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A CLUSTER OF BLACK HOLES AT THE GALACTIC CENTER

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ABSTRACT

If the stellar population of the bulge contains black holes formed in the final core collapse of ordinary stars with $M \gtrsim 30\ M_\odot$, then about 25,000 stellar mass black holes should have migrated by dynamical friction into the central parsec of the Milky Way, forming a black hole cluster around the central supermassive black hole. These black holes can be captured by the central black hole when they randomly reach a highly eccentric orbit due to relaxation, either by direct capture (when their Newtonian peribothron is less than 4 Schwarzschild radii) or after losing orbital energy through gravitational waves. The overall depletion timescale is ~ 30 Gyr, so most of the 25,000 black holes remain in the central cluster today. The presence of this black hole cluster would have several observable consequences. First, the low-mass, old stellar population should have been expelled from the region occupied by the black hole cluster as a result of relaxation, implying a core in the profile of solar-mass red giants with a radius of ~ 2 pc (i.e., 1′). The observed central density cusp (which has a core radius of only a few arcseconds) should be composed primarily of young ($\lesssim 1$ Gyr) stars. Second, flares from stars being captured by supermassive black holes in other galaxies should be rarer than usually expected because the older stars will have been expelled from the central regions by the black hole clusters of those galaxies. Third, the young ($\lesssim 2$ Gyr) stars found at distances ~ 3–10 pc from the Galactic center should be preferentially on highly eccentric orbits. Fourth, if future high-resolution K-band images reveal sources microlensed by the Milky Way's central black hole, then the cluster black holes could give rise to secondary ("planet-like") perturbations on the main event.

Subject headings: black hole physics — Galaxy: center — Galaxy: kinematics and dynamics

1. INTRODUCTION

The measurement of proper motions and radial velocities of stars within the central parsec of the Galaxy has led to the conclusion that a black hole of mass $(3.0 \pm 0.3) \times 10^6\ M_\odot$ is present in the center (Eckart & Genzel 1997; Ghez et al. 1998; Genzel et al. 2000). There is also increasing evidence that massive black holes are found in the centers of other galaxies (Richstone et al. 1998).

The central region of the Galaxy is also peculiar because the relaxation time among stars can be shorter than the age of the Galaxy, owing to the high density. The process of relaxation leads to a stellar cusp, which has a density profile $\rho \propto r^{-7/4}$ when all the stars have the same mass (Bahcall & Wolf 1976). Several interesting physical processes take place among the stars in this cusp: stars can come close enough to physically collide with each other, and they can also come sufficiently close to the black hole to be tidally disrupted or swallowed (e.g., Frank & Rees 1976; Lightman & Shapiro 1977; Quinlan, Hernquist, & Sigurdsson 1995; Sigurdsson & Rees 1997).

One of the consequences of the relaxation is that the most massive objects will sink to the center of the stellar cusp. Among an old stellar population, the most massive objects should be black holes formed in the final core collapse of massive stars. We assume in this paper that most massive stars with $M \gtrsim 30\ M_\odot$ produce black holes, most with a mass of 7 $M_\odot$ (Bailyn et al. 1998). The high mass of these black holes implies that their dynamical friction time to move to the center of the Galaxy is shorter than a Hubble time over a much larger volume than the one where ordinary stars have a short relaxation time (Morris 1993). We will find in § 2 that this should lead to the formation of a cluster of stellar black holes around the central supermassive black hole (hereafter "Sgr A*"), and that other stars are ejected from the region occupied by this cluster. In § 3 we discuss the rate at which the black holes in this cluster are captured by Sgr A*, and we find that most of the black holes should still be present in the cluster. Several observable consequences of the presence of this black hole cluster are discussed in § 4.

2. CLUSTER FORMATION

The deprojected light profile in the inner kiloparsec of the Galaxy scales as $r^{-1.8}$ (Becklin & Neugebauer 1968; Mezger, Duschl, & Zylka 1996), while the predicted profile around a massive black hole ($r \lesssim 1$ pc) scales as $r^{-7/4}$ (Bahcall & Wolf 1976). For simplicity, we therefore adopt a density profile $\rho(r)$,

$$\rho(r) \propto r^{-7/4} . \tag{1}$$

From the model fit of Genzel et al. (2000) to the velocity dispersion data, we find that the total mass within $r_0 = 1.8$ pc is 2 $M_{\rm cbh}$ (see their Fig. 17), where

$$M_{\rm cbh} = 3 \times 10^6\ M_\odot \tag{2}$$

is the mass of Sgr A*. Hence, the total distributed mass inside 1.8 pc is $M_{\rm cbh}$, and the density profile is

$$\rho_*(r) = \frac{5}{16\pi} \frac{M_{\rm cbh}}{r_0^3} \left(\frac{r}{r_0}\right)^{-7/4} , \quad r_0 \equiv 1.8 \text{ pc} . \tag{3}$$

We assume that this density profile is entirely composed of stars, brown dwarfs, and stellar remnants. To calculate the mass fraction of black holes, $\eta_{\rm bh}$, we use the following initial mass function. For the range $M > 1\ M_\odot$, we adopt a Salpeter law $dN/dm \propto m^{-\alpha}$ with $\alpha = 2.35$. For 0.7 $M_\odot$ <

