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A COMPARISON OF GENETIC ALGORITHMS, PARTICLE SWARM OPTIMIZATION AND THE DIFFERENTIAL EVOLUTION METHOD FOR THE DESIGN OF SCANNABLE CIRCULAR ANTENNA ARRAYS

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Abstract—A comparison between different modern population based optimization methods applied to the design of scannable circular antenna arrays is presented in this paper. This design of scannable circular arrays considers the optimization of the amplitude and phase excitations across the antenna elements to operate with optimal performance in the whole azimuth plane (360°). Simulation results for scannable circular arrays with the amplitude and phase excitation optimized by genetic algorithms, particle swarm optimization and the differential evolution method are provided. Furthermore, in order to set which design case could provide a better performance in terms of the side lobe level and the directivity, a comparative analysis of the performance of the optimized designs with the case of conventional progressive phase excitation is achieved. Simulation results show that

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differential evolution and particle swarm optimization have similar performances and both of them had better performance compared to genetic algorithms when all algorithms are allowed equal computation time.

1. INTRODUCTION

Modern optimization techniques have aroused great interest among the scientific and technical community in a wide variety of fields recently, because of their ability to solve problems with a non-linear and non-convex dependence of design parameters.

Several new optimization techniques have emerged in the past two decades that mimic biological evolution, or the way biological entities communicate in nature. Some of these algorithms have been used successfully in many electromagnetism and antenna problems with many constraints and non-linear processes. The most representative algorithms include Genetic Algorithms (GA) [1–5], Particle Swarm Optimization (PSO) [11–13], and the method of Differential Evolution (DE) [19–21]. In the electromagnetism research area, the use of these optimizers is most popular for antenna synthesis. To date, different GA, PSO and DE algorithms have been successfully applied to different problems including antenna design [1, 13, 19] and the array synthesis.

In this paper, a comparison of GA, PSO and DE for the design of scannable circular antenna arrays is presented. The purpose and contribution of this paper is to present a comparative evaluation of GA, PSO, and DE in the performance to design scannable circular antenna arrays. In this case, we study the behavior of the array factor for the design of scannable circular arrays considering the optimization of the amplitude and phase excitations across the antenna elements, for a maximum performance in terms of the side lobe level and the directivity in a scanning range of [0°, 360°].

To the best of the authors knowledge no performance comparison of GA, PSO, and DE applied to design of scannable circular arrays, has been presented previously. The performance of a circular array could be improved substantially, with respect to the circular array with the conventional progressive phase excitation, if the amplitude and phase excitations are set or optimized in an adequate way. Depending on the performance improvement that we could get (in terms of the side lobe level and the directivity), this information could be of interest to antenna designers.

The remainder of the paper is organized as follows. Section 2 states the problem of designing scannable circular arrays and a description of the objective function used by the evolutionary algorithms. A short description of the evolutionary algorithms is included in Section 3, and in Section 4 a performance comparison between the algorithms for the design of scannable circular arrays is illustrated. Finally, the summary and conclusions of this work are presented in Section 5.

2. DESIGN OF SCANNABLE CIRCULAR ANTENNA ARRAYS

In the application of evolutionary optimization techniques for designing antenna arrays, it has been considered the design of different phased array structures. Among antenna array configurations, the phased linear array is the most common form employed in cellular and personal communication systems (PCS) [22]. However, 360° scanning of the radiation beam can be obtained by combining a few linear arrays whose sector scans add to give the desired 360° scan. This could result in objectionably high costs, i.e., the array cost, the control complexity, and the data processing load are increased. Unlike the linear array, the performance of the circular array has not been extensively studied.

In the study of circular arrays, it has been considered the pattern analysis [23], and the pattern synthesis [24–26] of arrays with uniform separation. The design of non-uniform circular arrays optimizing a single objective for side lobe reduction is dealt with in [27]. A closely related work is the one proposed by Su and Ling [28] where the array beamforming problem in the presence of obstacles is dealt with using a genetic algorithm. They solve the design problem by optimizing the current excitation and the array's elements positions. Here we optimize the excitation current amplitude and phase perturbations. In their approach they need to fix a desired radiation pattern where as our approach does not need to provide such a desired pattern since it searches for the optimum unknown one. Next, the problem of designing scannable circular arrays is formulated.

