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A comparison of electricity market designs in networks

Andreas Ehrenmann and Karsten Neuhoff

The Cambridge-MIT Institute

Massachusetts Institute of Technology
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Environmental Policy Research

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Andreas Ehrenmann and Karsten Neuhoff

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Abstract

In the real world two classes of market designs are implemented to trade electricity in transmission constrained networks. Analytical results show that in two node networks integrated market designs reduce the ability of electricity generators to exercise market power relative to separated market designs. In multi node networks countervailing effects make an analytic analysis difficult.

We present a formulation of both market designs as an equilibrium problem with equilibrium constraints. We find that in a realistic network, prices are lower with the integrated market design.

1 Introduction

If electricity is traded in areas with significant transmission constraints, then two basic design options may be used. First, in the nodal pricing approach or integrated market design, a centralized system operator collects location-specific energy bids and then clears the market for the entire region according to a well defined protocol. Such a design, as implemented for example in New England and PJM (Pennsylvania, New Jersey Maryland and neighbouring states), ensures that different locational markets are automatically arbitraged and the network is used efficiently.

This design is mainly criticized for requiring a large degree of co-ordination. The degree of co-ordination can be reduced at the expense of efficiency if individual nodes are aggregated into zones, as in the Nordic countries referred to as market splitting or zonal pricing. The second basic approach to deal with significant transmission constraints are physical rights, which implies a separate market for transmission and energy. Physical transmission rights are defined between areas and traders must own the these rights to trade and transmit energy between areas. If a transmission networks is meshed, such that more than one path exists between two nodes in the network, then it exhibits the physical phenomena of loop flows. In the presence of loop flows transmission rights should be allocated centrally, to ensure that they are mutually compatible. Decentralized allocation of such rights either must be excessively conservative (which will make inefficient use of the system), as currently experienced in continental Europe, or must create opportunities to game the system, as experienced in the California crisis.

In a competitive market without uncertainty, the integrated market approach will result in the same generation dispatch and prices as an approach based on centrally- allocated physical transmission rights. This follows because Bohn, Caramanis and Schweppe (1983) showed that the methodology underlying integrated market prices (nodal prices) results in welfare maximizing dispatch, and Chao and Peck (1996) proved that the physical-rights-based approach also results in welfare maximizing dispatch in a situation of competition and no uncertainty provided that the welfare problems are identical and have a unique solution.

Harvey, Hogan and Pope (1996) suggested that the separated market design will exhibit inefficiencies when there is uncertainty, which can be empirically shown by the example of the German-Dutch interconnector (see Neuhoff 2003).

These claims to efficiency require that generators bid competitively (at short-run marginal costs), but in a privatised industry this is not guaranteed, particular given current levels of concentration.

The aim of this paper is to assess which market design performs better in the context of market power by generators.

Like all models we have to abstract from certain features of the real market designs to allow for the implementation. For example, New York and New England have an integrated energy and transmission market that also includes marginal loss calculations. Furthermore, they have like PJM a multi-settlement system with integrated day-ahead and balancing market. The possibility to implement these additional features in a consistent way can be seen as an additional benefit of an integrated market design, particularly because failure of a consistent treatment of day ahead and balancing market was one of the critical issues in California. However, we believe that abstracting from these effects is feasible in first order, but are excited to see and are working on expanding the analysis to include question of balancing markets, transmission contracts and forward contracting (see e.g. Kamat and Oren 2003).

Analytical models show that in a two-node network and in meshed networks with market power at one node, integrating energy and transmission markets reduces prices and improves welfare. This can be explained by two effects. First, if transmission markets are separated from energy markets, the allocation of transmission capacity in the network to export to or import from specific regions is determined before the stage of the energy spot market. The bids of generators to energy spot markets change the allocated transmission capacity only in expectation, not in realization. Net-imports are less responsive to output changes of generators, net demand elasticity is reduced and generators exercise more market power.

Second, if traders arbitrage the different energy markets after obtaining the physical transmission rights, they still have the option to only use a fraction of these rights. However, they must commit themselves to a certain quantity when they submit bids to the energy spot markets of the two regions they wish to arbitrage. As electricity can not be stored, the traders must submit a balanced schedule to the system operator, and therefore they must make sure that they buy the same amount of electricity in one market that they sell in the other market. In continental Europe, the energy spot markets are auctions which clear virtually simultaneously; and so traders cannot condition their bid in one market on the outcome of the other market. As a result, traders must submit a very high- priced buy bid and a very low- priced sell bid in the respective markets to ensure that both bids will be accepted. Subsequently, the traders are only obliged to pay the market clearing price, but the effect is that the amount of energy transmitted is not conditional on the prices in the energy spot markets. Therefore, generators are not exposed to price responsiveness in transmitted energy.

However, these general analytical results are hard to generalize to market power at more than two nodes of a complex network, nor to generators which own assets at more than one node of the network. For example, in the integrated market design, an increase of output by a generator at node A could also decrease the prices and therefore revenues of this generator at node B (Cardell, Hitt and Hogan 1997). This implies that the integrated market design also provides incentives for generators to reduce output relative to a separated market design.

To assess the relative importance of these effects, we decided to implement a numerical model for a realistic network configuration. We chose the Benelux countries Belgium and the Netherlands, with a reduced representation of the neighbouring states, Germany and France.

This paper presents a numerical implementation of the integrated market design. We also give a representation of the separated market design that deviates from previous models in that it includes net-demand elasticity of the competitive fringe. This allows for comparison of the results for both market designs.

Hobbs, Metzler and Pang (2003) show that endogenous and exogenous arbitrage, which corresponds to integrated and separated markets, are equivalent under an tacit assumption: strategic generators assume that transmission prices - defined as the price differences between different nodes - do not change in response to their output decision. This assumption might be appropriate to represent a situation where strategic generators have limited information, but frequently large generators are perceived as well-informed. The effect of the assumption of constant transmission prices is, that generators decide on their profit maximising output quantity in the believe that their output increase in any one zone leads to increased exports to all other zones. They assume that they face the demand responsiveness of the entire network, and will therefore exercise less market power than if they are aware of transmission constraints. This model approach therefore understates the exercise of market power in separated energy and transmission markets.

