (1+\alpha-\gamma\,\delta)} Y_t \qquad \text{and} \qquad \pi = 1 + \alpha - \gamma\,\delta \tag{9}$$

Using (9) and (7) we obtain

$$z_{t+1} = \frac{\beta}{\lambda^2 (1+\alpha-\gamma\,\delta)} Y_{t+1} = \frac{\beta}{\lambda^2 (1+\alpha-\gamma\,\delta)} \left((1+\alpha-\gamma\,\delta) Y_t - \frac{\beta}{\lambda^2} Y_t^2 \right)$$

$$= \frac{\beta}{\lambda^2} Y_t - \frac{\beta^2}{\lambda^4 (1+\alpha-\gamma\,\delta)} Y_t^2$$

On the other hand, using (8) and (9) we obtain

$$z_{t+1} = \pi\, z_t\, (1-z_t) = (1+\alpha-\gamma\,\delta) \left[\frac{\beta}{\lambda^2 (1+\alpha-\gamma\,\delta)} Y_t \right] - (1+\alpha-\gamma\,\delta) \left[\frac{\beta}{\lambda^2 (1+\alpha-\gamma\,\delta)} Y_t \right]^2$$

$$= \frac{\beta}{\lambda^2} Y_t - \frac{\beta^2}{\lambda^4 (1+\alpha-\gamma\,\delta)} Y_t^2$$

Thus we have that iterating $y_{t+1} = (1+\alpha-\gamma\,\delta)\, y_t - \frac{\beta}{\lambda^2} y_t^{\,2}$ is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using $z_t = \frac{\beta}{\lambda^2 (1+\alpha-\gamma\,\delta)} Y_t$ and $\pi = 1 + \alpha - \gamma\,\delta$

It is important because the dynamic properties of the logistic equation (8) have been widely analyzed (L i and Y o r k e (1975), M a y (1976)).

Consider the first order, nonlinear difference equation

$$z_{t+1} = \pi z_t (1 - z_t) \qquad z_t \in [0,1] \tag{8}$$

The steady-state points, z^e, are obtained by solving

$$z^e - \pi z^e (1 - z^e) = 0 \tag{10}$$

Symplifying gives

$$z^e \left(\frac{1-\pi}{\pi} + z^e \right) = 0$$

The two steady-state points are

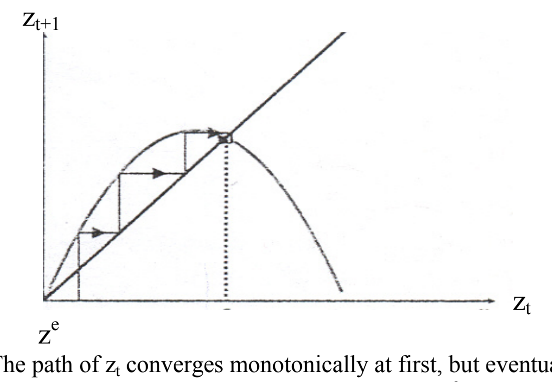
$$z^e = 0 \quad \text{and} \quad z^e = \frac{\pi - 1}{\pi} \qquad (11)$$

A strictly positive steady-state equilibrium exists only if $\pi > 1$. If $\pi \leq 1$, then the steady states are 0 and negative , respectively.

It is important to evaluate the derivative of equation (8) at the points z^e We obtain

$$\frac{d\,z_{t+1}}{d\,z_t} = \pi - 2\pi\, z_t = \begin{cases} \pi & \text{at } z^e = 0 \\ 2-\pi & \text{at } z^e = \dfrac{\pi - 1}{\pi} \end{cases}$$