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$m < 1\ M_\odot$, we adopt $\alpha = 2$ from Zoccali et al. (2000). For $0.05\ M_\odot < m < 0.7\ M_\odot$, we adopt $\alpha = 1.65$ by correcting the result of Zoccali et al. (2000) for binaries according to the adjustment of Gould, Bahcall, & Flynn (1997) and by extending the power law beyond the last observed point at $0.15\ M_\odot$. We cut off the mass function at $m \sim 0.05\ M_\odot$ in accordance with the preliminary indications from microlensing (Han & Gould 1996). We assume that the vast majority of stars in the central region are old, so that essentially all progenitors with masses 1–8 $M_\odot$ have become 0.6 $M_\odot$ white dwarfs, those with masses 8–30 $M_\odot$ have become 1.4 $M_\odot$ neutron stars, and those with masses 30–100 $M_\odot$ have become 7 $M_\odot$ black holes (although young stars are present in the Galactic center, their contribution to the total stellar mass is not significant enough to alter the mass fraction in stellar remnants). We then find

$$\eta_{\rm bh} = 1.6\%, \quad \langle m \rangle = 0.23\ M_\odot\ , \tag{4}$$

where $\langle m \rangle$ is the mean mass of the population.

Once this population is formed, the black holes will sink toward the center on the dynamical friction timescale (Binney & Tremaine 1987)

$$t_{\rm df}^{-1} = (\ln \Lambda)\, \frac{4\pi G \rho G m_{\rm bh}}{v^3} \int_0^v du\, 4\pi u^2 f(u)\ , \tag{5}$$

where $m_{\rm bh} = 7\ M_\odot$ is the mass of the black hole, v is its velocity, $f(u)$ is the velocity distribution of the ambient stars, and the integral gives the fraction of ambient stars with speeds below v. For a Gaussian velocity distribution with dispersion σ, and a typical black hole speed $v^2 = 3\sigma^2$, the value of the integral is 0.54. For the Keplerian part of the potential ($r < r_0$), $\ln \Lambda = \ln (M_{\rm cbh}/m_{\rm bh}) = 13$. For $r > r_0$, ln Λ rises slightly, but we ignore this in the interest of simplicity. We evaluate $t_{\rm df,0}$, the fiducial dynamical friction time at r_0, as

$$t_{\rm df,0} = 1.4\ {\rm Gyr} \tag{6}$$

and note its scaling in the two regimes:

$$t_{\rm df} = t_{\rm df,0}\left(\frac{r}{r_0}\right)^{1/4} \quad (r < r_0),$$
$$t_{\rm df} = t_{\rm df,0}\left(\frac{r}{r_0}\right)^{17/8} \quad (r > r_0)\ . \tag{7}$$

Hence, after a time t, all the black holes that were originally within a radius r will collect in a cluster near the center, where r is given by integrating equations (7),

$$\frac{r}{r_0} = \left(\frac{t}{4t_{\rm df,0}}\right)^4 \quad (t < 4t_{\rm df,0}),$$
$$\frac{r}{r_0} = \left[\frac{17}{7}\left(\frac{t}{t_{\rm df,0}} - 4\right) + 1\right]^{8/17} \quad (t > 4t_{\rm df,0})\ . \tag{8}$$

This implies an infall rate of black holes

$$\frac{dN_{\rm bh}}{dt} = \frac{5\eta_{\rm bh} M_{\rm cbh}}{4t_{\rm df,0}\, m_{\rm bh}}\left(\frac{t}{4t_{\rm df,0}}\right)^4 \quad (t < 4t_{\rm df,0})\ ,$$
$$\frac{dN_{\rm bh}}{dt} = \frac{10\eta_{\rm bh} M_{\rm cbh}}{7t_{\rm df,0}\, m_{\rm bh}}\left[\frac{17}{7}\left(\frac{t}{t_{,0}} - 4\right) + 1\right]^{-7/17} \quad (t > 4t_{\rm df,0})\ . \tag{9}$$

If we assume that the bulge formed at a time $t_{\rm bulge} \sim 10$ Gyr, then from equation (8), all the black holes within a radius $r_{\rm df} = 5$ pc will have migrated to the center by now, in agreement with the estimate by Morris (1993). The cumulative total and current rate of precipitation are therefore

$$N_{\rm bh} \sim 2.4 \times 10^4, \quad \frac{dN_{\rm bh}}{dt} \sim 2.9\ {\rm Myr}^{-1}\ . \tag{10}$$

In other words, provided that our assumption of the fraction of massive stars that form black holes in their final core collapse is correct, we must conclude that *a large number of stellar black holes, with a total mass of* $\sim 5\%$ *of the Sgr A* mass, have migrated to the center, and, unless they have subsequently been captured by Sgr A*, they should have formed a cluster of black holes in the center of the stellar cusp.*

As the black holes precipitate, they start dominating the total density in some central region at some point, and then the low-mass stars are expelled from this region over a relaxation time. Assuming that most of the energy is lost from the cluster by direct capture of black holes near the center, the black holes should also relax to a density profile proportional to $r^{-7/4}$, for which the outward energy flow is constant. As this energy flow is transmitted to the low-mass stars outside the black hole cluster, the cluster will need to expand and push out the low-mass stars. To derive the relative density of the black hole profile compared to the empirically normalized stellar profile, we invoke the steady state energy flow condition between two species of stars A and B, of mass m_A and m_B, which dominate the total density ρ_A and ρ_B at radii r_A and r_B, respectively. The total energy at radius r is proportional to $\rho\sigma^2 r^3$, and the relaxation time is proportional to $\sigma^3/(\rho m)$. Therefore, the constant energy flow condition yields

$$\frac{\rho_A \sigma_A^2 r_A^3}{\sigma_A^3/(\rho_A m_A)} = \frac{\rho_B \sigma_B^2 r_B^3}{\sigma_B^3/(\rho_B m_B)}\ . \tag{11}$$

