

# A Chunk-Based Resource Allocation Scheme for Downlink MIMO-OFDMA Channel using Linear Precoding

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**Abstract**—This paper presents a novel chunk-based resource allocation scheme for MIMO-OFDMA multiuser downlink channel. In chunk-based resource allocation, a number of contiguous subcarriers of each OFDM symbol are considered as a chunk and resource allocation is performed on chunk-by-chunk basis. Herein, an optimization problem is formulated that aims to maximize system sum rate under an average power constraint per chunk when Zero Forcing Beamforming (ZFB) is used for inter-user interference elimination within chunks. Under this framework, a low-complexity resource allocation algorithm is presented that exploits frequency and space correlation of wireless channels and jointly solves the problems of chunk allocation and power allocation. Simulation results show that the proposed algorithm performs closely to the optimal solution of the examined problem.

**Keywords**—Resource Allocation, OFDMA, MIMO, SDMA

## I. INTRODUCTION

**D**irty Paper Coding (DPC) is the optimal transmission technique for multiuser MIMO Downlink [1]. However, DPC is still considered as a theoretical performance bound, mainly because of its high complexity. In [2], [3], [4] and [5], linear processing Resource Allocation (RA) solutions were presented for narrowband channel where ZFB and greedy user selection have been employed. All of them perform closely to the optimal possible and have significantly lower complexity than DPC. Moreover, they can be easily extended to wideband wireless systems where OFDMA is used to improve throughput and transmission robustness. In such systems, the set of available subcarriers is grouped into chunks and RA is performed on chunk basis. In contrast with subcarrier-based RA, chunk-based RA may lead to notable reduction of signaling and coding overhead [6].

Success of chunk-based RA is based on the fact that the coherence bandwidth is significantly larger than subcarrier spacing. As a result, adjacent subcarriers of each OFDM symbol may be highly correlated and face similar propagation conditions. However, the majority of the proposed RA policies exploit naively this point. In the most cases, chunk allocation is performed based on simple representative values for each chunk like middle subcarrier channel or mean

channel quality across its subcarriers [8], [9]. Generally, it seems that existing RA schemes do not consider either the subcarrier diversity or the inter-subcarrier correlation within each chunk. A qualitative description of such quantities over different channel models was presented in [10], [11]. The results therein indicate worthwhile performance benefit if chunk-based RA is designed to take advantage of the inherent inter-subcarrier correlation rather than ignore it or smooth it by a mean operator.

In MIMO-OFDMA wireless communications systems, the number of active users is much larger than the number of simultaneously served users. Hence, user scheduling plays a key role in overall system's performance. This paper, is focused on user scheduling when RA is performed on chunk basis in order to maximize the aggregate system throughput. Even if such a quantity doesn't explicitly support user QoS, it consists a critical performance metric from a system perspective, as the time evolves. In [4], [5], a low-complexity user selection approach was proposed for narrowband channel that is based on spatial correlation between channels of different users. In this paper, the spatial-based user selection criterion is revised and it is extended in wideband scenarios to take account both space (antenna) and frequency (inter-subcarrier) correlation. This way, a user selection scheduling is proposed that exploits both spatial and frequency correlation. Moreover, when it is combined with ZFB, a highly efficient chunk-based RA scheme is formed both in performance and complexity. Because of its low complexity, the proposed scheme constitutes a competitive, feasible solution for broadband wireless systems [12], [13].

The remainder of the paper is organized as follows: In Section II, the model under consideration is introduced and the addressed problem is formulated. Optimal, linear processing scheme is reviewed briefly in Section III followed by the description of the suboptimal proposed scheme. Simulation results are provided in Section IV and concluding remarks are given in Section V.

*Notation:* In the following, lowercase bold letters denote column vectors and bold uppercase denote matrices.  $(\cdot)^T$  denotes transpose,  $(\cdot)^\dagger$  the conjugate transpose,  $\|\cdot\|$  the

Euclidean norm of a vector and  $tr(\cdot)$  the trace of a matrix. Set difference and set cardinality are denoted by  $(\cdot) \setminus (\cdot)$  and  $|\cdot|$ , respectively.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. System Model

The downlink of a multiuser OFDMA-based system is considered consisting of a single Base Station (BS) with  $T_x$  antennas and  $K$  single-antenna users, such that  $K \gg T_x$ . BS antenna elements are separated based on the half of the carrier wavelength. Thus, all  $T_x$  wireless channels between BS and each user are considered uncorrelated. Moreover, it is assumed that each user estimates perfectly its own channel and feeds it instantly back to the BS. Thus, BS has full Channel Side Information (CSI) each time it performs RA.