Consider a circular antenna array of N antenna elements uniformly spaced on a circle of radius a in the x-y plane. The array factor for the circular array shown in Figure 1, considering the center of the circle as the phase reference, is given by [29]

$$AF(\phi, \mathbf{I}) = \sum_{n=1}^{N} I_n \exp\left[jka\left(\cos(\phi - \Delta\phi_n) - \cos(\phi_0 - \Delta\phi_n)\right)\right] \quad (1)$$

where $\Delta\phi_n = 2\pi(n-1)/N$ for $n = 1, 2, \ldots, N$ is the angular position of the nth element on the x-y plane, $ka = Nd$, i.e., $a = Nd\lambda/(2\pi)$, $\mathbf{I} = [I_1, I_2, \ldots, I_N]$, I_n represents the amplitude excitation of the nth

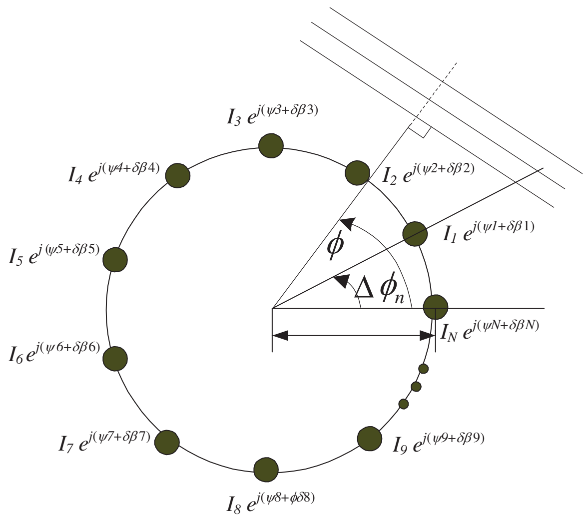


Figure 1. Array geometry for an N element uniform circular array with inter-element spacing d.

element of the array, ϕ_0 is the direction of maximum radiation and ϕ is the angle of incidence of the plane wave.

In this case, the array factor with phase excitation is created by adding in the appropriate element phase perturbations, $\mathbf{P} = [\delta\beta_1, \delta\beta_2, \ldots, \delta\beta_N]$, $\delta\beta_i$ represents the phase perturbation of the ith element of the array, such that

$$AF\left(\phi, \mathbf{I}, \mathbf{P}\right) = \sum_{n=1}^{N} I_n \exp\left\{j\left[\varphi_n + \delta\beta_n\right]\right\} \tag{2}$$

where $\varphi_n = ka[\cos(\phi - \Delta\phi_n) - \cos(\phi_0 - \Delta\phi_n)]$.

The idea of adding perturbations in the conventional array factor is that the optimization algorithm searches possible optimal phase excitations in angles near the direction of desired maximum gain. The optimization process developed in this paper for generating arrays that have radiation patterns with maximum performance in terms of the side lobe level and the directivity will be based on (2).

It is important to mention that as the center of the circle is taken as the phase reference in the array factor, it is considered a symmetrical excitation for the optimization process, i.e., the excitation would be given in the next way $I_1\delta\beta_1, \ldots, I_{N/2}\delta\beta_{N/2}$, $I_{N/2+1}\delta\beta_{N/2+1} = conj(I_1\delta\beta_1)$, $\ldots$, $I_N\delta\beta_N = conj(I_{N/2}\delta\beta_{N/2})$. Note that we will have $N/2$ amplitude and phase excitations.

The design problem is formulated as to minimize the next objective function

$$\begin{aligned} Of &= \left(\left|AF\left(\phi_{SLL}, \phi_0, \mathbf{I}, \mathbf{P}\right)\right| / \max\left|AF\left(\phi, \phi_0, \mathbf{I}, \mathbf{P}\right)\right|\right) \\ &\quad + (1/DIR(\phi, \phi_0, \mathbf{I}, \mathbf{P})) + \left|\phi_0 - \phi_0^{ind}\right| \end{aligned} \tag{3}$$

