A comparative study of initial basic feasible solution methods for transportation problems

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Abstract

In this research three methods have been used to find an initial basic feasible solution for the balanced transportation model. We have used a new method of Minimum Transportation Cost Method (MTCM) to find the initial basic feasible solution for the solved problem by Hakim [2]. Hakim used Proposed Approximation Method (PAM) to find initial basic feasible solution for balanced transportation model and then compared the results with Vogel's Approximation Method (VAM) [2]. The results of both methods were noted to be the same but here we have taken the same transportation model and used MTCM to find its initial basic feasible solution and compared the result with PAM and VAM. It is noted that the MTCM process provides not only the minimum transportation cost but also an optimal solution.

Keywords: Transportation problem, Vogel's Approximation Method (VAM), Maximum Penalty of largest numbers of each Row

1. Introduction

The transportation problem is a special linear programming problem which arises in many practical applications in other areas of operation, including, among others, inventory control, employment scheduling, and personnel assignment [1]. In this problem we determine optimal shipping patterns between origins or sources and destinations. The transportation problem deals with the distribution of goods from the various points of supply, such as factories, often known as **sources**, to a number of points of demand, such as warehouses, often known as **destinations**. Each source is able to supply a fixed number of units of the product, usually called the **capacity** or **availability** and each destination has a fixed demand, usually called the **requirements**. The objective is to schedule shipments from sources to destinations so that the total transportation cost is a minimum.

There are various types of transportation models and the simplest of them was first presented by Hitchcock (1941). It was further developed by developed by Koopmans (1949) and Dantzig (1951). Several extensions of transportation model and methods have been subsequently developed. In general, the Vogel's approximation method yields the best starting solution and the north-west corner method yields the worst. However, the latter is easier, quick and involves the least computations to get the initial solution [5]. Goyal (1984) improved VAM for the unbalanced transportation problem, while Ramakrishnan (1988) discussed some improvement to Goyal's Modified Vogel's Approximation method for unbalanced transportation problem [6].

Adlakha and Kowalski (2009) suggested a systematic analysis for allocating loads to obtain an alternate optimal solution [7]. However, the study on alternate optimal solutions is clearly limited in the literature of transportation with the exception of Sudhakar VJ, Arunnsankar N, Karpagam T (2012) who suggested a new approach for finding an optimal solution for transportation problems [8].

2. Transportation problem and General Computational Procedures

The transportation model of LP can be modeled as follows:

$$Minimize \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, x_{ij} \quad (Total\ transportation\ cost)$$

$$Subject\ to \qquad \sum_{j=1}^{n} x_{ij} = a_i \quad (Supply\ from\ sources)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad (Demand\ \ from\ destinations)$$

$$x_{ij} \geq 0, \quad for\ all\ i\ \ and\ \ j;$$

where Z : Total transportation cost to be minimized.

C_{ij} : Unit transportation cost of the commodity from each source i to destination j.

x_{ij} : Number of units of commodity sent from source i to destination j.

a_i : Level of supply at each source i.

b_j : Level of demand at each destination j.

Supply $\left(\sum_{i=1}^{m} a_i\right)$ = Demand $\left(\sum_{i=1}^{m} b_i\right)$.

NOTE: Transportation model is balanced if Supply $\left(\sum_{i=1}^{m} a_i\right)$ = Demand $\left(\sum_{i=1}^{m} b_i\right)$.

Otherwise unbalanced if $Supply \left(\sum_{i=1}^{m} a_i\right) \neq Demand \left(\sum_{i=1}^{m} b_i\right)$.

The total number of variables is *mn*. The total number of constraints is *m+n*, while the total number of allocations (*m+n–1*) should be in feasible solution. Here the letter m denotes the number of rows and n denotes the number of columns.

Solving Transportation Problems

The basic steps for solving transportation model are:

Step 1 - Determine a starting basic feasible solution. In this paper we use any one method NWCM, LCM, or VAM, to find initial basic feasible solution.
Step 2 - Optimality condition - If solution is optimal then stop the iterations otherwise go to step 3.
Step 3 - Improve the solution**.** We use either optimal method: MODI or Stepping Stone method.

Table 1: Transportation array

DESTINATIONS

		D1		D2		…….. ………..	Dn		Supply a_i
Sources	S_1	C_{11}	x_{11}	C_{12}	x_{12}	……..	C_{1n}	x_{1n}	a_1
	S_2	C_{21}	x_{21}	C_{22}	x_{22}	……..	C_{2n}	x_{2n}	a_2
	. . . .	……..		……..		……..	……..		……..
	S_m	C_{m1}	x_{m1}	C_{m2}	x_{m2}	……..	C_{mn}	x_{mn}	a_m
Demand b_j		b_1		b_2		……..	b_n		Balanced model $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

2. **Methodology**

The following methods are always used to find initial basic feasible solution for the transportation problems and are available in almost all text books on Operations Research [5].

