Accepted Manuscript

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PII: S1359-8368(12)00738-X
DOI: http://dx.doi.org/10.1016/j.compositesb.2012.10.037
Reference: JCOMB 2166

To appear in: *Composites: Part B*

A comparative study of genetic algorithms for the multi-objective optimization of composite stringers under compression loads

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Abstract

Optimization methods are close to become a common task in the design process of many mechanical engineering fields, specially those related with the use of composite materials which offer the flexibility in the design of both the shape and the material properties and so, are very suitable to any optimization process. While nowadays there exist a large number of solution methods for optimization problems there is not much information about which method may be most reliable for a specific problem. Genetic Algorithms have been presented as a family of methods which can handle most of engineering problems. However, starting from a common basic set of rules many algorithms which differ slightly from each other have been implemented even in commercial software packages. This work presents a comparative study of three common Genetic Algorithms: Archive-based Micro Genetic Algorithm (AMGA), Neighborhood Cultivation Genetic Algorithm (NCGA) and Non-dominate Sorting Genetic Algorithm II (NSGA-II) considering three different strategies for the initial population. Their performance in terms of solution, computational time and number of generations was compared. The benchmark problem was the optimization of a T-shaped stringer commonly used in CFRP stiffened panels. The objectives of the optimization were to minimize the mass and to maximize the critical buckling load. The comparative study reveals that NSGA-II and AMGA seem the most suitable algorithms for this kind of problem.

Keywords:

A. Carbon fibre; C. Finite element analysis (FEA); C. Numerical analysis; Optimization; Genetic algorithm

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1. Introduction

The use of optimization methods in the design of structural components has been growing in the last years and becoming a usual step in the mechanical engineering workflow of many companies, specially those focused on aircraft/aerospace composite structures whose characteristics frequently meet the paradigm of a standard multiobjective optimization problem. For this reason, a large amount of optimization strategies ([1–5] among others) are available in the literature nowadays.

A structure of special interest which has been the object of optimization routines are composite panels stiffened with stringers. The optimization of the set panel-stringer is of high interest since this kind of structure is widely used in the aircraft industry. For them, Genetic Algorithms (GAs) [6], a family of evolutionary algorithms, have been succesfully used, as reported in a large number of publications [7–11] among others. A case of special interest reported in the scientific literature is the optimization of the stacking sequence of composite laminates, for which GA have been used successfully [12, 13]. However, in situations where the stacking sequence cannot be considered as a design variable but a imposed requirement, the minimization of the weight is achieved with geometrical parameters [14, 15]. In that case, what makes different the optimization of composite structures from other materials is the use of failure mode based failure criteria such as Puck's [16] and LaRC [17]. These are in fact a set of failure criteria which assign a different index for the different failure modes under consideration. When they are included in optimization routines as non-smooth discontinuous constraints, the resulting optimization problem is very specific of composite materials, as can be concluded from some works analysing the effect of different failure criteria in the optimal solution [18–20].

The original formulation of GAs is based on the concept of natural evolution: the survival of the fittest member, i.e., the better adapted members have more possibilities to transmit their characteristics to future generations. The translation of this strategy into an algorithm is performed by means of three operators:

- *Selection operator* which selects individuals with high fitness to form the mating pool.
- *Crossover operator* which permits the exchange of some characteristics between two or

more members of the mating pool. Two individuals, called parents, exchange some characteristics to generate two new members, called children.

- *Mutation operator* is implemented to save the process of losing genetic information during crossover. Random changes are applied in some individuals during the mutation process to preserve diversity in the population.

Although these three operators are the basis of a GA, there exist a large number of variations which implement different encodings, different selection operators, different methods for mating pairs or different strategies for mutation [21]. The behaviour of a specific GA depends on the studied problem [22, 23] and the design variables [24], for this reason, some previous experience or some comparative analysis is needed for selecting one GA out of a set of implemented GAs. Some comparative studies of evolutionary algorithms with different industrial cases have been already carried out, [25, 26] for example. These studies reveal that the best GA is different for each kind of problem.

