

A COLLISION WARNING ALGORITHM FOR REAR-END COLLISIONS

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ABSTRACT

The purpose of this paper is to develop an experimentally-based rear end collision warning algorithm for the situation where two vehicles are initially traveling at the same speed in the same direction when the lead vehicle begins to brake. The full variety of initial conditions of vehicular motion are analyzed to determine the proper collision warning algorithm. The analysis shows that knowledge of traveling speed, headway, and lead-vehicle deceleration is sufficient to determine the type of relative motion. This can be coupled with warning logic and principles of vehicle dynamics to produce warning algorithms. An approach to warning implementation is suggested that avoids the difficult problem of estimating the lead-vehicle deceleration; using instead other measurable quantities such as range and range-rate.

INTRODUCTION

The pre-crash situation of two vehicles initially traveling at the same speed in the same direction when the lead vehicle begins to brake has been investigated by others. None of these investigations to date, however, have provided a complete, experimentally based algorithm that applies to all variations of the inter-vehicular dynamics. One recent experiment using the Iowa Driving Simulator (IDS) tested driver responses to a stationary vehicle in the lane of travel (ref. 1). Half of the drivers in this experiment were provided with collision warning advice and the other half were not.

This collision warning was based on a presumed driver/vehicle model which consisted of the following premises:

- Issue a warning to the driver at a time based on initial speed so that a constant-deceleration

stop could be completed in time to bring the following vehicle to a stop at a distance of 6.67 ft. behind a stopped lead vehicle.

- Assume a constant following-vehicle deceleration of $0.75g$ (the value used in the IDS tests), where $g = 32.2 \text{ ft/sec}^2$,
- Assume a driver delay of 1.5 seconds between collision warning and brake activation.

The results from this experiment suggest that a collision warning would be effective in reducing the number of such collisions (see Appendix A for details). This warning could also be used to activate a warning when both vehicles are initially moving. However, as will be shown later, these warnings would not be timely. Although the relationship itself is not appropriate for situations where both vehicles are moving, the logic behind such a warning may still be useful.

To achieve the purpose of this report there are two objectives: 1.) Extend the results of the Iowa experiment to driving situations where both vehicles are initially moving at the same speed to cover all possibilities, and 2.) determine suitable warning logic for each situation. No experimental database such as that discussed above for stationary vehicles exists for situations where both vehicles are moving at the beginning of an imminent crash.

This study has three parts. The first part is the separation of conditions at the onset of lead-vehicle braking into sets which correspond to distinct types of relative motion between the vehicles. The second part is to develop the mathematical formulas which can be used to activate a warning for each of these three types of distinct relative motion. In the third part, the results of the first two parts are combined to form a description of

the conditions at which an imminent collision warning should be activated.

NOTATION AND DYNAMICS

For this analysis, consider two vehicles initially moving at the same speed in the same direction (e.g., they are platooning). Both vehicles are assumed to initially be traveling with the absolute speed, V_0 , and separated by a distance, or range, of R_0 . The relationship of the two vehicles and the corresponding dynamic variables are shown in Figure 1, with the notations below.

Variables and Constants

R	range(ft)
X	position(ft)
V	absolute speed(ft/sec)
t	time(sec)
d	deceleration(ft/sec ²)
dR/dt	range rate(ft/sec)
g	standard acceleration of gravity
C	smallest headway
T _h	initial headway(sec)

Subscripts

0	initial, when $t = 0$
B	brakes on
F	following
L	lead
S	stopped
W	warning

The relative stopping dynamics begin at time $t = 0$, when the lead vehicle first applies its brakes and begins a constant deceleration to zero speed. The application of the lead-vehicle brakes determines the location of the reference for the measurement of all positions - this reference point is the location of the following-vehicle front bumper at the instant that the lead vehicle begins to brake. Time $t = 0$ also sets the initial headway range, R_0 . Hence, the position of the vehicles is indicated by the Cartesian coordinate, X , measured with respect to the fixed inertial reference at time $t = 0$. Note that X_F is measured to the following-vehicle front, and X_L to the lead-vehicle rear.

At an appropriate time $t = t_w$, a warning is issued to the driver of the following vehicle advising of a potential collision with a lead vehicle in the lane ahead. In keeping with the logic above, the time of the warning is based on the assumption that it will take the following driver 1.5 sec. to apply the brakes and that the driver will create a deceleration level of $0.75g$. The driver of the following vehicle then brakes at a constant deceleration. The time at which the following-vehicle brakes are applied is

$$t_{FB} = t_w + 1.5 \text{ sec.} \quad (1.)$$

However, before the warning is issued, the lead vehicle may or may not be at a stop in the lane ahead. In fact, it is necessary to first determine this condition in order to determine the proper time

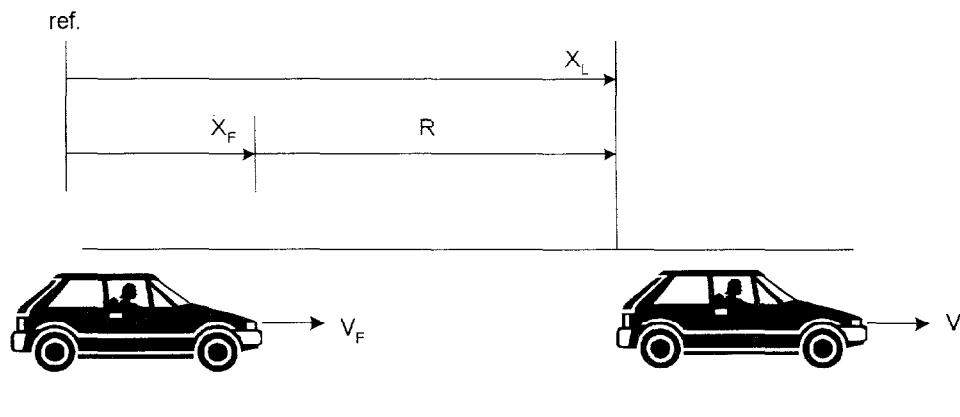


Figure 1. Two Vehicle Platooning Geometry.

for the collision warning to be issued to the following vehicle.

For all times after $t = 0$, the two vehicles continue their motions until they collide or are both stopped. At all times, the range, R , and the range rate, dR/dt , have the following relations:

$$R = X_L - X_F, \quad (2.)$$

$$dR/dt = V_L - V_F. \quad (3.)$$

TYPES OF RELATIVE MOTION

The pattern of relative motion between the two vehicles is completely determined by the initial conditions at the onset of braking by the lead vehicle, at the time $t = 0$. At this time, the three parameters which set the pattern of relative motion are the initial speed (V_0 , the same for both vehicles), the initial headway between the two vehicles, ($T_h = R_0/V_0$), and the level of deceleration taken by the lead vehicle (d_L , presumed to be a constant but unknown value in the present analysis). Given these conditions, only three types of relative motion are possible. These relative motions are:

1. The warning is issued to the following vehicle after the lead vehicle stops. This is the case with a large initial headway.
2. The warning is issued before the lead vehicle stops, and the lead vehicle stops *before* the following vehicle stops.
3. The warning is issued before the lead vehicle stops, but the lead vehicle stops *after* the following vehicle stops.

An example of Relative Motion 1 is shown in Figure 2, where it is plotted in both Cartesian coordinates (V, t) for each vehicle, and relative range coordinates (R and dR/dt). Note that the range plot is parametric in time, t , with $t = 0$ at the highest point where the curve intersects the range axis. Range/range-rate plots of this type were first introduced by Fancher (ref. 2) and will provide an insightful means to present the collision warning metrics later in this report.

Relative Motion 1 may be understood by considering the events shown along the time axis in Figure 2. This sequence of events is initiated by the lead vehicle applying its brakes at $t = 0$ and uniformly decelerating to a stop at $t = t_{LS}$. However, the initial headway is large enough so that the following vehicle does not receive a warning until

some time, t_w , well after the lead vehicle has stopped. Following the warning, it takes the following driver 1.5 seconds to apply the brakes at time t_{FB} and then uniformly brake to a stop at time t_{FS} .

An example of Relative Motion 2 is shown in Figure 3. In this case, the events are again started by the lead vehicle first applying its brakes, but now the lead vehicle comes to a stop *after* the warning is issued and *before* the following vehicle stops. Note that this includes a period during which the two vehicles are braking at the same time.

An example of Relative Motion 3 is shown in Figure 4. Here the lead vehicle comes to a stop *after* the following vehicle. This situation will give rise to the near approach of the two vehicles as the following vehicle decelerates rapidly to avoid the collision. The smallest headway occurs at time t_c when the relative speed between the two vehicles is zero. Note that this motion can only occur if the following vehicle has a greater deceleration than the lead vehicle.

We now focus our attention on the two boundaries between the above three types of relative motion in order to further clarify the motions. These boundaries are a critical element in determining the equations governing when the warning should be issued.