The Initial Basic Feasible Solutions Methods are:

(i) Column Minimum Method (CMM)
(ii) Row Minimum Method (RMM)
(iii) North West-Corner Method (NWCM)
(iv) Least Cost Method (LCM)
(v) Vogel's Approximation Method (VAM)

The Optimal Methods used are:

(i) Modified Distribution (MODI) Method or u-v Method
(ii) Vogel's Approximation Method (VAM)

3. Initial Basic Feasible Solution Methods and Optimal Methods

There are several initial basic feasible solution methods and optimal methods for solving transportation problems satisfying supplying and demand.

Initial Basic Feasible Solution Methods

We have used following three methods to find initial basic feasible solution of the balanced transportation problem:

- Vogel's Approximation Method (VAM)
- Proposed Approximation method (PAM)
- Minimum Transportation Cost Method (MTCM)

For optimal methods we have used the Modified Distribution (MODI) Method and the Stepping Stone Method

Vogel's Approximation Method (VAM)

This method provides a better starting solution than the North West Corner rule and Least Cost Method. VAM generally yields an optimum or close to optimum solution.

Algorithm

Step 1. Compute penalty of each row and a column. The penalty will be equal to the difference between the two smallest shipping costs in the row or column.

Step 2. Identify the row or column with the largest penalty and assign highest possible value to the variable having smallest shipping cost in that row or column.

Step 3. Cross out the satisfied row or column.

Step 4. Compute new penalties with same procedure until one row or column is left out.

Note: Penalty means the difference between two smallest numbers in a row or a column.

Proposed Approximation Method (PAM)

This method provides a better starting solution than the North West Corner rule and Least Cost Method. The PAM generally yields optimum solution or close to optimum solution.

Algorithm

Step 1. Identify the boxes having maximum and minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.

Step 2. Identify the boxes having maximum and minimum transportation cost in each column and write the difference (penalty) against the corresponding column.

Step 3. Identify the maximum penalty. If it is along the side of the table, make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.

Step 4. If the penalties corresponding to two or more rows or columns are equal, select the box where allocation is maximum.

Step 5. No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete the two or column is assigned zero supply (or demand).

Step 6. Calculate fresh penalty cost for the remaining sub-matrix as in step 1 and allocate following the procedure of previous step. Continue the process until all rows and columns are satisfied.

Step 7. Compute total transportation cost for the feasible cost for the feasible allocations using the original balanced transportation cost matrix.

Minimum Transportation Cost Method (MTCM)

This method provides a better starting solution than the North West Corner rule and Least Cost Method. The MTCM generally yields optimum solution or close to optimum solution.

Algorithm

Step 1. Make the table balanced. Compute penalty of each row. The penalty will be equal to the difference between the two largest shipping costs in the row.

Step 2. Identify the row or column with the maximum penalty and assign possible value to the variable having smallest shipping cost in that row. If two or more rows corresponding equal penalty then select the cell with minimum cost of that maximum penalty row.

Step 3. Cross out the satisfied row or column.

Step 4. Write the reduced table and compute new penalties with same procedure until one row or column is left out. Determine the total minimum cost of occupied cells satisfying m+n-1allocations.

Note: Penalty means the difference between two largest numbers in a row.

Optimal Method

Modified Distribution (MODI) Method

This method always gives the total minimum transportation cost to transport the goods from sources to the destinations.

Algorithm

1. If the problem is unbalanced, balance it. Setup the transportation tableau
2. Find a basic feasible solution.
3. Set $u_1 = 0$ and determine $u_i's$ and $v_j's$ such that $u_i + v_j = c_{ij}$ for all basic variables.
4. If the reduced cost $c_{ij} - u_i - v_j \geq 0$ for all non-basic variables (minimization problem), then the current BFS is optimal. Stop! Else, enter variable with most negative reduced cost and find leaving variable by looping.
5. Using the new BFS, repeat steps 3 and 4.

4. The Numerical Problem

We have used three methods to find an initial basic feasible solution for the balanced transportation problem [4]. The problem was developed by Hakim [2]. Consider the transportation problem presented in Table 2 - where there are 4 sources, 6 destinations; the cost is given in the cells, and the supply and demand given in bottom and right hand end row and column respectively in Table 2.