In chunk-based RA, a set of  $L$  contiguous subcarriers is properly combined into one chunk and the resources are allocated on a per-chunk basis. In time axis, a chunk is constituted by a group of contiguous OFDM symbols that are not significantly vary (called frame). If  $N$  data subcarriers exist per OFDM symbol, there are totally  $C = \lceil N/L \rceil$  chunks. When SDMA is used within each chunk, more than one users may share the same subcarriers and the transmitted signals are combinations of the corresponding symbols. Generally, inter-subcarrier correlation between subcarriers  $m$  and  $n$  of the same user can be described by [7]

$$r_{m,n} = \frac{1}{\sqrt{1 + \left(\frac{d(m-n)Df}{B_c}\right)^2}} \quad m, n = 1, \dots, N, \quad (1)$$

where  $B_c$  is the coherence bandwidth,  $d$  is a decay factor that specifies the amount of correlation and  $Df$  is the frequency separation between two consecutive subcarriers. Under the assumption of homogeneous fading, eq. (1) can be used to characterize channel correlation (in frequency) of all the users. If the wireless channel between BS and user  $k$ ,  $k = 1, \dots, K$ , in subcarrier  $n$ ,  $n = 1, \dots, N$ , is denoted by  $\mathbf{h}_{n,k} = [h_{n,k,1} \dots h_{n,k,T_x}]^T$  and the normalized vector  $\mathbf{w}_{n,k} = [w_{n,k,1} \dots w_{n,k,T_x}]^T$  maps the corresponding signal to  $T_x$  transmit antennas within subcarrier  $n$ , the general transmission model within chunk  $c$ ,  $c = 1, \dots, C$ , is

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{W}_n \mathbf{s}_n + \mathbf{z}_n, \quad n = (c-1)L + 1, \dots, cL, \quad (2)$$

where  $Q_c$ , is the subset of users selected for transmission within all subcarriers of chunk  $c$ , the vector  $\mathbf{y}_n = [y_{n,1} \dots y_{n,|Q_c|}]^T \in \mathbb{C}^{|Q_c| \times 1}$  contains all the received signals,  $\mathbf{x}_n = \mathbf{W}_n \mathbf{s}_n \in \mathbb{C}^{T_x \times 1}$  is the transmitted signal in subcarrier  $n$ ,  $\mathbf{H}_n \in \mathbb{C}^{Q_c \times |T_x|}$  is the matrix of channels  $\forall k \in Q_c$  and  $\mathbf{W}_n \in \mathbb{C}^{T_x \times |Q_c|}$  is the corresponding beamforming matrix. The uncorrelated entries of  $\mathbf{s}_n \in \mathbb{R}^{Q_c \times 1}$  contain the symbols destined to users in  $Q_c$  and  $\mathbf{z}_n = [z_{n,1} \dots z_{n,|Q_c|}]^T \in \mathbb{C}^{|Q_c| \times 1}$  represents  $|Q_c|$  i.i.d. samples of circularly symmetric complex Gaussian additive

noise with zero mean and variance  $\sigma^2$ . The received signal of user  $k \in Q_c$  is given by

$$y_{n,k} = \mathbf{h}_{n,k}^T \mathbf{w}_{n,k} s_{n,k} + \sum_{i \in Q_c, i \neq k} \mathbf{h}_{n,k}^T \mathbf{w}_{n,i} s_{n,i} + z_{n,k}, \quad (3)$$

where the second term is the undesirable interference caused by the simultaneous transmission to more than one user. When ZFB is used, symbols to each user are encoded independently and the beamforming vectors are selected in a way that the zero interference condition is active, meaning  $\mathbf{h}_{n,i}^T \mathbf{w}_{n,j} = 0$  for every two different users  $i$  and  $j$ , which transmit simultaneously. The total beamforming matrix within subcarrier  $n$  is  $\mathbf{W}_n = \mathbf{H}_n^\dagger (\mathbf{H}_n \mathbf{H}_n^\dagger)^{-1}$  and the SINR for each  $k \in Q_c$  is

$$SINR_{n,k} = \frac{\|\mathbf{h}_{n,k}^T \mathbf{w}_{n,k}\|^2 p_{n,k}}{\sigma^2}, \quad (4)$$

where  $p_{n,k} = \mathbb{E}[s_{n,k} s_{n,k}^\dagger]$  is the transmit power of user  $k \in Q_c$ . The transmission rate of each user  $k \in Q_c$  is given by

$$R_k^c = \sum_{n=(c-1)L+1}^{cL} R_{n,k} = \sum_{n=(c-1)L+1}^{cL} \log_2(1 + SINR_{n,k}).$$

