# A Common Vision for Undergraduate Mathematics

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Higher education is under intense public scrutiny. Revenue streams are diminishing. Mathematics is often at the center of discussions, lambasted for low student success rates. Change is coming, and the mathematical sciences community must come together to ensure that the change happens coherently and in a mathematically sound way. More mathematicians must become involved, working with colleagues from partner disciplines to modernize curricula and adopt evidence-based active learning strategies in mathematics classrooms.

In May 2015, we attended a remarkable workshop at the American Statistical Association headquarters in Alexandria, Virginia. What was truly extraordinary was the genuine collaboration of members of the five professional societies concerned with undergraduate teaching in the mathematical sciences: the American Mathematical Association of Two-Year Colleges (AMATYC), the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM). Workshop attendees represented not only the five mathematical sciences associations but also partner STEM disciplines, higher education advocacy organizations, and industry.

Workshop participants came together to develop a Common Vision of the pressing need to modernize undergraduate programs in the mathematical sciences, with particular attention to the first two years. The main themes emerging from the workshop are that all of us in the mathematical sciences community face similar challenges, that the status quo is unacceptable, and that the most effective solutions will involve cooperation and collaboration. Research on "collective impact" [8] suggests that, in achieving significant and lasting change in any area, a coordinated effort supported by major players from all existing sectors is more effective than an array of new initiatives and organizations. Common Vision encourages such action by highlighting existing efforts and draws on the collective wisdom of a diverse group of stakeholders to articulate a shared vision for modernizing the undergraduate mathematics program.

Photo courtesy of Linda Braddy.

**Common Vision participants hard at work.**

As the Common Vision project developed, another movement was afoot. With initial support from Carnegie Corporation of New York, Phillip Griffiths founded *Transforming Post-Secondary Education in Mathematics* (*TPSE Math*)[1] to help coordinate the mathematics community at a high level. *TPSE Math* has organized a national meeting and four regional meetings of mathematicians and higher education administrators to frame the challenges our community faces. *TPSE Math's* ability to convene all the stakeholders in this process is crucial to future success. Using *TPSE Math's* findings and reports together with the Common Vision final report, the next step is to bring together department leaders from the mathematical sciences to organize and address these challenges,

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[1] www.tpsemath.org/

*Women are almost twice as likely as men to choose not to continue beyond Calculus I*

capitalizing on the power of collective action. Both of these initiatives have now been endorsed by CBMS.[2]

The Common Vision project began with an evaluation of where the five societies stand with respect to our curricula in the mathematical sciences. The main focus was the seven undergraduate curricular guides endorsed by the five societies. Common Vision leaders[3] prepared a draft of the project report, *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025,* assessing the commonalities among the seven guides. There are four categories that all the guides agree will require further action: (1) curriculum, (2) course structure, (3) workforce preparation, and (4) faculty development. These are of course interdependent. Any particular challenge may fit into more than one of these categories. Moreover, some of these are not under the exclusive control of faculty members, and so substantial change will require the participation of others. Improving teaching and learning requires well-coordinated efforts by multiple stakeholders, including faculty, higher education administrators, employers, professional associations, and funding agencies.

There are many shocking data points about our enterprise of teaching undergraduate mathematics. For example,

> • Women are almost twice as likely as men to choose not to continue beyond Calculus I, even when Calculus II is a requirement for their intended major [5].
>
> • In 2012, 19.9 percent of all bachelor's degrees were awarded to underrepresented minority students (9.5 percent to African American, 9.8 percent to Hispanics). However, only 11.6 percent of mathematics bachelor's degrees were awarded to underrepresented minority students (4.9 percent to African American, 6.4 percent to Hispanics).[4]

Walker [15] noted that while there has been some improvement since Stiff and Harvey [13] called the mathematics classroom one of the most segregated places in the United States, upper-level mathematics classes remain predominantly white and Asian, and these statistics point to the fact that underrepresentation of women and racial and ethnic minority students in advanced mathematics courses is an ongoing problem. As Figure 1 shows, underrepresented minority students do not have the same access to advanced mathematics courses in high school as do their white peers.