Making use of the Kepler potential relation $\sigma^2 \propto r^{-1}$, this implies

$$\frac{\rho_A}{\rho_B} = \left(\frac{r_A}{r_B}\right)^{-7/4}\left(\frac{m_A}{m_B}\right)^{-1/2}\ . \tag{12}$$

Thus, the mass density of black holes in the central region is *below* that implied by extrapolating equation (3), by a factor $(\langle m \rangle/m_{\rm bh})^{1/2}$, that is, $\rho_{\rm bh}(r) = 0.18\rho_*(r)$. Hence, if all of the black holes precipitated over the lifetime of the Galaxy from a radius of 5 pc remain in the cluster at present, the cluster should extend over a radius $r_{\rm bh}$,

$$r_{\rm bh} = \left(\frac{\eta_{\rm bh}^2 m_{\rm bh}}{\langle m \rangle}\right)^{2/5} 5\ {\rm pc} = 0.7\ {\rm pc}\ . \tag{13}$$

The timescale to achieve this expansion is the relaxation time in the expanded (lower density) cluster, which is ~ 3 times longer than the dynamical friction timescale because of the lower density.

In comparison, the core radius of the K-band surface brightness profile around the Galactic center is about 0.15 pc (Mezger et al. 1996), although the faint, presently unresolved stars (with $K \gtrsim 17$) appear to have a larger core radius of ~ 1 pc (Philipp et al. 1999). We will discuss the expected relation between $r_{\rm bh}$ and these core radii in § 4.

3. RATE OF CAPTURE OF THE BLACK HOLES

In the previous section we found that about 24,000 stellar black holes should have migrated to the center of the stellar cusp around Sgr A*. We now address the question of the rate at which these black holes will be removed from the cluster by coalescing with Sgr A*. If this rate is low, most of the black holes should be in the cluster at present. If the rate is high enough, then many fewer black holes will be present, and a balance between the rate at which black holes are precipitating in the cluster by dynamical friction and the rate at which they are being captured should be established.

The dominant process by which black holes will be eliminated is by a random walk into a highly eccentric orbit as their orbits change over the relaxation timescale, from which they can be captured by Sgr A*. This process was first studied by Frank & Rees (1976) and Lightman & Shapiro (1977). In the case of stars, tidal disruption can eliminate them from the cluster once they come close enough to Sgr A*; obviously, orbiting black holes will be eliminated only when they are swallowed by Sgr A*, possibly after having lost orbital energy by emitting gravitational waves.

Before describing in more detail the mechanism by which black holes are captured, we need to discuss the process of orbital diffusion by which black holes will migrate into the eccentric orbits from which they can be captured.

3.1. *Orbital Diffusion*

A black hole can be captured by Sgr A* from an orbit of any semimajor axis, provided that its peribothron $q = a(1 - e)$ (where a is the semimajor axis and e is the eccentricity of the orbit) is sufficiently small. This will lead to a distribution of black holes in phase space that is strongly depleted at eccentricities very close to unity, and diffusion of black holes will take place toward orbits of decreasing peribothron. In order to investigate quantitatively this black hole migration, we first evaluate the diffusion tensor in velocity space. We sketch the derivation here and leave the details and the justifications for the various approximations to the Appendix.

The diffusion equation is given by

$$\nabla_v \cdot \boldsymbol{j} + \frac{\partial f}{\partial t} = 0, \quad j_k \equiv -\sum_l \kappa_{kl} \frac{\partial f(\boldsymbol{v}, \boldsymbol{r}, t)}{\partial v_l} , \tag{14}$$

where f is the phase-space density, $\boldsymbol{j}$ is the "velocity current density," and κ_{kl} is the diffusion tensor. By symmetry, $\kappa_{kl} = \mathrm{diag}\,(\kappa_\perp, \kappa_\perp, \kappa_r)$, where $\kappa_\perp$ and κ_r are the components of κ perpendicular and parallel to the radial direction. In general, the diffusion tensor depends on the spatial position and the velocity. A useful physical interpretation of the diffusion tensor components is that, over a small interval of time δt, the rms change in the velocity of a black hole in a direction perpendicular to its initial velocity is equal to $(2\kappa_\perp\,\delta t)^{1/2}$. The total rms change in any direction is therefore $[2\,\mathrm{Tr}(\kappa)\delta t]^{1/2}$ (where "Tr" means the trace), and the relaxation time is of order $v_{\rm esc}^2/[2\,\mathrm{Tr}\,(\kappa)]$.