### B. Problem Formulation

This paper focuses on the maximization of the system aggregate sum rate under an average transmit power constraint per chunk. Such a constraint has important practical value since it results in more uniform spread of the available power over the entire bandwidth. Specifically, the addressed problem is:

$$\begin{aligned} \max_{Q_c, p_{n,k}} \sum_{c=1}^C \sum_{k \in Q_c} R_k^c \quad s.t. \quad (5) \\ \sum_{n=(c-1)L+1}^{cL} tr(\mathbf{R}_n^x) \leq P_{chunk}, \forall c = 1, \dots, C, \\ |Q_c| \leq T_x, \forall c = 1, \dots, C. \end{aligned}$$

where  $P_{chunk}$  is the power constraint and  $\mathbf{R}_n^x = \mathbb{E}[\mathbf{x}_n \mathbf{x}_n^\dagger]$  is the covariance matrix of the transmitted signal in subcarrier  $n$ . Even if the above formulation may be used to describe more generic models, the interest herein is focused only in homogeneous case, where users' channels are statistically identical. The problem of eq. (5) summarizes a user selection and a power allocation problem that, generally, should jointly be carried out.

## III. OPTIMAL AND SUBOPTIMAL SOLUTIONS

### A. Optimal Solution

The problem of eq.(5) is not convex because sets  $Q_c$ ,  $c = 1, \dots, C$  are unknown. Optimal solution can be specified by searching over  $\sum_{i=1}^{T_x} \binom{K}{i}$  different set of users with cardinality up to  $T_x$  within each chunk. For each candidate set, the throughput is specified by waterfilling  $P_{chunk}$  across the corresponding effective channels  $c_{n,k}(Q_c) =$

$\{[(\mathbf{H}_n(Q_c)\mathbf{H}_n(Q_c)^\dagger)^{-1}]_{k,k}\}^{-1}$ , where  $\mathbf{H}_n(Q_c)$  is the part of  $\mathbf{H}_n$  that corresponds to users  $k \in Q_c$ ,  $n = (c-1)L + 1 \dots cL$ . The rate of user  $k$ ,  $\forall k \in Q_c$  is

$$R_k^c(Q) = \sum_{n=(c-1)L+1}^{cL} [\log_2(\mu c_{n,k}(Q_c))]^+, \forall c = 1, \dots, C$$

where  $\mu \in \mathbb{R}$  is obtained by solving the water-filling equation  $\sum_{n=(c-1)L+1}^{cL} \sum_{k \in Q_c} \left[ \mu - \frac{1}{c_{n,k}(Q_c)} \right]^+ = P_{chunk}$  [14]. Clearly, exhaustive search over all the different valid sets of users leads to a prohibitory high complexity for values of  $K$  and  $T_x$  with practical interest. If effective channels of each set of candidates are computed in  $\mathcal{O}(T_x^3)$  time (a  $T_x \times T_x$  channel inversion is required per each candidate set), the overall complexity per is  $\mathcal{O}\left(LT_x^3 \sum_{i=1}^{T_x} \binom{K}{i}\right)$  per chunk. In the following subsection, a suboptimal solution is described which is characterized by linear complexity w.r.t. the number of users and performs closely to the Upper Bound (UB) of eq. (5).

### B. Proposed Solution

The proposed Frequency-Space Correlation (FSC) algorithm is based on a balanced examination of spatial and inter-subcarrier (frequency) correlation between couples of different users. The normalized correlation between channels of users  $i$  and  $j$  in subcarrier  $n$  is given by  $\rho_{i,j}^n = \frac{\|\mathbf{h}_{n,i}^\dagger \mathbf{h}_{n,j}\|}{\|\mathbf{h}_{n,i}\| \|\mathbf{h}_{n,j}\|}$ ,  $0 \leq \rho_{i,j}^n \leq 1$ . In FSC, each set  $Q_c$  is gradually formed by appending users with low average spatial correlation to the already selected ones. Generally, for a given amount of power, the higher the value of  $\rho_{i,j}^n$  is, the lower the sum rate of users  $i$  and  $j$  becomes when they transmit simultaneously within the same subcarrier  $n$ . Hence, it is highly probable that low correlated grouped users would show higher aggregate throughput. However, it is not always true that the addition of the lowest correlated user to the already selected ones leads to the maximum increase of sum rate. As it was observed in [4], [5] it is more efficient if a pool of users with low average correlation to the members of  $Q_c$  is formed and the one that leads to the maximum increase of sum rate, if any, is selected. In FSC, this pool of users is determined by exploitation of inter-subcarrier correlation since two users with low  $\rho_{i,j}^n$  will have low  $\rho_{i,j}^m$  for  $m \neq n$  within the same chunk with high probability.