A good choice when using GAs for the optimization of composite stiffened panels is a GA specifically designed for them, for example [27] and [28]. However, most of engineers are not familiar with the implementation of such algorithms and a commercial software with the most common GAs already implemented is a recommended option to carry out the optimization. In that case, a comparison of the most used GAs is a necessity for the choice as well.

The solution of the multi-objective optimization problem is linked to the concepts of dominance and non-dominance. When an individual is non-dominated it is a member of the Pareto's front, which is the set of possible optimal solutions. A candidate to solution **A** dominates candidate **B** if the conditions of Eq. 1 are fulfilled. On the other hand, if the Eq. 2 is satisfied **A** and **C** are considered non-dominated candidates.

$$f_i(\mathbf{A}) \prec f_i(\mathbf{B}) \leftrightarrow \Big(f_1(\mathbf{A}) < f_1(\mathbf{B})\Big) \wedge \Big(f_2(\mathbf{A}) < f_2(\mathbf{B})\Big) \tag{1}$$

$$f_i(\mathbf{A}) \sim f_i(\mathbf{C}) \leftrightarrow \Big((f_i(\mathbf{A}) \nleq f_i(\mathbf{C})\Big) \wedge \Big(f_i(\mathbf{A}) \ngeq f_i(\mathbf{C})\Big) \tag{2}$$

In this paper a comparative study of composite stringers under compression loads with three different GAs is carried out. The chosen three, implemented in software Isight™ [29], are: Archive-based Micro Genetic Algorithm (AMGA) [30], Neighborhood Cultivation Genetic Algorithm (NCGA) [31] and Non-dominate Sorting Genetic Algorithm II (NSGA-II) [32]. The main differences between these GAs are listed below:

- *NSGA-II*: After the creation of the parent population, sorting based on the non-dominance is used. A fitness (equal to non-domination level) is fixed in each solution. The best individuals of this ranking are used to create the new population using the selection, crossover and mutation operators.

- *AMGA*: This algorithm uses a small population size and creates an external archive with the best solutions obtained, which is updated every iteration. AMGA employs the concept of the non-dominance ranking of NSGA-II and it creates the parent population from the archive with the method of SPEA2 [33]. The mating pool is a derivation of the binary tournament selection method of NSGA-II. The use of the archive permits to obtain a large number of non-dominated points at the end of the simulation. AMGA is a GA highly based in NSGA-II.

- *NCGA*: A neighborhood crossover mechanism is added in the normal mechanisms of GAs which it improves the crossover operator. The pair of individuals to perform crossover is not randomly chosen, but the individuals who are close each other in the objective space are selected.

A T-shape stringer is used as a benchmark because of its simple geometry with only two design variables (subsection 2.1) and because of its real-life interest in the design of stiffened panels. A preliminary study of the stringer is performed (subsection 2.3) which permits to know the approximated optimal result. These structures are used for their compression behaviour with low weight. For this reason, the objectives are both the maximization of the critical buckling load (P_{cr}) and the minimization of the stringer mass (m). In these cases, P_{cr} normally is most important for these structures and their design is in function of it. Then, in the optimization process is prioritized the P_{cr} than the mass (details in section 3). Therefore, the previous optimal result is compared with the optimization results (section 4) to know the reliability of the GA. Finally, a GA is proposed to use in the solution of similar multi-objective optimization problems.

2. Benchmark problem

2.1. Specimen

In this study a composite material T-shape stringer has been analysed under compression load (Fig. 1). This geometry was selected since it provides both simplicity to run a benchmark and real life engineering interest.

[Figure 1 about here.]

The stringer is made from AS4/8552 pre-preg whose properties are described in Table 1. Stacking sequence is $[0/90/0_2/\pm 45]$ for the stringer base and $[\pm 45/0_2/90/0]_S$ for the stringer rib.

[Table 1 about here.]

