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A COMPARISON OF MODELS FOR DUCTILE FRACTURE PREDICTION IN FORGING PROCESSES

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Abstract

The possibility of predicting ductile fracture plays an important role in the design of components by forging processes. Experimental observations showed that the nucleation, growth and coalescence of voids are the mechanisms that control the initiation and propagation of fracture and that these mechanisms are influenced in different ways by factors like the hydrostatic stress, the equivalent stress or by the maximal principal stress. Many ductile fracture indicators, based on some or all of those factors, are available and used in many practical situations in the design of those components. In this work a comparative work of many of those criteria was undertaken. Different criteria were chosen amongst the more popular ones and from different groups, in which they may be classified, namely those based on micromechanics and those based on the geometry of voids or their growth mechanisms. The criteria based on the Continuous Damage Mechanics, in which a coupling between plastic deformation and material degradation is taken into account and that include different damage evolution descriptions for traction or compressive stress states, give a more correct and clear localization for the fracture initiation site.

Key words: Ductile fracture indicators, forging processes

1. INTRODUCTION

The possibility of anticipating the development of ductile fracture is an important issue in the design of parts obtained in forging processes. In particular the possibility of including reliable models in the numerical simulation of these processes that can predict the occurrence of ductile failure is of utmost importance as computer modelling plays a decisive role in the design, optimization and innovation of forging processes, involving more and more complex strain paths.

To reach that goal with success besides the obvious need of an efficient and reliable numerical model it is also necessary the adequate choice for a ductile fracture model. The utilisation of ductile fracture models was initially suggested, many decades ago, when a criterion based on the total plastic work was formulated by Freudenthal (1950). Some years later Kachanov (1958) proposed an alternative which has evolved to what is known nowadays as the Continuous Damage Mechanics. Since then many criteria have been put forward in one or the other of those two directions. In this work a comparison of some of those different criteria is performed. In particular a ductile fracture criterion based on the Lemaitre´s damage model and on previous work of the present authors in this field is shown to behave better than the others criteria.

2. DUCTILE FRACTURE INDICATORS

Ductile damage criteria intend to describe at the macro or meso scale the effect of phenomena occurring at the microscopic level, whether with the re-

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course of experimental data or through physical/mathematical models. Generally it is accepted that those criteria should take into account:

- the deformation path, because the current stress/strain state is not enough to characterise the damage state, (Cockcroft and Latham, 1968; Norris *et al.*, 1978; Atkins, 1981; Atkins and Mai, 1985);
- the hydrostatic stress, σ_H, because ductility grows rapidly as σ_H decreases, (Hancock and Mackenzie, 1976; Norris *et al.*, 1978; Oyane *et al.*, 1978; Lemaître, 1985; Mudry, 1985; Tai and Yang, 1987);
- an adequate ratio of stresses, namely the triaxiality stress ratio, σ_H / σ_{eq}, in which σ_{eq} is the equivalent stress, so that the general state of plasticity and fracture may be better described, (Mudry, 1985; Hancock & Mackenzie, 1976).

Therefore, a ductile fracture criterion could be expressed in a general form as:

$$I_\Diamond = \int_0^{\bar{\varepsilon}^{\mathrm{P}}} \left\langle \Diamond \left(\sigma_H, \sigma_{eq}, \ldots \right) \right\rangle \mathrm{d}\bar{\varepsilon}^{\mathrm{p}} \qquad (1)$$

where $I_\Diamond$ represents the fracture indicator, $\left\langle \Diamond \left(\sigma_H, \sigma_{eq}, \ldots \right) \right\rangle$ represents a certain fracture criterion, $\bar{\varepsilon}^{\mathrm{p}}$ is the equivalent plastic deformation and $\langle . \rangle$ indicates that the integration in only made on the positive component of the integrating function.

The ductile fracture criteria may be classified in two big groups, namely those based on micromechanics and those based on the growth of defects. Some of those criteria are briefly described in the next sections.

