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A COMPARATIVE OVERVIEW OF MODAL TESTING AND SYSTEM IDENTIFICATION FOR CONTROL OF STRUCTURES

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Abstract. This paper presents a comparative overview of the disciplines of modal testing used in structural engineering and system identification used in control theory. A list of representative references from both areas is given, and the basic methods are described briefly. Recent progress on the interaction of modal testing and control disciplines is discussed. It is concluded that combined efforts of researchers in both disciplines are required for unification of modal testing and system identification methods for control of flexible structures.

Modal testing in the field of structures means the process of measuring signals produced by a structure and identifying modal parameters (damping, frequencies, mode shapes and modal participation factors). System identification in the field of controls means the process of measuring signals produced by a system and building a model to represent the system for control design. Techniques to identify a model from measured data typically contain two steps. First, a family of candidate models is chosen and then the particular member in this family is determined which satisfactorily describes the observed data based on some error criterion. If the identified model is a linear model in state space representation, the eigensolution of the model provides eigenvalues and eigenvectors that, in turn, determine modal parameters for structures. Correlation between the fields of modal testing and system identification for controls is evident.

The area of modal testing is a well-developed discipline with strong experimental foundations [1-76]. The area of system identification for controls is well-developed with solid theoretical and methodological foundations [77-170]. While the development of each individual area continues, there is a need to provide a comprehensive yet coherent unification of the areas. Active control of flexible structures will require the combined efforts of researchers in both disciplines. Among these challenges is control of large space antennas and platforms. The areas of modal testing and system identification encompass a multitude of approaches, perspectives and techniques whose interrelationships and relative merit are difficult to sort out. As a result, it is difficult for a nonspecialist to extract the fundamental concepts. It may take considerable effort to gain enough intuition about a particular technique to be able to use it effectively in practice.

The objective of this paper is to present an overview of the parallel historical development of modal testing used in structural engineering and system identification used in control theory. A list of principal references is provided for studying the similarities and differences among the many approaches in both areas.

MODAL TESTING

This section contains a synopsis of the field of modal testing. The following three items are provided: (1) a concise, yet complete, chronology of key developments that have occurred over the 40-year history of modal testing, (2) a brief summary of currently used approaches, and (3) a chronological reference list of key publications.

The subject of modal testing has evolved continuously since the 1940's, and an extensive literature has been generated (for example, note the bibliographies on pp 1659-1734 in Proceedings of the 4th International Modal Analysis Conference, February 1986). It is beyond the scope of this paper to discuss all this activity. Interested readers are referred to these references, or to several other more-complete overviews which have been written recently [51,53,64,74,75], for additional information.

Chronology of Key Developments. The chronology is divided into three separate eras: pre-1970, 1970-1979 and 1980-present. The start of the

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second and third eras accompanied significant improvements in computer technology. The second era began with the widespread introduction of minicomputers (e.g., PDP-11) and the third with the availability of much larger and faster computers for laboratory data analysis (e.g., VAX or mainframes).

In the first era of modal testing, from approximately 1940 through 1969, analog techniques were used almost exclusively. Laboratory computers were not yet available, and mainframe computers were used only rarely for data analysis. Much of the earliest work occurred in the aircraft industry, where testing was conducted to check the accuracy of calculated normal modes used in flutter and dynamic loads predictions. The two most significant contributions from this period were the works of Kennedy and Pancu [1], who introduced "circle fitting' for decomposing frequency response functions (FRF's) into the constituent modes, and Lewis and Wrisley [2], who described a systematic approach for tuning individual modes using multiple shakers and apportioned sinusoidal excitation. These two techniques, with numerous variations, were used in the majority of modal tests conducted prior to 1970. Testing was very time consuming, however, and required considerable practice and skill for success. All laboratory equipment was analog, and most data analysis was performed by hand.