BOUNDARY ANALYSIS

Boundary 1-2 divides Relative Motions 1 and 2, and occurs when a warning is issued at the instant of time that the lead vehicle comes to a stop. This means that the total distance traveled by the following vehicle after the lead vehicle begins to brake at $t = 0$ equals the initial separation, plus the distance needed by the lead vehicle in coming to a stop, minus a 2 meter final separation. In equation form this is

$$X_{FS} = R_0 + \frac{1}{2} V_0^2 / d_L - 6.67 \quad (4.)$$

However, from another point of view, the distance traveled by the following vehicle also consists of three parts, these being the distance traveled while the lead vehicle is stopping, the distance traveled during the 1.5 sec. delay in brake application, plus the distance traveled while stopping itself at constant deceleration. This is

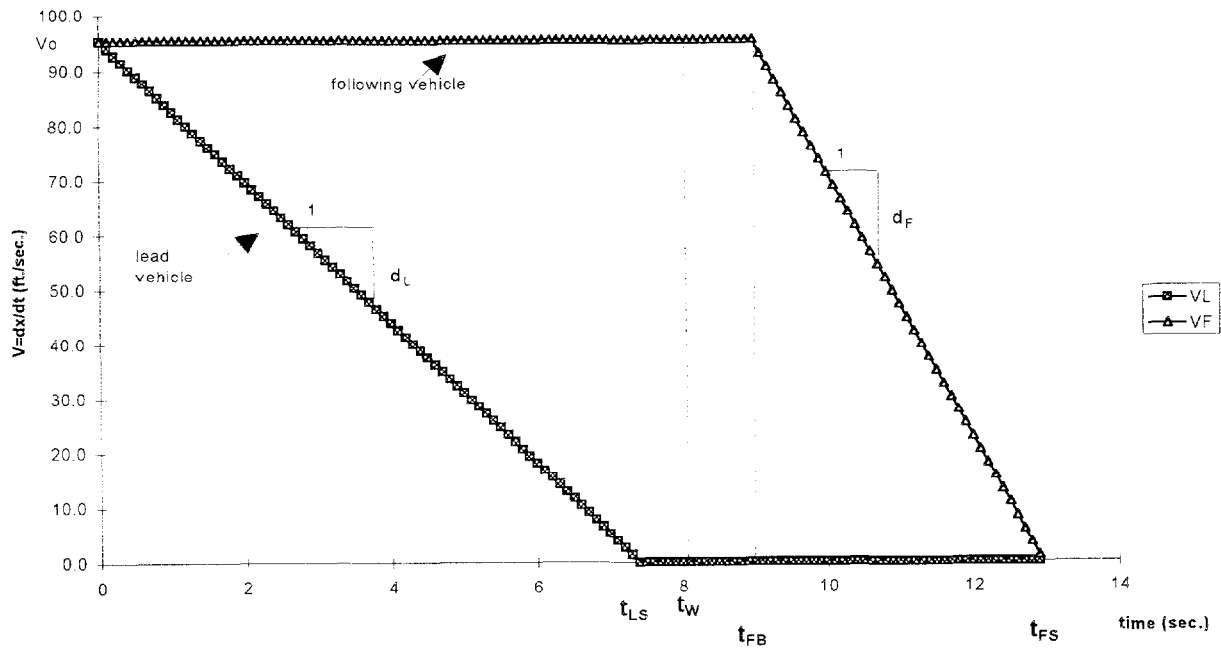


Figure 2(a). Velocity Plot. Example of Relative Motion 1 - Lead Vehicle Stops Before Warning.

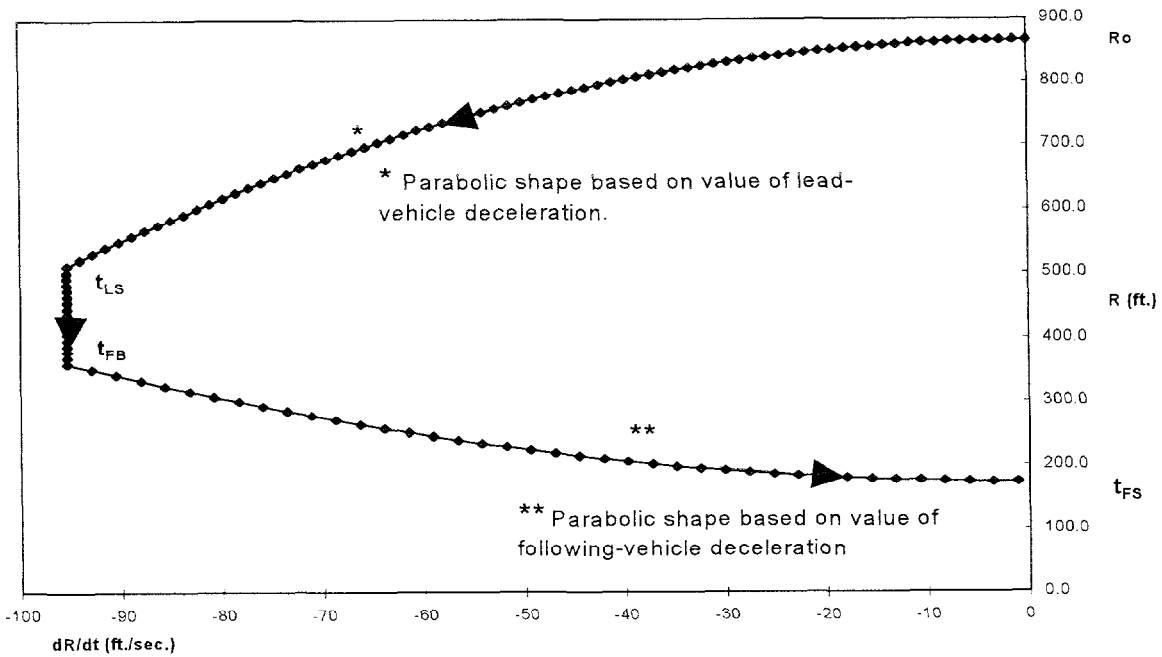


Figure 2(b). Range, Range Rate Diagram. Example of Relative Motion 1 - Lead Vehicle Stops Before Warning.

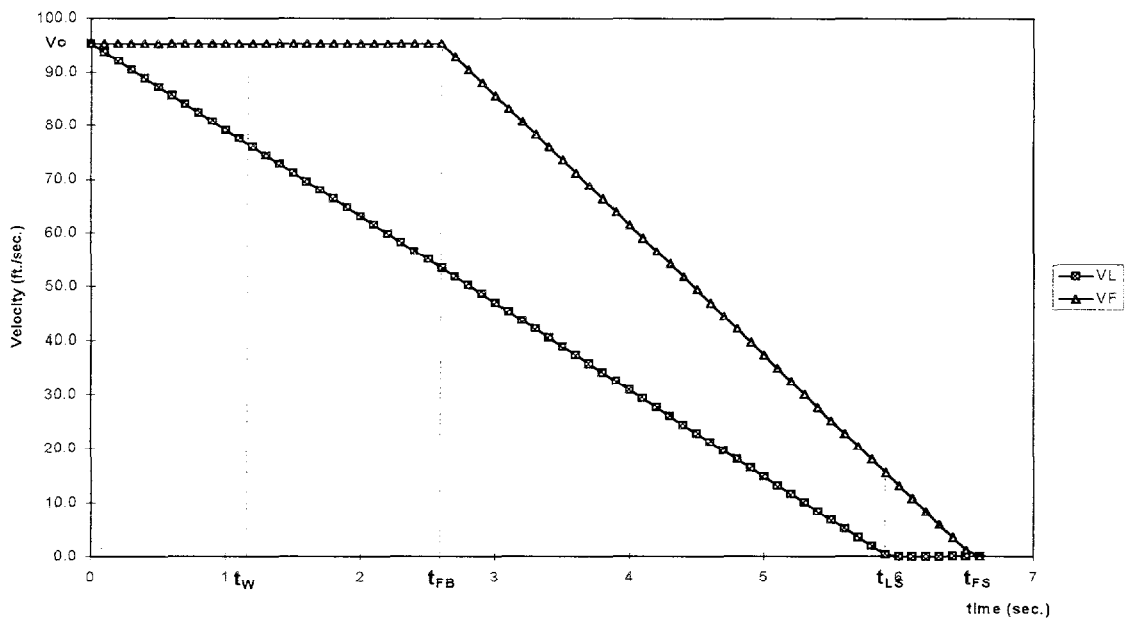


Figure 3(a). Velocity Plot. Example of Relative Motion 2- Lead Vehicle Does Not Stop Before Warning.

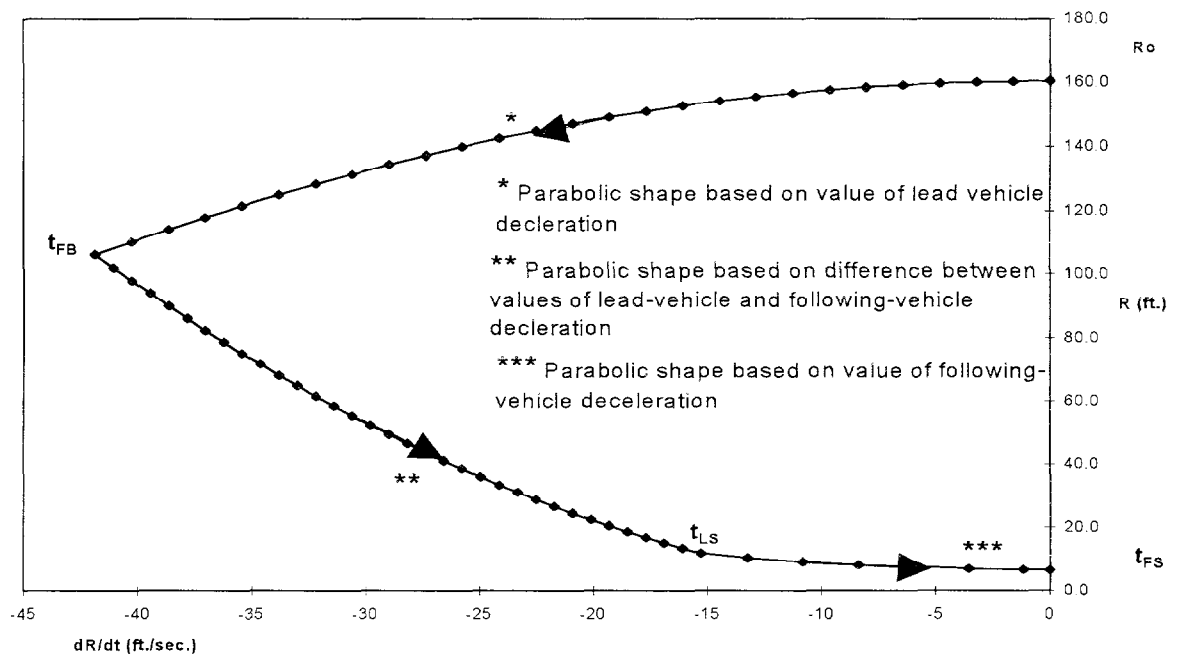


Figure 3(b). Range, Range Rate Diagram. Example of Relative Motion 2- Lead Vehicle Does Not Stop Before Warning.