In our numerical results the prices in the integrated market design were always lower than in the separated market design. The benefit of importing demand elasticity outweighs the disadvantages due to cross-holding.

This market model is of interest from two different perspectives: From the economic perspective regulators are interested in the optimal choice of market design. The ranking with regard to the behaviour in the presence of market power provides an argument in favor of the integrated market design. From the mathematical perspective, the integrated market design is an instance of an Equilibrium Problem with Equilibrium Constraints (EPEC) which is a recent field of research.

Hobbs, Metzler and Pang (2000) presented a multi- leader/-follower market model in accordance with Cardell, Hitt and Hogan (1997). They calculate oligopolistic price equilibria for generalized DC networks, using the supply functions of conjectural variation. Strategic generators can decide either on slope or intersect of their bid functions for each location. The optimization problem for each generator is a two- stage game, in which the generator anticipates the transmission allocation and market clearing by the Independent System Operator's (ISO). The problems which result for each player are of the MPEC type (Mathematical Programs with Equilibrium Constraints) and are therefore non-convex and non-differentiable. The authors search for an equilibrium by solving the generators' problems sequentially, using the last bid of the competitors as the input parameter.

Also related to the topic of this paper is the debate of nodal versus zonal pricing. In both separated energy and transmission markets and integrated markets, the aggregation of individual nodes to zones either requires a more conservative definition of transmission capacity or increases the opportunity for generators to exercise market power (Harvey and Hogan 2000); aggregation results in inefficient dispatch and causes generators to make incorrect location decisions. The comparison of integrated and separated markets in this paper is based on the same level of aggregation in both designs. For the integrated energy and transmission markets, a shift to smaller zones or nodes is feasible, while a design with separate energy and transmission rights would loose liquidity at each node and would require increasing complexity in transmission contracts.

The paper is structured as follows: In section two we use a three-node network to analytically determine the market equilibrium in both designs and to provide the intuition for some of the differences. In section three we provide general formulations for the integrated and section four for the separated market design as an equilibrium problem with equilibrium constraints. In section five we use a realistic network of the Benelux countries to compare the market designs and we conclude in section six.

2 A three node network

To provide an intuition of how the two discussed market designs affect the behaviour of the strategic generators, we start with the a simple network as frequently used in the literature.

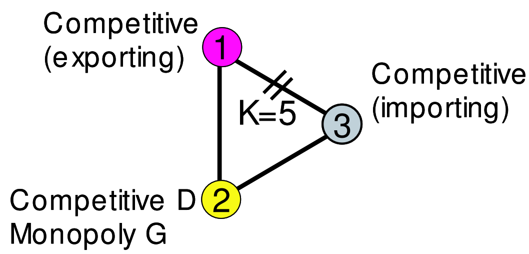

Figure 1: Three node network

In this three- node network only the transmission line between node 1 and 3 is constrained and of capacity K. Defining node 3 as the swing bus, we must determine the fraction γ_i (the power distribution factors) of flows from node 1 and 2 that crosses the constrained link $\overline{13}$. According to Kirchhof's laws, flows are split between all feasible paths proportional to the inverse of the resistance on these paths. For energy delivery from node 1 to node 3 the direct link has half the distance of the path via node 2. Therefore physical laws imply that two thirds of the energy pass along the direct path. The power distribution factor for exports t_1 from node 1 to node 3 cross link $\overline{13}$ is $\gamma_1 = \frac{2}{3}$. According to the same logic, $\gamma_2 = \frac{1}{3}$ of exports t_2 from node 2 to node 3 cross link $\overline{13}$.

We assume markets at node 1 and 3 and demand at node 2 are competitive. Net production at each node I is q_i and positive for exports and negative for imports. The net-demand curve is assumed to be linear:

$$p_i = -a_i - b_i q_i. \tag{1}$$

One strategic generator with output s_2 is located at node 2 and is assumed to have no variable costs of production.

In the first market design energy and transmission markets are separated. The transmission quantities are determined in the transmission auction which precedes the energy spot markets. Therefore generators choose which output quantities to bid into the energy spot market assuming fixed amounts of transmitted energy. In the second market design energy and transmission markets are integrated. The strategic generator knows that his output decisions will influence the allocation of transmission rights by the SO. This is represented by a Stackelberg model, where the strategic generator(s) are the Stackelberg leader which anticipate the reaction of the follower (- the system operator) - to their output choices. This problem will be later formulated as EPEC. The leaders (generators) include the optimality condition and constraints of the follower (system operator) into their profit function

2.1 Separated Market design

In the separate market design traders bid in the transmission auction. The system operator allocates transmission rights to make most efficient use of the network. Traders then use their transmission rights to arbitrage energy spot markets. Chao and Peck(1996) show that this design results in a welfare optimal allocation in a competitive setting. Therefore we can also represent it using a single welfare maximising function for the auctioneer. If a strategic generator decides how much energy to offer in the energy spot market, then the strategic generator influences net demand at his production node. In a world of complete information traders correctly anticipate the output decision of the strategic generator. They act as if demand at the corresponding node is reduced by the output quantity which will be offered by the strategic generator.

