A COMPARATIVE ANALYSIS OF TWO DISTANCE MEASURES IN COLOR IMAGE DATABASES

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ABSTRACT

Euclidean distance measure has been used in comparing feature vectors of images, while cosine angle distance measure is used in document retrieval. In this paper, we theoretically analyze these two distance measures based on feature vectors normalized by image size and experiment with them in the context of color image database. We find that the cosine angle distance, in general, works equally well for image databases. We show, for a given query vector, characteristics of feature vectors that will be favored by one measure but not by the other. We compute k-nearest neighbors for query images using both Euclidean and cosine angle distance for a small image database. The experimental data corroborate our theoretical results.

1. INTRODUCTION

Vector model is widely used for capturing and storing objects in information retrieval systems. One important issue to consider in using the vector model is the choice of appropriate distance measure that provides the metric for finding similarity between two objects. Several distance measures, such as the L1 metric (Manhattan Distance) and the L2 metric (Euclidean distance), have been proposed to compare the similarity of feature vectors. In content-based image retrieval systems Euclidean distance is commonly used to determine similarities between a pair of images. In document retrieval systems, on the other hand, distance measure based on cosine angle is more commonly used for similarities between two documents [1][2]. Even though both the Euclidean and the cosine angle based distances coincide when the components of the feature vectors are normalized by the norm of the vector, they differ when they are normalized otherwise. In image processing applications components of a feature vector (e.g., color histogram) are usually normalized by the size of the image and as a result, the Euclidean and the cosine angle based distances produce different results. The contribution of this paper is to show when one distance measure performs differently, and when they perform similarly for feature vectors normalized by size. We present a theoretical approach to compare these two distance measures and corroborate the theory with experimental results performed on feature vectors for color images.

2. THE COMPARISON OF TWO DISTANCE MEASURES

2.1. Related work

One way of comparing the distance measures is to see their retrieval performance in terms of precision and recall based on a particular image database [3]. Different aspects on how to choose a distance measure have also been studied. One concern in choosing a particular distance measure is the impact of computational overhead on system performance. When feature vectors are large, some distance measures may consume more computing resources than the others. One possible approximation of the Euclidean distance measure is proposed in [4]. On the other hand, it is also important to choose a similarity measure that is consistent with human ideas of similarity. The authors of [5] have proposed a similarity measure based on noise distribution of the image set.

2.2. The retrieval experiment

We start our comparison of Euclidean distance and cosine angle distance based on the retrieval results from our image database, which contains 650 photographic color images, derived from the web and IMSI master clips. Our experiment is based on both QBIC feature vectors [6] and color histograms proposed in [7]. For color histograms, we use a new histogram generation technique based on HSV color space where each pixel in an image contributes either its hue or its intensity based on its saturation.

Our results show that two distance measures give us different ordering in the retrieval results. Euclidean distance has better precision and recall for some query images and angle is better for some other query images.

In the next subsection, we describe the mathematical model we have used for analysis. The analysis shows the relationship between the distance measures and the variance of the components of the feature vectors (we use the name "variance of feature vector" or "variance" in the following sections). The detailed experimental results are presented in section 3.

2.3. The mathematical analysis

The analysis described in this subsection is based on geometrical properties of feature vectors derived from image hyperspace. To make feature vectors comparable to each other, each vector is normalized by its corresponding image size. Assuming there are n components in a feature vector, the formula $v_k = V_k \Big/ \sum_{i=1}^{n} V_i$ ($1 \le k \le n$) gives a normalized feature vector v for the original vector V. Each normalized vector in the image database can be regarded as a point on the hyperplane $x_1+x_2+\ldots+x_n = 1$ in a vector space which has n dimensions. There are two special points in the hyperspace. The *centroid* C is defined as the point: (1/n, ...,1/n) on the hyperplane, where n is the number of dimensions and the *origin* O is the point: (0,...,0).