2.2. Virtual test

To carry out the optimization, a virtual test was modelled, using ABAQUS$^{\mathrm{TM}}$ (Fig. 2). A compression load is applied on an end of the stringer and clamped by the other end. This compression load is applied by means of pottings, metallic elements where the stringer can be introduced and fixed with resin (Fig. 2). A potting only permits the displacement of the stringer base in X-axis and Y-axis in stringer rib. In the middle of the specimen a damaged zone was introduced to simulate the effects of an impact. This damaged zone is located in the stringer rib, in the middle of the specimen and it is modelled by reducing in a 50% the values of E_{xx} and X_{C}. The location of the damaged zone and the amount of properties reduction were obtained in a previous study [34]. It is added to simplify the finite element analysis (FEA) and to set the region where the first ply failure will appear. LaRC failure criteria is applied only in damaged zone to reduce computation time because it is known that the first ply failure will appear in the previously damaged zone. The elements used in mesh are S4 shell type (4-node shell element with full integration).

[Figure 2 about here.]

2.3. Preliminary study

A preliminary study aiming to determine the influence of design variables in the principal objective, P_{cr} and to obtain an approximated optimal solution was carried out. This results will be used to compare the performance of the analysed algorithms.

Individuals with different dimensions of the stringer base length (L_{B}) and the stringer rib length (L_{S}) were distributed in design space and FEA was run for each individual. A design was considered unfeasible if the specimen damage started.

P_{cr} was calculated with the expression:

$$P_{\text{cr}} = RF \cdot \lambda \tag{3}$$

where RF is reaction force supported by the stringer and λ is the first stringer eigenvalue.
Once all distributed cases were executed the influence of each design variable was analysed. As shown in Fig. 3 P_{cr} grows directly proportional to L_{B} until $L_{\text{B}} \simeq 29$ mm, when it starts to decrease. On the other hand, P_{cr} decreases inversely proportional to L_{S} (Fig. 4). This is because P_{cr} is dependent of λ, which is related to the vibration mode. At the same time, the vibration modes are dependent on the inertia. In our system of reference, the lowest inertia is I_{yy} and, for this reason, the specimen rotates respect to Y-axis. An increment of L_{B} generates an increment of I_{yy}, so the P_{cr} grows as well. When $L_{\text{B}} \simeq 29$ mm the vibration mode changes and λ decreases, and so does the P_{cr}.

[Figure 3 about here.]

[Figure 4 about here.]

When P_{cr} is plotted against L_{B} and L_{S} (Fig. 5) a peak is observed. This peak indicates the highest P_{cr}, that is the approximated optimal solution. This previous optimal solution has the values L_{B} approximately between 28 and 29 mm and L_{S} between 21 and 22 mm.

[Figure 5 about here.]

3. Multi-objective optimization

The two objectives of the optimization problem are to maximize P_{cr} and to minimize m, that is $f_1(\mathbf{x}) = -P_{\text{cr}}$ and $f_2(\mathbf{x}) = m$. The design variables are the length of the base (L_{B}) and the rib (L_{S}) of the stringer.

The optimization problem is defined as:

$$\begin{aligned} \text{Minimize} \quad & F_{\text{obj}}\left(f_1(\mathbf{x}), f_2(\mathbf{x})\right) \\ \text{Subject to} \quad & g(\mathbf{x}) > 0 \\ & 20 \leq \mathrm{x}_i \leq 30 \qquad i = 1, 2 \end{aligned} \tag{4}$$

where $\mathbf{x} = (L_{\text{B}}, L_{\text{S}})$, $g(\mathbf{x}) = 1 - FI(\mathbf{x})$ and $FI(\mathbf{x})$ is the LaRC failure index.

Subsequently, the objective function (F_{obj}) is described:

$$F_{\text{obj}} = \sum \left(\frac{f_i(\mathbf{x}) \cdot w_i}{s_i} \right) \tag{5}$$

where $f_i(\mathbf{x})$ are the different objectives, w_i and s_i the weight and scale factors for each objective, respectively. To give priority to P_{cr} the values of the weights w_1 and w_2 are set 0.7 and 0.3, respectively.

