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A comparative study of the influence of curvature on the boundary layers and on the inner region of wall jets

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In this paper, the differences in the curvature effects on turbulent boundary layers and on the inner region of turbulent wall jets are discussed. Both convex and concave curvature effects are considered. The comparison shows that the magnitudes of the various turbulent components are altered very differently for the two class of turbulent shear flows. In the boundary layers, it is mainly the centrifugal instability through which the curvature effects manifest themselves, whereas for wall jets there is the additional influence of the turbulence levels in the outer region which caps the inner region. Though, both the boundary layer and the inner region of a wall jet have very similar mean velocity distribution, the turbulence characteristics and their response to curvature effects are seen to be much different.

The influence of curvature (convex or concave) has to be considered in many practical situations like flow over wing surfaces, turbine blades, curved ducts and channels. The flow concerned could be either a boundary layer flow or a wall jet flow. In the case of the former the velocities increase from a zero value to the free stream velocity, whereas in the latter case, the velocity increases from a zero value to a maximum in the inner region[1,2]. Beyond this, the velocities decrease in the outer region. Thus the boundary layer-like inner region profile is capped by a large jet-like outer region where the turbulence levels are quite high. Though the shape of the velocity profiles in the boundary layer and the inner region of a wall jet are similar (as shown in Fig. 1), their properties are not, due to the conditions mentioned above.

Boundary layer flows on curved surfaces (both convex and concave) and two-dimensional wall jets on curved surfaces have been studied by quite a few investigators[2,3]. It would be interesting to see the comparative influence of curvature on a turbulent boundary layer and on the inner region of a turbulent wall jet, which are a set of typical shear flows with different boundary conditions. Further, it would be interesting to look into this aspect for the case of a three-dimensional wall jet also. It is observed that for three-dimensional wall jets on flat surfaces the turbulence levels are in general higher than those for two-dimensional wall jets on flat surfaces. It would be interesting to see the influence of curvature on the turbulence characteristics on the three-dimensional wall jets and compare the same with the corresponding cases for boundary layers[1] and two-dimensional wall jets[2,3]. With this in view, results have been obtained for a three-dimensional wall jet on a convex surface. The main objective of this paper is to focus attention on the various components of the turbulent Reynolds stresses for these two classes of shear flows, i.e., boundary layers and the inner region of wall jets.

In what follows, the experimental set-up, the measurement technique adopted for obtaining the turbulent quantities for a three-dimensional wall jet on a plane and on a convex surface are de-

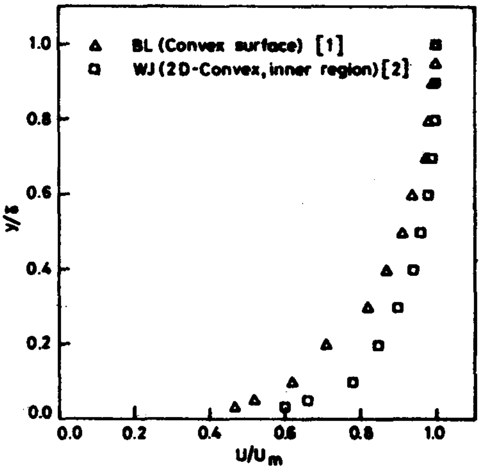

Fig. 1—Normalized mean velocity profiles

scribed. Then, an integral picture of the earlier results and the present results are given to bring out the curvature effects.

Experimental Procedure

The turbulent intensity measurements for a three-dimensional wall jet, both on plane surface and on convex surface have been performed using the jet tunnel facility of the Fluid Mechanics Laboratory, Indian Institute of Technology, Madras. Fig. 2a shows the schematic diagram of the jet tunnel and the position of the plane wall. At the end of the settling chamber, an orifice plate conforming to Indian Standard[4] (IS 2952, part I) made of mild steel of 5 mm thickness with a 10 mm diameter orifice is fixed. A smooth polished flat plate of size 1.45 m × 2.05 m and 18 mm thick, made of teak wood was used to generate the plane wall jet. The plate was fixed vertically and abuting with the orifice plate. A traversing mechanism (make: Aerodynamische Versuchsanstalt, Gottingen) was used for traversing the various probes. This has arrangement for movement in the three mutually perpendicular directions and rotation of the probe could be easily accomplished about a vertical axis and about the axis of the probe holder. Dial gauge of least count 0.01 mm with magnetic base has been used to make the measurements nearer to the wall, up to 1 mm. Beyond 1 mm, the scale on the displacement apparatus was used to locate the probe in various positions.