Assume that  $\mathcal{U} = \{1, \dots, K\}$  denotes the set of all  $K$  users. The FSC RA algorithm is the following:

#### A) Initialization:

- Set  $Q_c = \emptyset$ ,  $\forall c = 1, \dots, C$ ,

#### B) First User Selection:

- For each chunk,  $c = 1, \dots, C$ , user  $k_c^* = \arg \max_{k \in \mathcal{U}} R_k^c$  is found.
- Set  $Q_c = \{k_c^*\}$ ,  $SR_c = R_{k_c^*}^c$  and  $count = 1$ .

C) *Iteration Step (User Set Completion)*: While  $count < T_x$ , the following steps are performed for each chunk  $c = 1, \dots, C$ .

- $\rho_{i,k}^n$ ,  $n = (c-1)L + 1, \dots, cL$ , is computed for each  $i \in Q_c$  and  $k \in \mathcal{U} \setminus Q_c$ . Let  $Cor_k^n = \frac{\sum_{i \in Q_c} \rho_{i,k}^n}{|Q_c|}$  be the average correlation within subcarrier  $n$  of chunk  $c$  between already selected users and candidate user  $k$ ,  $k \in \mathcal{U} \setminus Q_c$ .
- A group of candidate users, let  $\mathcal{A}_c$ , is formed that contains the users with the smallest  $Cor_k^n$  in each subcarrier of the chunk. Each member  $a \in \mathcal{A}_c$  is temporarily added to  $Q_c$  and the sum rate of the chunk is computed via waterfilling on the effective channels. The member of  $\mathcal{A}_c$ , let  $a^*$ , that leads to the maximum sum rate is selected if its insertion to  $Q_c$  increases the previous sum rate of the chunk. In such a case, the sum rate is renewed accordingly,  $Q_c = \{Q_c \cup a^*\}$ ,  $count = count + 1$  and step C) is repeated. Otherwise,  $count = T_x$  and the process is terminated.

D) *Output*: Sets  $Q_c$ ,  $c = 1, \dots, C$ .

Given that the processing across chunks can be done in parallel, algorithm's time complexity may be described on chunk basis. The complexity within chunk  $c$ ,  $c = 1, \dots, C$  is mainly due to the computation of the average correlation  $Cor_k^n$ ,  $\forall k, k \in \mathcal{U} \setminus Q_c$  and  $\forall n = (c-1)L + 1, \dots, cL$ , and the required matrix inversions when each candidate user is temporarily added to the transmission set. Generally, the computation of  $\rho_{i,j}$  can be done within time  $\mathcal{O}(T_x)$  for each given pair of users  $(i, j)$ ,  $\forall i, j = 1, \dots, K$ . As a result, all average correlations  $Cor_k^n$ ,  $\forall k, k \in \mathcal{U} \setminus Q_c$  and  $\forall n = (c-1)L + 1, \dots, cL$  can be computed in  $\mathcal{O}(LKT_x)$  per iteration of step C). When the group of candidates  $\mathcal{A}_c$  (with cardinality up to  $L$ ) has been formed, effective channels within each subcarrier can be computed in time  $\mathcal{O}(T_x^2)$  by using blockwise matrix inversion identity over the effective channels of the previous iteration [2], [15]. Hence, the complexity of step C) is  $\mathcal{O}(LKT_x + L^2T_x^2)$  and the overall complexity for each chunk is  $\mathcal{O}((LKT_x + L^2T_x^2)T_x)$ . Normally, algorithm's complexity is specified by the first term since  $K > LT_x$ .

So far, to the best of the authors knowledge, chunk-based RA is performed using a single, representative value for each chunk, e.g., the mean value of subcarriers quality within each chunk [8], [9]. Thus, for comparison reasons a chunk-based RA scheme that follows the conventional way is used below. Specifically, a Middle Subcarrier (MS) based RA scheme is applied that employs the efficient approach in [4] within the middle subcarrier of each chunk to specify the transmission set of the chunk. As can be seen by eq. (1), the middle subcarrier of each chunk shows the highest sum (frequency) correlation with the other subcarriers of the same chunk. Hence, it is highly probable that a user transmission set with

high sum rate in the middle subcarrier will have high overall sum rate in the chunk. Interestingly, this low complexity solution performs very competitively as long as strong inter-subcarrier correlation exists in the system. The complexity of MS algorithm is  $\mathcal{O}(KT_x^2)$ . In Table I, time complexity of all mentioned algorithms is summarized.

