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Abstract:

The *order fill rate* is less commonly used than the *volume fill rate* (most often just denoted *fill rate*) as a performance measure for inventory control systems. However, in settings where the focus is on filling customer orders rather than total quantities, the order fill rate should be the preferred measure. In this paper we consider a continuous review, base-stock policy, where all replenishment orders have the same constant lead time and all unfilled demands are backordered. We develop exact mathematical expressions for the two fill-rate measures when demand follows a compound renewal process. We also elaborate on when the order fill rate can be interpreted as the (extended) *ready rate*. Furthermore, for the case when customer orders are generated by a negative binomial distribution, we show that it is the size of the shape parameter of this distribution that determines the relative magnitude of the two fill rates. In particular, we show that when customer orders are generated by a geometric distribution, the order fill rate and the volume fill rate are equal (though not equivalent when considering sample paths). For the case when customer inter-arrival times follow an Erlang distribution, we show how to compute the two fill rates.

Keywords:

Backordering, continuous review, compound renewal process, inventory control, negative binomial distribution, service levels.

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1. Introduction

Service levels are used in inventory control systems for performance evaluation and in target setting as substitutes for shortage costs that are hard to estimate. A review of standard service level measures and their relationships to shortage costs and different inventory control policies is provided by Schneider (1981). One of the most commonly used performance measures in inventory control is the *volume fill rate* (*VFR*), defined as the fraction of total demand that can be satisfied from inventory without shortages (Silver et al., 1998, p. 245). It is most often just denoted fill rate; in Silver et al. (1998) it is also referred to as P_2. Somewhat less common as a performance measure is the *order fill rate* (*OFR*), specified as the fraction of complete orders that can be filled directly from inventory (sometimes also identified as the *line fill rate*).

The motivation for our interest in this service measure is that we have recently been involved in a logistical analysis for a major Danish company operating as a provider of spare parts to various industries. The company used a base-stock inventory policy and was considering how to set the base-stock levels in order to provide adequate customer service. Through our discussions with the logistics managers we realized that the service measure of main concern to the company was the *OFR*, as defined above. To determine the base-stock levels the company used a standard heuristic method, assuming that lead-time demand can be described by a normal distribution. However, demands are often low frequent and the individual customer orders are erratic to such a degree that this assumption is not very satisfactory. Furthermore, even if the normal distribution approximation did hold, it was also clear that the method in use was not consistent with the preferred *OFR* service measure.

In most inventory control models as well as in some case studies, e.g. Ward (1978), the Poisson process is used to characterize arrivals of customer orders. However, by examining historical demand data from the company we found that a Poisson process sometimes gave a very poor representation of the observed arrival patterns. This is also noted by Smith and Dekker (1997) in connection with spare parts' demand. Hence, what was needed was to develop a decision-support tool for the company that could offer flexibility with respect to modeling the inter-arrival times as well as the order-size distributions. To this end, we developed a computer program that computes the base-stock level required to achieve a pre-specified level of the *OFR* under the assumption that inter-arrival times of customer orders are Erlang distributed, and that customer order sizes follow a binomial, a Poisson or a negative binomial distribution. The particular distributions for a certain item are to be fitted from historical demand data. This computer program is now in the process of being integrated with the company's Business Warehouse system.

In this paper, we generalize from the case study and derive formulas for the *OFR* and the *VFR* under the assumption that demand follows a compound renewal process. We also investigate the relationship between the two fill rates when customer order sizes can be represented by a negative binomial distribution with shape parameter s and probability parameter ρ. It turns out that whether s is above, equal to or below 1 determines if the *VFR* is above, equal to or below the *OFR* for any given base-stock policy. Thus, when customer order sizes follow a geometric distribution (the case when $s = 1$), the two service measures are equal, although their actual sample paths are different, in general. This special case is of particular practical interest, since Johnston et al. (2003) recently found empirical evidence to support it.

