

A Class Of Finite Difference Schemes For Interface Problems With An HOC Approach

H. V. R. Mittal¹, Jiten C. Kalita², Rajendra K. Ray^{1*}

¹*School of Basic Sciences, Indian Institute of Technology Mandi, India.*

²*Department of Mathematics, Indian Institute of Technology Guwahati, India.*

SUMMARY

In this paper, we propose a new methodology for numerically solving elliptic and parabolic equations with discontinuous coefficients and singular source terms. This new scheme is obtained by clubbing a recently developed Higher Order Compact (HOC) methodology with special interface treatment for the points just next to the points of discontinuity. The overall order of accuracy of the scheme is at least second. We first formulate the scheme for one-dimensional (1D) problems and then extend it directly to two-dimensional (2D) problems in polar coordinates. In the process, we also perform convergence and related analysis for both the cases. Finally, we show a new direction of implementing the methodology to 2D problems in cartesian coordinates. We then conduct numerous numerical studies on a number of problems, both for 1D and 2D cases including the flow past circular cylinder governed by the incompressible Navier-Stokes (N-S) equations. We compare our results with existing numerical and experimental results. In all the cases our formulation is found to produce better results on relatively coarser grids. For the circular cylinder problem, the scheme used is seen to capture all the flow characteristics including the famous von-Kármán vortex street. Copyright © 2010 John Wiley & Sons, Ltd.

Received ...

KEY WORDS: Interface; HOC scheme; Navier-Stokes equation; discontinuous coefficients; non-uniform grids; von-Kármán vortex street.

1. INTRODUCTION

Flow problems involving discontinuous coefficients and singular source terms still pose a tough challenge in the field of computational fluid dynamics (CFD). Such problems generally have non smooth solutions with jumps across lower dimensional interfaces. Standard finite difference approximations fail to yield correct numerical solutions of such problems because of the fact that the Taylor's expansions, which provide the platforms for such approximations, are not valid for non smooth functions. Of late, there has been a spur of interest in developing numerical methods for computing multiphase flows and multi-physics problems with interfaces. Such problems arise in numerous branches of science and engineering such as biochemical processing, free surface flow, drop deformation, solid mechanics, porous media flow, formation of gas bubbles in liquid, mining etc. For example, in problems involving dissimilar materials, at the interfaces, the material properties (elastic moduli, permeability, conductivity, etc) are discontinuous.

The last few decades have seen several approaches for numerically solving the interface equations [18, 19, 20, 31, 39, 40, 45]. Most of the earlier numerical works on such problems involved mainly the use of immersed boundary (IBM) or immersed interface (IIM) methods on uniform grids and

*Correspondence to: School of Basic Sciences, Indian Institute of Technology Mandi, India