As shown in Figure 2.1, given a query image point Q (q_1, q_2, ..., q_n), the Euclidean distance between Q and any other point P (p_1, p_2, …, p_n) is the edge |PQ| and the cosine angle distance is cos(∠POQ). We name ∠POQ as angle A and cos A as the cosine angle distance between P and Q. The variance of vector P is given by the formula $\sigma^2 = |PC|^2 / n$, where the number of dimensions n is fixed. The following theorems show the mathematical relation between variance and the cos angle distance cos A. The proof of Theorem 1 is given in Appendix.

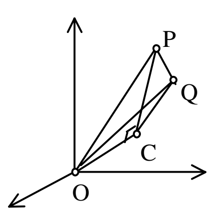


Figure 2.1

Theorem 1: Given a query vector Q on the hyperplane ($x_1+x_2+\ldots+x_n=1$), for any point P that has a fixed Euclidean distance from Q, the cosine angle distance cos A will be the maximum when the variance σ^2 of P reaches its highest possible value $(|CQ| + |PQ|)^2 / n$. If $|PQ| \ge |CQ|$, cos A monotonically decreases as σ^2 decreases. If $|PQ|<|CQ|$, cos A decreases when $(|CQ| - |PQ|)^2/n \le \sigma^2 \le (|CQ|^2 - |PQ|^2)/n$ and increases when $(|CQ|^2 - |PQ|^2)/n \le \sigma^2 \le (|CQ| + |PQ|)^2/n$.

Based on Theorem 1, we can see that if Euclidean distance of two vectors v1 and v2 are the same, the one with a greater variance value of its components will be most likely ranked higher by angle distance measure.

In real applications, it is seldom the case that the Euclidean distances of two feature vectors are exactly the same. Theorem 2 below handles this problem and shows what happens when two feature vectors have the same variance values but different Euclidean distances. The proof of Theorem 2 is given in the Appendix.

Theorem 2: Given a query point Q on the hyperplane ($x_1+x_2+\ldots+x_n=1$), for any point P1 that has a fixed variance value σ^2, the cosine angle distance cos A will decrease when Euclidean distance |PQ| increases.

Theorem 2 shows that if the variances of all feature vectors are fixed, the Euclidean distance and cosine angle distance will always give us the same retrieval results. Based on Theorem 1 and 2, we see that when Euclidean distance and cosine angle distance give different retrieval results, the variances of the vectors must be different. The feature vector that has a higher variance value will most likely be ranked higher by Angle distance. Experimental results given in the next section corroborates our analytical results.

3. THE EXPERIMENTAL RESULTS

The following results are based on the experiments described in section 2.2. Table 3.1 and 3.2 show that, on the average, the retrieval quality of Euclidean distance is almost the same as that of the cosine angle distance. However, the feature vectors that ranked higher by Angle distance have a higher average variance value then those ranked higher by Euclidean distance. Table 3.3 and 3.4 show the average variance values of feature vectors ranked 1-5, 1-10 and 1-25 in the retrieval results by Euclidean distance and by cosine angle distance. We see that the average value of variance is higher for those ranked by cosine angle distance than those by Euclidean distance, which corroborates our analytical results.

Table 3.1 Average precision and recall of QBIC feature vector:

Distance Measure	Precision			Recall		
	5	10	25	5	10	25
Euclidean	0.580	0.430	0.240	0.379	0.547	0.721
Cosine	0.560	0.420	0.236	0.366	0.530	0.715

Table 3.2: Average precision and recall of HSV histogram vector:

Distance Measure	Precision			Recall		
	5	10	25	5	10	25
Euclidean	0.820	0.630	0.336	0.520	0.781	0.983
Cosine	0.820	0.640	0.324	0.520	0.785	0.952

Table 3.3 Average variances of QBIC feature vectors:

Distance Measure	QBIC		
	5	10	25
Euclidean	0.00251	0.00249	0.00233
Cosine angle	0.00299	0.00286	0.00294