The rest of the paper is organized as follows. In Section 2 we specify mathematical expressions for the two service measures considered, i.e. the *OFR* and the *VFR*, respectively. Next, in Section 3 we comment on when the *OFR* can be identified as the (extended) ready rate. We also question the relevance of the ready rate as a service measure, when customer order arrivals do not follow a Poisson process. Section 4 contains the derivation of our main result on the relative magnitude of the two fill rates, when customer order size follows a negative binomial distribution. In Section 5 we then show how to compute the two service measures when inter-arrival times are Erlang distributed, thus indicating how we have developed the decision support tool for the company referred to above. Section 6 contains numerical comparisons and sensitivity analyses of the two fill rates. In a simulation study we also highlight the case with geometrically distributed customer order size ($s = 1$). We demonstrate that, although the two service measures are equal in terms of their mean values, their actual sample paths may differ (even though they are strongly positively correlated). Finally, Section 7 contains some concluding remarks.

2. Specification of the two service measures

First, we specify the compound renewal process used to model the demand process. The customer order inter-arrival times are assumed to be independently and identically distributed as a positive and continuous random variable T. In addition, the customer order sizes are independently and identically distributed as the positive, integer-valued random variable J. For any instant τ at which a demand occurs, let the random variable D_t represent the aggregate demand in the time interval $[\tau - t, \tau)$. Note that D_t and the demand at time instant τ (represented by the random variable J) are independent.

Now, assume that we apply a continuous review, base-stock policy with base-stock level S, where each replenishment order has a fixed lead time L and all unfilled demands are backordered. The base-stock policy is assumed simply because of its general popularity in practice, and because of its use in the particular company that motivated this study. It should be recalled that even if the base-stock policy might be the optimal policy in the special case of a compound Poisson demand process, this is not the case in general for a compound renewal demand process.

Under these assumptions, when a customer order arrives, with probability $P(D_L = n)$ there will at the time of the arrival be a net inventory of $S - n$. With probability $P(J \le S - n)$ the whole customer order can then be delivered immediately. Hence, the order fill rate, *OFR* for a given base-stock level S is obtained as

$$OFR(S) = \sum_{n=0}^{S-1} P(D_L = n) P(J \le S - n) . \qquad (1)$$

Equivalently, this can be written as

$$OFR(S) = \sum_{n=1}^{S} P(D_L \le S-n)P(J=n) \,, \tag{2}$$

which is how it appears in Song (1998, Eq. 4), where it is denoted the *item* fill rate (in her terminology *order* fill rate is used to denote the fill rate of customer orders for a group of items.) The service measure *OFR* also appears in Rosling (2002, Eq. 5). For the case when the random variable T is a constant, we could interpret the random variable J as the demand per period. Then *OFR* is equal to the service measure α_{per} in Tempelmeier (2000, Eq. 1).

The volume fill rate, *VFR*, service measure can be derived by applying a similar reasoning. When a customer order arrives, there will at the time of the arrival with probability $P(D_L = n)$ be a net inventory of $S - n$. Then

$$\sum_{j=1}^{S-n} jP(J=j) + (S-n)P(J \ge S-n+1) = \sum_{j=1}^{S-n} (j-S+n)P(J=j) + S - n \tag{3}$$

is the expected amount of that particular order that is delivered immediately. Therefore, the *VFR* is

$$VFR(S) = \frac{\sum_{n=0}^{S-1} P(D_L = n)(\sum_{j=1}^{S-n}(j-S+n)P(J=j) + S - n)}{E[J]} \,. \tag{4}$$

When the random variable T is a constant and if we then again interpret the random variable J as the demand per period, the *VFR* is equal to the service measure β in Tempelmeier (2000; Eq. 3).

In concluding this section we note that for the special case when $P(J{=}1) = 1$, i.e., $J \equiv 1$, it (trivially) follows that

$$OFR(S) = VFR(S) = P(D_L \le S-1) \,. \tag{5}$$

3. Relationship to the ready rate service measure

The *ready rate* service measure is defined as the fraction of time when the net inventory is positive; see Axsäter (2000, p. 57) and Silver et al. (1998, p. 245). For the pure Poisson process, i.e., if $P(J{=}1){=}1$ and T is exponentially distributed, it is well known, see Axsäter (ibid., p. 59) and Silver et al. (ibid., p. 246), that the *VFR* and the ready rate are the same. From (5) it then follows that for this case the *OFR* is also identical to the ready rate. Moreover, we note that when the demand process is a compound Poisson process, the *OFR*, as defined in our paper, can be interpreted as an *extended* ready rate. This observation is due to the PASTA property (Wolff, 1982) and to the fact that when applying the *OFR* one takes into account that it is not enough just to have a positive inventory, but that it must also be buffered against possible different customer order sizes.