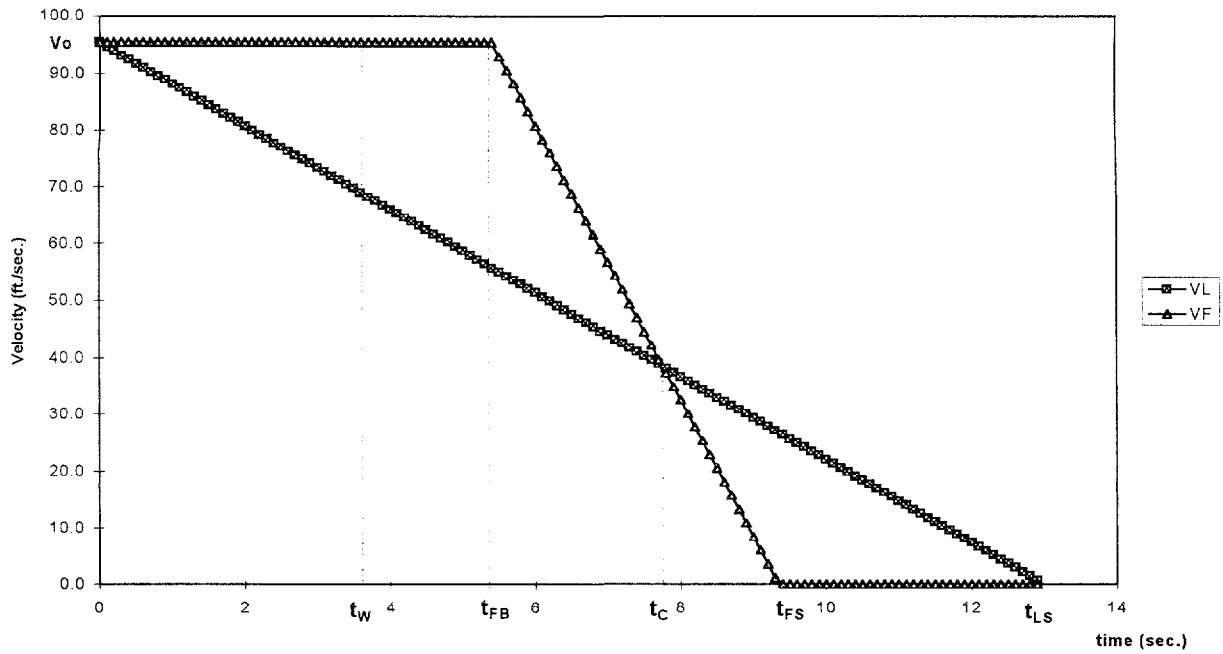


Figure 4(a). Velocity Plot. Example of Relative Motion 3 - Following Vehicle Stops Before Lead.

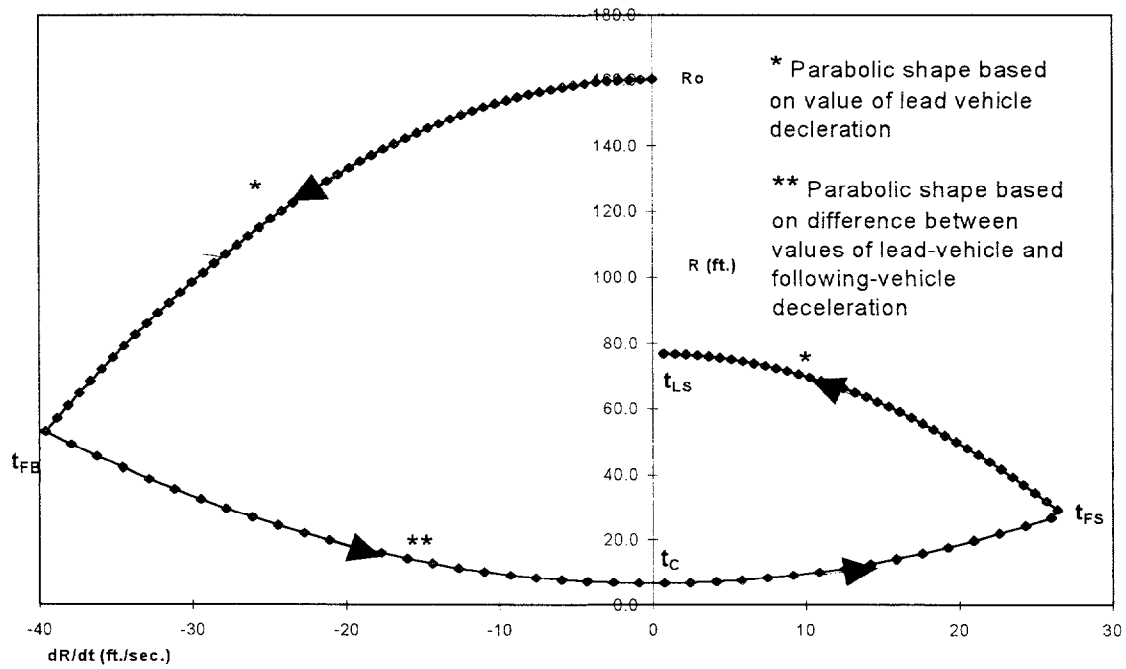


Figure 4(b). Range, Range Rate Diagram. Example of Relative Motion 3 - Following Vehicle Stops Before Lead.

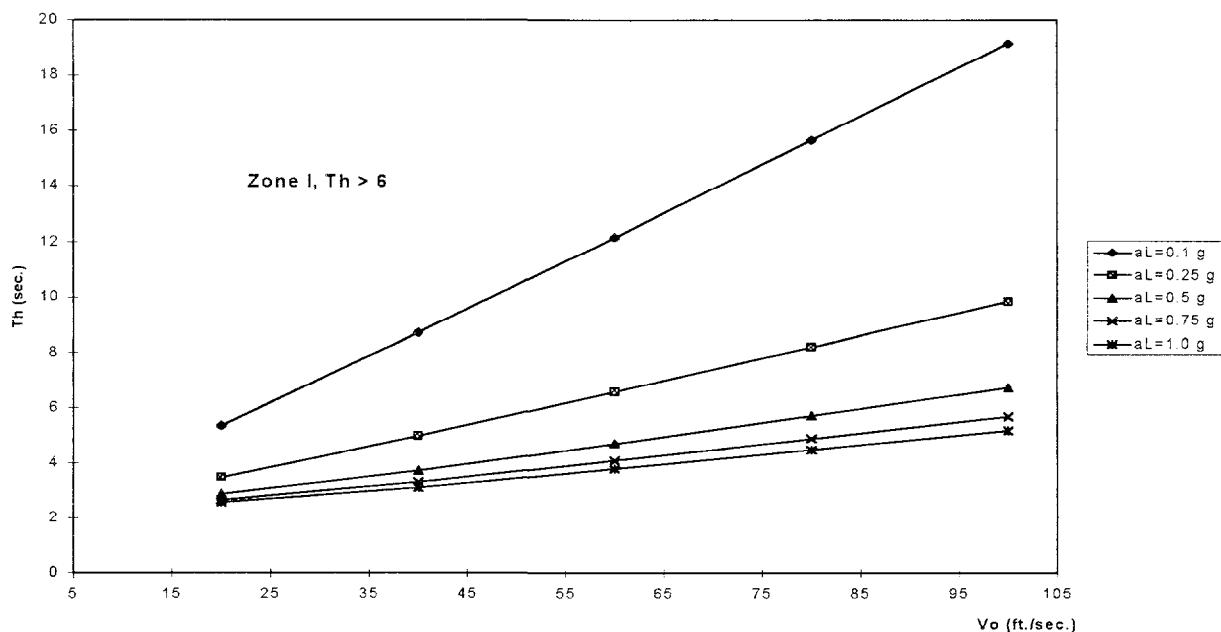


Figure 5. Plot of Boundary 1-2 Initial Conditions (Zone I as a function of a_L).

$$X_{FS} = V_0^2 / d_L + 1.5V_0 + \frac{1}{2} V_0^2 / d_F \quad (5.)$$

When Eqns. 4 and 5 are combined and solved for the initial headway ($T_h = R_0/V_0$), we find that the expression is

Initial headway,

$$T_h = \frac{1}{2} V_0 (1/d_L + 1/d_F) + 6.67/V_0 + 1.5 \quad (6.)$$

(The details of this derivation are included in Appendix B). So, given that $d_F = 0.75g$, the initial conditions (T_h versus V_0) may be plotted for various values of d_L , as is done in Figure 5.

The meaning of Figure 5 is that, for a given value of lead-vehicle deceleration, d_L , if the initial conditions of velocity and headway are plotted on this diagram and create a point above the line for that deceleration, then the lead vehicle will be stopped before a warning needs to be issued. In this case, the criteria for a stopped vehicle ahead should be used -- this will be called Warning Criteria 1, and will be developed in the next section of this report. However, if the initial conditions are at a point below the corresponding deceleration line, then a warning is needed before the lead vehicle comes to a stop -- these conditions will lead to Warning Criteria 2 to be developed below.