Lack of high school access does not allow us to yield responsibility for helping students find pathways to success in college mathematics. Courses in the first two years of college must be designed to address the disconnect between high school and advanced mathematics. The mathematical sciences community must redouble its efforts to attract, prepare, and retain a diverse student population.

The guides that Common Vision looked at identified a need for the introduction of multiple entry points and pathways into and through the mathematics curriculum. These pathways should align with the needs of students from other STEM and non-STEM fields to help increase success rates and decrease the time to completion of college credit-bearing courses. In terms of curriculum, these pathways should include data science, modeling, and computation. In addition, multiple pathways that provide new curricular formats afford students the opportunity to engage in problem-solving skills needed for their careers. These opportunities can increase student interest and help them understand the value and role of the mathematical sciences in society. The need for multiple pathways must be tempered by the fact that calculus remains central to many disciplines. Completing at least one calculus course is required for most STEM degrees, so pathways that lead to calculus must be maintained. Students should take calculus only when they are ready, and the calculus they

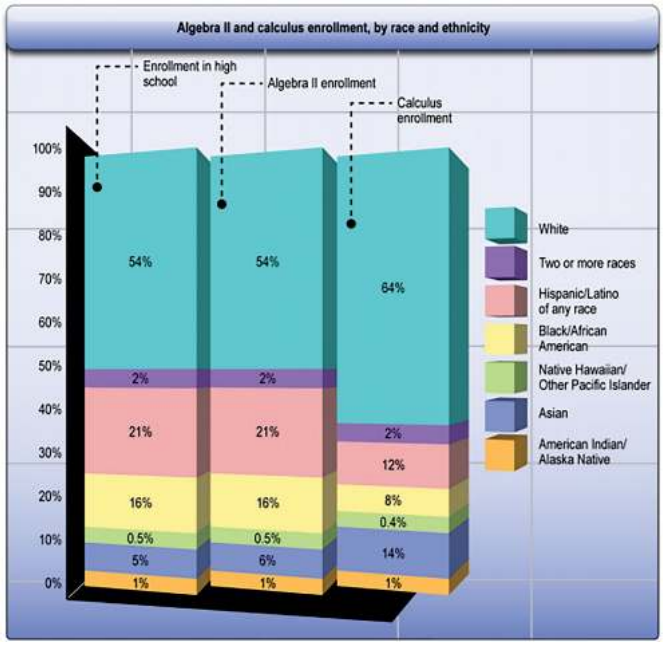


Photo courtesy of US Department of Education, Office for Civil Rights, Civil Rights Data Collection, 2011–12.

**Figure 1: Underrepresented minority students do not have the same access to advanced mathematics courses in high school as do their white peers.**

[2] www.maa.org/sites/default/files/pdf/common-ground/CBMSLetterofSupport.pdf

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[4] www.nsf.gov/statistics/nsf07308/content.cfm?pub_id=3633&id=2

take should be attentive to the needs of partner disciplines. Common Vision workshop participants agree that faculty and administrators should implement systemic changes that expand opportunities and remove barriers to success for all students.

Observations pointing to serious failures on our part are not new. The well-known report *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (NRC, 1989) addresses "the urgent national need to revitalize mathematics and science education [and] examines mathematics education as all one system, from Kindergarten through graduate school." What is different now is the national attention focused on teaching and learning in the mathematical sciences. The mathematical sciences are in the national spotlight in part because mathematical competencies can lead to higher-paying jobs and thus can play a profound role in students' economic mobility. There is substantial support for reforming undergraduate instruction in the mathematical sciences from influential actors: the White House Office of Science and Technology Policy, the National Academies, and the Association of American Universities, among others.

