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A combined boundary element and finite strip solution - BSM

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Abstract

This paper presents an approach to a solution based on combining the BEM and the Finite Strip Method, taking the advantages of both. The finite strip method is installed into the boundary element method by expanding the unknown parameters in terms of a trigonometric series and evaluating the unknown coefficients of this series. By this method a reduction in computational efforts needed to solve the given problem is achieved, compared with other numerical techniques. It is noted that the finite strip solution gives us a reduction of a semi dimension in the mesh generation and the boundary element method reduces one dimension. A combination of the boundary element method and the finite strip method, so called the Boundary Strip Method - BSM, creates a new powerful numerical method with two advantages over other numerical methods, the first one is a short computation time and the other is a reduction of one and a half dimensions in the mesh generation. Laplace equation was chosen as a test case for the method and the solution is compared with solution of the same problem using both boundary element and finite element solutions.

1 Introduction

The paper suggests a coupled use of the boundary element method and the method of the finite strip [1], in order to define the response of continuous media to various boundary conditions. Each of the two methods has its own advantages as compared with other numerical techniques [2,3]. Those advantages mean a quicker and more exact computation. The combined approach presented here was applied to problems in thermodynamics and mechanics,

which illustrate the possible future uses of the method.
In spite of the fact that the subject belongs to the area of numerical solutions, it should be emphasized that before using the numerical technique, there is a stage of physical formulation, which is the preparation of the physical problem towards a numerical solution. This is the pre-processing stage, after which the numerical solution by one of the common methods becomes realizable.
The main part of time spent by the design engineer during the numerical analysis is the transformation of the continuous medium physical problem into a desecrate problem which allows for the numerical solution. This transform includes the mesh generation which could be an elaborate problem by itself [4,5], especially when the medium carries a complicated geometry, e.g. a panel which consists of sub domains and holes. To be more specific, a plate with two holes in it as described in figure 1. The figure illustrates meshing for a given geometry by several methods. In figure 1a a finite element grid is presented. This method needs a dense mesh. The finite strip method necessitates a meshing reduced by half a dimension and lower density as seen in figure 1b. Application of the boundary element method reduces a whole dimension of the grid, as compared with the finite element method, see figure 1c, since the problem is solved only on the boundaries.

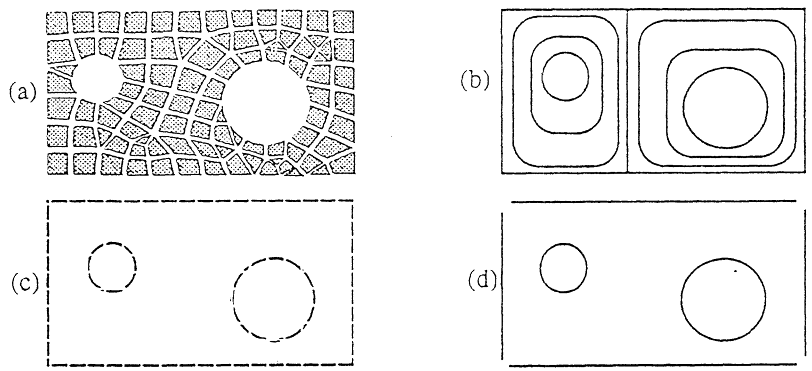

Figure 1: Shape meshing using different kinds of numerical methods.
(a) Finite element mesh (b) Finite strip mesh
(c) Boundary element mesh (d) Boundary strip mesh

A discussion about 3-D problem leads to similar consequences. The idea presented here of coupling the BEM and the finite strip method leads to a new numerical method, defined as BSM (Boundary Strip Method). As was mentioned before, the advantages of both methods may lead to a reduction of the mesh by one and a half dimensions, which in turn simplifies considerably the pre processing of the problem (figure 1d). The proposed method enables treating fully or partially each geometrical entity as a "boundary strip", obtained automatically by any CAD system, which becomes an ordinary device. By using this approach, the preparation time of transforming the problem from CAD into the analysis stage reduces to zero. This means that no human interference is needed in moving from the design stage into the computational one.

2 Formulation

Examination of the BSM was done here by using Laplace equation to illustrate steady thermal fields in bodies of various configurations, flow in porous media etc. In order to formulate the problem domain, the boundary Γ of which is combined of several sections. In some of these sections the natural condition exists, while in the others, the essential condition should be satisfied.

$$\begin{array}{lll l} \nabla^2 u = 0 & \text{in} & \Omega & \\ u = \bar{u} & \text{on} & \Gamma_u & : \text{essential condition} \\ q = \bar{q} & \text{on} & \Gamma_q & : \text{natural condition} \end{array} \tag{1}$$

$\bar{u}$ and $\bar{q}$ are prescribed functions, n is an outward normal to the boundary Γ, which includes all boundaries, $\Gamma = \Gamma_u + \Gamma_q$, $\Gamma_u = \sum_j \Gamma_{u_j}$, $\Gamma_q = \sum_j \Gamma_{q_j}$.