where ϕ_{SLL} is the angle where the maximum side lobe is attained, i.e., the first component evaluates the side lobe level (SLL) of the array factor generated, DIR the directivity of the array factor, ϕ_0^{ind} is the direction of the main beam for the array factor generated by an individual or a possible solution, and ϕ_0 could be considered as the desired direction of maximum radiation for the array factor. In this case, it is considered the behavior of the array factor for the scanning range of $-180° \leq \phi_0 \leq 180°$ with an angular step of 30°. It must be noted that if we select the number of antenna elements as $N = 12$ and there is a constant angular separation of 30° between different beam steering angles, the optimal excitation for a particular beam steering angle should also apply to other beam steering angles, by simply substituting (I_n, $\delta\beta_n$) into (I_{n+1}, $\delta\beta_{n+1}$) when the beam steering angle is increased by 30°. As a result, only one optimization is needed for all beam steering angles instead of the 12 trials to cover 360°. The rationale behind the use of metaheuristics to solve this optimization problem has to do with the nonlinearity and the many local optima of the objective function given by (3). Furthermore these approaches do not have any restriction in cases where we need to increase the complexity of this objective function.

The next section presents the evolutionary optimization algorithms to be evaluated when they are applied to this design problem.

3. THE EVOLUTIONARY OPTIMIZATION ALGORITHMS

As already being pointed out, the objective of this paper is to present a comparative evaluation of GA, PSO, and DE for the design of scannable circular arrays. Therefore, the main characteristics and the procedure for each algorithm are described in the next subsections.

3.1. Genetic Algorithms

By analogy with natural selection and evolution, in classical GA the set of parameters to be optimized (genes) defines an individual or potential solution X (chromosome) and a set of individuals makes up the population, which is evolved by means of the selection, crossover, and mutation genetic operators. The optimization process used by the GA follows the next steps.

The genetic algorithm generates individuals (amplitude excitations and phase perturbations of the antenna elements). The individuals are encoded in a vector of real numbers, that represents the amplitudes, and a vector of real numbers restrained on the range (0, 2π), that represents the phase perturbations of the antenna elements.

Each individual generates an array factor of certain characteristics of the side lobe level and the directivity. Then, the genetic mechanisms of crossover, survival and mutation are used to obtain better and better solutions. The genetic algorithm evolves the individuals to a global solution that generates an array factor with minimum side lobe level and maximum directivity in the steering direction.

3.2. Particle Swarm

One of the main drawbacks of the previous evolutionary algorithm is their lack of memory, which limits the search and convergence ability of the algorithms. In GA, the concept of memory relies on elitism, but there is no stronger operator to propagate accurate solutions in a faster way. However, the PSO algorithm emerges as a powerful stochastic optimization method inspired by the social behavior of organisms such as bird flocking or fish schooling, in which individuals have memory and cooperate to move towards a region containing the global or a near-optimal solution.

Particles within the swarm move influenced by its current position, its memory and by the cooperation or social knowledge of the swarm [30], using only one operator, the so-called velocity operator. Let us suppose a swarm of K particles, in which each particle $X_K = (x_{k1}, \ldots, x_{kD})$ representing a potential solution (amplitude excitations and phase perturbations of the antenna elements) is defined as a point in a D-dimensional space. The limits of the parameters x_{kd} to be optimized define the search space in D-dimensions. Iteratively, each particle k within the swarm flies over the solution space to a new position X_K with a velocity $V_K = (v_{k1}, \ldots, v_{kD})$, both updated along each dimension d, by the following:

$$v_{kd} = wv_{kd} + c_1 r_1 (pbest_{k,d} - x_{kd}) + c_2 r_2 (gbest_d - x_{kd}), \; v_k \leq v_{d,\max} \forall d \quad (4)$$

$$x_{kd} = x_{kd} + v_{kd} \Delta t \quad (5)$$

where w is known as the inertial weight, c_1 and c_2 are the acceleration constants and determine how much the particle is influenced by its best location (usually referred as memory, nostalgia or self-knowledge) and by the best position ever found by the swarm (often called shared information, cooperation or social knowledge), respectively. Moreover, r_1 and r_2 represent two separate calls to a random number function $\mathbf{U}[0,1]$, $v_{d,\max}$ is the maximum allowed velocity for each particle used as a constraint to control the exploration ability of the swarm and usually set to the dynamic range of each dimension [31], and Δt is a time-step usually chosen to be one. The detailed interpretations of these step terms may be found in [30].

In short, the PSO algorithm requires fewer lines of code than GA and is easier to implement. Another advantage of PSO against GA is the small number of parameters to be tuned. In PSO, the population size, the inertial weight and the acceleration constants summarize the parameters to be selected and tuned, whereas in GA the population size, the selection, crossover and mutation strategies, as well as the crossover and mutation rates influence the results.