However, for the case when inter-arrival times are not exponentially distributed, we would like to caution against using the ready rate as a service measure. This is illustrated by the following small numerical example. Let T be uniformly distributed between *2* and *18* and let $L = 1$. Assume that $P(J=1) = 0.5$ and $P(J=2) = P(J=3) = 0.25$. Then $P(D_L = 0) = 1$. It is now easy to specify how the three service measures *OFR*, *VFR* and the ready rate (*RR*) depend on *S*. The results are summarized in Table 1.

<Table 1 about here>

Of course, the numerical example is somewhat constructed, but nevertheless the results in Table 1 indicate that the ready-rate service measure might be quite misleading when customer order arrivals do not follow a Poisson process.

4. Relationship between the two fill-rate service measures

We now explore the relationship between the two fill-rate service measures for the case when J follows a delayed negative binomial distribution. The distribution is termed 'delayed', because it is specified for the positive integers rather than for the non-negative integers. Hence, it has the probability mass function

$$P(J = j) = \frac{(s + j - 2)!}{(j-1)!(s-1)!}(1-\rho)^s \rho^{j-1} \ , \ j = 1, 2, \ldots \qquad (6)$$

Note that s need not be integer valued, since the factorials in (6) can then be interpreted as gamma functions (see Zipkin, 2000, p. 452).

From (1) and (4) we can derive the following expressions

$$1 - OFR(S) = \sum_{n=0}^{S-1} P(D_L = n) P(J \geq S - n + 1) + \sum_{n=S}^{\infty} P(D_L = n) \qquad (7)$$

and

$$1 - VFR(S) = \frac{\sum_{n=0}^{S-1} P(D_L = n) \sum_{j=S-n+1}^{\infty} \left(j - S + n\right) P(J = j)}{E[J]} + \sum_{n=S}^{\infty} P(D_L = n) \,. \qquad (8)$$

The inequality $OFR(S) \geq VFR(S)$ is then equivalent to

$$E[J] \sum_{n=0}^{S-1} P(D_L = n) P(J \geq S - n + 1)$$

$$\leq \sum_{n=0}^{S-1} P(D_L = n) P(J \geq S - n + 1) \sum_{j=S-n+1}^{\infty} \frac{(j - S + n) P(J = j)}{P(J \geq S - n + 1)} \,. \qquad (9)$$

Before proceeding with the comparative analysis, we interpret the inner summation on the right-hand side of (9), i.e.

$$\frac{\sum_{j=S-n+1}^{\infty}(j-S+n)P(J=j)}{P(J\geq S-n+1)}. \tag{10}$$

The expression in (10) specifies the expected number of units recorded on the backorder list, given that we know the order will incur a backorder. If the stochastic variable *J* had been associated with the time duration of an "activity" (for instance a service time) we would have interpreted this expression as the expected remaining duration of the activity, given that we have already observed that the activity has been in duration for *S-n* time units. This analogy provides the intuition why our analysis concludes that it is the size of the form parameter *s* that determines the relative magnitude of *OFR(S)* and *VFR(S)*. It follows from the fact that when *s = 1*, *J* is geometrically distributed. The geometric distribution is the discrete "counterpart" of the exponential distribution and from elementary queuing theory (see e.g. Hillier and Lieberman, 2001, Ch.17) we know that when the duration is exponentially distributed, the expected remaining duration of an ongoing activity is equal to the expected duration of that activity.