The commercial software Isight™, with several optimization methods implemented, was used to solve the multi-objective optimization problem of Eq. 4. This software implements Eq. 5 which is used as a post-processing to extract the optimal solution from the Pareto front delivered by the GAs. Isight™ permits to link ABAQUS™ with the chosen optimization method and to calculate the P_{cr} for each individual. ABAQUS™ analyses the different geometries (individuals) computed for the optimization method. RF, λ, m and FI of the individuals are calculated by ABAQUS™ and P_{cr} by Isight™. Each GA has the same scheme. The used computer is a HP Compaq dx2400 Microtower with an Intel® Core™ 2 Quad CPU Q8200 with 2.33GHz, 4GB of RAM, MS Windows XP Professional x64 Edition, Isight™ 5.5 and ABAQUS™ 6.9-3.

Once the optimization scheme was designed the different GAs were executed with different initiation modes. These modes set how the initial population is generated:

- *Distributed population* (DP): Equally spaced points in the design space are created.
- *Random* (R): A cloud of random cases is generated.
- *Initial solution* (IS): The starting initial population is a random cloud near to an initial geometry. For the analysed case it was set $L_{\mathrm{B}} = 24$ mm and $L_{\mathrm{S}} = 25$ mm.

The GA parameters are fixed to analyse each GA with the same conditions. The values of parameters are listed below:

- *Number of generations*: 25
- *Generation size*: 16 individuals
- *Selection rate*: 50%
- *Crossover probability*: 90%
- *Mutation probability*: 50%

These parameters generate 400 individuals for each GA and each initiation mode. AMGA is an exception, since it needs a different initial generation. For this reason, the value of initial population of AMGA is 40. This modification forces to change the number of generations to 24 to obtain the same approximated number of cases. On the other hand, Isight$^{\mathrm{TM}}$ does not permit the IS mode with NCGA. Because of the fact that the GAs have a random component, related to crossover and mutation operators, each GA and each initiation mode was executed five times.
The executions for each GA and initiation mode are performed in random order to reduce the effect that other processes running in the computer might have on the results of the computational experiment.

4. Results and Discussion

The comparison of the different algorithms is performed in terms of: obtained solution, computational time and number of generations to obtain the optimal. When an optimal individual does not improve after a specific generation, it is considered that this generation has reached the optimum.

The obtained results are listed in Table 2.

[Table 2 about here.]

All values L_{B} and L_{S} of the Table 2 are in agreement with the previous study, except four individuals. These four individuals, all in NCGA and DP mode (iterations 1, 2, 3 and 4), obtain a lower value of F_{obj} than the individuals of other GAs and initiation modes. A priori, this fact indicates that NCGA is the GA with the worst results, particularly with DP mode.

The mean, median and standard deviation were calculated for each GA and each variable (Table 3). This table shows that there are non-significant differences between the GAs for time variable, since the differences of mean are lower than 1%. Then, the mean of F_{obj} in NCGA is 2.44% and 2.26% lower than AMGA and NSGA-II respectively. Again, NCGA delivers different and lower results of the F_{obj}. However, AMGA and NSGA-II have a similar result with 0.18% of difference. NSGA-II achieves the best result of number of generations which is 9.91% lower than to AMGA, which occupies the second place. On the other hand, NCGA obtains a number of generations 2.83% lower than AMGA and 7.28% greater than NSGA-II.

[Table 3 about here.]

To determine what statistical test is the most accurate to handle all data, the data type needs to be identified. The Kolmogorov-Smirnov test is used to determine the normality of the data (each GA and each initiation mode independently). This test concluded that all the sets of data are non-normal populations. In this situation, a non-parametric test is recommended. Furthermore, as reported in [35], non-parametric tests are specially useful for the analysis of evolutionary algorithms, in this case GAs. The Mann-Whitney U-test (also known as Wilcoxon rank sum test) was used to compare the data. The null hypothesis of the Mann-Whitney test is that compared populations have identical distributions with equal median, against the alternative of different medians. This test has to be applied by facing the data two by two which leads to face each GA to the others. This process was repeated in each comparison variable. The results of the Mann-Whitney test are in Table 4, where $=$ is null hypothesis acceptance and $\neq$ is null hypothesis rejection.

[Table 4 about here.]