The auctioneer maximizes residual welfare, taken the Cournot bids of strategic generators as fixed (see also Smeers and Wei, 1999):

$$W_r = \max_{q_i,t_i} \sum_i \frac{b_i}{2} q_i^2 + a_i q_i. \tag{2}$$

The maximizsation is subject the network energy balance:

$$\sum_i t_i = 0, \tag{3}$$

local energy balance:

$$q_i + s_i = t_i, \tag{4}$$

and transmission constraint:

$$\gamma_1 t_1 + \gamma_2 t_2 \leq K. \tag{5}$$

For simplicity, we assume that the transmission constraint is binding so we can use the equality sign in (5). We also assume that capacity and non-negativity constraints are satisfied. Using (4) to eliminate t_i in (3) and (5) q_1 and q_3 can be written as function of q_2:

$$\begin{aligned} q_1 &= \frac{K - \gamma_2 (q_2 + s_2)}{\gamma_1}, \\ q_3 &= \frac{-K + (\gamma_2 - \gamma_1)(q_2 + s_2)}{\gamma_1}, \end{aligned}$$

allowing us to express the optimization problem (2) as a function of the remaining choice variable q_2. To simplify the calculations we subsequently set parameter values $a_1 = b_3 = 0$ (note that all the a_i and b_i are negative):

$$W_r = \max_{q_2} \frac{b_1}{2} \left(\frac{K - \gamma_2 (q_2 + s_2)}{\gamma_1} \right)^2 + a_2 q_2 + \frac{b_2}{2} q_2^2 + a_3 \frac{-K + (\gamma_2 - \gamma_1)(q_2 + s_2)}{\gamma_1}.$$

This is a concave function in q_2 with a unique maximum. The FOC gives the optimal competitive output choice:

$$q_2 = \frac{\gamma_2 b_1 K - \gamma_2^2 b_1 s_2 - \gamma_1^2 a_2 - a_3 (\gamma_1 \gamma_2 - \gamma_1^2)}{\gamma_2^2 b_1 + \gamma_1^2 b_2}.$$

The system operator uses (4) to determine how much transmission capacity to allocate to the different links:

$$t_2 = s_2 + q_2 = \frac{\gamma_2 b_1 K + \gamma_1^2 b_2 s_2 - \gamma_1^2 a_2 - a_3 (\gamma_1 \gamma_2 - \gamma_1^2)}{\gamma_2^2 b_1 + \gamma_1^2 b_2}. \tag{6}$$

The strategic generator at node 2 takes the t_i as given in his optimisation decision. His maximization problem is:

$$\Pi = \max_{s_2, q_2, p_2} p_2 s_2, \tag{7}$$

subject to the local energy balance (4) and the competitive bids of local net demand (1). The two constraints allow the expression of the two choice variables q_2 and p_2 as a function of s_2, such that the optimisation problem reads:

$$\Pi = \max_{s_2} \left(b_2\left(s_2 - t_2\right) - a_2\right) s_2, \tag{8}$$

and is solved:

$$s_2 = \frac{a_2}{2b_2} + \frac{t_2}{2}. \tag{9}$$

In equilibrium, traders correctly anticipate the output choice s_2 of the strategic generator when bidding in the transmission auction. Otherwise, there would either be an profitable opportunity for a new entrant or existing traders would leave the market due to losses. Therefore, also the welfare maximising system operator, which is equivalent to the competitive traders, correctly anticipates the output decision of the strategic generator when determining the allocation of export rights t_2. This implies that (6) and (9) have to be simultaneously satisfied. This gives us the equilibrium output quantity of our generator[1] :

$$s_2 = \frac{\gamma_2 b_1 K + \gamma_2^2 \frac{b_1}{b_2} a_2 - \left(\gamma_1\gamma_2 - \gamma_1^2\right) a_3}{2\gamma_2^2 b_1 + \gamma_1^2 b_2}. \tag{10}$$

2.2 Integrated energy and transmission Markets

In the Stackelberg game, the leader (generator) continues to maximize his profit function (8) subject to local energy balance (4) and competitive local demand response (1). The

[1]Substituting parameter values for demand $a_1 = 0$, $b_1 = -2$, $a_2 = -8$, $b_2 = -1$, $a_3 = -10$, $b_3 = 0$ and network $K = 5$, $\gamma_1 = \frac{2}{3}$, $\gamma_2 = \frac{1}{3}$ gives the following equilibrium prices and quantities:

$$s_2 = 8.25, \quad q_1 = 3.25, \quad q_2 = .25, \quad q_3 = -11.75,$$
$$p_1 = 6.5, \quad p_2 = 8.25, \quad p_3 = 10.$$

difference is that t_2 is no longer fixed but determined by the follower (system operator) as a function of the strategic output choice of the generator s_2.

We have already solved the followers' reaction function $t_2(s_2)$ and determined the optimal allocation of transmission rights as a function of s_2 in equation (6). Therefore, we need only substitute $t_2(s_2)$ for t_2 in the profit function of the strategic generator (8):

$$\Pi = \max_{s_2} \left(b_2 \left(s_2 - \frac{\gamma_2 b_1 K + \gamma_1^2 b_2 s_2 - \gamma_1^2 a_2 - a_3 \left(\gamma_1\gamma_2 - \gamma_1^2\right)}{\gamma_2^2 b_1 + \gamma_1^2 b_2} \right) - a_2 \right) s_2, \tag{11}$$

and calculate the FOC to obtain the optimal output choice s_2 :

$$s_2 = \frac{\gamma_2 b_1 K \gamma_2^2 + \frac{b_1}{b_2} a_2 - \left(\gamma_1\gamma_2 - \gamma_1^2\right) a_3}{2\gamma_2^2 b_1}. \tag{12}$$

The nominator in (12) and (10) is identical, but the denominator is larger in (10).[2] This shows that the production of the strategic generator is larger in the integrated market design. The intuition behind this result is, that the generator no longer faces only local demand response, but also the response of the network. To calculate this demand slope, we differentiate the price at node 2, as expressed in the parentheses of equation (11), with respect to the output choice s_2 of the strategic generators:

$$-\frac{\partial p_2}{\partial s_2}|_{integrated} = -b_2 \frac{1}{1 + \frac{b_2\gamma_1^2}{b_1\gamma_2^2}} < -b_2 = -\frac{\partial p_2}{\partial s_2}|_{separated}.$$

Provided $b_1, b_2 < 0$. Integrating the energy and transmission markets implies that prices are changing less with output changes, or that effective demand is more responsive to price changes. Higher effective demand elasticity is the main driver in mitigating market power and can be obtained at low costs by choosing the appropriate integrated market design.