Table 2: Example problem

	Destinations						
Sources	**1**	**2**	**3**	**4**	**5**	**6**	**Supply**
1	**1**	**2**	**1**	**4**	**5**	**2**	**30**
2	**3**	**3**	**2**	**1**	**4**	**3**	**50**
3	**4**	**2**	**5**	**9**	**6**	**2**	**75**
4	**3**	**1**	**7**	**3**	**4**	**6**	**20**
Demand	**20**	**40**	**30**	**10**	**50**	**25**	**175**

Solution

Three methods have been used here to find initial basic feasible solution of the above problem and these are presented in turn.

Vogel's Approximation Method (VAM)

Using VAM the final solution is presented in the Table 3.

Table 3: Solution using VAM

Sources	1	2	3	4	5	6	Supply
1	20		10				30
2			20	10	20		50
3		20			30	25	75
4		20					20
Demand	20	40	30	10	50	25	175

Therefore the total transportation cost determined by the Vogel's Approximation Method is:

Minimize Z = (20) (1) + (10) (1) + (20) (2) + (10) (1) + (20) (4) +(20) (2) + (30) (6)
+ (25) (2) + (20) (1)

= 20 + 10 + 40 + 10 + 80 + 40 + 180 + 50 + 20
= 450

Solution Proposed Approximation Method (PAM)

The final solution completed using PAM is presented in the Table 4.

Table 4: Solution by PAM

Sources	1	2	3	4	5	6	Supply
1			30				30
2				10	40		50
3	20	20			10	25	75
4		20					20
Demand	20	40	30	10	50	25	175

The total transportation cost by Proposed Approximation Method can be given as:

Minimize Z = (30) (1) + (10) (1) + (40) (4) + (20) (4) + (20) (2) +(10) (6) + (25) (2)
+ (20) (1)

= 30 + 10 + 160 + 80 + 40 + 60 + 50 + 20
= 450

Minimum Transportation Cost Method (MTCM)

The total transportation cost by Minimum Transportation Cost Method is given in Table 5.

Table 5: Solution by MTCM

Sources	1	2	3	4	5	6	Supply
1	20		10				30
2			20		30		50
3		40			10	25	75
4				10	10		20
Demand	20	40	30	10	50	25	175

The total transportation Cost by MCTM is given as:

Minimize Z = (20) (1) + (10) (1) + (20) (2) + (30) (4) + (40) (2) +(10) (6) + (25) (2)
+ (10) (3) + (10) (4)

= 20 + 10 + 40 + 120 + 80 + 60 + 50 + 30 +40
= 450

Modified Distribution (MODI) Method
We have found total minimum transportation cost using MODI method by taking initial basic feasible solution obtained by Minimum Transportation Cost Method (MTCM). The final solution is shown in Table 6.

Table 6: Solution using MODI

Sources	1	2	3	4	5	6	Supply
1	20		10				30
2			20	10	20		50
3		40			10	25	75
4					20		20
Demand	20	40	30	10	50	25	175

The total transportation cost by Modified Distribution Method is given as:

Minimize Z = (20) (1) + (10) (1) + (20) (2) + (10) (1) + (20) (4) + (40) (2) + (10) (6)
+ (25) (2) + (20) (4)

= 20 + 10 + 40 + 10 + 80 + 80 + 60 + 50 +80
= 430

Table 5: A comparison of the methods - VAM, PAM and MTCM and MODI

Initial Basic Feasible methods	Value of the objective function	Minimum Value (cost)
VAM	450	Same Result
PAM	450	Same Result
MTCM	450	Same Result
MODI Method	430	Optimal or minimum cost

The cost of transportation shows that the:

(i) Minimum Transportation Cost Method (MTCM), Vogel's approximation method (VAM), and Proposed Approximation Method (PAM) provide the same result, not optimal but close to optimal;

(ii) In MTCM, we have used penalty of maximum numbers of each row yet not of each column;

(iii) In VAM and PAM, the penalty of smallest numbers of each row and column are applied;

(iv) In MTCM, the penalty of each row makes the problem simple, easy and takes a short time in calculation; and

(v) In VAM and PAM, the penalty of each row and column makes the problem lengthy and the calculation time is longer.

5. Conclusion

As transportation problem is a special linear programming problem having many practical applications in other areas of operations, including, among others, inventory control, employment scheduling, and personnel assignment as mentioned earlier. Here in our research work we have used three methods, The Minimum Transportation Cost Method (MTCM), Vogel's Approximation Method (VAM) and Proposed Approximation Method (PAM). These were used to find an initial basic feasible solution for the transportation balanced model. The results are noted to be the same. It is important to note that we have used only penalty of each row of maximum numbers that is a simpler option and thus takes much less time in the calculation. In contrast, other methods using maximum penalty of smallest numbers of each row and column makes the problem lengthy and the calculation takes longer. Moreover, the method presented here is simpler in comparison of other presented methods earlier and can be easily applied to find the initial basic feasible solution for the balanced and unbalanced transportation problems.

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