Boundary 2-3 as shown in Figure 6 occurs when the two vehicles stop at the same instant of time. For this case, a similar development to that for Eqn. 6 may be used to find the expression for the headway - this gives

$$T_h = \frac{1}{2} V_0 (1/d_L - 1/d_F) + 6.67/V_0 \quad (7.)$$

Eqn. 7 is plotted in Figure 6 and derived in Appendix C. Again, for a given level of lead vehicle deceleration, d_L , when the initial conditions of velocity and headway are at a point above the corresponding deceleration line, then the lead vehicle stops first. If the initial condition point is below the corresponding deceleration line, then the following vehicle stops first. The equations for these warning criteria are developed in the next section of this report.

Figs. 5 and 6 are aspects of a single three-dimensional relationship between V_0 , T_h , and d_L . In order to clarify this relationship, consider the example shown in Figure 7 for the case of $d_L = 0.5 g$. Here the two boundary lines from Figs. 5 and 6 for this case of deceleration are plotted on the same axes of V_0 and T_h , thus separating the V_0 - T_h space into three zones. Also shown in Figure 7 is the location of points corresponding to a constant value of range

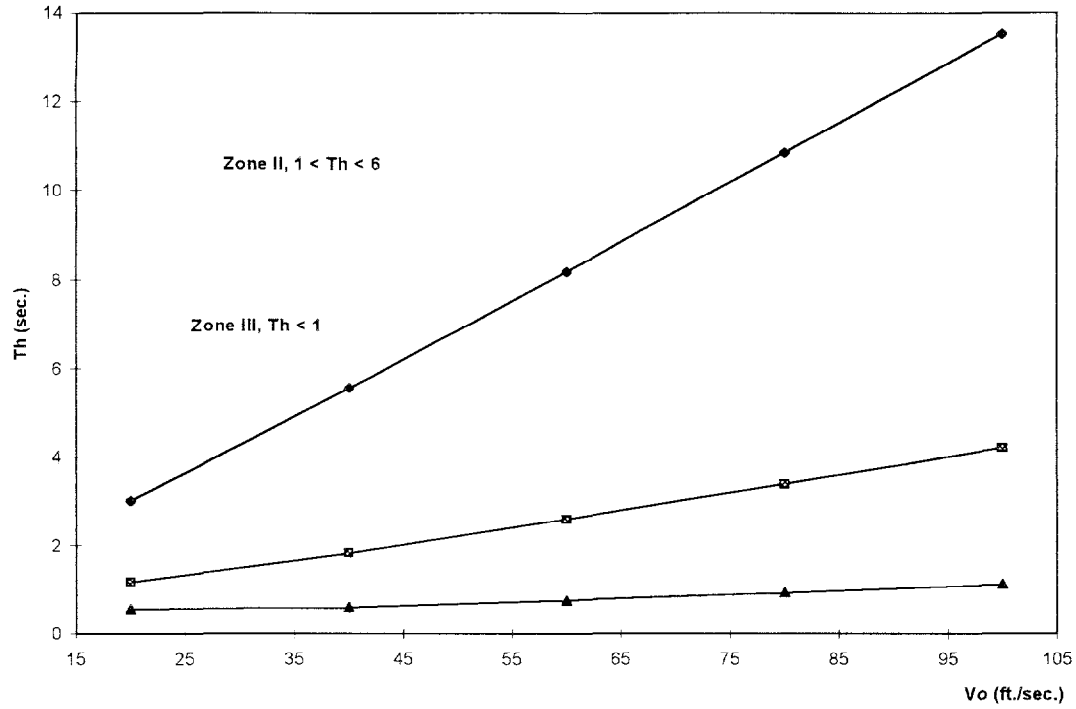


Figure 6. Plot of Boundary 2-3 Initial Conditions.

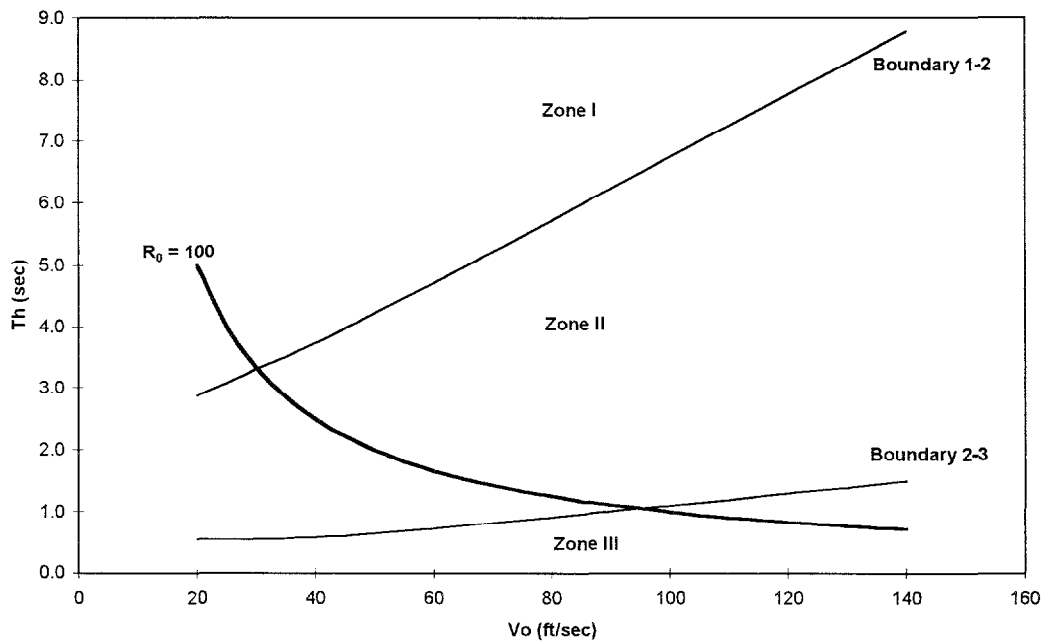


Figure 7. Example Initial Conditions Plot for $d_L = 0.5g$.

of 100ft. The three zones shown in the figure are each governed by different equations for warning criteria which are derived below. Zone I is for the cases where the warning is issued after the lead vehicle stops, Zone II is for cases where a warning is issued before the lead vehicle comes to a stop and the lead vehicle stops before the following vehicle, while Zone III is for the cases when the lead vehicle stops last.

CRITERIA FOR ISSUING A WARNING

Zone I This zone is for those situations where the lead-vehicle is stopped before a warning is needed, leading to Warning Criteria 1. Recall that $t = 0$ occurs when the lead vehicle first applies its brakes; however, since it is likely to be difficult or impossible to tell when the lead vehicle first applied its brakes (very large headway cases), we must rely on a simple range criteria for this zone based on the expected stopping distance of the following vehicle. That is, the warning range, R_w , must be based on the deceleration distance of the following vehicle, plus a 1.5 sec. lag to apply the brakes, plus the required 6.67 ft. safety margin. In equation form (using constant values as stated) this is

$$R_w = \frac{1}{2} V_0^2 / d_F + 1.5V_0 + 6.67$$

When $d_F = 0.75g$, this becomes

$$R_w = V_0^2 / 48 + 1.5V_0 + 6.67 \quad (8.)$$

which is the warning criteria used in the stationary vehicle Iowa experiment. Details of this experiment and the effectiveness of this warning are provided in Appendix A.

Zone II For situations where the lead vehicle is still stopping when the warning must be given, we must use another analysis, leading to Warning Criteria 2. Again, $t = 0$ starts the analysis and sets the reference for all distances, with $X = 0$ measured from the position corresponding to the following-vehicle's front bumper when $t = 0$. After $t = 0$, the lead vehicle uniformly brakes to a stop. However, for this zone, the following vehicle has a large enough initial headway that it continues on for a short while at constant speed until it reaches the warning time, t_w , at the warning range, R_w .

In this situation, the final location of the following vehicle, X_{FS} , is equal to the sum of the distance traveled before the warning, plus that after the warning before the brakes are applied, plus that to stop at constant deceleration. The final location of the lead vehicle, X_L , is equal to the initial headway plus the distance for it to decelerate. We can relate these to the required 2 meter, or 6.67 ft., separation required at the end of braking by the relationship

$$X_F = 6.67 - X_L \quad (9.)$$

Substituting for the appropriate terms allows us to find the warning time, range, and range rate, as is shown in Appendix D

$$t_w = \frac{1}{2} V_0 (1/d_L - 1/0.75) + (T_h - 1.5) - 6.67/V_0 \quad (10.)$$

$$R_w = R_0 - \frac{1}{2} d_L t_w^2 \quad (11.)$$

$$dR_w/dt = -d_L t_w \quad (12.)$$

Zone III In this zone, the warning is based on the closest approach of the two vehicles, which occurs while they are both still moving, thus leading to Warning Criteria 3. For these cases, the following vehicle stops before the lead vehicle, which is only possible due to the greater deceleration of the following vehicle. Now the warning criteria is based on the recognition that at some point during the braking maneuver, both vehicles will again be traveling at the same (slower) velocity at that time when they are in the nearest proximity (see Figure 4). Now, if the value of closest proximity is set equal to 6.67 ft., the resulting equations can be solved for the corresponding values of time, range, and range rate at which a warning should be issued--this is done in Appendix E. Thus,

$$t_w = \left\{ \frac{(0.75 - d_L)/0.75}{2(V_0 T_h - 6.67) / \{d_L(1 - d_L/0.75)\}} \right\}^{1/2} - 1.5 \quad (13.)$$

Equations (12.) and (13.) also hold for Zone III as well as Zone II.

It is now possible to overlay the three warning criteria developed above onto the three plots of range/range-rate previously shown in Figs. 2, 3 and 4. Examples of this are shown in Figure 8 for each of the three different zones. In order to generate these plots, it was necessary to pick specific values for the governing parameters d_L , T_h , and V_0 .

Note that the relative motion curves are parametric in time, with $t = 0$ at the highest intersection of the plot with the range axis. The collision warning should be given at the time that the relative motion plot first intersects the warning curve.