[2] The previous parameter values give the following equilibrium prices:

$$s_2 = 16.5, \quad q_1 = 0.5, \quad q_2 = -2.5, \quad q_3 = -14.5,$$
$$p_1 = 1, \quad p_2 = 5.5, \quad p_3 = 10.$$

3 Formulation of the Integrated Market as an Equilibrium Problem

The integrated market design corresponds to a two- stage game, in which the strategic generators submit quantity bids to an ISO. They anticipate the allocation of transmission capacity by the ISO in their optimisation problem. The usual method prescribed for the ISO is to assume that bids of the generators and demand side are competitive and to clear the market such that social welfare is maximised. The demand and competitive fringe bids at marginal cost into the energy market. We start with the stationarity conditions of the ISO, followed by the optimisation problems of the strategic generators. We conclude with a discussion of the solution algorithm.

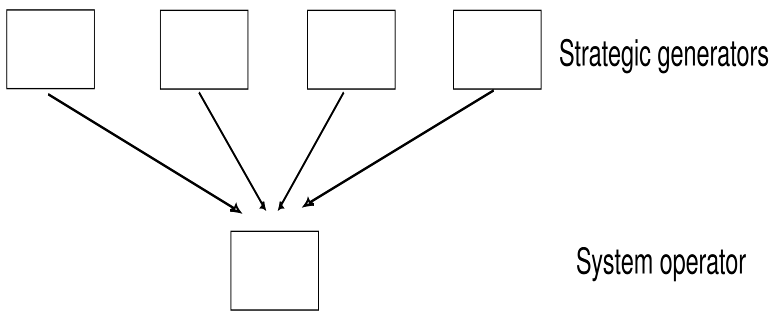

Figure 2: Integrated Market Design

3.1 Formulation of the Follower's problem (ISO)

We will use the following notation for the description of the problem. The network has I nodes i with M links m between these nodes. We model J strategic generation companies j, which can control generation on one or several nodes of the network. To allow for piecewise linear marginal cost schemes, we split the schemes in L sections l with linear cost segments.

In our formulation of the cost function $c(q) = max_l\{ca_l + cb_l q\},\ q \leq q^{max}$ we use L

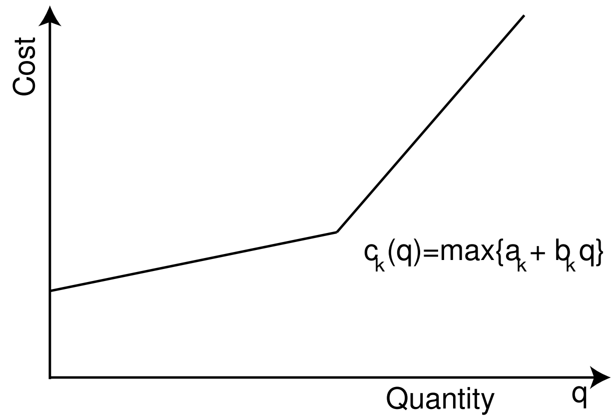

Figure 3: Cost Curves

variables q_l with $c(q_1, \ldots, q_L) = \sum_l ca_l + cb_l q_l, \quad q_{l-1}^{max} \leq q_l \leq q_l^{max}$ where $q_L^{max} = q^{max}$. Likewise, to represent piecewise linear cost schemes for competitive generation, their cost curves are split into $K-1$ linear sections $k = 2, \ldots, K$. The linear demand curve for each node is included as a negative competitive supply curve and is denoted by $k = 1$. The quantities of the demand and the competitive fringe are therefore q_k with $c(q_1, \ldots, q_K) = \sum_l a_k + b_k q_k, \quad q_{k-1}^{max} \leq q_k \leq q_k^{max}$ where $q_K^{max} = q^{max}$ and a_1 and b_1 are the parameters of the linear demand function and negative.

The exogenous parameters are denoted as:

ca_{ijl}	cost curve intercept, node i, strategic generator j, section l
cb_{ijl}	cost curve slope, node i, strategic generator j, section l
$capg_{ik}$	capacity limit fringe generator node i
$capg_{ijl}$	capacity limit generator j, node i, section l
a_{ik}	bid curve intercept, node i, fringe generator and demand
b_{ik}	bid curve slope, node i, fringe generator and demand
$\gamma_{i,m}$	% flow from node i over link m to swing bus (node 1)
$capk_m$	capacity link m
σ_k	$\sigma_1 = 1$ and $\sigma_k = -1$ for $k \neq 1$ for the different treatment of demand and competitive fringe.

And the variables are:

$s_{i,j,l}$ quantities strategic generator j, node i, section l

$q_{i,j,k}$ quantitiy, consumer or fringe generator, node i.

In the separated market design we have additionally:

t_i export quantity, node i

The ISO's objective is to allocate transmission capacity, such that the network is optimally used. This can be achieved by maximizing social welfare. For discussion of this topic, see Smeers and Wei (1999). The optimization problem of the ISO contains the energy conservation constraint, the capacity constraints of the transmission lines and the capacity constraints for the competitive fringe:

$$\begin{array}{llr} \max_{q_{ik}} & \sum_i q_{ik}(a_{ik} + \frac{b_{ik}}{2} q_{ik}) & \\ s.t. \sum_i \left(\sum_j s_{ij} + q_{ik}\right) & = 0 & p_1 \\ -capk_m & \leq \sum_i \gamma_{m,i} \left(\sum_j s_{ijl} + \sum_k q_{ik}\right) \leq capk_m & \rho_m, \delta_m \\ 0 & \geq \sigma_k q_{ik} \geq \sigma_k capg_{ik} & \lambda_{ik}, \mu_{ik} \end{array}$$

This is a quadratic optimization problem with a unique solution.