We assume that the unperturbed (by black hole capture) phase-space density, f_0, is a function only of the energy, and hence find, for a $\rho \propto r^{-\alpha}$ density profile in a Kepler potential,

$$f_0(\boldsymbol{u}, \boldsymbol{r}) = g\left(\frac{u^2}{v_{\rm esc}^2}\right) h(r), \quad g(x) \equiv (1 - x)^{\alpha - 3/2}\Theta(1 - x) , \tag{15}$$

$$h(r) \equiv \frac{(3 - \alpha)\alpha!}{8\pi^2(\alpha - 3/2)!(1/2)!} \frac{N_{\rm bh}}{(2GM_{\rm cbh}\, r_{\rm bh})^{3/2}} \left(\frac{r}{r_{\rm bh}}\right)^{3/2 - \alpha} , \tag{16}$$

where $v_{\rm esc}(r)$ is the local escape velocity, $N_{\rm bh}$ is the total number of black holes within a radius $r_{\rm bh}$, $M_{\rm cbh}$ is the central mass, and Θ is a step function. Thus, for $\alpha = 7/4$, $f_0 \propto r^{-1/4}(1 - v^2/v_{\rm esc}^2)^{1/4}$. In the Appendix, we show that for $\alpha = 3/2$, the velocity dependences of the parallel and perpendicular components of the diffusion tensor are

$$\kappa_\perp(v, r) = \left(1 - \frac{1}{5}\frac{v^2}{v_{\rm esc}^2}\right)\kappa_0(r), \quad \kappa_r(v, r) = \left(1 - \frac{3}{5}\frac{v^2}{v_{\rm esc}^2}\right)\kappa_0(r) , \tag{17}$$

where κ_0 is the (isotropic) diffusion coefficient at $v = 0$. We also justify in the Appendix using this velocity dependence as an adequate approximation for the case $\alpha = 7/4$, for which we find,

$$\kappa_0(r) = \frac{7}{6\sqrt{\pi}} \frac{(3/4)!}{(1/4)!} \frac{N_{\rm bh}(Gm_{\rm bh})^2 v_{\rm esc}^2}{(2GM_{\rm cbh}\, r_{\rm bh})^{3/2}} \left(\frac{r}{r_{\rm bh}}\right)^{-1/4} \ln \Lambda_1 , \tag{18}$$

The quantity $\ln \Lambda_1$ can be slightly smaller than $\ln \Lambda = \ln (M_{\rm cbh}/m_{\rm bh})$, as discussed in the Appendix, depending on the scale over which the diffusion takes place, because scatterings with large velocity changes do not contribute to the diffusion rate over a small range of velocities.

3.2. *Three Regimes of Capture*

Before identifying the condition for capture from highly eccentric orbits, it will be useful to compute the core radius, r_c, where the energy-loss time to gravitational radiation on a circular orbit is equal to the relaxation time. We define the relaxation time for circular orbits as $t_{r,0} = v_c^2/[2\,\mathrm{Tr}(\kappa)]$, where $v_c = (GM_{\rm cbh}/r)^{1/2}$ is the circular velocity. The energy loss by gravitational radiation for circular orbits is (Shapiro & Teukolsky 1983)

$$\frac{d \ln E}{dt} = \frac{64}{5} \frac{G^3 m_{\rm bh} M_{\rm cbh}^2}{c^5 r_c^4} . \tag{19}$$

Equating this to $t_{r,0}^{-1}$, we find

$$\begin{aligned} r_c &= 1.57\left(\frac{GM_{\rm cbh}}{c^2}\right)^{2/3} r_{\rm bh}^{1/3} \left(\frac{M_{\rm cbh}}{N_{\rm bh}\, m_{\rm bh} \ln \Lambda}\right)^{4/15} \\ &= 8.6\ \mathrm{AU} \left(\frac{M_{\rm cbh}}{3 \times 10^6\, M_\odot}\right)^{2/3} \left(\frac{r_{\rm bh}}{0.7\ \mathrm{pc}}\right)^{1/3} \\ &\quad \times \left(\frac{M_{\rm cbh}}{1.38 N_{\rm bh}\, m_{\rm bh} \ln \Lambda}\right)^{4/15} . \end{aligned} \tag{20}$$

This radius is extremely small compared to the cluster radius $r_{\rm bh}$. If black holes were captured by diffusing to orbits with $a \sim r_c$, and then losing energy by gravitational waves at low eccentricity, the capture rate would therefore also be extremely small. In reality, as we shall see now, black holes will be captured from a large range of radii, mostly from highly eccentric orbits. We must therefore define a relaxation timescale for these high-eccentricity orbits.

To do this, we first calculate the orbit-averaged rate of change of the peribothron $q \equiv a(1-e)$ in a highly eccentric orbit. From angular momentum conservation, we have $v_\perp^2 = (1+e)qGM_{\rm bh}/r^2 \to qv_{\rm esc}^2/r$. Hence,

$$P\,\frac{d\langle q\rangle}{dt} = \int_0^P dt\,\frac{2\kappa_\perp r}{v_e^2} = 2^{5/4}\ln\Lambda_1\,\frac{53}{33}\left(\frac{m_{\rm bh}}{M_{\rm cbh}}\right)^2 N_{\rm bh}\left(\frac{a}{r_{\rm bh}}\right)^{9/4} r_{\rm bh}\,. \quad (21)$$

We then define the eccentric relaxation time $t_{r,1}(a) = (d\langle \ln q\rangle/dt)^{-1}$. We now use $t_{r,1}$ to demonstrate that there are three regimes of capture: (1) for $a < a_{\rm trans}$, capture is dominated by gravitational radiation, (2) for $a_{\rm trans} < a < a_{\rm crit}$, it is dominated by direct capture, and (3) for $a > a_{\rm crit}$, capture falls off very rapidly and can be ignored.