Table 3.4 Average variances of HSV histogram vectors:

Distance Measure	HSV		
	5	10	25
Euclidean	0.00197	0.00188	0.00164
Cosine angle	0.00228	0.00229	0.00198

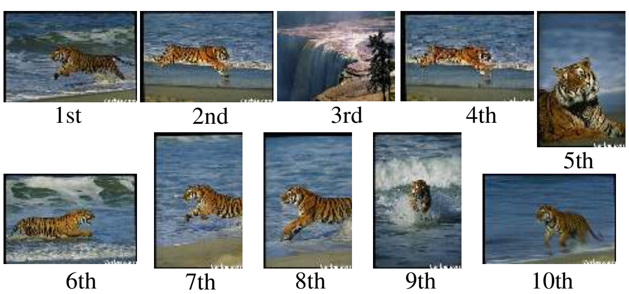


Figure 3.1 Query result by cosine angle distance using HSV

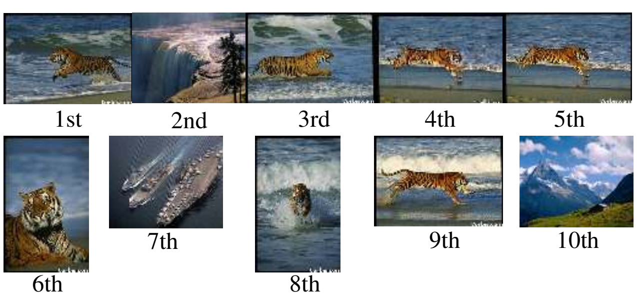
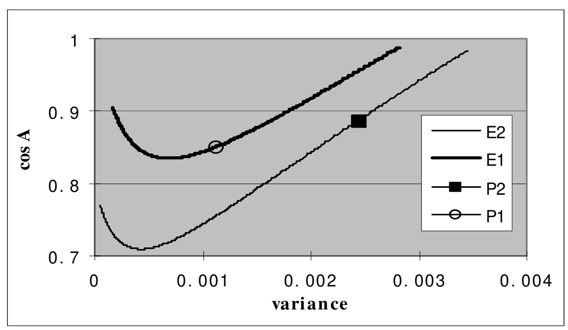


Figure 3.2 Query result by Euclidean distance using HSV

As we mentioned in section 2.2, the actual ranking results of Euclidean and cosine angle distance for a given query image are often different. We have proved by Theorem 1 and 2 that the different retrieval results by Euclidean distance and cosine angle distance are related to the variances of the components of the feature vectors. Figure 3.1 and 3.2 illustrates one typical query result where the first image is the query image. In this example, the cosine angle distance shows a better retrieval results than the Euclidean distance.

We pick two images from ranked list and make the following observation. The 7th image in Figure 3.1 (named as P2 in Figure 3.3) contains a tiger, and the 7th image in Figure 3.2 (named as P1 in Figure 3.3) contains ships. In Figure 3.3, the thinner curve is based on all points with the same Euclidean distance of value |PQ|=0.20587. The thicker curve is based on |PQ|=0.16086. We also have |CQ|=0.26393 (Please refer to Figure 2.1). From figure 3.3, we can see that P2 has a greater variance value than P1, while angle distance is favoring P2 but Euclidean distance is favoring P1, which corroborates our analytical results. The two curves have a shape conforming to Theorem 1. The two curves do not have any intersections, which conforms to Theorem 2.



4. CONCLUSIONS

We have compared Euclidean distance with cosine angle distance for feature vectors normalized by size, and have shown when one gives different results from the other. We corroborate our theoretical results with experimental data using content-based image retrieval. Though our experiment shows that Euclidean and cosine angle distance perform similarly, the property of angle distance presented in this paper may be exploited to achieve better retrieval quality in image databases.