The KKT-stationarity conditions of the ISO's optimization problem are:

$$(a_{ik} + b_{ik} q_{ik}) + p_1 - \sum_m \rho_m \gamma_{m,i} + \sum_m \delta_m \gamma_{mi} + \sigma_k \lambda_{ik} - \sigma_k \mu_{ik} = 0$$

$$\sum_i (\sum_{j,l} s_{ijl} + \sum_k q_{ik}) = 0$$

$$0 \geq -Capk_m - \sum_i \gamma_{m,i} \left(\sum_{j,l} s_{ijl} + \sum_k q_{ik}\right) \perp \rho_m \geq 0$$

$$0 \geq \left(\sum_i \gamma_{m,i} \left(\sum_{j,l} s_{ijl} + \sum_k q_i\right)\right) - Capk_m \perp \delta_m \geq 0$$

$$\sigma_k q_{i,k} \geq 0$$

$$\lambda_{ik} \geq 0$$

$$q_{ik} \lambda_{ik} = 0$$

$$-\sigma_k q_{ik} q_{ik} + \sigma_k capg_{ik} \geq 0$$

$$\mu_{ik} \geq 0$$

$$(-\sigma_k q_{ik} q_{ik} + \sigma_k capg_{ik}) \mu_{ik} = 0$$

3.2 Formulation of Leader's problem (Generator)

In the leader problem for the strategic generators, the quantity bids of their fellow strategic generators are taken as constant. Leaders anticipate the ISO response to their quantity bid. This is modelled by including the optimality conditions of the ISO's problem as constraints in the strategic generators' optimization problem. The strategic generators maximize profits. Profits consist of electricity sales at the nodes of production minus production costs. The first ten constraints are the KKT-conditions of the ISO optimization problem; the remaining two constraints are the non-negativity and capacity constraints of production.

$$\max_{s_{ijl},q_{ik},p_1,\rho_m,\delta_m,\lambda_{ik},\mu_{ik}} \left[\sum_i \left(\left(p_1 + \sum_m \gamma_{i,m}\left(-\rho_m + \delta_m\right)\right)\sum_l s_{ijl} - \sum_l \left(ca_{ijl} + \tfrac{1}{2}cb_{ijl}s_{ijl}\right)s_{ijl}\right)\right]$$

$$s.t \quad (a_{ik} + b_{ik}q_{ik}) + p_1 - \sum_m \rho_m\gamma_{m,i} + \sum_m \delta_m\gamma_{mi} + \sigma_k\lambda_{ik} - \sigma_k\mu_{ik} = 0$$

$$\sum_i(\sum_{j,l} s_{ijl} + \sum_k q_{ik}) = 0$$

$$0 \geq -Capk_m - \sum_i \gamma_{m,i}\left(\sum_{j,l} s_{ijl} + \sum_k q_{ik}\right) \perp \rho_m \geq 0$$

$$0 \geq \left(\sum_i \gamma_{m,i}\left(\sum_{j,l} s_{ijl} + \sum_k q_i\right)\right) - Capk_m \perp \delta_m \geq 0$$

$$\sigma_k q_{i,k} \geq 0$$

$$\lambda_{ik} \geq 0$$

$$q_{ik}\lambda_{ik} = 0$$

$$-\sigma_k q_{ik} + \sigma_k capg_{ik} \geq 0$$

$$\mu_{ik} \geq 0$$

$$\left(-\sigma_k q_{ik} + \sigma_k capg_{ik}\right)\mu_{ik} = 0$$

$$s_{ijl} \geq 0$$

$$s_{ijl} \leq capg_{ijl}$$

Note that electricity prices are calculated as nodal prices where the multiplier p_1 of the energy conservation constraint determines the price at the swing bus.

For the leader j, the optimization problem is a MPEC. MPECs are nonconvex and nondifferentiable optimization problems, and are therefore difficult to solve. For further details, see Luo et al. (1996).

3.3 Properties of EPECs

The integrated energy and transmission market and also the later described separated market design lead to Equilibrium Problems with Equilibrium Constraints, which are a special case of a generalized Nash game (GN)in which all leaders share the same complementarity constraints. Harker (1991) and Ehrenmann (2003) showed that GN-games can have non-isolated, multiple solutions. Oren (1997) showed that pure strategy solution do not necessarily exist. Possibly only mixed strategy equilibria exist. They are difficult to calculate in such complex games, but have been calculated for a simple network, e.g. by Borenstein, Bushnell and Stoft (2000).

Hu and Ralph (2001) gave an example for the non-existence of pure strategy equilibrium in in a three node network.

We followed the digitalisation approach of Hobbs, Metzler and Pang (2000) to solve the optimisation problem of each generator sequentially. In the separate market design the coordinated auction is a further step of the sequential optimisation. This equal representation is possible even so the coordinated auction precedes the output decisions of the generators. From the perspective of generators in the energy market both representations are identical as they take the bids of traders as given in the Nash representation of the one stage game as if they would have occurred in a preceding time step. From the perspective of traders in the transmission auction both representations are identically if the traders are competitive. They do not bid strategically in the transmission auction, therefore this causality can be ignored.

We implemented the different market designs in the modelling package GAMS and solved the MPECs via a vanilla SQP method (see Fletcher, Leyffer, Ralph and Scholtes). In our experiments, the sequence generated by a diagonalization converged for both market designs.

4 Formulation Separated Market

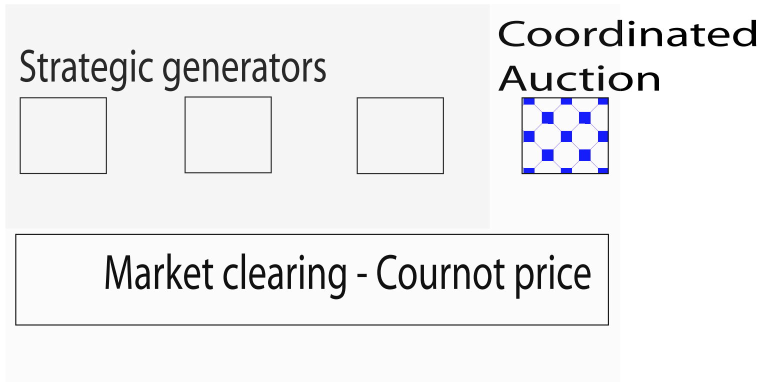

Figure 4: Separated Market Design

In the separated market design, the strategic generators do not anticipate a reaction in the volumes of transmission in response to their quantity bids, since the transmission auction takes place as a coordinated auction before generators submit their quantity bids.