A particle orbiting around a Schwarzschild black hole with semimajor axis a much greater than the Schwarzschild radius will be directly captured by the black hole if its peribothron q, computed by extrapolating the Newtonian orbit, is less than 4 Schwarzschild radii (e.g., Misner, Thorne, & Wheeler 1973). When $q = 8GM_{\rm cbh}/c^2$, the particle is actually brought to 2 Schwarzschild radii by relativistic effects, where the maximum of the effective radial potential is located; the particle has then overcome the angular momentum barrier and can directly fall into the black hole. Therefore, the phase-space density should drop to zero below the minimum peribothron

$$q_{\rm min} = \frac{8GM_{\rm cbh}}{c^2} \simeq 0.24\ {\rm AU}\,. \quad (22)$$

The black hole can also be captured at larger peribothron if the timescale to change the eccentricity by diffusion is longer than the timescale for losing its orbital energy by gravitational waves. For $1-e \ll 1$, the gravitational radiation decay rate is

$$\frac{d\ln E}{dt} = \frac{170}{3}(2q)^{-7/2}\,\frac{G^3 m_{\rm bh} M_{\rm cbh}^2}{c^5 a^{1/2}}\,. \quad (23)$$

We determine $q_{\rm min}$, the minimum peribothron that avoids capture, by setting $d\ln E/dt = t_{r,1}^{-1}$,

$$q_{\rm min}(a) = 0.35\left(\frac{8GM_{\rm cbh}}{c^2}\right)\left(\frac{M_{\rm cbh}}{1.73 N_{\rm bh} m_{\rm bh}\ln\Lambda_1}\right)^{2/5}\left(\frac{r_{\rm bh}}{a}\right)^{1/2}, \quad (24)$$

where the factor 1.73 is the ratio $M_{\rm cbh}/(N_{\rm bh} m_{\rm bh}\ln\Lambda_1)$ for the values we use in this paper, and for $\ln\Lambda_1 = 9.7$ (see Appendix). Hence, the transition from capture by gravitational radiation to direct capture occurs at $a_{\rm trans} = 0.35^2 r_{\rm bh} = 17{,}000$ AU.

Above some critical semimajor axis $a_{\rm crit}$, the diffusion of q over a single period exceeds $q_{\rm min}$ as given by equation (22) for direct capture. Hence, during each period, $P \sim a^{3/2}$, the black holes are captured with a probability that decreases with the semimajor axis as $(q_{\rm min}/a) \sim a^{-1}$, so that the capture rate falls off as $a^{-5/2}$. Since the mass within radius r increases as $r^{3-\alpha}$, captures from orbits with $a \gg a_{\rm crit}$ produce a negligible loss of black holes. We evaluate $a_{\rm crit}$ by setting $t_{r,1} = P$, and find

$$a_{\rm crit} = 0.41\ {\rm pc}\left(\frac{N_{\rm bh}}{2.4\times 10^4}\right)^{-4/9}\left(\frac{M_{\rm cbh}}{3\times 10^6\ M_\odot}\right)^{4/3} \times\left(\frac{m_{\rm bh}}{7\ M_\odot}\right)^{-8/9}\left(\frac{r_{\rm bh}}{0.7\ {\rm pc}}\right)^{5/9}. \quad (25)$$

3.3. *Capture Rate from the Loss Cylinder*

While κ is a function of both $\boldsymbol{v}$ and r, we will solve the diffusion equation at fixed r, and temporarily assume that κ is independent of $\boldsymbol{v}$. We will introduce variation in κ only when we evaluate the loss rate. This is a very good approximation, as we discuss in the Appendix. We focus first on the case of $a_{\rm trans} < a < a_{\rm crit}$, for which capture is typically direct as described by equation (22). Since $q_{\rm min}$ is independent of a, the captured orbits at fixed r are characterized by a cylinder in velocity space,

$$\frac{v_\perp}{v_{\rm esc}} \le \sqrt{\frac{q_{\rm min}}{r}} = 2\,\frac{v_{\rm esc}}{c}\,, \quad \frac{|v_r|}{v_{\rm esc}} \le 1\,, \quad (26)$$

where v_r and $\boldsymbol{v}_\perp$ are the radial and perpendicular components of the velocity. Note that this structure in velocity space is definitely a "loss cylinder" and not a "loss cone" as it is often described in the literature. Making use of yet another good approximation described in the Appendix, we solve the steady state diffusion equation (14), at fixed v_r:

$$f(\boldsymbol{v}_\perp; v_r) = f_0(v_r)\left\{1 - \frac{\ln[v_\perp^2/(v_{\rm esc}^2 - v_r^2)]}{\ln(q_{\rm min}/r)}\right\}. \quad (27)$$

Hence, the capture rate per unit volume, dC/dV, is given by

$$\frac{dC}{dV} = -\int_{\rm cyl\text{-}bnd} \boldsymbol{j}\cdot d\boldsymbol{A} = 8\pi\int_0^{v_{\rm esc}} dv_r\,\frac{\kappa_\perp(v_r) f_0(v_r)}{\ln(r/q_{\rm min})}\,, \quad (28)$$

where $d\boldsymbol{A}$ is the area element on the boundary of the capture cylinder ("cyl-bnd"). We evaluate this using equations (15) and (17),