2.1. Criteria based on micromechanics

The criterion of total plastic work, also known as Freudenthal's fracture criterion (Freudenthal, 1950; Gillemont, 1976), postulates that the initiation and propagation of a crack is dominated by a critical value of the absorbed plastic energy:

$$I_{wp} = W_{\mathrm{p}} = \int_0^{\bar{\varepsilon}^{\mathrm{pf}}} \sigma_{eq} \, \mathrm{d}\bar{\varepsilon}^{\mathrm{p}} \qquad (2)$$

where W_{p} is the specific total plastic work, $\bar{\varepsilon}^{\mathrm{p}}$ is the equivalent plastic strain and $\bar{\varepsilon}^{\mathrm{pf}}$ is the equivalent plastic strain at fracture.

The criterion of maximum plastic shear work was proposed in an attempt to reproduce the mechanism of chip formation in orthogonal metal cutting. Most of the theoretical models (Ernst and Merchant, 1941; Lee and Shaffer, 1951) assume that the large plastic deformations take place along shear planes which may constitute the main phenomenon for chip formation. The criterion is given by

$$I_\gamma = \int_0^{\gamma^{\mathrm{pf}}} \tau_{\max}^{\mathrm{xy}} \, \mathrm{d}\gamma_{\max}^{\mathrm{p\ xy}} \qquad (3)$$

where $\tau_{\max}^{\mathrm{xy}}$ is the maximum shear stress in plane xy, $\gamma_{\max}^{\mathrm{p\ xy}}$ is the maximum plastic shear strain (in plane xy) and γ^{pf} is the maximum plastic shear strain at fracture.

The criterion of equivalent plastic strain was proposed by Datsko (1966) and assumes that fracture is initiated when the equivalent plastic strain reaches a critical value as:

$$I_{\bar{\varepsilon}^{\mathrm{p}}} = \int_0^{\bar{\varepsilon}^{\mathrm{pf}}} \mathrm{d}\bar{\varepsilon}^{\mathrm{p}} = \bar{\varepsilon}^{\mathrm{pf}} \qquad (4)$$

where $\bar{\varepsilon}^{\mathrm{pf}}$ is the equivalent plastic strain at fracture.

2.2. Criteria based on models based on the growth of defects

The mechanism of nucleation, growth and coalescence of voids is commonly accepted to be the reason for ductile fracture. The criteria inspired on that assumption may be based on different physical aspects, like the geometry of the voids, on the void grow mechanism or in constitutive material models. Some of those criteria will be referred next.

2.2.1. Geometry of defects

In the criterion proposed by McClintock (1968) the material is assumed to be divided in quadrilateral elements containing elliptical cylindrical voids. The criterion may be expressed by

$$I_b = \int_0^{\bar{\varepsilon}^{\mathrm{pf}}} \left\{ \frac{\sqrt{3}}{2(1\text{-n})} \sinh\left[\frac{\sqrt{3}}{2} (1-\mathrm{n}) \frac{\sigma_a + \sigma_b}{\sigma_{eq}} \right] + \frac{3}{4} \left(\frac{\sigma_a - \sigma_b}{\sigma_{eq}} \right) \right\} \mathrm{d}\bar{\varepsilon}^{\mathrm{p}} \qquad (5)$$

where n is the exponent in the constitutive law $\sigma_y = \sigma_{y0} \left(\varepsilon_0 + \varepsilon^{\mathrm{P}} \right)^{\mathrm{n}}$ and σ_a, σ_b may be taken as the principal stresses.

Rice and Tracey (1969) established a criterion based on the analysis of the growth of spherical voids in a triaxiality stress state which is stated as

$$I_R = \int_0^{\bar{\varepsilon}^{pf}} 0.283 \exp\left(\frac{\sqrt{3}}{2}\frac{\sigma_H}{\sigma_{eq}}\right) d\bar{\varepsilon}^p \quad (6)$$

and was reported to behave well in metal cutting processes.

2.2.2. Growth mechanism

The criterion proposed by Cockcroft and Latham (1968) assumes that the maximum principal stress is the most relevant in the initiation of fracture. This criterion is therefore defined in terms of traction plastic work associated to the principal stress along the path of the equivalent plastic strain as:

$$I_{\sigma_1} = \int_0^{\bar{\varepsilon}^{pf}} \sigma_1 \, d\bar{\varepsilon}^p \quad (7)$$

Later Brozzo, De Luca and Rendina (1972), based on the evidence that ductility diminishes with the hydrostatic stress, included in the previous criterion an explicit dependence on σ_H as

$$I_{\sigma_1,\sigma_H} = \int_0^{\bar{\varepsilon}^{pf}} \frac{2\sigma_1}{3(\sigma_1 - \sigma_H)} d\bar{\varepsilon}^p \quad (8)$$

Norris *et al.* (1978) proposed an empirical criterion based only on the hydrostatic stress as

$$I_{\sigma_H} = \int_0^{\bar{\varepsilon}^{pf}} \frac{1}{(1\text{-}c_N \sigma_H)} d\bar{\varepsilon}^p \quad (9)$$

where c_N is a material constant. The authors claimed that the fracture indicator could be, if conveniently calibrated, used as a measure of fracture toughness.