After changing the summation index and introducing the new random variable *X*, (10) can be rewritten as

$$\frac{\sum_{i=a}^{\infty}(i-a)P(X=i)}{P(X\geq a)}+1\ , \tag{11}$$

where the random variable *X* is distributed as *J -1*. Hence, *X* follows a standard negative binomial distribution. Therefore, a sufficient condition for the comparative analysis of (9) concerns whether *H(a)*, given by

$$H(a)=\frac{\sum_{i=a}^{\infty}(i-a)P(X=i)}{P(X\geq a)}\ , \tag{12}$$

is greater than *E[X]* for all $a \geq 1$. Note that the numerator of the right-hand side in (12) is the first-order loss function. It then follows from Zipkin (2000, p. 452) that

$$H(a)=E[X]+a\frac{\frac{\rho}{1-\rho}P(X=a)-P(X\geq a+1)}{P(X\geq a)} \tag{13}$$

Therefore, investigating whether *H(a)* is greater or smaller than *E[X]* is equivalent to investigating whether the function $g(a) = \rho P(X \geq a) - P(X \geq a+1)$ is greater or smaller than zero for all $a \geq 1$. First, note that *g(a)* approaches zero as *a* approaches infinity.

Second, observe that from (6) it follows that $g(0) = (1-\rho)^s - (1-\rho)$, which is negative when $s > 1$, zero when $s = 1$, and positive when $s < 1$. Third, using (6) the first-order difference $\Delta g(a) = g(a+1) - g(a)$ is obtained as

$$\Delta g(a) = \rho P(X = a)\frac{s-1}{a+1} \ , \tag{14}$$

which means that $g(a)$ is increasing when $s > 1$, constant when $s = 1$, and decreasing when $s < 1$. Collecting these three observations proves the following theorem, which is our main result.

Theorem: Consider a continuous review, base-stock inventory policy, with base-stock level S, where all unfilled demands are backordered and all replenishment orders have a fixed lead time L. Then, for any compound renewal demand process in which the customer order size follows a (delayed) negative binomial distribution with shape parameter s and probability parameter ρ, it holds that $VFR(S) < OFR(S)$ when $s < 1$, $VFR(S) = OFR(S)$ when $s = 1$ and $VFR(S) > OFR(S)$ when $s > 1$.

Observe that the equality of the two service measures in the case $s=1$ is between their expectations and not regarding their corresponding sample paths. The example to follow in Section 6 illustrates this. The implication of this observation is that empirically observed estimates of the two service measures will be different in general. A similar observation can of course be made for the cases $s < 1$ and $s > 1$.

5. The case of Erlang distributed inter-arrival times

In order to make the derived service measures computable, one needs to be able to specify the probability distribution of the lead-time demand. We show how this can be accomplished when the customer order inter-arrival times are Erlang distributed with k phases and mean k/λ. This implies that the duration of each phase is exponentially distributed with mean $1/\lambda$. Define $J^{(m)} = \sum_{r=1}^{m} J_r$ where $J_1, J_2, \ldots$ are independent and distributed as J, and $J^{(0)} = 0$. Then it follows from Rosling (2002, Eq. 4) and Cox (1962, Eq. 4, p. 37) that

$$P(D_L = x) = \begin{cases} e^{-\lambda L}\sum_{i=0}^{k-1}\frac{(\lambda L)^i}{i!}, & x = 0, \\ e^{-\lambda L}\sum_{m=1}^{x} P(J^{(m)} = x)\sum_{i=0}^{k-1}\frac{(\lambda L)^{mk+i}}{(mk+i)!}, & x > 0. \end{cases} \tag{15}$$

Note that when J follows a (delayed) negative binomial, (delayed) binomial or (delayed) Poisson distribution, as assumed in our logistical analysis for the case company, then it is straightforward to specify the probability distribution of $J^{(m)}$ and to develop an algorithm for computing the fill-rate service measures as functions of the base-stock level S. Note also that in the case $k = 1$, i.e. when demands are generated from a compound Poisson process, then (15) can be further simplified by use of the recurrence scheme due to Adelson (1966).

6. Numerical results

First, we conduct a sensitivity analysis in order to investigate the magnitude of difference between the two fill-rate service measures. Note that the expected value of the random variable J specified in (6) is

$$E[J] = \frac{s\rho + 1 - \rho}{1 - \rho} \tag{16}$$

and that its variance is

$$V[J] = \frac{s\rho}{(1-\rho)^2} \ . \tag{17}$$

Thus for any choice of s and ρ, the fraction

$$\frac{V[J]}{E[J]-1} = \frac{1}{1-\rho} \tag{18}$$

is always greater than 1 and does not depend on s. It is the lexis ratio (Law and Kelton, 1991, p. 359) for the random variable $J–1$, which is the standard (non-delayed) representation of a negative binomially distributed random variable. Therefore, it is not surprising that if one conducts a sensitivity analysis in which the parameter ρ is fixed, then, because the lexis ratio stays the same, the difference between the fill-rate service measures will not change dramatically when the parameter s is varied. This kind of sensitivity analysis is presented in Table 2.