$$\frac{dC}{dV} = \frac{35}{16\pi^{3/2}}\,\frac{(3/4)!}{(1/4)!}\,\frac{\ln\Lambda_1}{\ln(r/q_{\rm min})}\,\frac{N_{\rm bh}^2(Gm_{\rm bh})^2}{(2GM_{\rm cbh} r_{\rm bh})^{3/2}}\left(\frac{r}{r_{\rm bh}}\right)^{-2} r_{\rm bh}^{-3} = \frac{6\kappa_0}{v_{\rm esc}^2}\,\frac{n(r)}{\ln(r/q_{\rm min})}\,, \quad (29)$$

where $n(r) = 5N_{\rm bh}/(16\pi r_{\rm bh}^3)(r/r_{\rm bh})^{-7/4}$ is the number density of black holes at radius r, and $6\kappa_0/v_{\rm esc}^2$ is of the order of the relaxation time. Integrating over volume, the total capture rate within some maximum radius $r_{\rm max}$ (determined below) is

$$C = k\,\frac{\ln\Lambda_1}{\ln(r_{\rm max}/q_{\rm min})}\,N_{\rm bh}^2\left(\frac{m_{\rm bh}}{M_{\rm cbh}}\right)^2\frac{r_{\rm max}}{r_{\rm bh}}\,\frac{2\pi}{P_{\rm bh}}\,, \quad (30)$$

where $k = (2\pi)^{-1/2}(35/8)(3/4)!/(1/4)! \sim 1.76$, $P_{\rm bh}$ is the orbital period at $a = r_{\rm bh}$, and where we have made the evaluation treating $\ln(r/q_{\rm min}) \to \ln(r_{\rm max}/q_{\rm min})$ and $\ln\Lambda_1$ as constants. Notice that for $r_{\rm max} = r_{\rm bh}$, the capture rate increases as $N_{\rm bh}^{4/5}$ as black holes are added to the cluster, because $P_{\rm bh} \propto r_{\rm bh}^{3/2} \propto N_{\rm bh}^{6/5}$.

As described already by Frank & Rees (1976), the result we have reached in equations (29) and (30) shows that *the capture rate of the black holes is essentially proportional to the relaxation time, and the fact that* $q_{min}/r \ll 1$ *increases the time required for the black hole to find a capture orbit only logarithmically*. The reason is that, over a relaxation time, an orbiting black hole will totally change its eccentricity not only as a result of some large deflection in a close encounter, but also because of many small deflections that will change the eccentricity by very small amounts, allowing the black hole to effectively sweep over all possible eccentricities and find the very narrow range of eccentricity where it can be captured. However, this is no longer true for $a > a_{crit}$: when q is brought below q_{min} by the random deflections, the black hole will most likely miss being captured unless it happens to be at peribothron.

The maximum radius r_{max} of integration of the capture rate in equation (30) must therefore be of order a_{crit}. For a highly eccentric orbit, the time-averaged radius is $\langle r \rangle = (3/2)a$. We therefore adopt $r_{max} = (3/2)a_{crit}$. Since capture is dominated by black holes near r_{max}, we evaluate $\ln(r/q_{min})$ there and find

$$\ln(r_{max}/q_{min}) = \ln\frac{3c^2 a_{crit}}{16GM_{cbh}} \simeq 13.2 . \qquad (31)$$

At the same time, the term $\ln\Lambda_1$ can be approximated as (see Appendix)

$$\ln\Lambda_1 \simeq \ln\Lambda - \frac{1}{4}\ln\frac{c^2 r_{max}}{8GM_{cbh}} \simeq 9.7 . \qquad (32)$$

Hence, the ratio of logarithms in equation (30) is $\ln\Lambda_1/\ln(r_{max}/q_{min}) \sim 0.73$. We are finally able to evaluate the capture rate explicitly,

$$C \simeq \frac{N_{bh}}{30\ \mathrm{Gyr}} . \qquad (33)$$

Since this timescale is much longer than a Hubble time, most of the 24,000 black holes that have entered the cluster are still in it and have not been captured. Hence, the actual radius of the black hole cluster is close to our initial estimate given by equation (13).

Note that we have everywhere used the direct capture formula (22) to calculate q_{min}, rather than the gravitational radiation formula (24), which applies at $r < a_{trans} \sim 17{,}000$ AU. Recall, however, that q_{min} only enters into the logarithm term. The capture rate from the inner part of the cluster is therefore higher than we have assumed, but not dramatically. Notice that since q_{min} is a function of a in equation (24), the loss structure in velocity space is not a cylinder as in the case of direct capture (see eq. [26]). Rather, this structure is fatter near $v_r \sim 0$ and narrower near $v_r \sim \pm v_{esc}$.

4. OBSERVABLE CONSEQUENCES

The large number of black holes that move to the center by dynamical friction have several observable effects, which we now discuss.

The most important effect is that the stars that formerly resided in the region that is now occupied by the cluster of black holes are ejected into orbits at larger radius. Therefore, any old stellar population of mass m should have a very large core radius, given by $r_{cs} = r_{bh}(m_{bh}/m)^{2/7} \sim 1$–2 pc. This core radius ($\sim 40''$) is much larger than the value expected from stellar collisions alone, which would only produce a core at the radius ~ 0.03 pc where the orbital velocities are comparable to the escape velocities from the stellar surfaces. The most straightforward test of our model is therefore to measure the distribution of low-mass stars and find out whether this very large core is indeed present.