Having verified that the previous criterion did not properly describe fracture in deep drawing and forging Atkins (1981) introduced an explicit dependence on the deformation path as

$$I_L = \int_0^{\bar{\varepsilon}^{pf}} \frac{1 + 1/2L}{(1\text{-}c_A \sigma_H)} d\bar{\varepsilon}^p \quad (10)$$

where c_A is a material parameter and L is the ratio between the maximum and minimum plastic strain increments.

2.2.3. Material behaviour

Oyane *et al.* (1978) developed a material model for metal powders and porous metals which included an explicit dependence of the von Mises yield function on the hydrostatic stress, the apparent density of the porous material and the matrix density. Adapting and extending the basic theory they proposed a criterion for ductile fracture for porous and dense materials (Oyane *et al.*, 1978 and 1980). The model assumes that fracture initiates when the volumetric deformations reach a critical value and takes the final form:

$$I_{\varepsilon_v} = \int_0^{\bar{\varepsilon}^{pf}} \left[1 + \frac{1}{A_{Oy}}\left(\frac{\sigma_H}{\sigma_{eq}}\right)\right] d\bar{\varepsilon}^p \quad (11)$$

where A_{Oy} is a material parameter.

The models based on the theory of Continuous Damage Mechanics may give an important insight into the analysis of fracture initiation and even to its evolution until failure (François, 1985; Murakami, 1990). In many situations even the damage variable has been used as failure indicator (François, 1985; Benallal *et al*., 1989; Murakami, 1990; Min *et al*., etc.). Cescotto and Zhu (1995) concluded that the damage variable is the only criterion that may predict the site of fracture initiation.

Lemaître (1986) has suggested a criterion based on a critical value of the elastic energy release rate that could characterise the fracture initiation as:

$$-Y_c = \frac{\sigma_{eq}^2}{2E(1-D)^2}\left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right] \quad (12)$$

where D is a damage variable and $-Y_c$ is the elastic energy release rate with damage at fracture.

In the same direction Tai and Yang (1987), assuming that the property that controls the initiation and evolution of a crack may be treated as a constitutive property of a material proposed a new fracture criterion based on the nucleation, growth and coalescence of voids as:

$$V_D = \int_{\bar{\varepsilon}^{po}}^{\bar{\varepsilon}^{p}} \left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right] d\bar{\varepsilon}^p \quad (13)$$

where V_D is the critical damage parameter.

The observation of Lemaitre of the adequacy of using an energy measure as a ductile fracture criterion, and the characteristics that such a criterion should have, guided Vaz Jr. (1998) and Vaz Jr *et al.* (2001) to propose a fracture indicator based on the total damage work as

$$I_{WD} = \int_0^t (-Y)\dot{D} \, dt \quad (14)$$

or

$$I_{WD} = \int_0^{D_c} (-Y)\mathrm{d}D = \int_0^{D_c} \frac{\sigma_{eq}^2}{2\mathrm{E}(1-D)^2}\left[\frac{2}{3}(1+\nu)+ 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right]\mathrm{d}D \qquad (15)$$

where I_{WD} represents the critical damage parameter.

It is noteworthy that in this definition, which is consistent with the thermodynamics of irreversible processes, the parameter of energy release rate with damage, $-Y$, contains the representation of the damage state through the variable D, and takes also in consideration the hydrostatic stress through the triaxility factor σ_H / σ_{eq}.