Individual competitive arbitrageurs will not influence the market results and therefore only look at the price levels, not at the price changes they are able to induce. They are not interested in the reaction of generators to their arbitrage decisions as long as they make zero losses in equilibrium.

Marginal revenues by definition internalize the effect of output changes on price levels. Therefore, in the separated market design, we can treat the Cournot game between the strategic generators and the coordinated auction as 'on the same level'. Due to the capacity constraints on the competitive generators, the net demand (which is demand minus competitive generation) is not differentiable in the points at which the capacity or non-negativity constraints become active. In the mathematical formulation, this is represented by mixed complementarity constraints. The resulting optimization problems are again of

MPEC type. Furthermore, the non-negativity constraints of competitive generation imply that the net demand functions are not necessarily convex, and so the solution, is not necessarily unique.

This problem of non-concavity can be avoided by assuming that the competitive generators submit bids that are not responsive to price changes. Then the competitive generators' output decision can be modelled on the same level as the strategic generators' output decision and the allocation of transmission capacity in the coordinated auction (Hobbs 2001). This approach is likely to understate the effect of the net demand responsiveness provided by competitive generation.

As the strategic generators take the allocation of the transmission rights t_i as given, they no longer optimize over the shadow price of transmission constraints (ρ, δ). Instead, we introduce local spot prices p_i.

The optimization problem of generator j then becomes:

$$\max_{p_{ijl}, q_{ik}, p_i, \lambda_{ik}, \gamma_{ik}} \sum_i \sum_l (p_i s_{ijl}) - \sum_i \sum_l (ca_{ijl} + \frac{1}{2} cb_{ijl} s_{ijl}) s_{ijl}$$

$$\begin{aligned} s.t. \quad & a_{ik} + b_{ik} q_{ik} + p_i + \sigma_k \lambda_{ik} - \sigma_k \mu_{ik} = 0 \\ & \textstyle\sum_{l,j} s_{i,j,l} + \sum_k q_{ik} - t_i = 0 \\ & \sigma_k q_{ik} \geq 0 \\ & \lambda_{ik} \geq 0 \\ & q_{i,k} \lambda_{ik} = 0 \\ & -\sigma_k q_{ik} + \sigma_k capd_{ik} \geq 0 \\ & \mu_{ik} \geq 0 \\ & \mu_{ik}(-\sigma_k q_{ik} + \sigma_k Capd_{ik}) = 0 \\ & s_{ijl} \geq 0 \\ & -s_{ijl} + capg_{ijl} \geq 0 \end{aligned}$$

The coordinated auction determines the export quantity t_i in order to maximise social welfare:

$$\max_{t,q_i,p_i} \sum_{i,k} -\sigma_k \left(a_{i,k} + \frac{1}{2} b_{i,k} q_{ik} \right) q_{ik},$$

subject to the energy balance (line 1) in the network and (line 2) at each individual node. Constraints (line 3) are the market clearing condition for competitive generation and demand with the (line 4-6) non-negativity and (line 7-9) capacity constraints. Constraints (line 10-11) are the upper- and lower- line capacity constraints.

$$
\begin{aligned}
s.t. \qquad & \textstyle\sum_i t_i = 0 \\
& \textstyle\sum_{j,l} s_{ijl} + \sum_k q_{ik} - t_i = 0 \\
& a_{ik} + b_{ik} q_{ik} + p_i + \sigma_k \lambda_{ik} - \sigma_k \mu_{ik} = 0 \\
& \sigma_k q_{ik} \geq 0 \\
& \lambda_{ik} \geq 0 \\
& q_{i,k} \lambda_{ik} = 0 \\
& -\sigma_k q_{ik} + \sigma_k capd_{ik} \geq 0 \\
& \mu_{ik} \geq 0 \\
& \mu_{ik}(-\sigma_k q_{ik} + \sigma_k Capd_{ik}) = 0 \\
& Capk_m - \textstyle\sum_i \gamma_{m,i} t_i \geq 0 \\
& \textstyle\sum_i \gamma_{m,i} t_i + Capk_m \geq 0
\end{aligned}
$$

The resulting problem is of the EPEC type since the ISO and the strategic generators share the market clearing conditions of demand and competitive fringe as common constraints.

5 Numerical findings for a realistic network

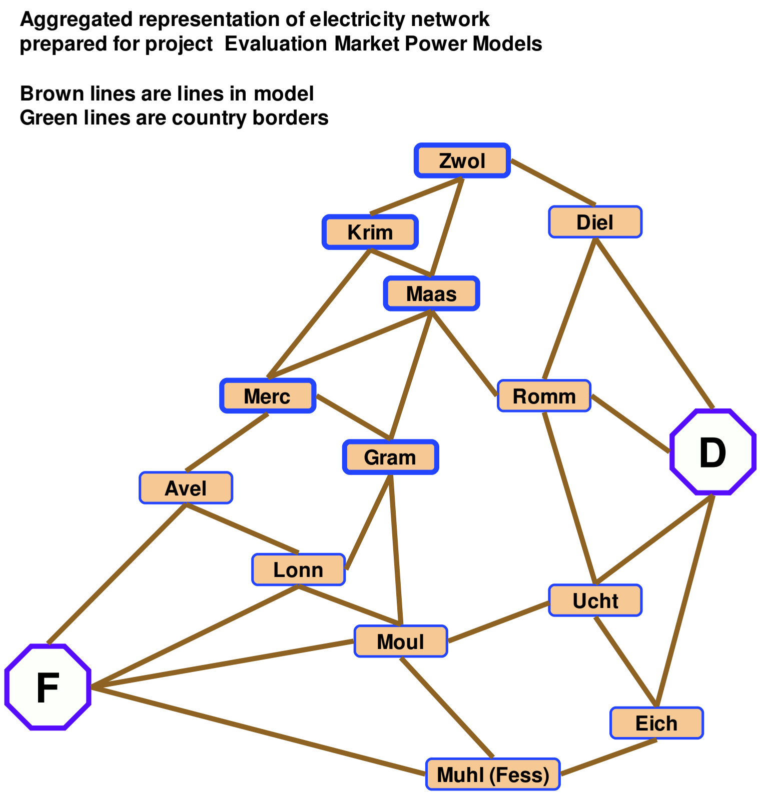

Figure 5: Network

The network consists of three nodes in the Netherlands, two nodes in Belgium and one node each in Germany and France with generation and demand. Further intermediate nodes without demand or generation are used to model the linearised DC network. The transmission constraints were summarized as 28 flow-gates. All flow-gates are characterized by an upper limit in MWh, and by power distribution functions characterizing the amount of energy transmitted to each node from the reference node Germany, passing via the flow-gate.