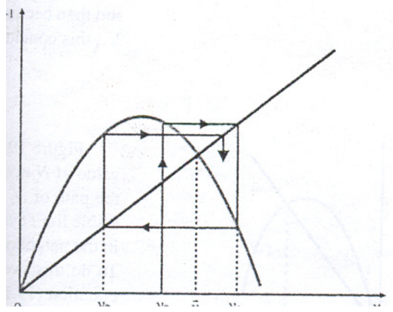
We can draw the phase diagram for this difference equation. The graph cuts through the 45^o line at the points 0 and $\frac{\pi - 1}{\pi}$. The graph peaks at z =1/2 where the slope is 0. If the second derivative of the function is negative (= - 2 π) then the function is concave. Whether the peak of the graph occurs to the left or right of the point $z^e = \frac{\pi - 1}{\pi}$ depends on whether π is larger or smaller than 2. If $1 < \pi < 2$ then the graph intersects the 45^o line to the left of the peak and the slope of the graph is positive at the stable steady – state point. If $\pi > 2$ then the slope of the graph is negative at the steady-state point. Figure 1. shows the phase diagram for the case in which $2 < \pi < 3$. This fulfils the condition for local stability. On the other hand, the slope of the graph is negative at the stable steady-state point.



F i g. 1. - The path of z_t converges monotonically at first, but eventually oscillates as it converges to the steady state z^e

Because the slope is negative but less than 1 in absolute value, z_t converges to the equilibrium value z^e from either direction within a neighborhood but the approach path will oscillate locally. As z_t approaches the neighborhood of z^e, the slope becomes negative, causing z_t to begin oscillating as it converges to the steady state. If $\pi \geq 3$ then the point $z^e = \frac{\pi - 1}{\pi}$ is no longer a stable steady state. An important characteristic of a hill shaped phase diagram is interesting dynamic behavior of z_t . Although it never converges to z^e , z_t oscillates within a bounded range and could even converge to a regular periodic behavior.

z_{t+1}



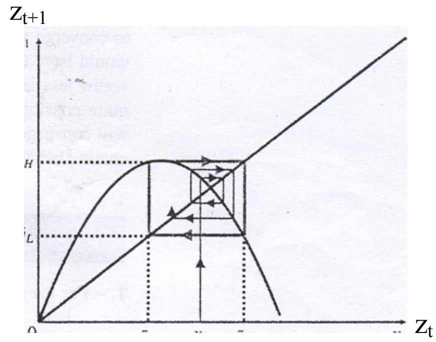
z_t

z^e

F i g. 2. - The phase diagram when $\pi = 3.5$

Figure 2. shows the phase diagram for the case of $\pi = 3.5$. It is known that f' (z_t) is negative in this case so that paths oscillate in the region of the stationary point. Besides, $f'(z_t)$ being smaller than -1 is outside the stable range. In this sense, paths do not converge to the steady stae point, z^e . They also do not diverge to zero or infinity. The reason is that f (z) is nonmonotonic, causing the phase curve to be hill – shaped. As a path diverges from z^e, z_1 is further away from z^e than was z_0 ; z_2 is even further away. Further, the path next runs into the positively sloped region of the phase curve, causing it to bounce back towards z^e.This returns the path immediately to the negatively sloped region of the phase curve, which leads once again to diverging oscillations.

If $1 < \pi < 3$ than z_t converges to the steady-state point and if $\pi \geq 3$ than z_t will not converge to the steady-state point but neither will it diverge endlessly. If π is slightly larger than 3, z_t converges to a stable limit cycle, two periods long. Namely, regardless of the starting value, z_t converges to a path that cycles back and forth in successive periods between two values. The system bifurcates at $\pi = 3$, which means that it changes from having one steady-state value, z^e , to having an equilibrium in which there are two values between which the path of z oscillates. If π is as large as about 3.5 , the two-period limit cycle itself becomes unstable and the system bifurcates again. Now, a stable limit cycle being four periods long.

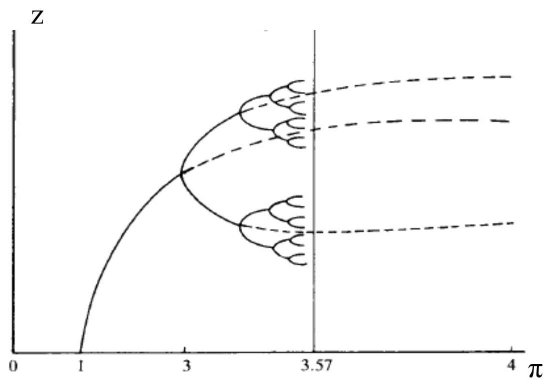


F i g. 3. - When $\pi = 3.2$. there is a stable limit cycle of two periods

Figure 3. shows the phase diagram for $\pi = 3.2$. At this value , we get a limit cycle of two periods.