The second era occurred during the 1970's, sparked by the introduction of laboratory minicomputer systems [15,19] and the fast Fourier transform algorithm to compute frequency response functions [18,30]. Compared with the classical method of slowly sweeping a sinusoidal signal to generate FRF's, these systems offered tremendous speed advantages. They were widely adopted by the modal testing community, with the exception of those organizations that had already made large investments in multiple-shaker sinusoidal testing equipment. The minicomputer in these laboratory systems was used not only to compute FRF's, but also for curve fitting the FRF data to estimate modal parameters. Many of the analysis techniques used during this period, however, were simply digital versions of techniques developed earlier (e.g., circle fitting or phase separation techniques). This situation began to change during the second half of the 1970's in conjunction with the availability of more powerful processors [31]. Also, the use of mainframe computers for data analysis was beginning to occur [34,35]. There was considerable hesitancy to use mainframes, however, because of the expense and because most analysis techniques in use at the time required considerable interaction with the user.

The third era of modal testing began around 1980, again in conjunction with improved computers and data acquisition equipment. One of the most significant changes occurred in the use of multiple-input random excitation [42], rather than the single-input random approach previously used most often. Multiple-input excitation provides several advantages: (1) it is consistent with multiple-reference (multiple-input) modal identification algorithms [49,55,60,61,67] which allow closely spaced modes to be better identified, (2) it minimizes troublesome shifts in frequency and mode shapes which can occur when exciters are moved to different points on a structure, and (3) it is significantly faster whenever several different locations and directions of excitation are needed to excite all the modes of interest, which is usually the case. In conjunction with multiple-input methods, improved methods of excitation [54,56], signal processing [65], frequency response calculation [59], and quick data analysis [66,69] have also been introduced. Renewed attention is being given to ARMA-type analysis techniques [72] which were studied years earlier [11,13,35] and generally thought to be too computationally extensive for the volume of data obtained in modal tests (often >20 modes and >100 measurements). Faster and more powerful computers now make these approaches more practical. Faster computers also permit nonlinearities to be better detected and identified, using new signal processing techniques [57]. Nonlinearities are also being better quantified using modern stepped-sine excitation techniques, in conjunction with new modal identification algorithms which can use unequally spaced frequency data [70]. Renewed interest is also occurring in the classical forced normal mode approach to modal testing. However, today the excitation tuning process is being computerized more than ever [76].

Current Methodology. As discussed above, modal testing methodology has changed considerably over the years -- and it continues to change. There are now many different ways to conduct testing and data analysis. Essentially all techniques work well with simple structures, yet significant differences occur when used on complex, built-up structures. It is still often difficult to deduce the exact source of these differences, however, because the "true" answers are unknown with experimental data. Better methods for comparing various identification techniques with complex data are needed [71], as are methods for conducting more realistic and thorough simulations of complex structural behavior to generate data for these studies. To a large degree, the techniques used most often today are those which have demonstrated repeatedly their tolerance of the complexities of real data.

Figure 1 provides an estimate of the general types of excitation and analysis now being used in the modal testing community in the United States (the percentage of multi-input tuned sine dwell testing is much higher in Europe). Multiple-input random excitation is now very popular. It is estimated that 45% of all laboratory modal tests are being conducted this way. Still very popular is the simpler, traditional single-input random approach (30%). The percentage of people using multi-input tuned sine dwell (forced normal mode) testing has been falling since around 1975 and is estimated to be about 10%. Of these, approximately one-half are now using computers to generate optimum tuning patterns. A small percentage of tests (5%) are also conducted using natural ambient excitation forces (e.g., wind on buildings, waves on offshore oil platforms), usually because artificial excitation is impractical. In many cases, the identification results from these tests are much more ambiguous than those from controlled-excitation tests.