Results of the test reflect that the time values are equal for all GA. Furthermore, an equal distribution is observed for F_{obj} in AMGA and NSGA-II, while different results are detected in NCGA. The lowest value of F_{obj} in NCGA (shown in Table 3) indicates that AMGA and NSGA-II are a good option to obtain a high and similar value of F_{obj}. On the other hand, an unequal distribution is obtained for the value of number of generations in AMGA and NSGA-II. Moreover, NCGA is similar to AMGA and NSGA-II. The values of Table 3 reveal that the number of generations for NCGA are approximately equidistant between AMGA and NSGA-II. For this reason, NCGA is similar to AMGA and NSGA-II but these are different among them. NSGA-II needs less generations to obtain the optimal. However, a high standard deviation indicates that a random component exists. Additionally, the initiation mode was studied. The distribution of the studied cases in each GA and each initiation mode was analysed and the optimum evolution as well. The most representative cases are shown in Fig. 6.

[Figure 6 about here.]

Fig. 6(a) depicts the lines of distributed cases and the fact that the initial optimal solution is close to the final solution. This means that a DP mode enables the GA to achieve a faster optimal solution. On the other hand, a R mode has an expected random distribution (Fig. 6(b)). A possible remote initial optimal solution is the problem of a R mode, which may delay the arrival at the optimum. Finally, the first optimal solution is usually further from the final optimum in IS mode (Fig. 6(c)). This last initiation mode is recommended to improve a previous result.

5. Conclusion

A process to compare three GAs for the solution of multi-objective optimization problem of a simple composite material structure has been presented. A T-shape composite stringer under compression loads has been used as a benchmark for three different GA: AMGA, NCGA and NSGA-II. Moreover, a preliminary study of the specimen has been carried out to demonstrate that all the GAs reach the optimal solution.

An analysis of the results aids to recognize the first differences between the GAs. Therefore, a lower value of F_{obj} is observed in NCGA. A non-parametric test (Mann-Whitney U-test) has been used to compare the equality or inequality of the results. This test evidences that the computing time is independent on the GA used for the calculation because all the time values are similar. This conclusion might be affected by the use of a reduced number of design variables. On the other hand, both the AMGA and the NSGA-II achieve a high and similar value of F_{obj}. The lowest number of generations is obtained by NCGA and NSGA-II.

Finally, the different initiation mode (DP, R and IS) has been analysed to appreciate the differences among them.

In conclusion, the results of F_{obj} and the number of generations indicate that the most recommended GAs for similar structural cases are NSGA-II and AMGA, because they give similar results.

Acknowledgments

The authors wish to acknowledge the Ministerio de Ciencia e Innovación for the funding of the project DPI2009-08048 and particularly to Universitat de Girona for the research grant coded as BR2011/02.

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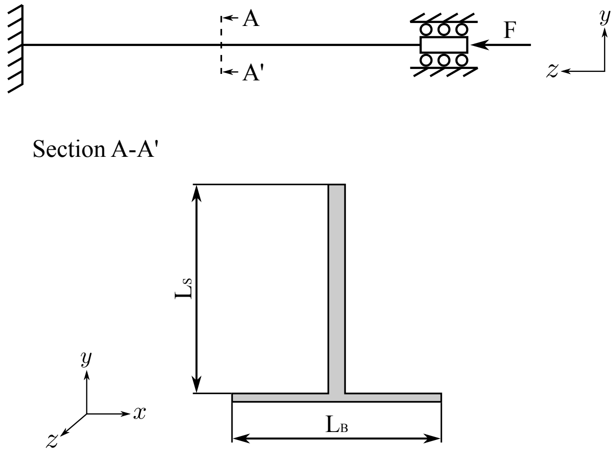


Figure 1: Stringer section and schematic representation of the test.

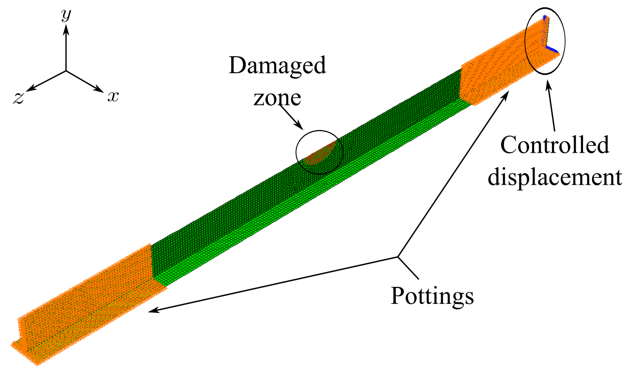


Figure 2: Benchmark problem.