The boundary integral for Laplace problem is:

$$c_i u_i + \int_\Gamma u q^* d\Gamma = \int_\Gamma q u^* d\Gamma \tag{2}$$

An expansion of the boundary unknown functions into a trigonometric series yields the formulation for the strip:

$$\begin{array}{lll} q = q_0 + \sum_{n=1}^{\infty} \left[q_n^1 \cos\frac{2\pi n\eta}{\Gamma_u} + q_n^2 \sin\frac{2\pi n\eta}{\Gamma_u} \right] & \text{on} & \Gamma_u \\ u = u_0 + \sum_{n=1}^{\infty} \left[u_n^1 \cos\frac{2\pi n\eta}{\Gamma_q} + u_n^2 \sin\frac{2\pi n\eta}{\Gamma_q} \right] & \text{on} & \Gamma_q \end{array} \tag{3}$$

with $u_0, u_n^1, u_n^2, q_0, q_n^1, q_n^2$ as the trigonometric series coefficients, η is a circumferential coordinate along the boundary in a 2-D problem.
A substitution of equation (1) in (2) results:

$$c_i u_i + \int_{\Gamma_u} \bar{u} q^* d\Gamma + \int_{\Gamma_q} u q^* d\Gamma = \int_{\Gamma_q} \bar{q} u^* d\Gamma + \int_{\Gamma_u} q u^* d\Gamma \tag{4}$$

Since the unknowns in equation (3) are given as an approximation by only finite number N_i of harmonies, with i as the strip number, then each strip has $2N_i+1$ unknowns, which are the series coefficients. In order to solve $2N_i+1$ unknowns, the basic solution has to be sampled in a number identical to the number of unknowns over the strip. This way the number of unknowns will be the same as the number of equations. The number of sampling points, n_{cp}, for ns, strips is:

$$n_{cp} = ns + 2\sum_{i=1}^{ns} N_i \tag{5}$$

Two possible forms, which depend on location of the sampling points, characterize equation (4). If the sampling point is over Γ_u strip, the term $c_i u_i$ is known, and if the sampling point is given over Γ_q strip, $c_i u_i$ is unknown. Thus we can write:

$$\int_{\Gamma_q} uq^* d\Gamma - \int_{\Gamma_u} qu^* d\Gamma = -c_i u_i + \int_{\Gamma_q} \bar{q} u^* d\Gamma - \int_{\Gamma_u} \bar{u} q^* d\Gamma \qquad \text{on} \qquad \Gamma_u$$

$$c_i u_i + \int_{\Gamma_q} uq^* d\Gamma - \int_{\Gamma_u} qu^* d\Gamma = \int_{\Gamma_q} \bar{q} u^* d\Gamma - \int_{\Gamma_u} \bar{u} q^* d\Gamma \qquad \text{on} \qquad \Gamma_q \tag{6}$$

The right hand side of equations (6) is known explicitly, while the left hand side which includes the unknowns is approximated by equation (3):

$$\int_{\Gamma_q} uq^* d\Gamma = u_0 \int_{\Gamma_q} q^* d\Gamma + \sum_{n=1}^{N_i} \left[u_n^1 \int_{\Gamma_q} \cos\frac{2\pi n\eta}{\Gamma_q} d\Gamma + u_n^2 \int_{\Gamma_q} \sin\frac{2\pi n\eta}{\Gamma_q} d\Gamma \right]$$

$$\int_{\Gamma_u} qu^* d\Gamma = q_0 \int_{\Gamma_u} u^* d\Gamma + \sum_{n=1}^{N_i} \left[q_n^1 \int_{\Gamma_u} \cos\frac{2\pi n\eta}{\Gamma_u} d\Gamma + q_n^2 \int_{\Gamma_u} \sin\frac{2\pi n\eta}{\Gamma_u} d\Gamma \right] \tag{7}$$

Hence the only integrations to be solved are those included in the basic solutions u* and q* and their harmonies. It is important to note that for some geometrical entities these integrations can be found analytically.