For the generation capacity, the following eight firms were considered as strategic generators, with production in one or several countries. Production in Belgium and the Netherlands is divided between the two or respectively three nodes in each country, according to the location of the generation plants. In Germany and France, production and demand are located at the national nodes D and F.

Germany	EOn, ENBW, RWE, Vattenfall, EdF
France	EdF, ENBW
Belgium	Electrabel
Netherlands	Essent, Nuon, E.ON, Electrabel

We assume that all these firms are bidding their entire output in the spot market with the sole objective to maximise profit. This assumption does not correspond to reality for two reasons. First, monopolists like EDF face the threat of regulatory intervention if they charge excessive prices and will therefore moderate their behaviour. Secondly, electricity generators sign long-term contracts, either in the form of explicit contracts with large customers or implicit contracts due to their vertical integration with the supply business. Due to these long-term contracts the exposure to the spot market is reduced and strategic generators face less incentive to exercise market power (Allaz and Vila 1993). Nevertheless, we think the results of our analysis are relevant, because the focus of our analysis is the comparison of two market designs and not the determination of absolute price levels. The remaining generation plants, which are not allocated to one of the mentioned companies, are assumed to bid their marginal cost curves into the spot market.

Variable cost of the generation plants is represented using a two-piece linear cost curve.

To create several different demand scenarios, the empirical load data for the summer and winter super peak are used and scaled with a scenario and location- specific random factor. With this method we created ten different demand scenarios, of which five are for a summer and five for a winter generation structure. Figure 6 shows the demand levels at 30 Euro/MWh. We assume that demand is linear. To determine the slope we assume that demand elasticity is 0.1 at the previously calculated demand for a price of 30 Euro/MWh.

With these parameter values we solve the equilibrium problems described in section three. For the integrated market design we solved iteratively the optimisation problems of the strategic individual generators with respect to the stationarity conditions of the ISO. Strategic generators are Stackelberg leaders with respect to the ISO, therefore their optimisation problem is constrained by complementarity conditions describing the stationarity conditions of the ISO. We use a NLP reformulation, suggested by Fletcher, Leyffer, Scholtes and Ralph (2002) in which we replace the complementarity $0 \leq f(z) \perp g(z) \geq 0$ by $f(z) > 0, g(z) \geq 0, f(z)^\top g(z) \leq 0$.

We use SNOPT SQP to solve the equilibrium problems. Figure 7 illustrates that the diagonalisation converges after very few iteration. However we expect, but so far did not observe, that changes in the starting values might lead to convergence against different equilibria.

The representation of the separated market design differs in two aspects. First, the ISO does not allocate transmission in reaction to the output decision of strategic generators, but before the spot market. This timing would suggest that he should now be modelled as

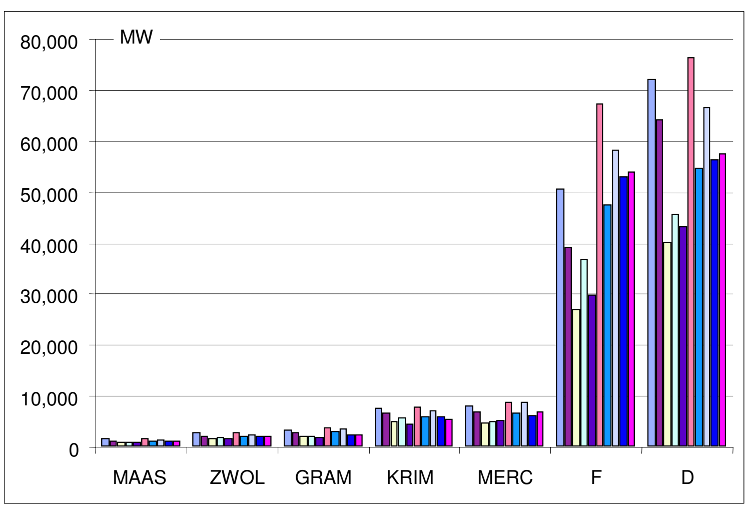

Figure 6: Load Scenarios

a leader. However, as the ISO does not act strategically he can be modelled at the same stage as strategic generators. We iterate the optimisation over all strategic generators and the ISO. Secondly, competitive fringe generators continue to submit bids conditional on the equilibrium price. They are modelled as followers in the optimisation problem and we therefore still have to solve a two-stage game.

The numeric results for the separated energy and market design are represented in Figure 8. The prices calculated for France are higher than in the other zones, which is due to EdF's monopoly position and binding import constraints from both Germany and Belgium. As discussed above, we did not model that in reality a dominant national generator could not increase the price to the calculated level without triggering strong regulatory interference. By contrast, Germany has the lowest price levels, since it has four strategic generators and a large share of competitive generation that provides a large responsiveness in net demand. The relative prices in the integrated market look similar, also here France exhibits the highest prices.

