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A class of group divisible designs

Jennifer Seberry University of Wollongong, jennie@uow.edu.au

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A CLASS OF GROUP DIVISIBLE DESIGNS

Jennifer Seberry

Let Q be the incidence matrix of a cyclic projective plane of order q, which exists whenever q is a prime power, then

$$QQ^{T} = qI + J, QJ = (q+1)J.$$

Let $Q = \sum_{d \in D} T^d$ where T is the matrix which is 1 in the (i,i+1) deD position (reduced mod q^2+q+1) for $1 = 1, \dots, q^2+q+1$. Then

$$Q^{2} = \sum_{d \in D} T^{2d} + 2 \sum_{\substack{e,f \in D \\ e \neq f}} T^{e+f}.$$

It is shown in Geramita and Seberry [3, Theorem 4.124] that Q^2 has entries 0,1,2. Let $C = \sum_{d \in D} T^{2d}$. Then D is a difference set. C satisfies

$$CC^{T} = qI + J, CJ = (q+1)J.$$

Let $W = Q^2 - J$ which has entries (0,1,-1). Let A be the matrix which is 1 where W is 1 and zero elsewhere. Let B be the matrix which is 1 where W is -1 and zero elsewhere. Then

$$W = A - B$$
 and $A + B = J - C$.

Now W and A+B satisfy $WW^{T} = AA^{T} + BB^{T} - AB^{T} - BA^{T} = q^{2}I,$ (A+B) (A+B)^T = $AA^{T} + BB^{T} + AB^{T} + BA^{T} = qI + (q^{2}-q)J.$

Hence

$$AA^{T} + BB^{T} = \frac{1}{2}(q^{2}+q)I + \frac{1}{2}(q^{2}-q)J,$$

$$AB^{T} + BA^{T} = \frac{1}{2}(q-q^{2})I + \frac{1}{2}(q^{2}-q)J.$$

So

 $\left(\begin{smallmatrix} A & B \\ B & A \end{smallmatrix}\right)$

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is a (0,1) matrix with q^2 elements in each row and column with inner product between any two rows 0 (if they are in different submatrices of the partition) or $\frac{1}{2}(q^2-q)$ (otherwise).

We refer the reader to [1,2] for the definition and some important properties of symmetric regular group divisible designs. Now we can say: Suppose q is a prime power; then there is a symmetric regular group divisible design with parameters

 $(v,b,r,k,\lambda_1,\lambda_2,m,n) = (2(q^2+q+1), 2(q^2+q+1),q^2,q^2,0,\frac{1}{2}(q^2-q),q^2+q+1,2).$

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University of Sydney Sydney, Australia.

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