3.3. Differential Evolution

One of the latest evolutionary computational techniques apart from the PSO is the differential evolution (DE) algorithm, in which, some individuals are randomly extracted from the solution population and geometrically manipulated [20], avoiding the destructive mutation of GA. The most prominent advantage of DE is its low computation time compared to that of GA, particularly in large antenna arrays. DE is an alternative to speed up the GA. Instead of small alterations of genes in GA mutation, DE mutation is performed by means of combinations of individuals [20].

First an initial population is formed in which the chromosomes have a Gaussian distribution. For each vector or solutions of the population $(N_p)X_i$, $i = 1, 2, \ldots, N_p$ of the G_{th} iteration, two new trial members, ε_{t1} and $\varepsilon_{t2,}$ are generated as follows:

$$\varepsilon_{t1} = \varepsilon_{r1}^{(G)} + F\left(X_i^{(G)} - \varepsilon_{r2}^{(G)}\right) \tag{6}$$

$$\varepsilon_{t2} = \varepsilon_{r1}^{(G)} + F\left(X_i^{(G)} - \varepsilon_{r3}^{(G)}\right) \tag{7}$$

where $F \in [0, 2]$ is a real constant factor range suggested in [21], which controls the amplification of the differential variation, and the integers $r_1, r_2, r_3 \in [1, N_p]$ are randomly chosen such that $r_1 \neq r_2 \neq r_3$. After the objective function evaluation, the best solution in the set $\{\varepsilon_i, \varepsilon_{t1}, \varepsilon_{t2}\}$ becomes the new member for the next iteration, ε_i^{G+1}. Some chromosomes in the new population occasionally generate array factors which are not physically realizable, and an adjusting process is needed [32]. Taking the best solution into account, a termination criterion is proposed by fixing a number of iterations without an improvement over this solution. Storn and Prince [33] explain the procedures involved at each step of this algorithm in detail.

The results of using these evolutionary algorithms for the design of scannable circular arrays are described in the next section.

4. EXPERIMENTAL SETUP AND RESULTS

The methods of GA, PSO and DE were implemented to study the behavior of the array factor for the scanning range of $0° \leq \phi_0 \leq 360°$ with an angular step of 30°. The number of antenna elements was set as $N = 12$, for a uniform separation of $d = 0.5\lambda$. In this case, we follow the literature and our previous results to set the parameters for each algorithm in attempt to make a fair comparison. We also give all algorithms the same computation time with equal computational resources.

In the case of PSO, we have set $c_1 = c_2 = 2.0$ suggested by [34] and [35] for the sake of convergence. To further accelerate the convergence, a time-varying inertial weight, w, is utilized and varies from 0.9 at the beginning to 0.4 toward the end of the optimization [36]. Researchers have found [34–36] that for an n-dimensional problem, the number of agents or particles should be at least comparable to n. In our case n is 24, 12 excitation amplitudes and 12 phase perturbations. In this case, the optimization is executed using a number of 150 agents for 1000 iterations to provide a better sampling of the solution space. The value of $v_{d,\max}$ in (4) is set as $0.9r$ where r is the difference between the maximum and minimum each decision variable can achieve.

For the case of GA and DE, we have set the proposed parameters based mainly on our previous experience in solving similar problems [37, 38]. The population size for DE is set to 50 while that of GA was set to 148 in order to have similar computation time to those of PSO and DE. Two point crossover along with standard single point mutation and ranking selection are used. In the DE the value of F in (6) and (7) is set to 0.5. The stopping criterion is, in both algorithms, 1000 iterations.

Now, the comparison of GA, PSO and DE is conducted in a statistical manner. Since the distribution of the best objective function values do not follow a normal distribution the Wilcoxon two-sided rank sum test [39] was used to compare the objective function mean values. Each algorithm was executed 50 times and the best result for each run is considered.

Tables 1–2 show the comparison results among the means of solution's quality for each pair of algorithms, we can say that at a

99% confidence level the means of PSO and GA as well as those of GA and DE are statistically different. However, PSO and DE are not. Based on these results we can conjecture that the standard GA performs poorly incomparison to newer approaches like PSO or DE.

Figure 2 illustrates the best array factor obtained in the 50 trials by a) GA, b) PSO and c) DE. The numerical values of the side lobe level, directivity, amplitude and phase perturbation distributions for the array factor shown in Figure 2 are presented in the Table 3.