Figure 9 shows the price change when shifting from the separated to the integrated market

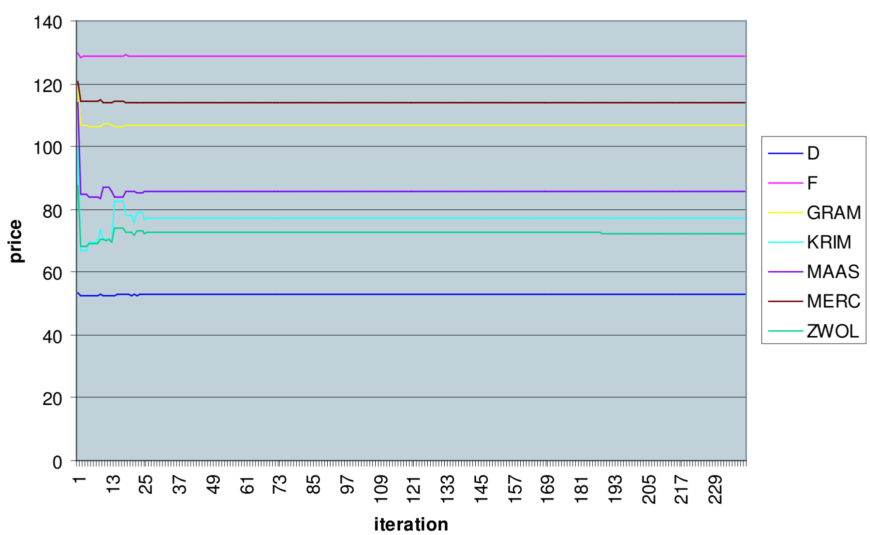

Figure 7: Prices after each iteration

design. In our examples prices are always higher in a separated market design. The lowest impact of integrating energy and transmission market is on price levels in France because transmission constraints are binding from both neighbouring countries represented in our model, Germany and Belgium. The flexibility of allocating transmission capacity provided by the integrated market design only allows limited readjustment and therefore only little additional demand responsiveness. By contrast, the nodes in the Benelux countries have large price changes because they are located in the middle of a meshed network.

Note, that the Netherlands are represented as three separate zones and in the separated market approach generators compete in small markets. In the integrated market approach a higher zonal resolution does not exclude competition with generators in neighbouring zones.

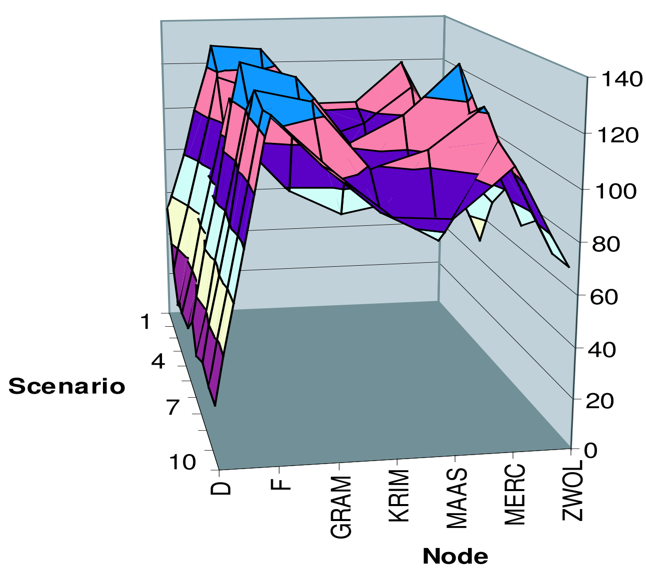

Figure 8: Price levels in Euro

6 Conclusions

Two types of market design exist for the allocation of transmission capacity in meshed electricity networks. In the separated energy and transmission market, physical transmission rights are allocated in a coordinated auction. Trading electric energy between regions requires the ownership of transmission rights.

In the integrated energy and transmission market, energy is traded locally and a system operator schedules energy flows between regions. Financial transmission contracts facilitate trading between regions.

In competitive markets without uncertainty, both designs produce identical market outcomes. If generators act strategically, integrating energy and transmission markets effectively induces demand elasticity. This should reduce the ability of strategic generators to exercise market power, and should therefore reduce prices. However, if companies own

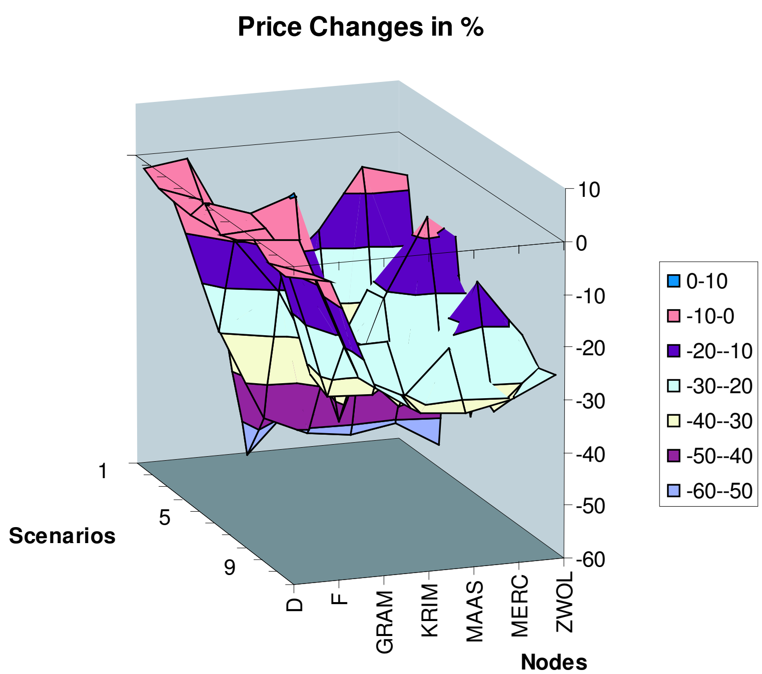

Figure 9: At all locations and scenarios prices reduce after integrating energy and transmission markets.

generation facilities at several nodes, integration also provides an incentive to increase the exercise of market power. The balance of these effects could not be determined analytically for realistic networks. We therefore implemented the two market designs as an Equilibrium Problem with Equilibrium Constraints in the modelling package GAMS. We applied our models to data representing the Benelux situation. Comparing the resulting prices, we observed that the ability of the integrated market design to 'import net demand elasticity' dominates and that prices were always lower in our test scenarios in the integrated market design. So far we have only ensured that bidding strategies of generators are stationary points, but have not examined whether finite deviations are profitable.

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