In October 2015 Michelle Obama unveiled a public-awareness campaign called Better Make Room to celebrate higher education and encourage students to pursue postsecondary education. Better Make Room will "leverage traditional and new-media platforms to celebrate student stories in the same way that we often celebrate celebrities and athletes," according to a White House fact sheet.[5] The campaign website urges students to "Get a better education, have a better career, imagine a better future and live a better life." Two-year colleges are a large, growing, and increasingly important component of the United States higher education system. Indeed, these institutions taught just over 45 percent of the student credit hours in the mathematical sciences in 2010 ([7], Table S.1). But,

> at community colleges across the country, the basic math requirement has been a notorious hindrance to advancement. More than 60 percent of all students entering community colleges must take what are called developmental math courses, according to the Carnegie Foundation for the Advancement of Teaching, but more than 70 percent of those students never complete the classes, leaving them unable to obtain their degrees. [3]

President Obama announced an initiative to mint five million more community college graduates by 2020, a remarkable goal given that just over 77,000 associate degrees and certificates were awarded in 2011–12 by community colleges [1]. According to *Two-Year Contributions to Four-Year Degrees* [11], 46 percent of all students who completed a degree at a four-year institution in 2013–14 had been enrolled at a two-year institution at some point in the previous ten years. To meet the goals for higher education laid out by the President and First Lady, we must transform mathematical sciences courses to be pathways toward STEM degrees.

[5]goo.gl/zw2n4I

Photo courtesy of Linda Braddy.

**Common Vision participants after the May 2015 workshop at ASA headquarters in Alexandria, VA.**

In our efforts, it is imperative that members of the mathematical sciences community understand the interconnectedness of the educational ecosystem. Roughly 3.6 million students entered college for the first time in the fall of 2008 and over the past six years have transferred 2.4 million times, "ricocheting between two- and four-year public and private colleges, often across state lines" [6]. The high mobility rate exacerbates challenges for students from lower economic sectors. According to the National Center for Public Policy and Higher Education [9], 44 percent of low-income students enroll in two-year college after high school, compared to 15 percent of high-income students. Because transferring is more common for two-year college students, lack of program coordination across multiple institutions impacts low-income students to a greater degree. These facts highlight the need to work across sectors within higher education, as well as across states, to improve student success. Understanding the differences of perspectives and points of view is critical.

The AMS is not new to the conversation on education. *Towards Excellence* [2] was a product of the AMS Task Force on Excellence in Mathematics Scholarship, funded in the 1990s by the Exxon Foundation and the National Science Foundation. At the Joint Mathematics Meetings in 1993, William (Brit) Kirwan, now chancellor emeritus of the University System of Maryland, said that the "AMS Task Force on Excellence must deal substantially with the issue of the quality of undergraduate mathematics education for all students, not just mathematics majors… [W]e must make it easier for senior mathematicians at research universities to take on with dignity, respect, and reward some of the challenging obligations facing the math community." Though these words were spoken almost a quarter century ago and we have made progress, our work requires substantial and ongoing energy. In 2014 the Coalition for Reform in Undergraduate STEM Education[6] recommended that "the faculty reward system must recognize and reward efforts by faculty members to use evidence-based pedagogy" and observed that "[s]easoned faculty members who are experienced teachers or innovators can be powerful champions for change, especially if they carry institutional clout or political capital."

[6]*Fry, C. (2014),* Achieving Systemic Change: A Sourcebook for Advancing and Funding Undergraduate STEM Education, *Washington, DC: The Coalition for Reform of Undergraduate STEM Education.*