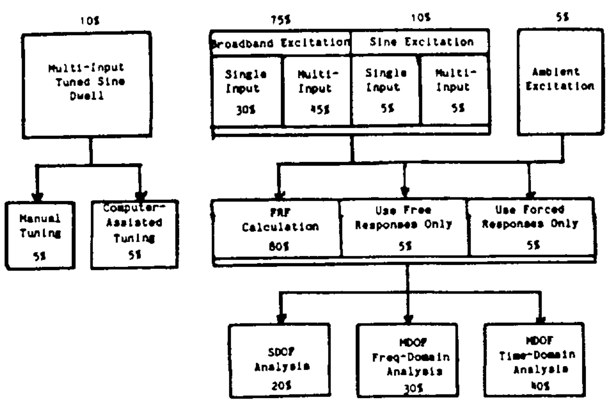

Figure 1. Current U.S. Modal Test Methodology Preferences (Estimated).

In the vast majority of testing (80%), frequency response functions are computed prior to using a modal identification algorithm. There are many reasons why FRF's are still generated so often, although there are now many identification algorithms available which can analyze free- or forced-response time histories directly. A significant factor is tradition: experienced modal testing personnel can deduce considerable information simply by observing frequency response functions. If time histories are processed directly by the modal identification algorithm, many traditional evaluation criteria are unavailable. The intermediate step of calculating FRF's will probably be skipped more often in the future as more experience and confidence is gained with the newer, direct analysis approaches.

For modal identification, multi-degree-of-freedom (MDOF) time-domain algorithms are being used the majority of the time (40%), followed by MDOF frequency-domain (30%), and faster single-degree-of-freedom (SDOF) algorithms (20%). In most cases, the data used by the MDOF time-domain algorithms are impulse response functions obtained by inverse Fourier transformation of FRF's. Only in a few isolated instances are free-response data being used directly. MDOF time-domain algorithms still appear to be unsurpassed in their ability to analyze wideband data with many modes, while their frequency-domain counterparts have been observed by some people to be more accurate over narrow frequency bands, or when damping is relatively high (e.g., greater than 5%).

SYSTEM IDENTIFICATION USED IN CONTROL THEORY

Since the mid-1960's the field of system identification has been an important discipline within the automatic control area. One reason is the requirement that mathematical models within a specified accuracy must be used to apply modern control methods. Another reason is the availability of digital computers that can perform complex computations. It is not the aim of this paper to present a detailed analysis of achievements in this continuously growing field or to give the state-of-the-art. See references [77-110] for more details.

Frequency and time domain methods give complementary views of many important problems in linear system theory and control theory. Sometimes, the two methods have been seen as rivals, particularly on issues of implementation and application to real systems. Historically, frequency domain methods dominated theory and practice of system identification in control engineering prior to the 1960's.

Frequency Domain Approach. Frequency domain identification in control engineering gained relevance with stability and design methods based on frequency response measurements. Frequency response estimation began with the technique known as transfer function analysis. The sinusoidal transfer function analyzer is recognized as a robust and practically useful non-parametric identification method. This is due to the intrinsic reliability with which the sinusoidal transfer function analyzer is able to reject low-frequency drift and harmonic distortion due to nonlinearity. However, the sinusoidal transfer function analyzer requires long test times to sequentially identify each relevant point on a frequency response curve. A more widely used class of techniques has been developed around digital spectral analysis and numerical Fourier transforms. The impact of the Fast Fourier Transform (FFT) developed by Cooley and Tukey [113] upon digital and analog spectral analysis was enormous. The FFT actually encompasses a whole family of algorithms, many of which are included in the Institute of Electrical and Electronics Engineers (IEEE) collected reprint written by Rabiner and Rader [117]. Modern digital spectral analysis, real time or off-line, is normally achieved by the direct method which takes the FFT of data blocks and then averages the resulting spectral estimates.

The direct estimation scheme can also be applied to determine estimates of frequency response functions by using a closed loop system design. The normal open loop system design may give biased results with physically unrealizable frequency response estimates. A closed loop system design may improve this situation, leading to gain and phase estimates which are well behaved, provided that careful attention is paid to windowing and aliasing. A more precise discussion of the accuracy of closed loop estimates is given in Davall [119], and Wellstead [122].