Higher values of π produce repeated bifurcations and hence, repeated doublings of the period of the limit cycles. At some values of π , the time paths do not have cycles of any length, although they continue to be bounded. This is said to be chaos. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

More precisely, as π increases through the range $3 < \pi < 3.57$, stable cycles of lengths 1,2,4.8,16,... are generated. As π increases further , $3.57 < \pi < 4$, an infinite number of bifurcations arise, leading to chaos (see Figure 4.)



F i g. 4. - The structure of the set of stable solution to (8) changes as the parameter π varies

It is obtained that : (i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$; (ii) For $1 < \pi < 3.57$ there exist fixed points the number of which depends on π; (iii) For $1<\pi< 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$; (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$; (v) For $3 < \pi < 4$ all solutions will continuously fluctuate; (vi) For $3.57 < \pi < 4$ the solution becomes "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Conclusion

The agricultural share of a country's total output generally declines in the process of economic growth. There are two main reasons which explain this tendency: firstly, the Engel's Law (Ernest Engel, 19th century), i.e. demand for food and agricultural products tends to be income-inelastic ; secondly, economic growth generates improvements in agricultural productivity. Further, labor is released from agriculture. Also, as incomes rise, industry's share in the gross domestic product (GDP) is increased.

This paper suggests conclusion for the use of the economic growth model in predicting the fluctuations of the gross domestic product (GDP) . The model (7) has to rely on specified parameters α , β , γ, λ and δ and initial value of the gross domestic product (GDP), y_0. But even slight deviations from the values of parameters α , β , γ, λ and δ and initial value of the gross domestic product (GDP), y_0, show the difficulty of predicting a long-term behavior of the gross domestic product (GDP).

A key hypothesis of this work is based on the idea that the coefficient $\pi = 1 + \alpha - \gamma \delta$ plays a crucial role in explaining economic stability , where α – the autonomous rate of the GDP growth , δ – the proportion of the gross domestic product assigned to agriculture, γ- the influence of the agricultural share of a total output (P/Y) on the GDP growth rate.

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HAOTIČAN MODEL EKONOMSKOG RASTA I UČEŠĆE POLJOPRIVRE U OUTPUTU

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R e z i m e

Učešće poljoprivrede u u ukupnom outputu generalno opada u procesu ekonomskog rasta. Postoje dva razloga koja objašnjavaju ovu tendenciju: prvo, Engelov zakon (Ernest Engel, 19ti vek), tj. tražnja za hranom i poljoprivrednim proizvodima ima tendenciju dohodne neelastičnosti; drugo, ekonomski rast generiše povećanje produktivnosti u poljoprivredi. Dalje, radna snaga se oslobadja iz poljoprivrede. Takodje, kako dohodak raste, učešće industrije se povećava u bruto domaćem proizvodu (GDP) .

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Ovaj rad sugeriše zaključak da se haotični modela ekonomskog rasta može predvidjati fluktuacije bruto domaćeg proizvoda (GDP) u zavisnosti od učešća poljoprivrede u bruto domaćem proizvodu (GDP). Model (7) se zasniva na specificiranim parametrima α , β, γ, λ i δ i početnoj vrednosti bruto domaćeg proizvoda (GDP) , y_0. Čak i malo odstupanje od zadate vrednosti parametara α , β , γ, λ i δ i početne vrednosti bruto domaćeg proizvoda (GDP) , y_0, pokazuje kako je teško predvidati dugoročno ponašanje bruto domaćeg proizvoda (GDP).
Ključna hipoteza ovog rada se zasniva na ideji na koeficijent $\pi = 1 + \alpha - \gamma \delta$ ima suštinsku ulogu u objašnjenju ekonomske stabilnosti, pri čemu je α – autonomna stopa rasta bruto domaćeg proizvoda (GDP) , δ – proporcija bruto domaćeg proizvod (GDP) namenjena poljoprivredi, γ- uticaj proporcije bruto domaćeg proizvod (GDP) namenjene poljoprivredi (P/Y) na stopu rasta bruto domaćeg proizvoda (GDP).

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