The curved plate employed (Fig. 2b) was prepared by mounting a smooth plywood sheet of size 1.22 m × 1.88 m and 3 mm in thickness on a wooden framework. The curved plate has an initial straight portion of 600 mm (i.e., 60 orifice diameters) and then a curved portion with a radius of curvature of 600 mm (Fig. 2b). The initial straight portion was provided so that a fully developed wall jet is obtained before it experiences curvature effects. It is known that a length of about 50 times the orifice diameter is required for similarity conditions to occur in a three-dimensional wall jet[5]. The curved plate also was arranged so that its front edge abutted to the orifice plate. A special fixture was designed and fabricated to enable the traverse of probes normal to the curved surface. The fixture was in turn attached to the traversing mechanism earlier mentioned.

The turbulent quantities both on the plane surface and on the convex surface have been measured using DANTEC hot wire anemometer (55 C) system with two channels having the necessary CTA bridges (56 C 17), linearizers (56 N 21), signal conditioners (56 N 20) and analog processor unit (56 N 23). The mean and rms components of the signal from the linearizer and the signal conditioner were measured by digital and rms voltmeters (56 N 22 and 56 N 25) respectively. A cross wire probe (55 P 61) operated in two planes (xy and xz) is made use of to obtain the various Reynolds normal and shear stresses. Here, x is in the direction of the flow, y is measured normal to

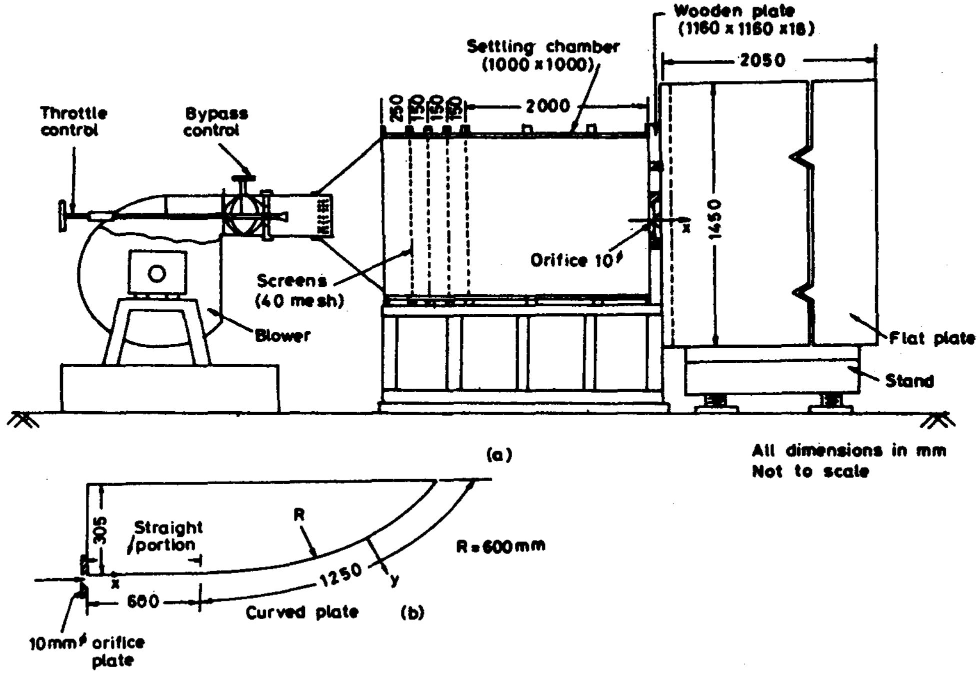

Fig. 2—Experimental set-up

the surface of the plate and z is the spanwise direction. u, v and w are the fluctuating turbulent components in the x, y and z directions respectively and $\tilde{u}$, $\tilde{v}$ and $\tilde{w}$ are the corresponding rms values.