3 The Program

The computer program gets a CAD input as a description of the geometrical entities that define the body for solving BSM and the loading functions acting on them. The program translates the geometrical input into strips with the

approximation needed for solution, and this way, the number of terms of the geometrical series is defined for each strip. The next step the in program creates a vector of sampling points over each strip according to the number of harmonies by which the solution over the strips is approximated.
It is possible to create for each sampling point a row of the matrix and one term in the right hand side vector, while integrating for the basic solutions u*,q* at the sampling points and for their harmonies. Last stage consists of the matrix inversion and its multiplication by the right hand side vector in order to solve the unknowns, which are the terms of the trigonometric series. Obviously, the solved unknowns on the boundary allows for a direct derivation of the interior field of the problem, within the given domain, without any further approximation. A flow chart of the program is shown in figure 2.

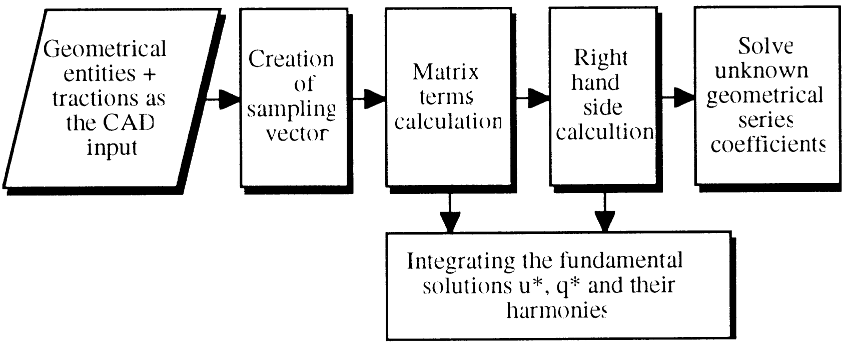

Figure 2: Flow chart for the BSM

4 Illustrations

In this section we solve three examples. The first example consists of a circular disk with a concentric hole, the nondimensional radius of the disk is R_o=10 and the radius of the hole is R_i=5. The tractions are defined by a constant essential boundary condition over the inner and the outer boundary, in this case the analytical solution is known as:

$$\frac{\partial u}{\partial n} = \frac{(u_i - u_o)}{r \ln(R_i/R_o)} \tag{8}$$

where R_i, R_o are the inner and the outer radii respectively. When a steady state heat transfer problem is considered, the essential condition is the temperature u and the natural condition is $\partial u/\partial n$, the heat flux per unit length of the primer at radius r, in the normal direction n. Thus u_i, u_o are the inner and the outer nondimensional temperatures. The solution to this problem by the boundary strip method is by using two closed boundary strips one for the outer boundary and one for the inner boundary. The harmonies order approximation on each strip can be zero since the solution on each strip is a constant, which yields in turn a set of two equations, with an exact solution as seen in figure 3.

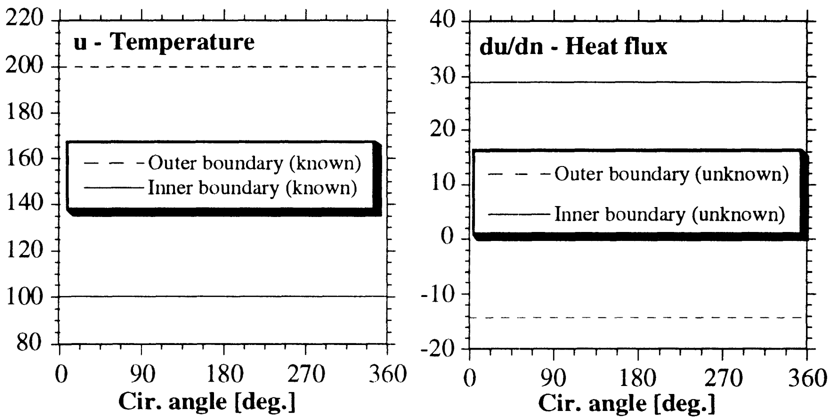

Figure 3: Knowns and unknowns variance along circumference

The second problem demonstrate a circular plate with two different eccentric holes inside it and the tractions on the boundaries are constant temperatures as described in figure 4a. In order to solve this problem we must define three closed strip, one strip for each circle. The harmonies approximation on each strip is of the tenth order. The solution of the problem is shown in figure 5a.

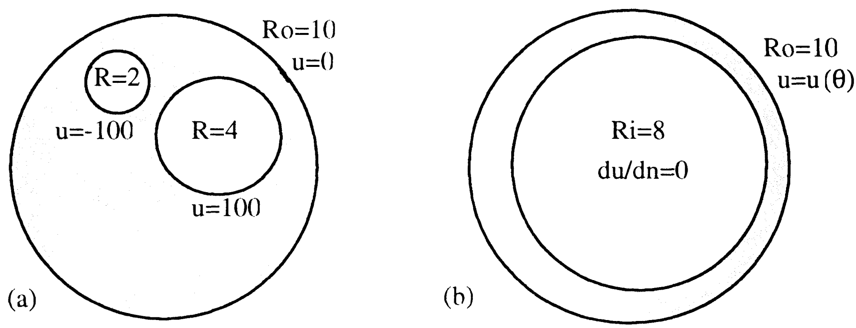

Figure 4 : Illustrations for problems with eccentric holes and different loading.
a : Scheme of second example. b : Scheme of third example.