One area in which AMS members are particularly well positioned to act is through their work with graduate students, who are important contributors to the teaching mission of many universities. Graduate students who pursue academic careers will be the faculty of the future, but historically we have provided very little training focused on preparing them to teach. While research universities play a unique and significant role in the preparation of these future faculty members, they in turn hire only a small proportion. According to *Towards Excellence* [2], fewer than 20 percent of new PhDs find employment at PhD-granting institutions. The remainder of those who pursue careers in higher education work at other types of institutions and spend much of their time teaching undergraduates. Vélez et al. [14] reported that 32 percent of 2012–13 PhD recipients found employment upon graduation in PhD-granting departments, but almost three-quarters of them were in postdoctoral—and hence not long-term—appointments. Attending to graduate student preparation for teaching has far-reaching benefits beyond the PhD-granting institutions at which they serve as teaching assistants, for instance, to the students at institutions where they will spend their careers and to the mathematical sciences community as a whole. There have been some promising examples of programs designed to help graduate students transition to careers as professors. One was the now defunct Preparing Future Faculty initiative, sponsored by the Pew Charitable Trust. This program worked with graduate students across disciplines to help them develop expertise in teaching and expose them to the professional environment at a variety of institutions, including two-year colleges, liberal arts institutions, and comprehensive universities. Two current relevant programs are the MAA's College Mathematics Instructor Development Source (CoMInDS)[7] and Project NExT.[8]

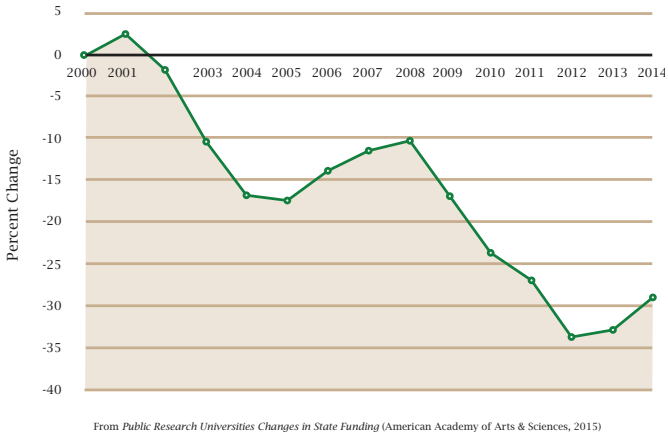
*A disturbing trend is the recent decline in state investment in public higher education*

A disturbing trend is the recent decline in state investment in public higher education (see Figure 2). About three-quarters of all undergraduate students were enrolled in the fall of 2009 in public institutions.[9] Simultaneously, many states are now tying funding to degree completion and are requiring an increase in the number of STEM graduates in order to secure state funding. Budget cuts surely affect many AMS members, and practical tactics for addressing decreasing resources and simultaneously increasing numbers of students are urgently needed.

[7] www.maa.org/programs/faculty-and-departments/cominds

[8] www.maa.org/programs/faculty-and-departments/project-next

[9] trends.collegeboard.org/sites/default/files/trends-2012-public-higher-education-expenditures-brief.pdf




From *Public Research Universities Changes in State Funding* (American Academy of Arts & Sciences, 2015)

**Figure 2: Percent change in state support for public higher education (all colleges and universities) per full-time equivalent student in constant 2014 dollars.**

The good news is the inspiration we found at the Common Vision workshop. Members of the mathematical sciences community face these challenges together, and we truly do have a Common Vision! We encourage you to join this collective action. Read the AMS blog on teaching mathematics.[10] Attend Joint Mathematics Meeting activities focused on teaching. Follow the TPSE Math movement. Visit the Common Vision website[11] and read our final report, which includes ideas generated at the workshop for moving forward, for implementing and adopting programs to incrementally improve mathematical sciences courses in the first two years of college.

Change is unquestionably coming to lower-division undergraduate mathematics, and it is incumbent on the mathematical sciences community to ensure that it is at the center of these changes, not on the periphery.

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## ABOUT THE AUTHORS

Photo courtesy of Melissa Totman.

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Photo courtesy of David Turner, Macalester College.

**Karen Saxe** is an active volunteer with several of the professional associations in mathematics, including the AMS, AWM, and MAA and has served as a vice president of the MAA (2014–16) and as the 2013–14 AMS/AAAS Science and Technology Policy Congressional Fellow. She currently serves on the Advisory Board for Transforming Post-Secondary Education in Mathematics (TPSE Math). Karen has been a resource in Minnesota on redistricting, consulting with city governments, and served on the Minnesota Citizens' Redistricting Commission, created to draw congressional districts following the 2010 census.