Results and Discussion

Curvature effects on boundary layer flows—The curvature parameter for the boundary layer is defined as δ/R, the ratio of the boundary layer thickness to the radius of curvature. It is taken as positive for convex surface and negative for concave surface and the results are presented in Figs 3-6. The value of δ/R are $+0.0741$ and -0.0728 for the convex and concave surfaces respectively[1]. (In these figures U_m is the free stream velocity for the boundary layer and the maximum velocity for the wall jet). In general, the turbulent quantities $\tilde{u}$, $\tilde{v}$, $\tilde{w}$ and uv are reduced due to convex curvature compared to those on a plane surface[6], whereas, they are increased on a concave surface (Figs 3-6). But the influence is not to the same degree for the different components.

Considering $\tilde{u}$ (Fig. 3), for $y/\delta \leqslant 0.20$, the levels are nearly same for the plane and the concave surface (curves C1 and C3; hereafter the curves are indicated as C1, C2, etc.); over the remaining part of the boundary layer there is a large and constant difference between the two. For the convex surface (C2), the $\tilde{u}$ values are considerably reduced in the lower half of the boundary layer be-

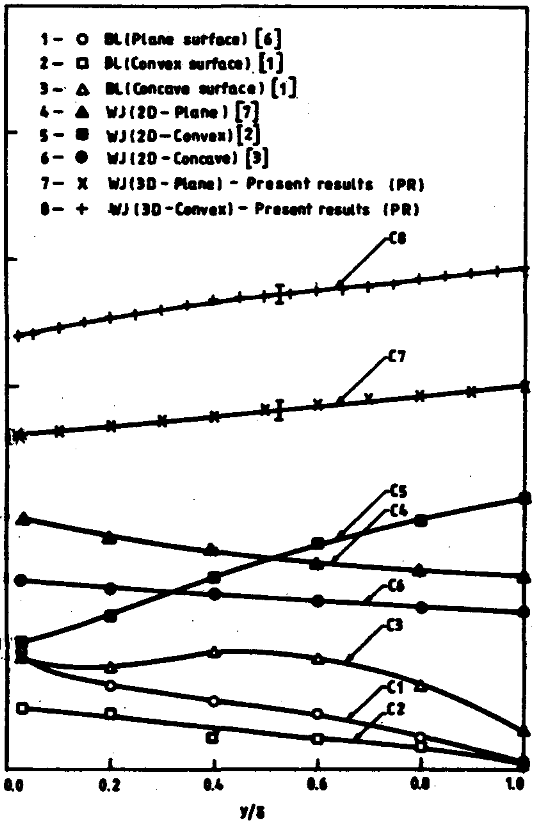

Fig. 3—Variation of $\tilde{u}$

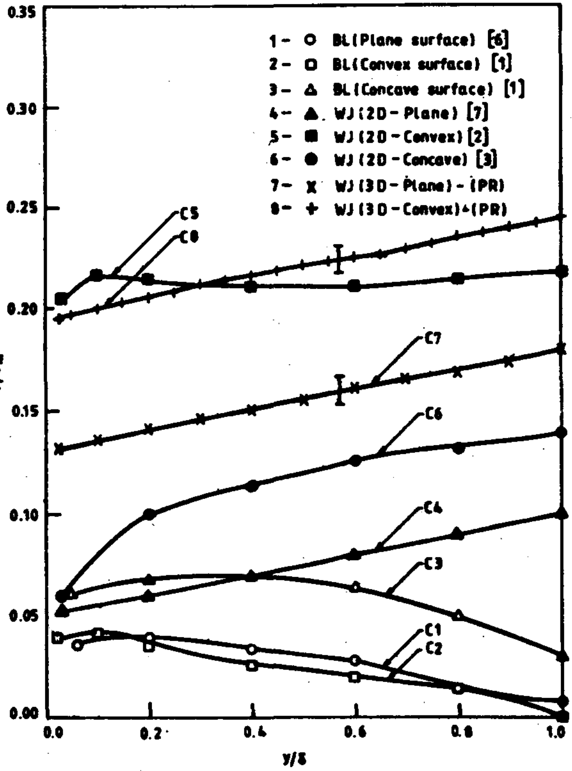

Fig. 4—Variation of $\tilde{v}$

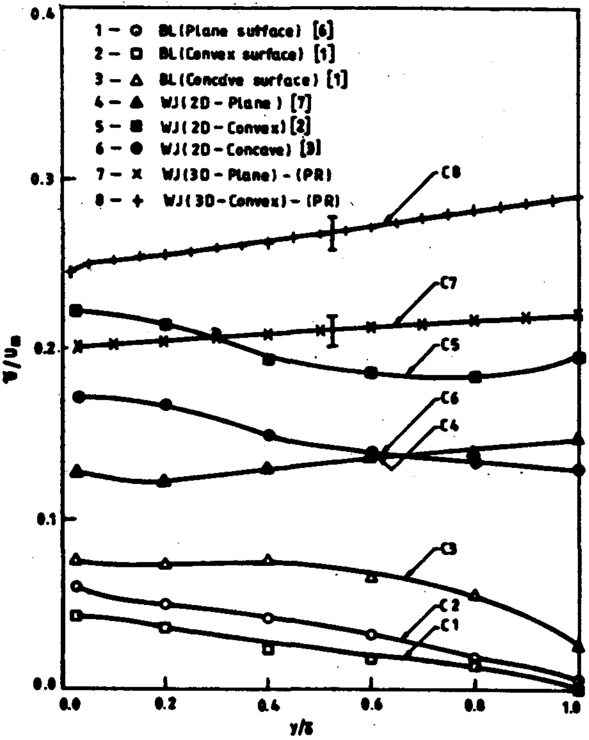

Fig. 5—Variation of $\tilde{w}$

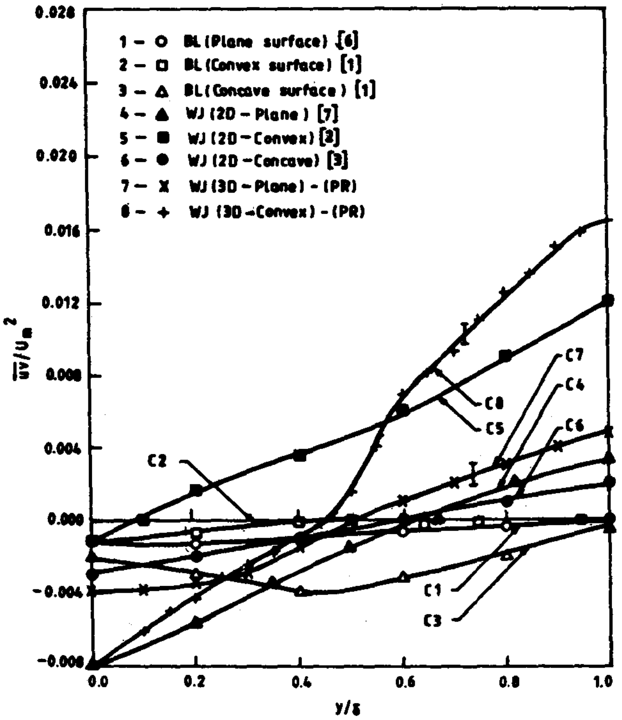

Fig. 6—Variation of $\overline{uv}$

yond which the differences decrease. In the outer 20%, there is only marginal difference between the two cases (Fig. 3; C1 & C2).