The warning curves shown in Figs. 8(a), (b), and (c) correspond to initial velocities of 30, 60, and 120 ft/sec respectively with a constant lead-vehicle deceleration of 0.5g. The headway was allowed to vary and was stepped through a range of values. At each value of headway a test was made to determine which zone of initial conditions existed through the relationship depicted in Figure 7. When the zone changed, the warning range and range-rate equations were changed to the warning formulae for that zone.

IMPLEMENTATION ISSUES

In application, a processor on board the following vehicle could continuously monitor the value of initial conditions on V_0 and T_h . In the event of sensed lead-vehicle deceleration, the value of d_L is estimated and added to the processing algorithm and would trigger two calculations. The first calculation would be a determination of which of the three zones, or types of relative motion, is occurring based on initial conditions of V_0 and T_h and the level of deceleration, d_L . This would be followed by a calculation of range and range rate at which a warning should be issued using the warning formulae that correspond to the type of relative motion, as derived above. This process is suggested by the overlay of the warning curves onto the relative motion plots shown in Figure 8. Although this implementation process is conceptually sound, it may not be practical due to the need to estimate the level of lead-vehicle deceleration. There are two computational issues here which are problematic: estimation of deceleration from samples of range and range rate data is a noisy process; and the calculation requires two or more samples, thus introducing delays of at least one sample period. However, these difficulties can be overcome by a simple, although previously unreported, change of perspective. The change is to use a d_L -based formulation instead of the T_h -based formulation shown thus far.

To accomplish this new formulation, the warning equations are generated by holding initial velocity and headway constant while allowing deceleration to vary. Graphically, this will create a warning range/range-rate plot which is parametric in lead-vehicle deceleration. While this seems like a slight difference from that above (parametric in headway), consider the advantage of storing a family of equations of warning criteria in terms of range versus range-rate for sets of initial conditions in velocity and headway. Now, since velocity and headway are easily measured values, the warning curves become look-ups which do not depend on lead-vehicle deceleration. Examples of such curves are shown in Figure 9. Here *each curve* is for constant values of initial velocity and headway for the full range of d_L . The stationary vehicle curve is included for reference. Thus, d_L never has to be estimated. This makes implementation vastly easier.

An example of the velocity-headway application is shown in Figure 10. In this case, the on-board processor could continuously calculate the warning criteria equations for range and range rate based on current values of velocity and headway. Then, any deceleration by the lead vehicle would produce a trajectory in range/range-rate that eventually intersects with the warning criteria. The point of the intersection would correspond to the unknown level of lead-vehicle deceleration. This means that the lead-vehicle deceleration does not have to be estimated regardless of the zone of motion. Thus, the benefits of using a warning algorithm that relies on knowledge of the lead-vehicle deceleration can be accomplished without actually doing the time-consuming computations that are necessary to produce an estimate. If there is a single important insight from this entire study, it is this latter point:

The advantages and benefits of using a warning algorithm based on knowledge of the level of deceleration of the lead vehicle can be achieved without actually having to compute an estimate of the lead-vehicle deceleration.

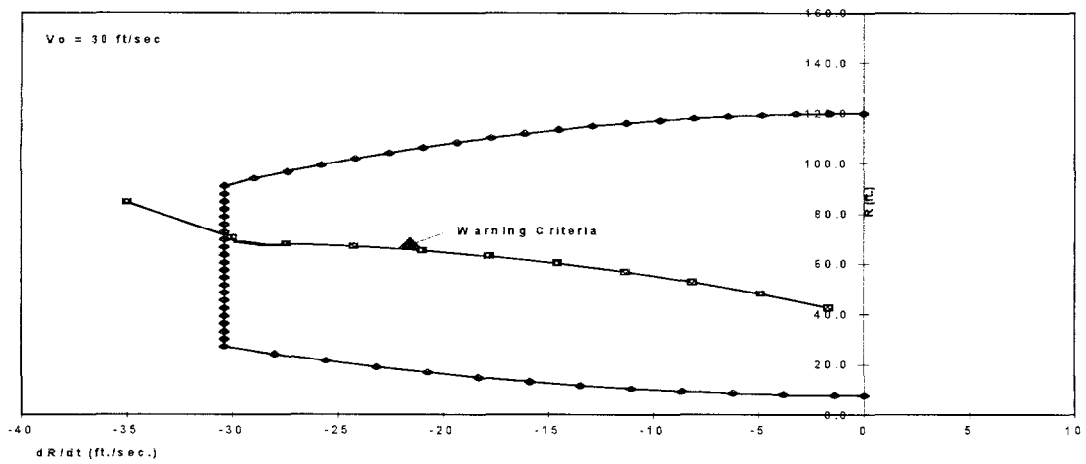


Figure 8(a). Examples of Warning Criteria Parametric in T_h for Relative Motion 1.

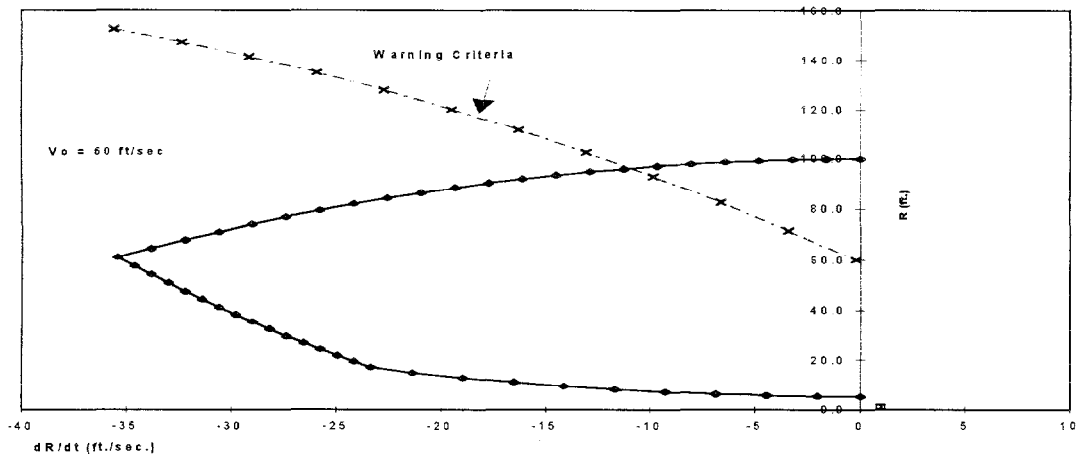


Figure 8(b). Examples of Warning Criteria Parametric in T_h for Relative Motion 2.

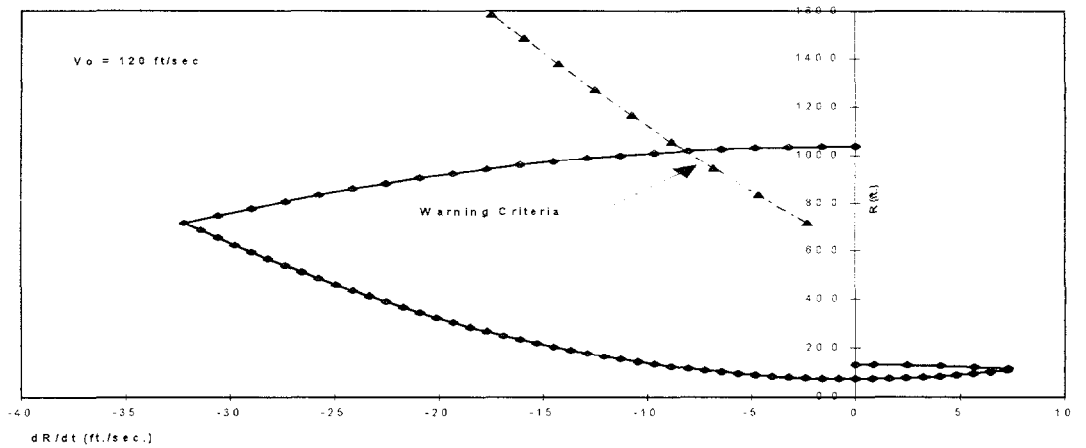


Figure 8(c). Examples of Warning Criteria Parametric in T_h for Relative Motion 3.

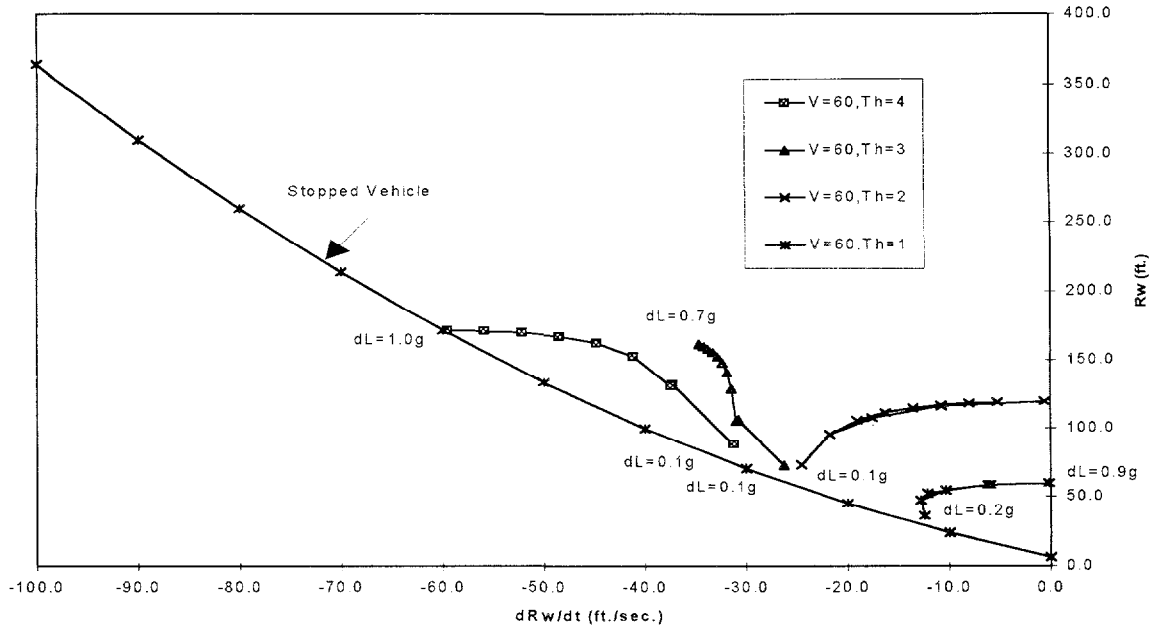


Figure 9. Warning Curves Parametric In Lead-Vehicle Deceleration.