As illustrated in the Figure 2 and the Table 3, the methods of particle swarm optimization and differential evolution presents a better performance in terms of the side lobe level with respect to genetic algorithms, maintaining very similar values for the directivity in the

Table 1. Mean value of the objective function for each algorithm obtained with 50 trials.

Algorithm	**Objective function mean value**
GA	0.8076
PSO	0.7557
DE	0.7601

Table 2. P value for the Wilcoxon two-sided rank sum test for the comparison of means.

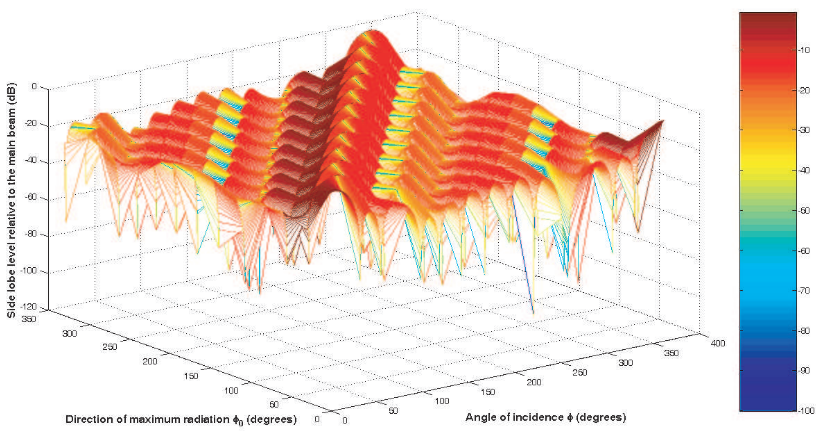
Comparison pair	**P-value**
PSO/DE	0.7854
PSO/GA	2.09 EXP-7
GA/DE	7.4477 EXP-10

Table 3. Numerical values of the side lobe level (SLL), directivity (DIR), amplitude and phase perturbation distribution for the array factor illustrated in Figure 2.

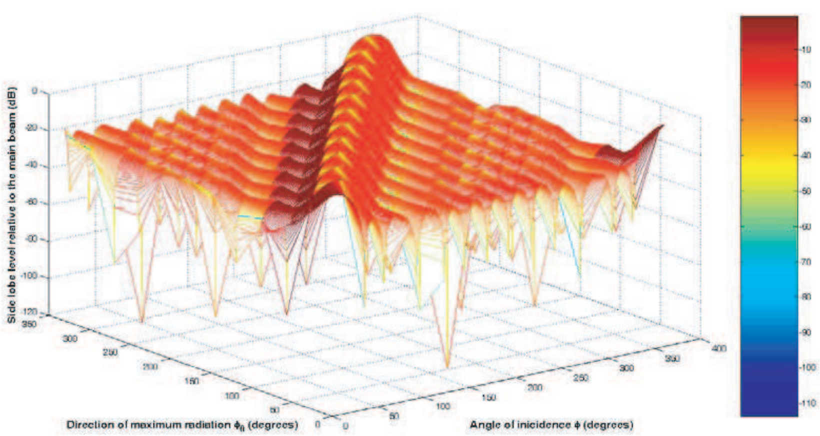
Design case	***SLL* (dB)**	***DIR* (dB)**	**Amplitude Distribution**	**Phase perturbation distribution (deg)**
GA	–14.22	11.48	9.0210, 6.8639, 6.3014, 6.7600, 8.5353, 13.9554	19.7432, –85.164, 5.3822, 79.2396, –23.966, 28.7819
PSO	–18.19	11.27	8.2588, 10.3593, 7.8372, 12.3624, 8.7257, 9.6441	–159.3256, 96.52, –177.3974, –99.7824, 160.56, –150.6
DE	–18.91	11.26	7.3104, 12.0239, 8.4663, 8.2479, 11.0431, 8.7574	–20.7692, 81.3383, 0.3238, –81.6377, 20.5736, –29.2867
Conventional case	–7.167	10.65	1, 1, 1, 1, 1, 1	0, 0, 0, 0, 0, 0

performance of these three optimization methods. In this case, it is observed a reduction in the side lobe level for DE and PSO of 4.6 dB and 4.0 dB with respect to GA.