Andrade Pires *et al.* (2001, 2003) and César de Sá *et al.* (2002) extended this criterion in order to take into account the effect of crack closure under compression as:

$$I_{WDN} = \int_0^{D_c} (-Y)\mathrm{d}D =$$

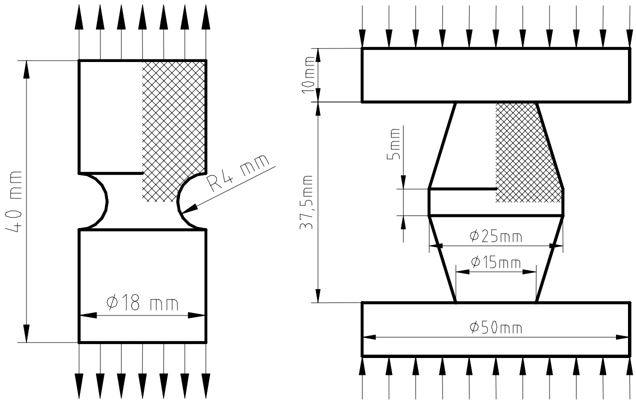
$$\int_0^{D_c} \left\{ \frac{-1}{\mathrm{E}(1\text{-}D)^2}\left[(1+\nu)\langle\boldsymbol{\sigma}\rangle:\langle\boldsymbol{\sigma}\rangle - \nu\langle\mathrm{Tr}\,\boldsymbol{\sigma}\rangle^2\right] - \frac{-h}{\mathrm{E}(1\text{-}hD)^2}\left[(1+\nu)\langle-\boldsymbol{\sigma}\rangle:\langle-\boldsymbol{\sigma}\rangle - \nu\langle\text{-Tr}\,\boldsymbol{\sigma}\rangle^2\right]\right\} c \qquad (16)$$

In this criterion the energy release rate with damage is expressed in terms of principal stresses allowing to treat differently damage evolution for traction or compression stress states by means of a crack closure parameter, h. This criterion, although computationally more expensive, allows for the consideration of more complex strain paths, therefore approximating better the real life forming processes.

3. ASSESSMENT AND COMPARISON OF THE DIFFERENT CRITERIA

Two examples will be used to assess and compare the different fracture indicators described. The geometry data, material properties and experimental conditions may be found in (Andrade Pires *et al.*, 2003). In Figure 1 a schematic representation of each of the two tests is depicted.

The first example refers to a tension test of an axisymmetric notched specimen of an aluminium alloy subjected to monotonic axial stretching. In this example the deformation is highest near the notch where the maximum value of the equivalent plastic takes place but fracture initiates at the centre of the specimen where the stress triaxiality ratio has its maximum value.



(a) (b)

Fig. 1. *Test examples. (a) tension test of an axisymmetric notched specimen; (b) upsetting test of an axisymmetric specimen.*

The second example is the upsetting test of an axisymmetric specimen of a lead alloy reported in Gouveia *et al.* (1996). Here fracture initiation occurs in the external surface near the equator where traction stresses are detected, revealing the importance of treating differently the damage evolution in traction or compression.

The results obtained with the different criteria described in section 2 are presented in the next sections.

3.2. Criteria based on micromechanics

The analysis of the two examples with the criteria based on the total plastic work, on the maximum plastic shear work or on the plastic effective strain revealed them as inadequate and should, therefore, be dismissed as indicators of fracture. In fact in both examples they indicate a wrong localization for ductile fracture initiation as seen in Figures 2 to 4: in the tension test they predict fracture near the notch and in the compression test they predict fracture in the interior of the specimen. These criteria tend to indicate fracture in regions where plastic deformation concentrates, which in many cases do not correspond to what really happens in practice.