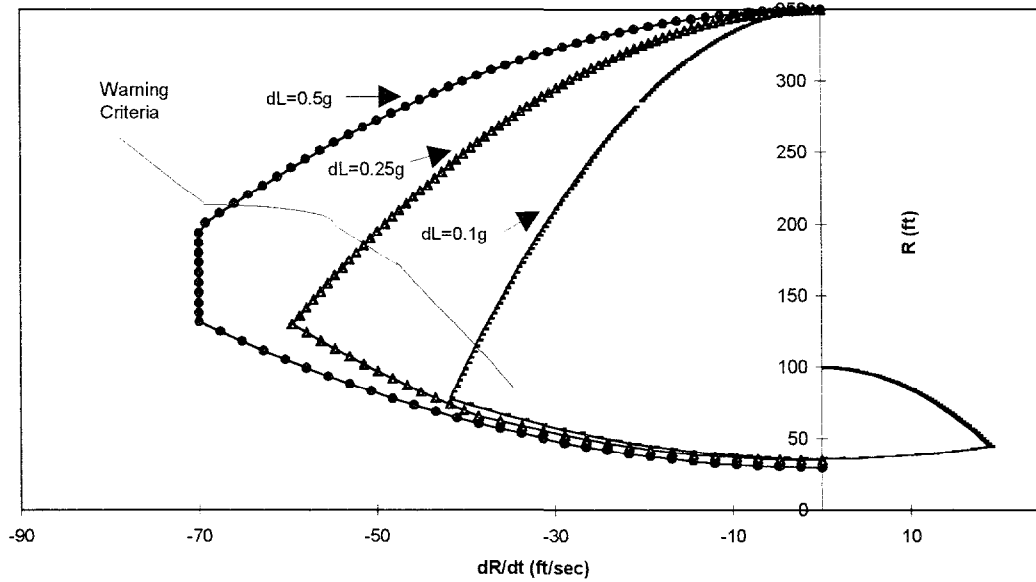


Figure 10. Example of the Warning Criteria for $V_0=70$ ft/sec, $T_h=5$ Relative Motion In All Zones.

CONCLUSIONS

This report develops criteria for issuing warnings to drivers when a rear-end crash with a lead vehicle which is initially moving. The approach used here is to extend previous logic on a warning criteria which has been shown to be effective in situations where the driver is confronted with a stationary vehicle in the travel lane. As such, analysis is given to the logic in situations where two vehicles are initially traveling at the same speed prior to braking by the lead vehicle. The analysis shows that there are three distinct types of relative motion that can result, and that each type can be determined by examining the initial conditions at the time that the lead vehicle begins to brake. The appropriate values of warning range and range rate can then be determined for each type of motion. Examples of the location of these criteria in a range/range-rate diagram are shown in Figure 8 for example initial condition sets.

Finally, these results are extended to implementation where it is shown that the lead-vehicle deceleration need not be known to create an effective driver warning system.

In evaluating these results, several questions are suggested. Is it practical to use straight-line approximations in the range/range-rate warning criteria as a simplification (similar to the control law used in some adaptive cruise control systems)? Also, in a human factors sense, does this extension of a criterion from a stationary vehicle situation to

moving vehicle situations create a basis for a meaningful and effective warning to the driver? Similarly questions of variation in driver reaction time as well as system noise effects and roadway conditions are potential areas for enhancement of these findings.

STS ACKNOWLEDGMENT

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APPENDIX A: THE STOPPED-LEAD-VEHICLE SCENARIO TEST

Introduction

The objective of this study was to investigate how drivers with and without rear-end collision warning systems react when purposefully distracted just when a stationary vehicle is revealed. The test was conducted using the Iowa Driving Simulator equipped with a driver warning system that provided auditory warnings based on two different warning criteria. A total of 30 subjects was split across three conditions with 10 subjects each. The two warning distances resulted from use of two values of driver reaction time for a hypothetical average driver.

Driver collision avoidance performance was compared to that in the baseline condition where no warning was present. Results showed that the collision warning system in the long warning condition showed shorter accelerator release reactions times, fewer crashes, and less severe crashes compared with both the baseline condition and the short warning condition. Experimental evidence suggests that the short warning condition may also have distracted drivers at the last moment after they had already begun to brake, resulting in some crashes. This is possibly a result of the warning display modality (auditory) and not a function of the timing of the warning.

Background

This was the second of two tests to be carried out on the Iowa Driving Simulator. The first test consisted of a scenario where a lead-vehicle was in motion at the time of the collision.

Warning Display

The primary warning alert was an auditory car horn icon.

Warning Algorithm

The warning algorithm used was the stopping distance algorithm of the form:

$$WD = V_F^2 / 2d_F + T_d V_F + R \quad (A-1)$$

where,

WD Warning distance compared with sensor range to the lead vehicle

V_F Following (host) vehicle absolute speed (measured)

d_F Following (host) vehicle deceleration (assigned)

T_d Warning time delay (assigned)

R Confidence interval (assigned).

WD is continuously calculated and compared to the measured headway between the host and lead vehicle. If the distance is less than the warning distance, a driver warning is activated.

For this study the following values were used:

V_F Subject vehicle speed

d_F 0.75 g's (7.35 m/s²)

T_d 1.0 seconds (short), 1.5 seconds (long)

R 6.67 ft.

The stopping distance algorithm functions to bring the host vehicle to a stop at a distance R (6.67 ft.) behind the lead vehicle, when the driver has a reaction time of T_d (1.0 seconds short, 1.5 seconds long) and decelerates at a constant rate of 0.75g (7.35 m/s²).

Experimental Procedure

To reduce anticipation of rear-end crashes, subjects were told that they would participate in a study to assess the fidelity of the simulator. Baseline subjects were given no further instructions. Subjects in the collision warning condition had the collision warning system explained to them before driving and then performed two "looming" maneuvers on a practice lead vehicle so they could see the collision warning function in operation. All participants were allowed a 5-minute practice drive.

After some initial driving along a freeway, the subjects came across a lead vehicle (a large truck.) The simulator scenario then "coupled": the subject vehicle with the truck at a 3.2 second headway. Once the vehicles were coupled, a digitized voice came over the vehicle's speakers and asked the driver to "press the button above the rear-view mirror until the red light comes on." Three hundred milliseconds after the driver pressed the button the truck swerved to the center lane and exposed a stopped passenger vehicle in the right lane. Corresponding to the swerve of the truck, the

collision warning display (auditory horn) was actuated using one of the two warning times.

Results and Discussion

Drivers chose many strategies besides just braking to avoid the stationary vehicle. Results showed that the collision warning system in the long warning condition showed shorter accelerator release reactions times, fewer crashes, and less severe crashes compared with both the baseline condition and the short warning condition. Experimental evidence suggests that the short warning condition also may have distracted drivers at the last moment, resulting in some crashes. This is possibly a result of the warning display modality (auditory) and not a function of the timing of the warning.