The maximum entropy method (MEM) of spectral analysis was originated by Burg [114] for analyzing geophysical data, and further developed by Ables [118] and Ulrych and Bishop [120]. The basic philosophy of the method is to construct spectral estimates that are consistent with all relevant data and are maximally non-committal with regard to unavailable data. The maximum entropy method generally gives superior spectral resolution to traditional methods at the expense of increased variability, and possibly erroneous splitting of spectral lines observed by Fougere, etc. [121].

Frequency domain identification, which, in the past, emphasized non-parametric identification, i.e., frequency response estimation, has lost popularity in recent years. This is due to the fact that current control synthesis and design tools require parametric system models such as a state space representation, stochastic difference equation or generalized regression model. If a spectral analysis or transfer function analysis experiment is conducted, least squares can be used to fit a parametric frequency response model by assembling a set of N-measured frequency response points and solving the unknown coefficients of the transfer function model [112,115,123]. However, these fitting methods may produce poor models which may not represent the underlying system well enough for controller design, particularly for complex structural dynamics problems. To a limited degree, these shortcomings are being investigated by current work on combined identification and control. Indeed, the pole-zero assignment technique developed by Wellstead, et al. [124] seems to offer qualities of robustness. The technique developed by Juang and Suzuki [168], which uses estimated frequency spectra to identify a state space model in modal space via system realization theory, also seems to offer a good model for control design.

Time Domain Approach. The time domain approach has dominated the control engineering literature on system identification over the past 20 years. In this section, a general description of commonly used linear system identification methods will be given. Time domain approaches are categorized according to the choice of model and the choice of identification criterion for evaluating the estimation quality. Basic methods are given in the references.

The origin of the least squares method can be traced to Gauss [125] who formulated the basic concept and used it practically for astronomical computation. Since then, it has been widely applied to many problems. The recursive algorithm to calculate the least squares estimate has apparently been found independently by several authors. The original reference seems to be Plackett [128]. An early and thorough treatment of the least squares method applied to dynamic system identification is given by Astrom [94]. The statistical background for stochastic approximation was developed by Robbins and Monro [129]. Stochastic approximation methods have also been derived by Sakrison [138]. Computational algorithms are based on stochastic gradient methods for linear regression models. In Ljung [161] the stochastic approximation approach is used to derive recursive identification algorithms for problems other than linear regression models.

The Kalman-Bucy filter [132] is a state estimator. It is Mayne [134] who draws attention to extending the Kalman-Bucy filter for parameter estimation of a state space model. The basic idea of the instrumental variable method (e.g., Kendal and Stuart [133] and Young [143]) is the generation of an extra signal, i.e., the instrumental variable, which is correlated with the useful signals of the process but which is uncorrelated with noise. This eliminates the bias error associated with least squares estimation. Recursive instrumental variable methods have been used extensively by Young [153].

The characteristics of noise corrupting the output of the system may not be well known. In the generalized least squares method, the parameter estimates may include estimates of noise parameters. Inspired by Clarke's algorithm [139] for generalized least squares analysis, a recursive method was suggested by Hasting-James and Sage [142]. The principle of extended least squares is that the calculation of the error between the true output and the estimated output is based on past estimates of system and noise parameters. The extended least squares algorithm was independently derived by Panuska [140] and Young [141] and widely used and rediscovered by Talmon and van den Boom [149]. In many practical problems of parameter estimation the problems arises of solving an overdetermined ill-conditioned set of algebraic equations. To circumvent this problem, the error covariance matrix can be propagated in a square root form so that the positive semi-definite nature of the error covariance is maintained to minimize the complexity of the statistical properties of the error estimates. A survey of square-root filtering techniques was given by Kaminski, et al. [146].