We have also done experiments on the same image database of 650 images with Manhattan (L1) distance measure. Our initial results show that, though the ranked ordering of images for a query using Manhattan distance is different from those by Euclidean and cosine angle, the average precision and recall are very similar to those of the Euclidean distance and cosine angle distance. We have seen in our experiment that the precision and recall of Manhattan distance is always better than those of the Euclidean distance and cosine angle distance in the first 1-5 results, but always worse in the first 1-25 results.

5. APPENDIX

Property 1: For every point P (p_1, p_2, …, p_n) on the hyperplane ($x_1+x_2+...+x_n=1$) other than centroid C, hyperangle ∠OCP = 90°.
Proof. Vector PC is (p_1 - 1/n, …, p_n - 1/n), and vector OC is (0 - 1/n, ... , 0 - 1/n). We have: cos ∠OCP = (PC•OC)/(|PC|×|OC|)

$$= ((p_1 + p_2 + \dots + p_n - 1) \times (-1/n)) / (|PC| \times |OC|)$$

We know $p_1+p_2+...+p_n=1$. So cos∠OCP=0 and ∠OCP=90°. □

Theorem 1 Proof. As shown in Figure 2.1, we know that ∠OCP and ∠OCQ are 90° from Property 1. In right ΔCOP, We have: $|OP|^2 = |CP|^2 + |CO|^2$ (1) $|OP| = \sqrt{|OP|^2 + |CO|^2}$ (2)

Based on ΔOPQ, we have the following:

$$\cos(A) = \cos(\angle POQ) = \frac{|OP|^2 + |OQ|^2 - |PQ|^2}{2|OP| \cdot |OQ|} \quad (3)$$

Combing Formula 1, 2 and 3 together, we have:

$$\cos(A) = \frac{|PC|^2 + |OC|^2 + |OQ|^2 - |PQ|^2}{2 \cdot |OQ| \cdot \sqrt{|PC|^2 + |OC|^2}} \quad (4)$$

Since |OC|, |OQ| and |PQ| are fixed, consider |PC| as the independence variable of the function. We can get the derivative function formula 5, in which *a* > 0:

$$f'(x) = \frac{|PC|^2 + |OC|^2 - |OQ|^2 + |PQ|^2}{a} \quad (5)$$

In right ΔCOP, We have: $|CQ|^2 = |OQ|^2 - |OC|^2$ (6)

Combing Formula 5 and 6, we have:

$$f'(x) = \frac{|PC|^2 - |CQ|^2 + |PQ|^2}{a} \quad (7)$$

Based on the derivative function in Formula 7 and the equation σ^2 = |PC| / n, we can see that if |PQ| ≥ |CQ|, cos A monotonously decreases as σ^2 decreases. If |PQ|<|CQ|, cos A decreases when $(|CQ| - |PQ|)^2/n \le \sigma^2 \le (|CQ|^2 - |PQ|^2)/n$ and increases when $(|CQ|^2 - |PQ|^2)/n \le \sigma^2 \le (|CQ| + |PQ|)^2/n$.

Since the only two local maximum values of cos A appears at where σ^2 equals $(|CQ| - |PQ|)^2$ / n or $(|CQ| + |PQ|)^2$ / n. It is clear from Figure 2.1 that cos A at $(|CQ| + |PQ|)^2$ / n will be greater than cos A at $(|CQ| - |PQ|)^2$/ n. Thus, the cosine angle distance cos A will be the maximum when σ^2 is $(|CQ|+|PQ|)^2$/n. □

Theorem 2 Proof. Since variance of the components of P σ^2 is fixed. Using equation σ^2 = |PC| / n , we know |PC| is also fixed. So |PC|, |OC| and |OQ| are all fixed in formula 4. We can treat |PQ| as the independence variable and get the following derivative function (a>0):

$$f'(x) = \frac{|CQ|^2 \cdot \cos^2(\beta) - |OQ|^2}{a} \quad (8)$$

f'(x) will always be less than zero. It means that when Euclidean distance |PQ| increases, the cosine angle distance cos A will decrease. □

6. REFERENCES

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