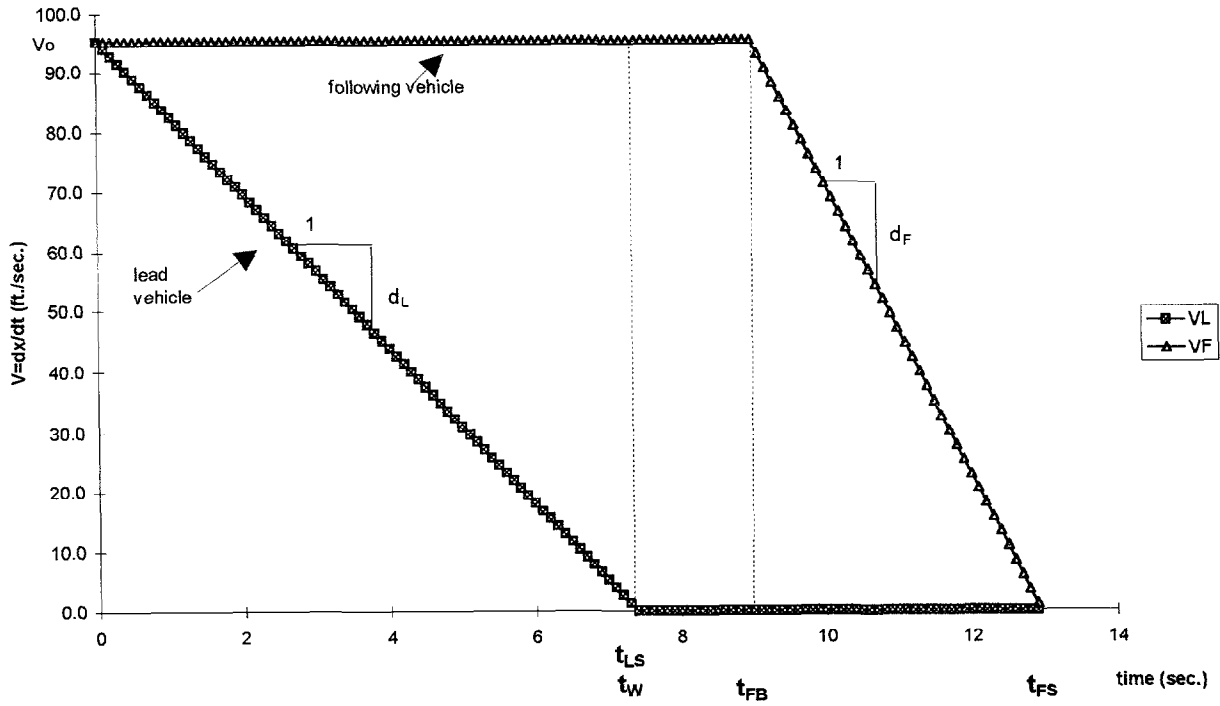
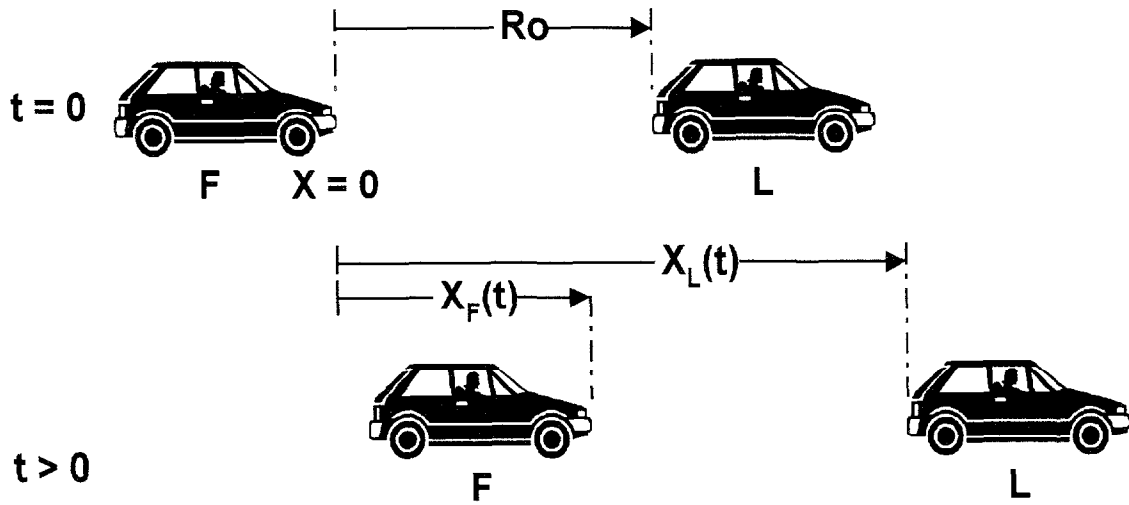
Conclusions

This study showed that the timing of a warning is important in the design of collision warning systems. Furthermore, data suggests the potential to provide a disbenefit to drivers if the warning alert is done improperly.

Reference

Daniel V. McGehee and Timothy I. Brown of The University of Iowa., Terry B. Wilson and Mike Burns of Sensor Technologies & Systems (Formerly Frontier Engineering), "Examination of Driver's Collision Avoidance Behavior in a Lead Vehicle Stopped Scenario Using a Front-To-Rear-End Collision Warning System," DOT/NHTSA Technical Report, Contract DTNH22-93-C-07326, September 19, 1997.

APPENDIX B: ANALYSIS FOR BOUNDARY 1-2



(a) Velocity Plot

Figure B-1 Timing of Events for Boundary 1-2.

Criteria

The lead vehicle comes to a stop before following vehicle begins to brake. A

warning is issued at the same time that the lead vehicle stops. The following vehicle begins to brake 1.5 seconds after the warning is activated.

Analysis

1) The warning time, t_w , is the value of first time at which $dX_L/dt = 0$, i.e.,

$$dX_L/dt > 0 \text{ for } t < t_w$$

$$dX_L/dt = 0 \text{ for } t > t_w$$

2) $X_F(t_{FS}) = X_L(t_w) - 6.67$ where t_{FS} is the time at which the following vehicle stops, i.e.,

$$dX_F/dt > 0 \text{ for } t < t_{FS}$$

$$dX_F/dt = 0 \text{ for } t > t_{FS}$$

By definition:

$$t_w = V_0/d_L$$

$$X_F(t_w) = V_0 t_w = V_0^2/d_L$$

Also, the final positions of the two vehicles are,

$$X_F(t_{FS}) = X_F(t_w) + 1.5V_0 + V_0^2/2d_F$$

$$X_L(t_w) = R_0 + V_0^2/2d_L$$

The relationship between the two final positions must be,

$$X_F(t_{FS}) = X_L(t_w) - 6.67$$

Substituting gives,

$$X_F(t_w) + 1.5V_0 + V_0^2/2d_F = R_0 + V_0^2/2d_L - 6.67$$

But, since,

$$X_F(t_w) = V_0^2/d_L$$

$$V_0^2/d_L + 1.5V_0 + V_0^2/2d_F = R_0 + V_0^2/2d_L - 6.67$$

Simplifying gives:

$$V_0^2/2[1/d_L + 1/d_F] + 1.5V_0 - R_0 + 6.67 = 0$$

However, $R_0 = V_0 T_h$ where T_h is the headway between the two vehicles before any braking, i.e., when $t < 0$. Then:

$$V_0^2/2[1/d_L + 1/d_F] + V_0[1.5 - T_h] + 6.67 = 0$$

is the relationship that describes Boundary 1-2. This expression can also be written as:

$$T_h = 1.5 + V_0/2[1/d_L + 1/d_F] + 6.67/V_0$$

APPENDIX C: ANALYSIS FOR BOUNDARY 2-3

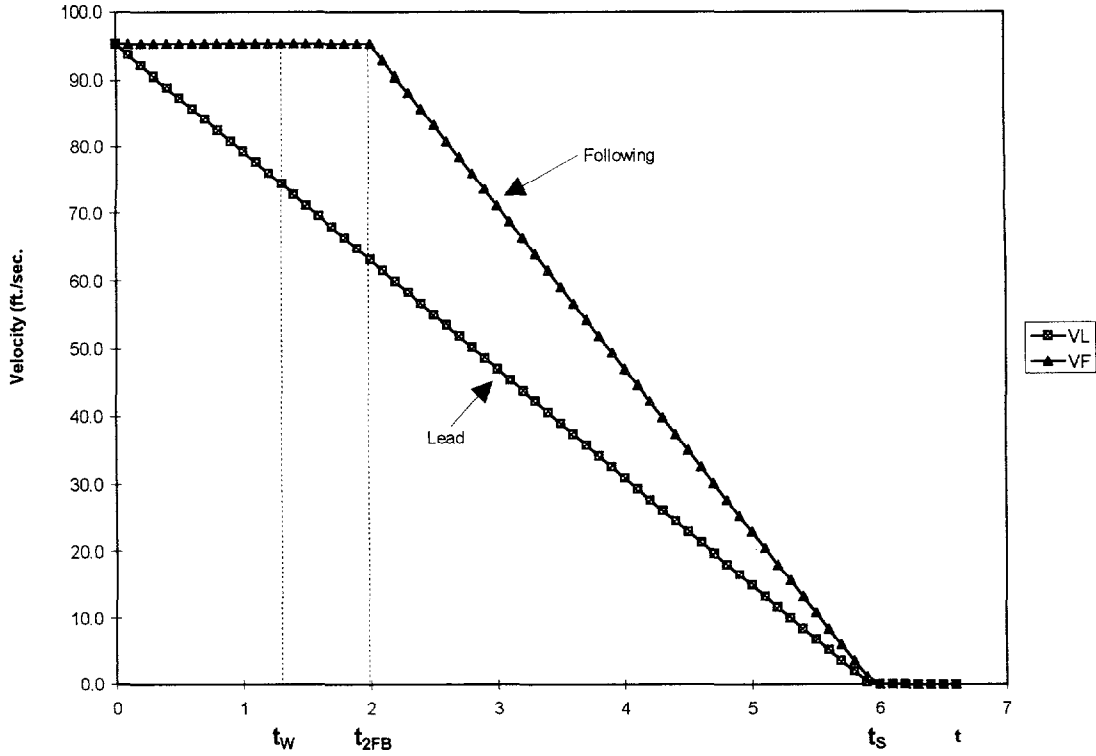


Figure C-1 Timing of Events for Boundary 2-3

$$X_F(t_W) = V_0^2/2[1/d_L - 1/d_F] + V_0[T_h - 1.5] - 6.67$$

Criteria

Both vehicles come to a stop at the same time.