<Table 2 about here>

In Table 2 the results are obtained from a renewal demand process with inter-arrival times that are Erlang distributed with $k=1$, i.e. demand follows a compound Poisson process. The value S_{OFR} indicates the smallest value of S which makes $OFR(S) \geq 0.98$ and the value S_{VFR} is correspondingly the smallest value of S which makes $VFR(S) \geq 0.98$. The results confirm how the relative magnitude of the fill rates depends on the parameter s, as stated in the theorem in Section 4. In particular, for case 5 (s = 1) we see that $S_{OFR} = S_{VFR}$ and $OFR(S_{RR}) = VFR(S_{FR})$. We also observe that as s increases, a small and increasing but not dramatic difference S_{OFR} - S_{VFR} appears. Note that owing to the discrete nature of the base-stock levels, $OFR(S_{OFR})$ is in general not exactly equal to 0.98, especially not for some of the lower base-stock levels.

Another sensitivity analysis to conduct in order to understand the behavior of the two fill rates is to keep $E[J]$ fixed while varying $V[J]$. The relevant values of the parameters s and ρ can then be computed from (17) and (18). As in the previous analysis, the demand process is compound Poisson. The result of this sensitivity analysis is presented in Table 3, organized similarly to Table 2.

<Table 3 about here>

As *V[J]* increases (when *E[J]* is fixed, this means that s approaches 0 and ρ approaches 1) we observe a significant discrepancy between the two service measures, as is exemplified by case *15* of Table 3. The result is intuitively reasonable, since in case *15* there are many small and a few large customer orders. This implies that in order to obtain a good *OFR* performance one does not need to set S that high, because the small orders count just as much as the large orders. However, in order to achieve a good *VFR* performance, one needs to have a larger S to cover also the larger orders which account for a large share of the total demand volume.

Finally, we focus on a specific item for which the two fill-rate service measures are equal. The item belongs to the sample provided to us by our case company and for this particular item a (delayed) geometric distribution (i.e. a negative binomial distribution with parameter $s = 1$) gave a fair fit to the observed order-size pattern. We also fitted the parameters k, λ and ρ to be $k \approx 1$, $\lambda \approx 0.3174$ (orders per day) and $\rho \approx 0.6229$. The lead-time L is *4* days, and in order to satisfy a target $OFR = VFR$ of *98%* it is required to let $S = 18$ giving $OFR(18) = VFR(18) \approx 0.9842$. We then simulated a base-stock inventory system, with the data given above. The system was simulated for *11000* days, and data for the two fill-rate service measures were collected after a *1000*-day run-in period. This experiment was replicated *50* times. The result of our simulation study is summarized in Table 4.

<Table 4 about here>

First, we note that both of the estimated service levels are close to their theoretical value of $VFR(18) = OFR(18)$, which are well within the respective *95%* confidence intervals. Second, the two fill-rate service measures are not equivalent, though indeed highly positively correlated with a correlation coefficient that has an estimated mean value of *0.9267* (and a *95%* confidence interval ranging from *0.8738* to *0.9580*). Third, the fill-rate service measures are evidently hard to estimate. Although we have effectively run our simulation experiments for more than *10000* days, which of course in any practical setting is meaningless (though meaningful from a statistical point of view), it is interesting to note that in *8%* of the cases for *OFR* and in *14%* of the cases for *VFR*, the target service level at *98%* is not attained. Thus, when setting base-stock levels based on mean service levels and then collecting performance, say after a year, one might very well see considerable deviations between the observed and the predicted performance. This calls for focusing not solely on the mean value of a service measure, but also on its higher order moments. Further discussion on this issue is found in, e.g. Chen and Krass (2001) and Thomas (2005).