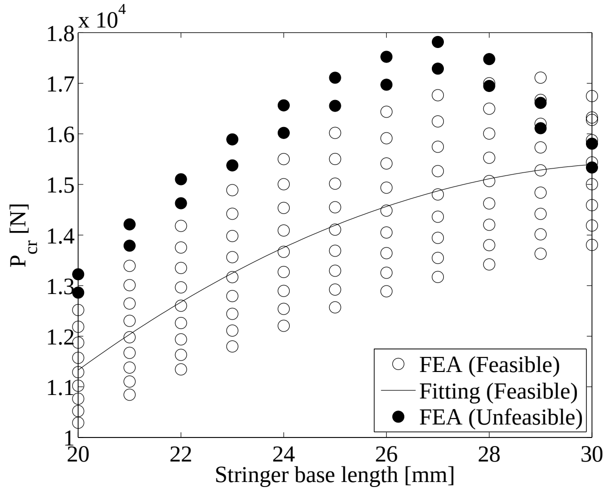


Figure 3: P_{cr} vs. L_{B}.

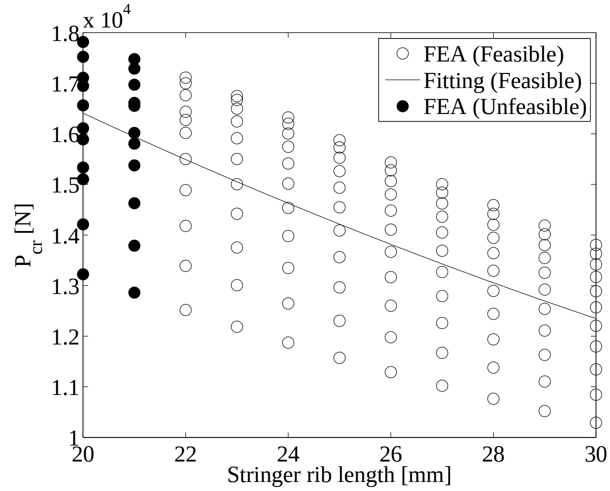


Figure 4: P_{cr} vs. L_{S}.

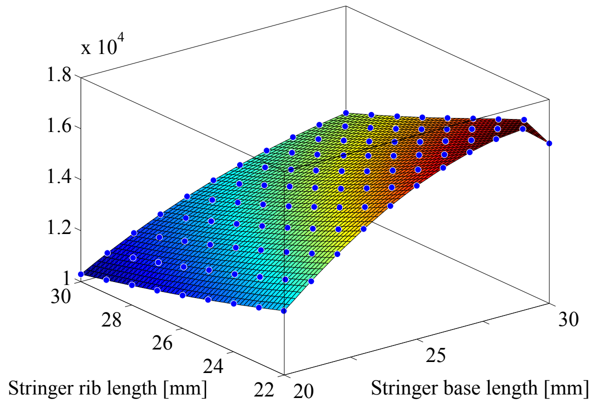
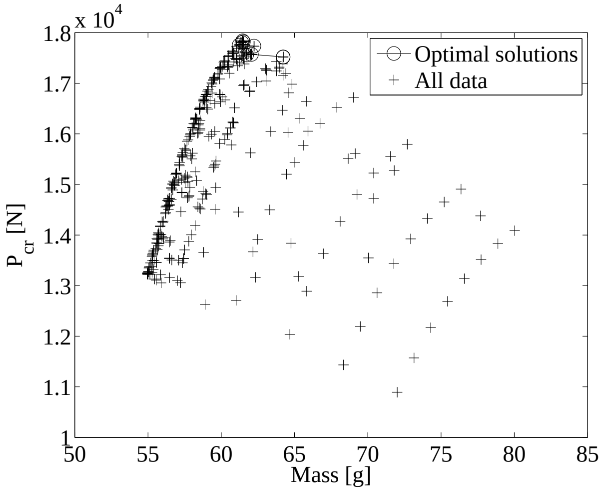
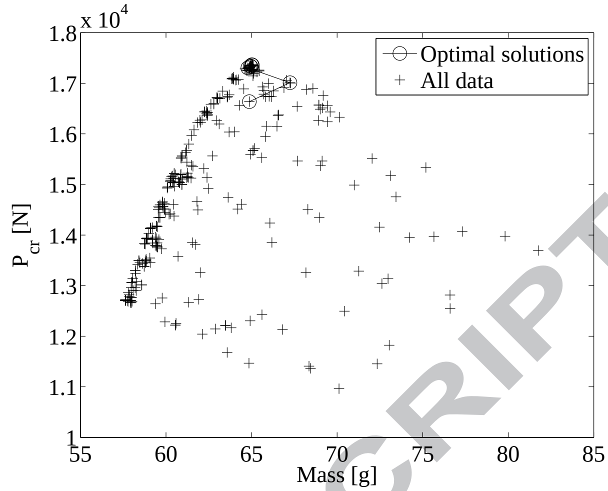