The values of $\tilde{v}$ are only marginally reduced due to convex curvature compared to that on a plane surface (Fig. 4; C1 & C2). But there is a large increase on the concave surface (C3), the difference remaining nearly the same over the entire height of the boundary layer.

In case of $\tilde{w}$ (Fig. 5), the effect of convex or concave curvature (C2 & C3) remains nearly constant across the entire boundary layer thickness but with the magnitude of the effect being larger for the concave compared to the convex.

The convex curvature reduces the turbulent shear stress ($\overline{uv}$) andl the value becomes zero at $y/\delta = 0.5$ and remains so in the remaining portion of the boundary layer (Fig. 6; C2). The influence of the concave curvature is comparatively stronger and the shear stresses are increased by three to four times than that for the plane surface (C1 & C3).

Curvature effects on the inner region of wall jets—The curvature parameter in this case (i.e., where only the inner region of the wall jets are considered) is defined here as δ/R where δ is the distance from the wall to the point where $U = U_m$, U_m being the maximum velocity in the wall jet. The results for two-dimensional wall jets on plane[7], convex[2] and concave[3] surfaces and three-dimensional wall jets on plane and convex surfaces are presented in Figs 3-6 along with those for boundary layers. This is done to facilitate the comparison between the two. The values of δ/R chosen (from the available earlier results) for two-dimensional convex[2] and concave[3] cases are $+0.06$ and -0.064 respectively which are nearly same as those for the boundary layers. The value of the curvature parameter for the three-dimensional case for which the results have been obtained is 0.07. This occurs at a value of $x/d = 160$.