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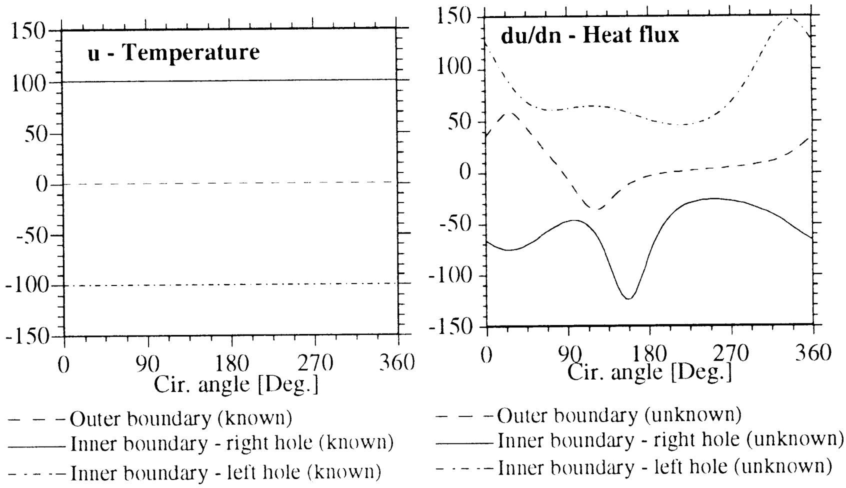
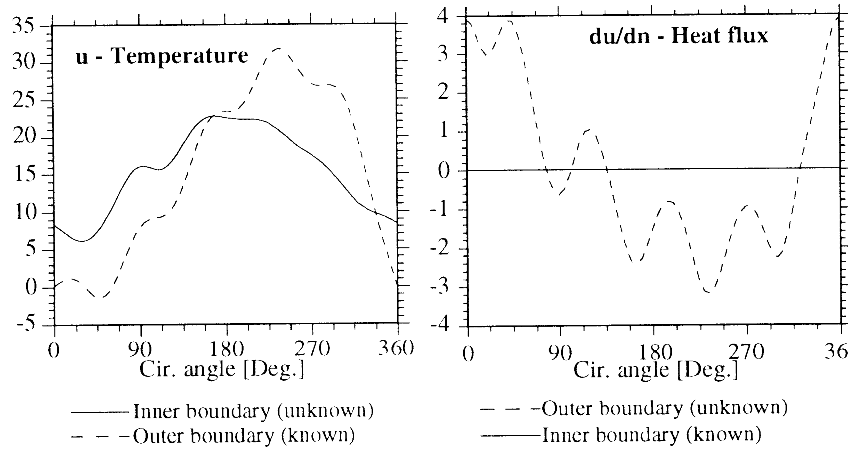


(a)



(b)

Figure 5 : Knowns and unknowns variance along circumference
a : Results of second problem. b : Results of third problem.

The third problem is of a full circular disk containing an eccentric hole (see figure 4b) with a noisy traction distribution on the outer boundary and an insulation on the inner boundary as shown in figure 5b. In this problem the solution is done using only two closed strips with ten harmonies in each, the harmonies approximation is a function of the higher frequency of the traction distribution. The solution obtained for this problem is shown in figure 5b.

5 Discussion

The few examples which were solved in last section showed the advantages of using the boundary strip method over other numerical method. From the illustrations it can be seen that a physical problem which demands a discretization using a dense meshing for one of the common methods such as the finite element, the finite strip or the boundary element, can be solved by practically avoiding any mesh generation. The boundary strip method leads to a considerably lower computation time with more accurate results. It is important to emphasize that the boundary strip method need no further mesh over the given mesh from the CAD system. For some geometrical entities we can write an analytical expression for the integrations which are needed in order to solve the physical problem, In that case there is no reduction in the accuracy of the solution as occurs when numerical treatment is used.

6 Remarks and Conclusion

The boundary strip method - BSM, is a new numerical method that can be a break throw in the linkage between the design and analysis stages. This method allows us to take a given geometrical data shape directly from the CAD system into the analysis stage, without any human processing. The boundary strip solution has been examined and proven as a powerful numerical method which allows solving some complicated physical problems, with a low computation time and a better accuracy.

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