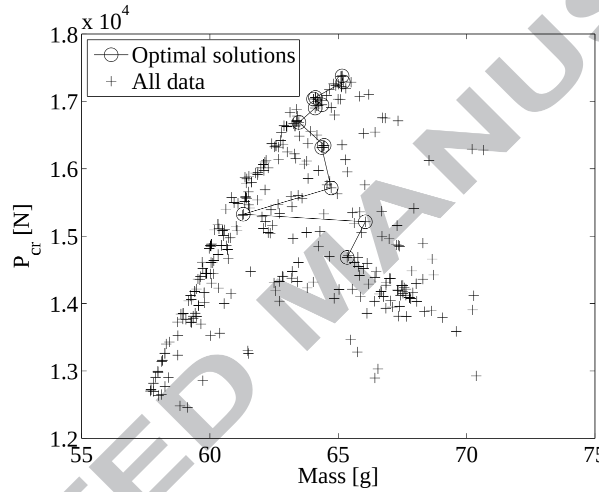
Figure 5: P_{cr} vs. L_{B} and L_{S}.



(a) AMGA in DP



(b) AMGA in R



(c) NSGA-II in IS

Figure 6: Evolution of the optimums.

List of Tables

Property	Value	Units	Description
E_{xx}	135	GPa	Young's modulus in fiber direction.
E_{yy}	9.6	GPa	Young's modulus in transversal fiber direction.
E_{zz}	9.6	GPa	Estimated $E_{\text{yy}} = E_{\text{zz}}$. (transversally isotropic material).
ν_{xy}	0.32	-	Poisson's modulus in XY plane.
ν_{xz}	0.32	-	Estimated $\nu_{\text{xy}} = \nu_{\text{xz}}$. (transversally isotropic material).
ν_{yz}	0.487	-	Poisson's modulus in YZ plane.
G_{xy}	5.3	GPa	Shear modulus in XY plane.
G_{xz}	5.3	GPa	Estimated $G_{\text{xy}} = G_{\text{xz}}$ (transversally isotropic material).
G_{yz}	3.228	GPa	Shear modulus in YZ plane.
X_{T}	2207	MPa	Longitudinal tensile strength.
X_{C}	1531	MPa	Longitudinal compressive strength.
Y_{T}	80.7	MPa	Transverse tensile strength.
Y_{C}	199.8	MPa	Transverse compressive strength.
$S_{\text{L}_{\text{UD}}}$	114.5	MPa	In-plane shear strength.
G_{IC}[1]	0.2839	N/mm^2	Fracture energy toughness in mode I.
G_{IIC}[2]	1.0985	N/mm^2	Fracture energy toughness in mode II.
ρ	$1.59 \cdot 10^{-9}$	T/mm^2	Density.

[1] Source: [37]
[2] Source: [38]

Table 1: AS4/8552 properties. Source: [36], unless otherwise stated.