Considering $\tilde{u}$ component for the two-dimensional case (Fig. 3), the levels increase monotonically on a convex surface (C5) across the inner layer. Starting from a value much lower than that for the plane surface, it reaches a sufficiently higher value than that for the plane case at $y/\delta = 1.0$ (C4 & C5). With the result, the levels are lower in the bottom half and higher in the outer half compared to the plane surface. Due to the concave curvature, the levels are reduced by about 25% uniformly across the inner layer (C4 & C6). Comparing the results for two-dimensional and three-dimensional wall jets, it is seen that even on a plane surface, the latter exhibits much higher values (C4 & C7). There is a further increase in the turbulence level due to the convex curvature effects (C8) on the three-dimensional wall jet. The increase is nearly 40% due to the latter effects (i.e., due to only curvature) and is almost constant across the entire width of the inner region (C7 & C8).

As for the $\tilde{v}$ component (Fig. 4), in the two-dimensional case, the levels are increased due to both convex and concave curvature (C5 & C6), but the increase due to the former is much larger than the latter. On the concave surface (C6), the magnitudes increase from a low value to a high value in the lower 20% of the inner region and remain nearly the same in the rest of the portion. This is much different from the variation observed for the $\tilde{u}$ component in Fig. 3. The $\tilde{v}$ component for three-dimensional wall jet on plane surface (C7) is nearly 50% more than that for the two-dimensional wall jet on plane surface (C4) similar to that observed for the $\tilde{u}$ component. There is a further increase of about 45% in the $\tilde{v}$ values for three-dimensional wall jet on convex surface (C8), due to curvature effects.

Curvature effects are seen to be slightly different in the case of $\tilde{w}$ component for two-dimensional wall jets (Fig. 5). On the convex surface (C5), the magnitudes increase compared to that on the plane surface (C4) similar to $\tilde{v}$ component. But on the concave surface (C6) the trend is opposite to that noticed for $\tilde{v}$ component, with the magnitudes decreasing from a large value at the wall towards the outer edge of the inner layer. As far as the three-dimensional wall jet is concerned, trends similar to those for $\tilde{u}$ and $\tilde{v}$ are observed in this case also. That is, the value of $\tilde{w}$ for a three-dimensional wall jet on plane surface (C7) is considerably higher than that for the two-dimensional wall jet on plane surface (C4) right across the width of the inner region. The levels increase further due to convex curvature effects (C7 & C8).

The most striking features are seen in the case of the shear stress component $\overline{uv}$ (Fig. 6). It is well established that the point of zero shear stress and the point of maximum velocity do not coincide both for two-dimensional and three-dimensional wall jets on a plane surface, unlike for the case of a boundary layer where the two points coincide[5] (C1, C4 & C7). For wall jets, the point of zero shear stress lies below the point of maximum velocity. The curvature effects are striking on this feature of the wall jets. For the two-dimensional case, due to the convex curvature (C5) the point of zero shear stress is pushed very close to the wall ($y/\delta \simeq 0.1$). Whereas , there is a lesser shift towards the wall due to concave curvature (C6). Further, the magnitudes are considerably different for both the cases compared to that for the plane surface (C4, C5 & C6). As far as the three-dimensional wall jet is concerned, the location of the point of zero shear stress for both plane and convex cases (C7 & C8) are nearly the same. However, the magnitudes are very much different. The curvature effects are seen to give rise to very large increase in the shear stress values, particularly towards the outer edge of the inner region.

Comparison between the curvature effects on boundary layers and on inner region of wall jets—Comparing first the case of boundary layers and two-dimensional wall jets, even on a plane surface, the turbulence levels in a boundary layer are lower than those in the inner region of a wall jet (C1 & C4). This is because the latter is capped by a large outer region where the turbulence levels are quite high, i.e., there is effectively a 'free stream' with large turbulence levels which interacts with the inner region, increasing the levels there also. Considering the influence of curvature, it is seen to be much different for boundary layers and the inner region of wall jets. For the former, the turbulence levels in general are reduced on convex surface and increased on concave surface compared to those on a plane surface (Figs 3-5; C1, C2 & C3). This is true for all the components of the normal stresses. But this is not so for the wall jets; the influence is more complex and the effects are also not uniform over the thickness particularly for the $\tilde{u}$ component on convex surface (Fig. 3). On the overall, the turbulent components on the convex surface (C5) are more than for those on the concave surface (C6) which is reverse of that observed for boundary layers (C2 & C3). Further, unlike for a boundary layer, the values of $\tilde{v}$ and $\tilde{w}$ for both concave and convex surfaces are larger than those on a plane surface (Figs 4 and 5; C4, C5 & C6).