The basic idea of maximum-likelihood estimation is to construct a function of the data and unknown parameters called the likelihood function. The likelihood function is essentially the probability density of observations. The estimate of parameters is then obtained as the parameter set which maximizes this function. The method of maximum likelihood was developed by Fisher [126,127] although the basic idea dates back to Gauss [125]. In the Bayesian approach, the parameters themselves are treated as a random variable. Based on observations of other random variables that are correlated with the parameter, information about its value can be inferred. Therefore, the parameter estimate is expressed in terms of the probability distribution conditioned by past history.

Ho [135] showed that the instrumental variable method, generalized least squares and extended least squares are closely related to stochastic approximation and Kalman-Bucy filtering. Actually, square-root filtering belongs to the same family, as well, but the computational concept differs essentially. For linear systems and Gaussian noise, the maximum-likelihood approach yields the same conditions for the parameter calculation as the least-squares approach. Although the preceding techniques have been widely used in the field of controls, formal direct application to modal parameter identification for flexible structures has been minimal. Two techniques which has been extended and applied for modal identification of structures -- minimum realization and lattice filtering -- are discussed in the following section.

Techniques Related to Modal Testing. In the field of controls, the process of constructing a state space representation using experimental data is called system realization. A minimum realization is a model with the smallest state space dimension among models realized that have the same input-output relations within a specified degree of accuracy. Minimum realization theory was originally developed by Ho and Kalman [137], using Markov parameters (pulse response functions). Questions regarding minimum realization from various types of input-output data and generation of a minimum partial realization were studied by Tether [144], Silverman [145], and Rossen and Lapidus [147]. Rossen and Lapidus [148] successfully applied Ho-Kalman and Tether methods to chemical engineering systems.

A common weakness of the preceding schemes is that the effects of noise were not evaluated. Among follow-up developments along similar lines, Kung [156] presented another algorithm in conjunction with the singular value decomposition technique to treat the presence of the noise. Under the interaction of structure and control disciplines, the Eigensystem Realization Algorithm (ERA) [61] was developed by Juang and Pappa for modal parameter identification and model reduction for dynamic systems from test data. Based on a similar approach, a frequency-domain ERA and a recursive ERA also were developed [168,170]. Thorough treatment of the effects of noise on the ERA-identified modal parameters was presented by Juang and Pappa [167]. Correlation of several modal testing methods was derived by Juang [169] via system realization theory.

Ladder or lattice filtering first appeared in the reviewed literature by Morf, et al. [155] as a recursive method for solving the linear least squares problem. The term "ladder" or "lattice" originated from the shape of the data flow diagram of the technique. Interpreted in terms of the corresponding state space realization, the ladder implementation uses a state vector that has a diagonal covariance matrix. Recursive identification using the ladder representation has been extensively studied by Lee and Morf [159] and Friedlander [88]. Recently, the algorithms have been applied to identification of flexible structures by Sundararajan and Montgomery [162,164] and Wiberg [165].

CONCLUDING REMARKS

The field of modal testing has expanded continuously over its 40-year history. This growth is largely associated with corresponding improvements in computer capabilities. These increases in computer capability have permitted more accurate and complete testing and data analysis to occur. Algorithms and approaches thought too extensive in the past are now feasible. Several new uses for modal test data have also evolved. In particular, experimental modal data are now being used frequently to directly solve the problem at hand, rather than only for refining a finite-element model, the traditional use in the aerospace community. These direct uses of test data include predicting the effects of physical changes to the structure using an experimental modal model and developing hybrid analytical/experimental models of overall system dynamics.

For active control of space structures, the experimental modal data can also be used directly for control law design, once the final configuration of the system is tested using the control sensors and actuators. This task will require complete and accurate identification of the system while in orbit and is motivating further technology improvements to ensure success. Complex, built-up structures, in particular, still pose a significant challenge to the best ground-based methodology now available. Success with large space structures will demand the combined efforts of the control and structural dynamics disciplines. The solid theoretical and methodological foundations from the control field should be combined with the extensive experimental knowledge from the modal testing field. Additional work is needed to better understand and correlate current techniques from both fields. A principal goal is to find a common basis to explain and to select from the myriad of possible techniques.

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