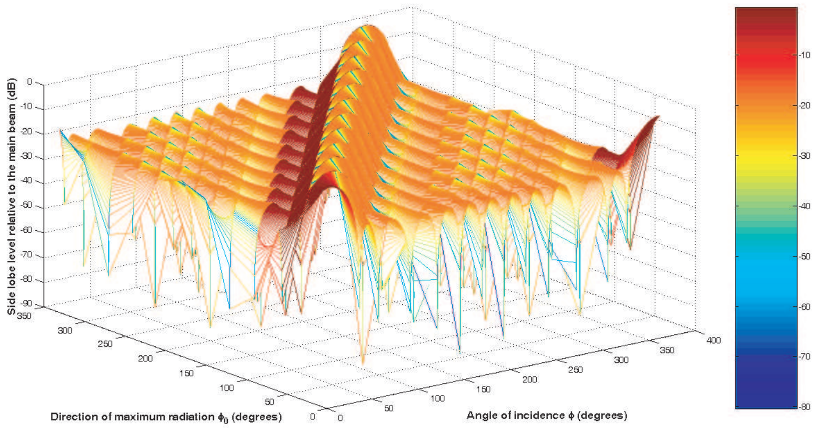
Now, if the optimization results of the three methods are compared with the conventional case of progressive phase excitation, the results of the side lobe level and the directivity for the optimized designs are really surprising. Figure 3 shows a comparison of the array factor for the conventional case and the optimized designs by GA, PSO and DE in the cut of $\phi_0 = 180°$. Observing the results, the conventional case of progressive phase excitation provides a $SLL = -7.16$ dB, and $DIR = 10.6$ dB. For the case of the optimized designs, it is obtained a



(a) GA



(b) PSO



(c) DE

Figure 2. The best array factor obtained in the 50 trials by GA, PSO and DE.

$SLL = -14.22$ dB and $DIR = 11.48$ dB for GA, a $SLL = -18.19$ dB and $DIR = 11.27$ dB for PSO, and a $SLL = -18.91$ dB and $DIR = 11.26$ dB for DE.

These values mean a substantial improvement in the performance of the array for the designs optimized by the methods of GA, PSO

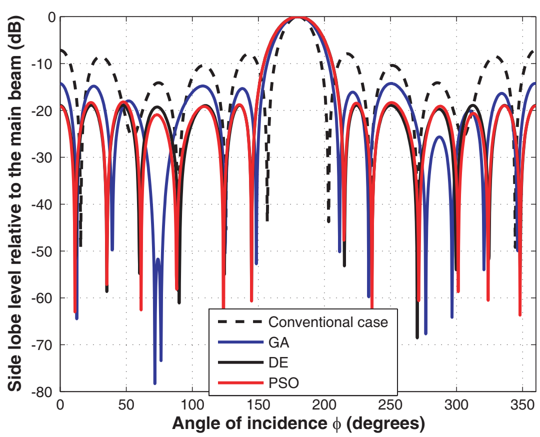


Figure 3. Comparison of the array factor for the conventional case and the optimized designs by GA, PSO and DE in $\phi_0 = 180°$.

and DE, with respect to the conventional case of progressive phase excitation, i.e., a substantial improvement was obtained in the sense of the side lobe level and an improvement of about 1 dB in the directivity, maintaining the same scanning range and the same aperture. The three evolutionary optimization methods efficiently computes a set of antenna element amplitude and phase excitations in order to provide a radiation pattern with maximum performance in the side lobe level and the directivity in the entire scanning range.

Notice that the results from the different methods are not the same, this is mainly because the algorithms do not guarantee convergence to the global optimum in finite time.

5. CONCLUSIONS

This paper illustrates how to model the design of scannable circular arrays with the optimization of the amplitude and phase excitations for improving the performance of the array in the sense of the side lobe level and the directivity considering a scanning range of [0°, 360°]. In this design problem, a performance comparison of three evolutionary optimization algorithms was achieved. The obtained results illustrates that the methods of differential evolution and particle swarm optimization present a better performance in terms of the side lobe level with respect to the genetic algorithms under equal computation time. Furthermore, the results illustrated that the optimization of the array could provide a substantial improvement in the side lobe level and an improvement of about 1 dB in the directivity, with respect to the conventional case of progressive phase excitation. These improvements in the performance of the array are achieved maintaining the same scanning range, i.e., in all azimuth plane (360°), and the same aperture.

Future work is aimed at studying the robustness of these methods when different numbers of antenna elements and other design variables are taken into account.

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