When three-dimensional wall jet on plane surface (C7) is considered, it is seen that all the components of turbulent normal stresses, i.e., $\tilde{u}$, $\tilde{v}$ and $\tilde{w}$ increase nearly by an amount of 40% compared to those for two-dimensional wall jets on plane surfaces (C4). This increase is attributed to lateral stretching of the eddies for the former case[5,9]. The convex curvature effects, further increase the turbulence levels (Figs 3-5; C7 & C8). It is interesting to see that this increase also is nearly the same for all the normal components. The curvature effects are strikingly stronger for the three-dimensional wall jets compared to that for two-dimensional wall jets (Here, only the convex curvature effects for the three-dimensional wall jet are presented as the results for the concave case are not available).

As mentioned earlier, the curvature effects on the turbulent shear stress component are striking in the case of both boundary layers and wall jets. In addition, the differences in the comparative effects between the two cases are significant. The concave curvature reduces the shear stress value over the entire thickness of the boundary layer in a non-uniform way (Fig. 6; C3); whereas, for a wall jet (C6), the shear stresses are increased over the lower half and slightly decreased in the upper half. The influence of convex curvature on uv is much more marked for a wall jet than for a boundary layer (Fig. 6; C5 & C2) indicating a stronger and a more complex interaction between the curvature and turbulent quantities for the former. However, it is surprising to observe that even in the case of a boundary layer the point of zero shear stress and the point of maximum velocity occur at different locations due to the effect of convex curvature. It is usually assumed that such a phenomenon occurs only in the case of a wall jet. But this appears to be not so; it can occur in the case of boundary layers also.

When flow takes place on a curved surface, if the conditions are such that the momentum increases in the direction of the radius of curvature, stable conditions prevail; otherwise instability occurs[1]. Elaborating further, if we consider a small fluid element at a particular distance y in the boundary layer on a convex surface, it is in equilibrium under the action of the centrifugal force and the force due to the normal pressure gradient. Suppose this element is displaced to a position $(y+dy)$, where the velocity is higher; it will still remember its earlier velocity in its previous position. But it experiences a force due to the normal pressure gradient which is higher than the centrifugal force corresponding to the earlier velocity. There will be a net downward force and the particle will be pushed back to its original position. Hence, boundary layer on a convex surface is stable with the resulting forces opposing the displacement of fluid particles from one layer to another. It is the reverse on a concave surface. Therefore, the turbulence levels can be expected to be reduced for boundary layer on a convex surface and increased on a concave surface, which is the reason for the trends seen in the results in Figs 3-6 for the boundary layer case.

The conditions are not this simple in the case of a wall jet on a curved surface, where inner region is capped by a large outer region with high turbulence. Considering the wall jet on a convex surface, though stable conditions exist within the inner region (because momentum increases in the direction of radius of curvature), centrifugal instability exists over the large outer region where the momentum decreases in the direction of the radius of curvature, as is obvious from the shape of the wall jet profile[9]. Secondly, the turbulence levels are also quite high in this region due to the entrainment and the jet-like character of the outer region. A combination of these two effects in the outer region, override the stabilizing influence of the convex curvature within the inner region and hence the turbulence levels increase in the inner region unlike for a boundary layer. Considering the wall jet on a concave surface, though unstable conditions exist within the inner region, stable conditions prevail in the large outer region. In spite of entrainment which is present in this case also, the stable conditions in the outer region are strong enough to interact with the unstable inner region and keep the turbulence levels low as observed in Figs 3-6.

Conclusions

Boundary layers and wall jets form two interesting and practically important class of shear flows. Though the shape of the boundary layer profile and the inner region of a wall jet are similar, they have considerably different characteristics, particularly when turbulent quantities are considered. The influence of convex and concave curvature on these properties are striking and in general opposing. Whereas the centrifugal instability is responsible for the effects observed in a boundary layer, for the wall jet, both the centrifugal instability and the turbulence levels in the outer region seem to be responsible for the effects seen. It would be interesting to control these parameters individually and observe the effects for the latter.

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