GA	Initiation	Iteration	P_{cr} [kN]	m [g]	L_{B} [mm]	L_{S} [mm]	F_{obj}	Time [min]	Generations
AMGA	DP	1	17.386	65.24	28.49	21.37	67.36	580.8	14
		2	17.356	65.41	28.53	21.44	67.16	585.6	23
		3	17.340	65.54	28.58	21.49	67.04	597.5	23
		4	17.359	65.16	28.36	21.39	67.25	586.8	23
		5	17.335	65.25	28.36	21.45	67.09	590.7	22
	R	1	17.350	65.41	28.51	21.45	67.13	586.4	23
		2	17.394	65.19	28.47	21.35	67.41	575.6	23
		3	17.365	65.02	28.28	21.36	67.32	583.6	20
		4	17.385	65.26	28.51	21.38	67.35	584.2	20
		5	17.355	65.73	28.78	21.49	67.06	581.1	23
	IS	1	17.342	65.55	28.59	21.45	67.04	574.1	24
		2	17.362	65.31	28.48	21.42	67.22	582.9	22
		3	17.353	65.35	28.48	21.43	67.16	576.5	21
		4	17.386	65.28	28.52	21.38	67.35	584.0	19
		5	17.344	64.91	28.15	21.36	67.24	581.6	18
NCGA	DP	1	17.198	66.70	29.04	21.89	65.98	586.0	24
		2	16.948	68.08	29.34	22.50	64.31	596.3	22
		3	16.617	68.94	29.02	23.13	62.40	581.4	20
		4	16.877	67.55	28.75	22.50	64.12	577.9	20
		5	17.156	66.24	28.58	21.86	65.91	581.1	22
	R	1	17.103	65.01	27.66	21.66	66.02	568.7	16
		2	17.303	65.35	28.34	21.50	66.91	579.2	22
		3	17.313	65.38	28.39	21.49	66.95	581.8	22
		4	17.112	64.62	27.45	21.55	66.17	580.9	20
		5	17.303	65.14	28.20	21.46	66.97	577.2	18
NSGA-II	DP	1	17.364	65.23	28.42	21.40	67.25	578.1	23
		2	17.361	65.33	28.58	21.42	67.21	575.6	19
		3	17.322	65.09	28.22	21.42	67.08	587.9	21
		4	17.357	65.38	28.51	21.44	67.17	581.8	21
		5	17.319	64.98	28.13	21.41	67.09	588.9	20
	R	1	17.363	65.50	28.62	21.45	67.16	581.1	21
		2	17.375	65.05	28.32	21.35	67.36	577.7	16
		3	17.301	64.99	28.10	21.43	67.00	579.4	14
		4	17.372	65.41	28.58	21.42	67.24	580.7	17
		5	17.370	65.51	28.65	21.44	67.19	592.1	10
	IS	1	17.377	65.15	28.40	21.37	67.34	570.7	19
		2	17.394	65.44	28.58	21.44	67.19	580.1	21
		3	17.213	64.45	27.56	21.40	66.73	578.2	20
		4	17.151	64.19	27.29	21.40	66.49	563.7	22
		5	17.265	64.71	27.84	21.41	66.91	577.3	22

Table 2: Summary of obtained results.

Variable	GA	Mean	Median	Stand. dev.
F_{obj}	AMGA	67.21	67.22	0.13
	NCGA	65.57	66.00	1.50
	NSGA-II	67.09	67.17	0.23
Time	AMGA	583.4	583.6	5.9
	NCGA	581.0	581.0	6.9
	NSGA-II	579.6	579.4	6.9
Number of generations	AMGA	21.2	22	2.7
	NCGA	20.6	21	2.3
	NSGA-II	19.1	20	3.5

Table 3: Statistics of the results.

		AMGA	NCGA	NSGA-II
F_{obj}	AMGA		$\neq$	$=$
	NCGA	$\neq$		$\neq$
	NSGA-II	$=$	$\neq$	
Time	AMGA		$=$	$=$
	NCGA	$=$		$=$
	NSGA-II	$=$	$=$	
Number of generations	AMGA		$=$	$\neq$
	NCGA	$=$		$=$
	NSGA-II	$\neq$	$=$	

Table 4: Mann-Whitney test results.