7. Concluding remarks

In this paper we have focused on the inventory performance measure known as the *order fill rate*. We have specified this measure mathematically and under quite general assumptions about the demand process, we have shown its relationship to the more commonly applied *volume fill rate*. From our experience with a case company, we have noted the importance of first choosing an appropriate service measure and then also being able to develop a methodology that computes inventory control parameters consistently with the chosen service measure. Otherwise, lack of consistency between the performance measure and the control policy might leave the logistics manager unable to take the right steps in order to try to rectify an inventory system that is out of balance.

However, even if these deliberations are settled appropriately, it should also be noted that due to their inherent stochastic properties, empirically observed service performance in general deviates from the targeted performance. This could call for further research emphasis on service measures by not only focusing on their mean values, but also on their variability, or even their complete distributions.

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Tables:

S	*OFR*	*VFR*	*RR*
0	*0*	*0*	*0*
1	*0.500*	*0.571*	*0.900*
2	*0.750*	*0.857*	*0.950*
3	*1*	*1*	*0.975*

Table 1. Relationship between the three service measures *OFR*, *VFR* and *RR* for a specific numeric example and varying base-stock levels *S*.

Case	s	S_{OFR}	$OFR(S_{OFR})$	$VFR(S_{OFR})$	S_{VFR}
1	*0.01*	*5*	*0.9947*	*0.9944*	*5*
2	*0.05*	*5*	*0.9878*	*0.9863*	*5*
3	*0.1*	*6*	*0.9910*	*0.9895*	*6*
4	*0.5*	*9*	*0.9872*	*0.9860*	*9*
5	*1*	*12*	*0.9862*	*0.9862*	*12*
6	*1.5*	*14*	*0.9811*	*0.9825*	*14*
7	*2*	*17*	*0.9836*	*0.9857*	*16*
8	*5*	*31*	*0.9808*	*0.9863*	*30*
9	*10*	*54*	*0.9803*	*0.9877*	*50*
10	*20*	*100*	*0.9807*	*0.9890*	*91*
11	*50*	*235*	*0.9802*	*0.9893*	*214*
12	*100*	*457*	*0.9800*	*0.9888*	*417*
13	*200*	*890*	*0.9800*	*0.9875*	*820*
14	*500*	*2141*	*0.9800*	*0.9852*	*2023*
15	*1000*	*4196*	*0.9800*	*0.9839*	*4022*

Table 2. Sensitivity analysis with respect to the shape parameter s.
For all items $k = 1$, $\lambda = 0.25$, $\rho = 0.5$ and $L = 4$.

Case	$V[J]$	S_{OFR}	$OFR(S_{OFR})$	$VFR(S_{OFR})$	S_{VFR}
1	12	53	0.9822	0.9895	48
2	13	53	0.9817	0.9891	49
3	14	53	0.9812	0.9887	49
4	15	53	0.9806	0.9883	49
5	16	53	0.9801	0.9879	49
6	17	54	0.9818	0.9889	50
7	18	54	0.9813	0.9885	50
8	19	54	0.9808	0.9881	50
9	20	54	0.9803	0.9877	50
10	50	61	0.9810	0.9856	58
11	100	71	0.9806	0.9814	70
12	200	89	0.9808	0.9743	95
13	500	129	0.9804	0.9535	168
14	1000	174	0.9801	0.9211	290
15	5000	286	0.9800	0.7168	1257

Table 3. Sensitivity analysis with respect to $V[J]$.
For all items $k = 1$, $\lambda = 0.25$, $E[J] = 11$, and $L = 4$.

Performance measure	***OFR***	***VFR***
Estimated mean	0.9841	0.9839
Standard deviation across replications	3.719E-03	4.437E-03
Minimum across replications	0.9728	0.9689
Maximum across replication	0.9922	0.9914
95% confidence interval, lower limit	0.9830	0.9827
95% confidence interval, upper limit	0.9851	0.9852
Target percentage	92%	86%
Correlation coefficient	0.9267	

Table 4. Summary results of simulation experiment.
For item with $k = 1$, $\lambda \approx 0.3174$, $s = 1$, $\rho \approx 0.6229$ and $L = 4$.

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