Table I  
ALGORITHMS COMPLEXITY

Upper Bound (UB)	$\mathcal{O}\left(CLT_x^3 \sum_{i=1}^{T_x} \binom{K}{i}\right)$
Frequency-Space Correlation (FSC)	$\mathcal{O}(CLT_x^2 K)$
Middle Subcarrier (MS)	$\mathcal{O}(CT_x^2 K)$

#### IV. SIMULATION RESULTS

As it was mentioned above, in all the presented simulations only small-scale fading is considered since path loss and large-scale fading effects are compensated by power control. The available bandwidth is 2.5 MHz, decay factor is  $d = 2$ ,  $N = 128$  (thus, subcarrier spacing is approximately 20 kHz) and the results are averaged over 10000 experiments. The performance of the proposed algorithm is strongly dependent on the existent inter-subcarrier correlation. In Fig. 1, the inter-subcarrier correlation of eq. (1) is depicted when  $B_c$  is equal to 0.25 MHz and 0.5 MHz. Moreover, three other correlation profiles are illustrated as references which correspond to practically consisted 6-path and 12-path Typical Urban (TU) Power Delay Profiles (PDPs) of COST model [16] and a uniform, 12-path PDP. Clearly, the curve of  $B_c = 0.5$  MHz corresponds to a strong inter-subcarrier correlation case while the curve of  $B_c = 0.25$  MHz corresponds to a medium inter-subcarrier correlation case. In wireless systems, the value of  $B_c$  is approximately related to the delay spread ( $T_d$ ) as ( $B_c = 1/(2\pi T_d)$ ) and outdoor typical delay spread values range in the area of a few  $\mu\text{sec}$ . Thus, usually, coherence bandwidth takes values up to some hundreds KHz.

In Fig. 2, the sum rate versus chunk size is depicted when SNR is 10 dB and  $B_c$  is equal to 250 and 500 KHz. Even if results are presented for only  $T_x = 4$  and  $K = 20$ , similar behavior was observed for higher values of antennas and/or users. It can be seen that FSC performs closely to the Upper Bound, with performance over than 95% for chunk sizes with practical interest, e.g.,  $L \geq 13$ . Typical chunk sizes are  $L = 24$  for WiMAX and  $L = 12$  for LTE [12], [13]. Even if MS performs better than FSC for small chunk sizes (where strong inter-subcarrier correlation exists), its performance is degraded rapidly as  $L$  increases. This is because MS exploits multiuser diversity only within a small portion and not over the entire available spectrum of the chunk. In the opposite, FSC's performance is smoothly degraded as the chunk size increases since both frequency (inter-subcarrier) and multiuser diversity are efficiently exploited by the way the pool of candidates is formed in each iteration step of the

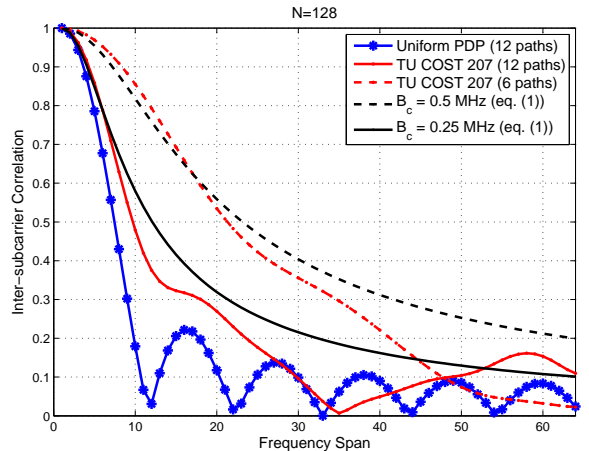


Figure 1. Inter-subcarrier Correlation.

algorithm. As a result, FSC may be a useful scheme even for the case of jointly managed groups of (smaller) chunks, especially when  $B_c$  is relatively high.