In fact, the bright ($K < 15$) stars in the inner Galaxy exhibit a power-law profile ($\alpha \sim 7/4$) all the way to $r \sim 0.1$ pc, where a core sets in (see Mezger et al. 1996; Genzel et al. 2000; Schmitt 1995). However, most of the observed stars in this small core may be young and of relatively high mass ($M \gtrsim 2\ M_\odot$); since the relaxation time at radius r_{bh} is $\sim 10^9$ yr (see § 2), stars younger than this would not have had enough time to be ejected by the black holes from the central region. Given the abundant observational evidence of recent star formation in the nuclear bulge (Lindqvist, Habing, & Winnberg 1992), and specifically within the central cluster dynamically dominated by Sgr A* (Krabbe et al. 1991; Blum, Sellgren, & Depoy 1996a, 1996b; Tamblyn et al. 1996; Genzel et al. 2000), the presence of young stars with a small core should be expected. Therefore, our model predicts that only the old stars should have a core radius as large as 1 or 2 pc.

This prediction of our model is strongly supported by the recent evidence that the K-band surface brightness profile around the Galactic center arising from faint stars, with $K > 17$, has a much larger core than the profile of the more luminous stars (Philipp et al. 1999). Low-luminosity stars have a greater contribution from low-mass, old stars than more luminous stars. In addition, it has long been known that the strength of the 2.3 μm CO index decreases in the inner $10''$ around Sgr A* (Sellgren et al. 1990; Haller et al. 1996), indicating a remarkable change in the stellar population. However, it is not clear at present whether this change of the CO index is a result of a change in the distribution of masses or ages of the stars or is due to the different environment in the Galactic center (for example, the envelopes of supergiants may be destroyed by the ambient radiation field).

The intermediate-age (~ 2 Gyr) and older stars ejected from the central cluster could be expected to be found up to a few r_0 from Sgr A* on "Oort cloud"–like orbits. That is, a star from this population would keep getting jolted by the black holes to a more and more eccentric orbit until diffusive encounters near apocenter drove it into an orbit with a pericenter just beyond r_{bh}. Narayanan, Gould, & Depoy (1996) list 16 presumably intermediate-age stars ($K_0 < 5$) at projected radii of 8–20 pc. If these come primarily from the central parsec, they should be preferentially on radial orbits. Of course, there has been recent star formation outside the central parsec as well. For example, Blum et al. (1996b) estimate the ages IRS 24 and IRS 23 at ~ 100 and ~ 200 Myr, respectively, and these both lie at a projected distance of 1.7 pc. Hence, not all the Narayanan et al. (1996) stars are necessarily ejected.

Another interesting consequence of the ejection of low-mass stars from the center is that the capture of ordinary stars by supermassive black holes could be much rarer than commonly believed. These captures should lead to bright optical flares (e.g., Rees 1988; Ulmer 1999), which could be found in supernova searches. A cluster of black holes should have formed around all the supermassive black holes in galactic centers by the dynamical friction process described here, and these clusters would reduce by a large factor the rate at which stars can come close enough to the black hole to be tidally disrupted. Of course, since there is a central density cusp containing at least young stars around Sgr A*, there will be some tidal captures, but the absence of old stars should reduce the number of expected flares. In addition, for galaxies such as ellipticals that are poor in neutral gas, one would not expect continuous star formation near the central black holes. Consequently, after the black hole cluster had ejected all the old stars, no young stars would form to replace them. For these galaxies, flares from stellar captures could be extremely rare.

We mention also microlensing of a background bulge star as another potentially observable effect, although requiring a large improvement in sensitivity and resolution of infrared imaging over our current capabilities. If a bulge star at a distance 2 kpc behind the center could be observed, the angular Einstein radius of Sgr A* would be $b = 0''.8$, corresponding to a linear size of $r_E = 0.03$ pc. The two images of the star could therefore be comfortably resolved. The star would typically take several hundred years to complete the "microlensing event" by Sgr A*. These two images would then have a microlensing optical depth to be lensed by one of the cluster black holes, $\tau \sim (N_{bh} m_{bh}/M_{cbh})(r_E/r_{bh})^{5/4} A \sim 10^{-3} A$, where A is the magnification of the image. Imaging down to $K = 21$, one should on average find about one background star at 2 kpc within an Einstein radius of Sgr A*, and about 100 similar stars within the same angular separation, which would be mostly located inside a core radius of ~ 1 pc (for which the Einstein radius of Sgr A* would be only $\sim 0''.02$). Of course, the identification of the two images of a star lensed by Sgr A* would be of enormous interest by itself; despite the large number of orbiting black holes in the cluster, the expected rate of the "planet-like" events is still low, and several lensed stars would need to be identified to have a good chance of detecting the short events over a period of ~ 10 yr.

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APPENDIX

DIFFUSION TENSOR IN A KEPLER POTENTIAL

Here we calculate the diffusion tensor κ for the general case of a $\rho \propto r^{-\alpha}$ density profile in a Kepler potential, assuming that the velocity distribution is isotropic. Recall that the phase-space distribution is given by equations (15) and (16). We begin our evaluation at $\boldsymbol{v} = 0$, where the diffusion tensor, κ_0, is isotropic. A single encounter with another black hole at speed u changes the velocity by $\Delta v = 2Gm_{bh}/(bu)$. Hence the time-averaged growth of $\langle v^2 \rangle$ is

$$\frac{d\langle v^2\rangle}{dt} = \int 2\pi b\, db \int d^3u \left(\frac{2Gm_{bh}}{bu}\right)^2 u f_0(u, r) = \gamma(\alpha) v_{esc}^2 \left(\frac{r}{r_{bh}}\right)^{3/2-\alpha} , \quad \text{(A1)}$$

where

$$\gamma(\alpha) \equiv \frac{2(3-\alpha)\alpha!}{(\alpha - 1/2)!(1/2)!} \frac{N_{bh}(Gm_{bh})^2}{(2GM_{cbh} r_{bh})^{3/2}} \ln \Lambda_1 , \quad \text{(A2)}$$

and where Λ_1 is the ratio of maximum to minimum impact parameters, which we evaluate below.