Also note that

$$X_F(t_W) = V_0 t_W$$

and

$$t_W = t_s - (t_s - t_{FB}) - (t_{FB} - t_W)$$

$$t_W = t_s - V_0/d_F - 1.5$$

and since $t_s = V_0/d_L$,

$$t_W = V_0[1/d_L - 1/d_F] - 1.5$$

and

$$X_F(t_W) = V_0^2[1/d_L - 1/d_F] - 1.5V_0$$

Equating the expressions for $X_F(t_W)$ we have:

$$V_0^2[1/d_L - 1/d_F] - 1.5V_0 = V_0^2/2[1/d_L - 1/d_F] + V_0[T_h - 1.5] - 6.67$$

Combining terms:

$$V_0^2/2[1/d_L - 1/d_F] + V_0 T_h + 6.67 = 0.$$

Analysis

If the vehicles are 6.67 ft. apart when they come to rest, they must satisfy the end conditions:

$$X_F(t_s) = X_L(t_s) - 6.67$$

$$X_L(t_s) = R_0 + V_0^2/2d_L$$

$$X_F(t_s) = X_F(t_W) + 1.5V_0 + V_0^2/2d_F$$

Substituting the first two equations into the last gives:

$$X_F(t_W) + 1.5V_0 + V_0^2/2d_F = R_0 + V_0^2/2d_L - 6.67$$

Solving for $X_F(t_W)$ and noting that $R_0 = V_0 T_h$ gives

This is the relationship between the initial conditions for Boundary 2-3. Solving for T_h we have:

$$T_b = V_0/2[1/d_L - 1/d_F] + 6.67/V_0$$

APPENDIX D: WARNING CRITERIA FOR ZONE 2

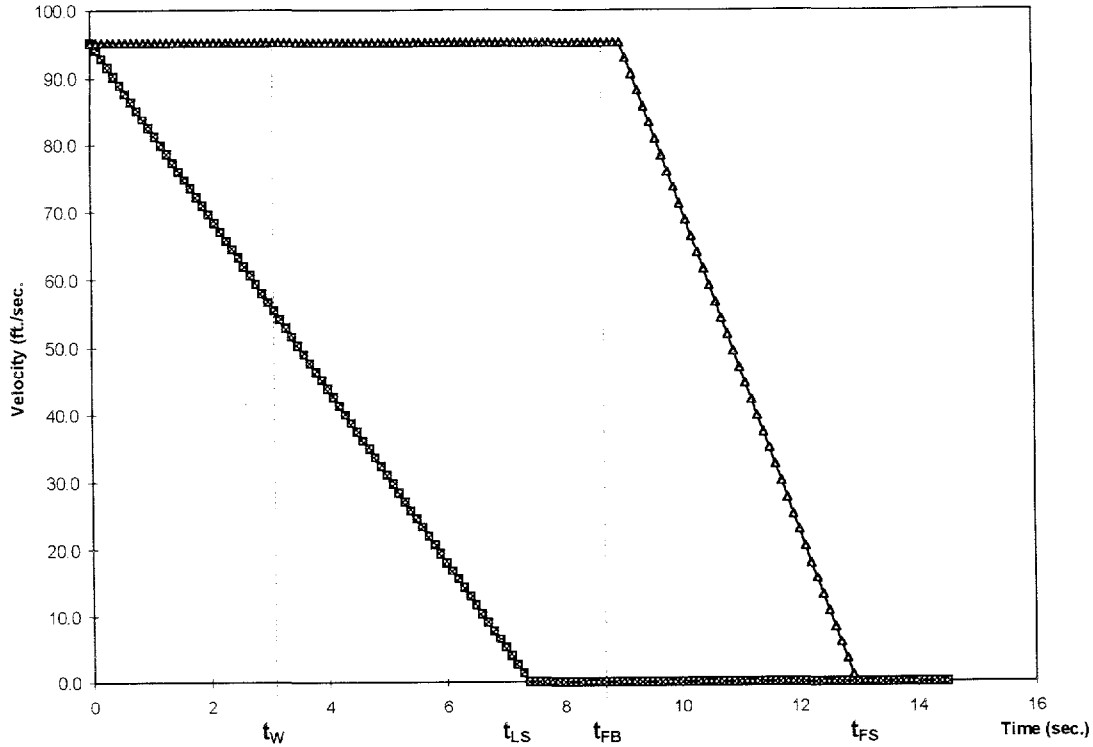


Figure D-1. Timing of Events for Zone 2 Motions

Criteria

Lead vehicle comes to a stop before following vehicle, with the warning issued while the lead vehicle is braking.

Analysis

At the end of the motion, the vehicles are assumed to be separated by 6.67 ft.:

$$X_F(t_{FS}) = X_L(t_{LS}) - 6.67$$

And their final positions are

$$X_F(t_{FS}) = V_0 t_w + V_0 (t_{FB} - t_w) + V_0^2/2d_F$$

$$X_L(t_{LS}) = R_0 + V_0^2/2d_L$$

Substituting into the first equation,

$$V_0 t_w + 1.5V_0 + V_0^2/2d_F = R_0 + V_0^2/2d_L - 6.67$$

Solving for t_w gives

$$t_w = V_0/2[1/d_L - 1/d_F] - 1.5 + 1/V_0[R_0 - 6.67]$$

But $R_0 = V_0 T_h$ therefore,

$$t_w = V_0/2[1/d_L - 1/d_F] + [T_h - 1.5] + 6.67/V_0,$$

The corresponding values of R_w and dR_w/dt at $t = t_w$ are:

$$R_w = R_0 - d_L t_w^2/2$$

$$dR_w/dt = -d_L t_w$$

APPENDIX E: WARNING CRITERIA FOR ZONE 3

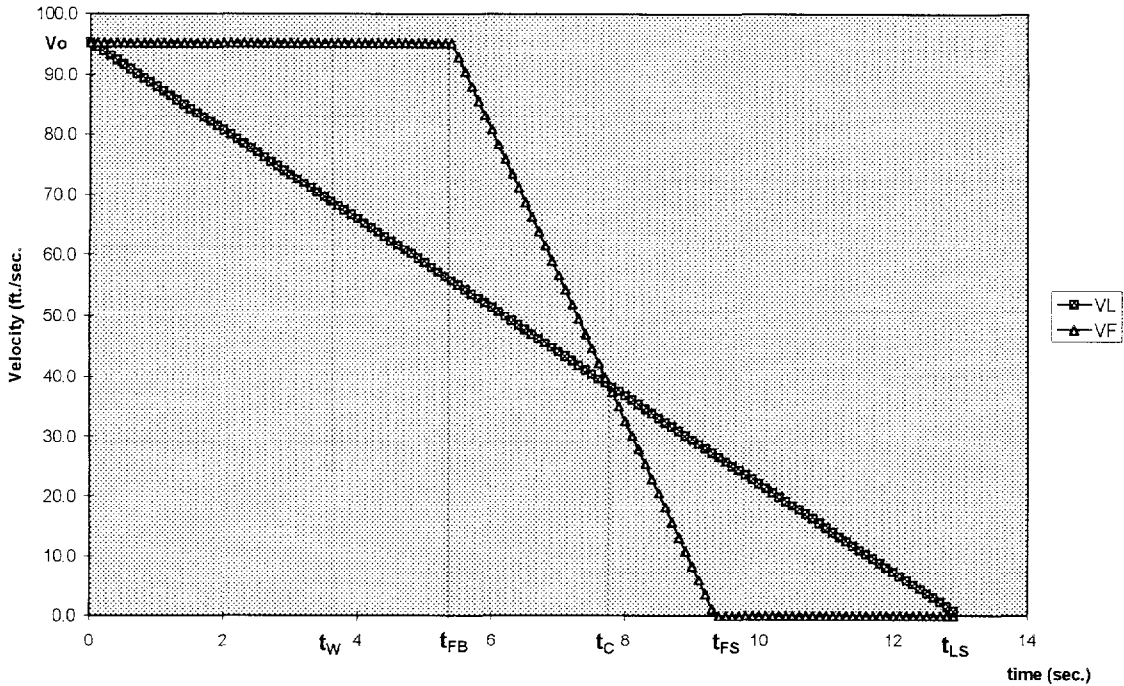


Figure E-1 Timing of Events for Zone 3 Motions

Criteria

Following vehicle comes to a stop before lead vehicle. The closest approach occurs while the two vehicles are still in motion.

Analysis

Here, the closest approach of the two vehicles occurs at time t_c and requires the following relationships:

$$X_F(t_c) = X_L(t_c) - 6.67 \quad (E-1)$$

$$dX_L(t_c)/dt = dX_F(t_c)/dt, \quad (E-2)$$

Also note that the closest approach occurs at the time when the range rate, dR/dt , changes sign,

$$dR/dt < 0 \quad 0 < t < t_c$$

$$dR/dt > 0 \quad t > t_c$$

The position of the two vehicles at the closest approach is:

$$X_L(t_c) = R_0 + V_0 t_c - (d_L/2)t_c^2$$

$$X_F(t_c) = V_0 t_c - (d_F/2)(t_c - t_{FB})^2$$

Substituting into eqn. E-1 above,

$$V_0 t_c - (d_F/2)(t_c - t_{FB})^2 = R_0 + V_0 t_c - (d_L/2)t_c^2 - 6.67 \quad (E-3)$$

Further, the speed equations for the two vehicles at the critical time may be written as,

$$dX_L(t_c)/dt = V_0 - d_L t_c$$

$$dX_F(t_c)/dt = V_0 - d_F(t_c - t_{FB})$$

which may be equated at the critical time using eqn. E-2 above,

$$V_0 - d_L t_c = V_0 - d_F(t_c - t_{FB})$$

Rearranging this equation gives,

$$t_c (d_F - d_L) = t_{FB} d_F$$

and

$$t_{FB} = [(d_F - d_L)/(d_F)] t_c \quad (E-4)$$

Substituting this into eqn. E-3 and simplifying gives,

$$-d_F/2[t_c - ((d_F - d_L)/(d_L))t_c]^2 = R_0 + V_0 t_c - (d_L/2)t_c^2 - 6.67$$

$$-d_F/2[d_L t_c/d_F]^2 = R_0 - 6.67 - (d_L/2)t_c^2$$

$$[d_L/2 - d_F/2(d_L/d_F)^2]t_c^2 = R_0 - 6.67$$

$$d_L/2 [1 - d_L/d_F]t_c^2 = R_0 - 6.67$$

$$\text{and } t_c = [2(R_0 - 6.67)/\{d_L(1 - d_L/d_F)\}]^{1/2}$$

Substituting into the eqn. E-4, and using

$$t_W = t_{FB} - 1.5$$

gives,

$$t_W = [(d_F - d_L)/d_F] [2(V_0 T_h - 6.67)/\{d_L(1 - d_L/d_F)\}]^{1/2} - 1.5$$

For $0 \leq t \leq t_2$, including $t = t_W$

$$dR/dt = dX_L/dt - dX_F/dt = (V_0 - d_L t) - V_0$$

$$dR/dt = -d_L t$$

So, at the time of warning,

$$R_W = R_0 - (d_L/2)t_W^2$$

$$dR_W/dt = -d_L t_W$$