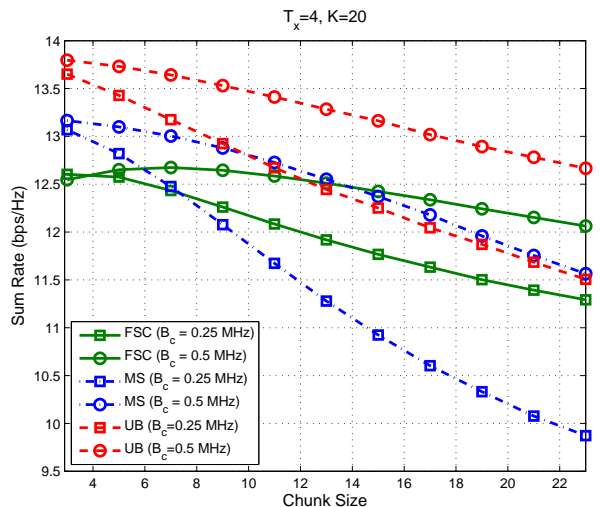


Figure 2. Sum Rate vs Chunk Size.

In Fig. 3 the influence of the coherence bandwidth over the sum rate is depicted. As it was mentioned earlier,  $B_c$  varies according to the channel model but typically it is not higher than a few hundreds of KHz [13]. For that area, FSC RA offers significant performance benefits versus middle subcarrier-based RA, under the cost of limited increase in complexity as it is shown in Table I. For example, when  $L = 15$  and  $B_c = 0.3$  MHz, FSC offers a 0.6 bps/Hz improvement over MS performance.

In Fig. 4, FSC performance is depicted versus SNR and it is compared with the performance of MS for  $K = 100$ ,  $B_c = 0.25$  MHz and  $T_x$  equal to 4 and 8. It is well



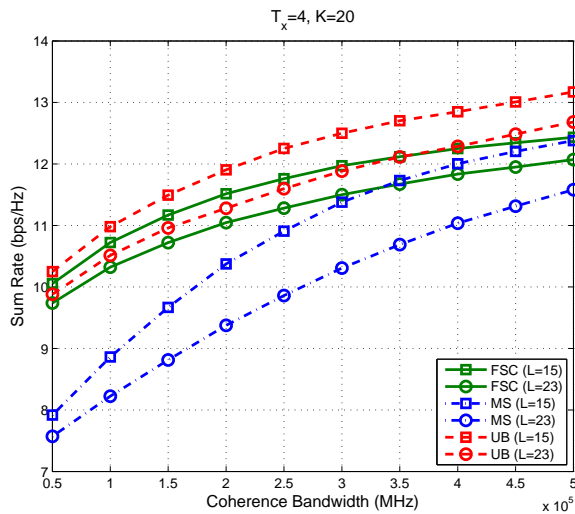


Figure 3. Sum Rate vs Coherence Bandwidth.

known that ZFB doesn't perform well in low SNR regime. Hence, both FSC and MS perform poorly in low SNR's [17]. However, as SNR increases the performance is almost linear to SNR and the benefits of exploiting subcarriers' diversity by FSC becomes clear, especially as the chunk size increases and space diversity is enhanced by the usage of more transmit antennas.

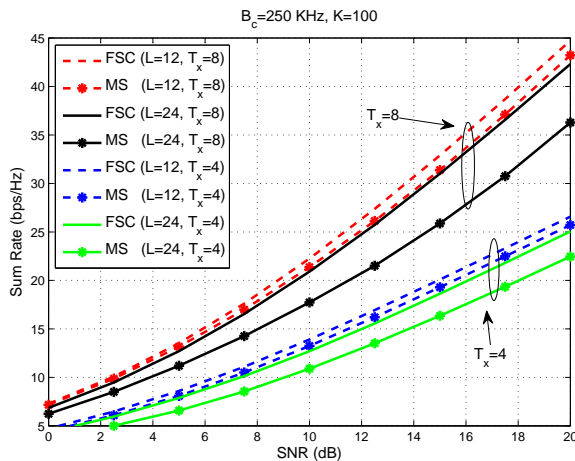


Figure 4. Sum Rate vs SNR.

## V. CONCLUSIONS AND FUTURE WORK

In this paper, a low-complexity chunk-based RA algorithm is presented for OFDMA MIMO Downlink. The algorithm aims to maximize the throughput of the system by efficiently exploiting frequency and spatial correlation of wireless channels of the users. Simulation results have demonstrated that nonnegligible gain is achieved when the system is optimized

on chunk basis. Moreover, that the proposed RA scheme performs closely to the optimal possible solution. Future work includes the performance analysis of the proposed scheme and its integration into cross layer solutions for video streaming applications over broadband wireless systems.

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