If the diffusion equation is written (for isotropic κ_0) as $\kappa_0 \nabla_v^2 f = \partial f/\partial t$, then

$$\kappa_0 = \frac{1}{6} \frac{d\langle v^2\rangle}{dt} , \quad \text{(A3)}$$

which, combined with equation (A2), gives κ_0.

Next, we evaluate $\kappa_\perp(v)$ and $\kappa_r(v)$ for the special case of $\alpha = 3/2$ for which the phase-space density (eqs. [15] and [16]) is independent of both radius and velocity and consequently the problem is tractable analytically. The transverse velocity diffusion is given by

$$\frac{d\langle v_\perp^2\rangle}{dt} = \frac{\gamma(3/2)}{2} \int_{-v_{esc}}^{v_{esc}} du_r \int_{\theta=0}^{\theta_{max}(u_r)} \left(\frac{1+\cos^2\theta}{2}\right) \frac{(u_r+v)^2 d\tan^2\theta}{|u_r + v| \sec\theta} , \quad \text{(A4)}$$

where $\cos\theta_{max}(u_r) \equiv |u_r + v|/(v_{esc}^2 + v^2 + 2u_r v)^{1/2}$, and $\gamma(3/2)$ is defined by equation (A2). For the total velocity diffusion $d\langle v_\perp^2\rangle/dt$, we find a similar expression, but with $(1 + \cos^2\theta)/2 \to 1$. We evaluate these expressions and find

$$\frac{d\langle v^2\rangle}{dt} = \gamma(3/2) v_{esc}^2 \left(1 - \frac{1}{3}\frac{v^2}{v_{esc}^2}\right), \qquad \frac{d\langle v_\perp^2\rangle}{dt} = \frac{2}{3}\gamma(3/2) v_{esc}^2 \left(1 - \frac{1}{5}\frac{v^2}{v_{esc}^2}\right). \quad \text{(A5)}$$

We therefore conclude that $\kappa_{\perp}(v)/\kappa_0 = 1 - 0.2(v/v_{esc})^2$ and $\kappa_r(v)/\kappa_0 = 1 - 0.6(v/v_{esc})^2$. These results apply to $\alpha = 3/2$, but we adopt them for $\alpha = 7/4$ as well. If the true coefficient for $\alpha = 7/4$ is $0.2(1 + \epsilon)$ rather than 0.2, then this introduces an error into the capture formula (30) of only $(1/6)\epsilon$. Since ϵ is likely to be small, this correction is negligible.

We now justify two other approximations made in the capture calculation in § 3. First, in equation (27), we effectively considered the diffusion coefficient as fixed on *slices* of the velocity sphere perpendicular to the cylinder and passing through v_r. In fact, it is fixed on spherical shells of constant speed. This "plane-parallel" approximation is justified by two related considerations. First, κ varies very slowly, only by a factor 1.25 over the entire velocity sphere. Second, most of the depleted region of velocity space is relatively near the cylinder, so that the error in κ made by the plane-parallel approximation is extremely small.

Next, the boundary condition for the solution to the diffusion equation (27) sets the phase-space density f at the edge of the velocity sphere ($u = v_{esc}$) equal to the unperturbed density f_0 at $u = v_r$. Strictly speaking, if the boundary condition is set at ($u = v_{esc}$), then one should use $f_0(v_{esc})$. However, for the $\alpha = 7/4$ profile, this is zero. (Our approximation would be exact for $\alpha = 3/2$.) The basis for our approximation is that the density returns essentially to f_0 long before the edge of the velocity sphere, at which point f_0 is not much different from $f_0(v_r)$.

Finally, we evaluate $\Lambda_1 = b_{max}/b_{min}$, the ratio of the maximum to minimum impact parameters. This can also be written as $\Lambda_1 = \epsilon_{max}/\epsilon_{min}$, where $(\epsilon_{min} v_{esc}, \epsilon_{max} v_{esc})$ is the range of impulses relevant to the problem at hand. For general relaxation $\epsilon_{max} = 1$ and $\epsilon_{min} \sim 2Gm_{bh}/(rv_{esc}^2) = m_{bh}/M_{cbh}$. However, while the harder scatters all contribute to the *overall* relaxation, they do not contribute to diffusion into the loss cone because the black hole simply "jumps over" the capture cylinder. Unfortunately, it is not trivial to identify exactly what the largest allowed jumps should be: while jumps larger than $(q_{min}/r)^{1/2} v_{esc}$ do not directly lead to capture, they do help maintain the overall velocity profile given by equation (27). Note that the profile differs significantly from the background for many *e*-foldings. If the jumps larger than $(q_{min}/r)^{1/2}$ did not contribute at all, then $\Lambda_1 = (q_{min}/r)^{1/2}\Lambda$. In view of the distribution's slow approach to the background level near the capture cylinder, we adopt $\Lambda_1 = \Lambda(q_{min}/r)^{1/4} = 9.7$.

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