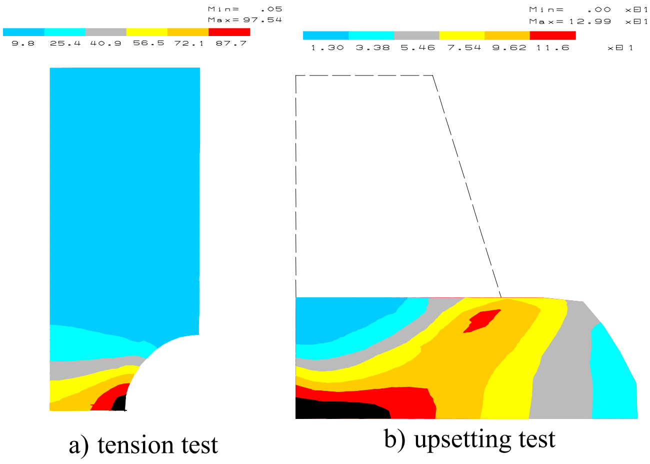


Fig. 2. *Criterion of total plastic work (Freudenthal, 1950; Gillemont, 1976).*

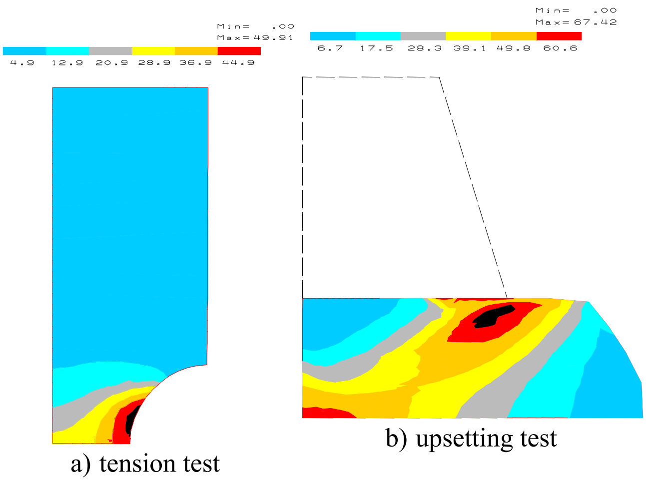


Fig. 3. *Criterion of maximum plastic shear work.*

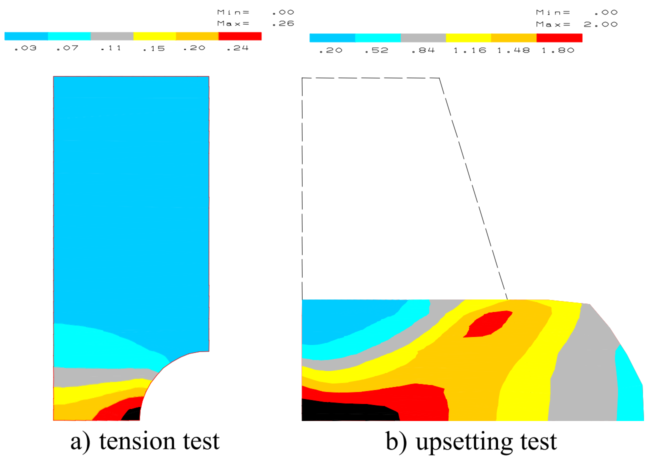


Fig. 4. *Criterion of equivalent plastic strain (Datsko, 1966).*

3.3. Criteria based on models based on the growth of defects

3.2.1. Geometry of defects

The two criteria based on the geometry of defects which were tested (McClintock,1968 and Rice and Tracey, 1969), as seen in Figures 5 and 6, predict the localization of fracture initiation inside the specimen in the tension test but in a very diffuse way, as the fracture zone extends in a large region. For the case of the compression test the criterion of McClintock (1968) shows the same feature as the fracture zone becomes very large, Figure 5, whilst the criterion of Rice and Tracey (1969) wrongly predicts the fracture initiation at the centre of the specimen, Figure 6.

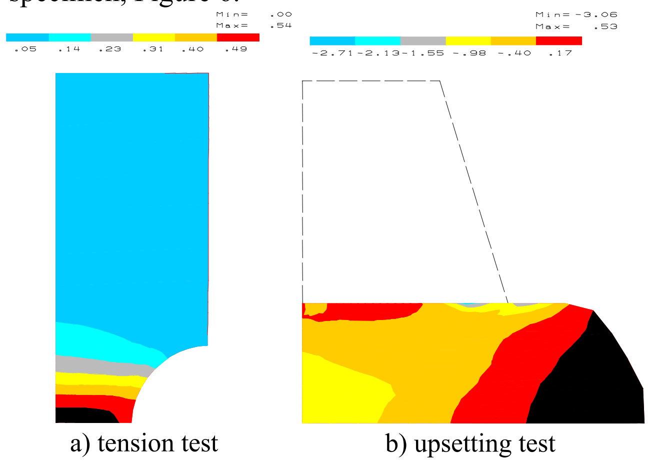


Fig. 5. *Criterion based on the geometry of defects – McClintock (1968).*

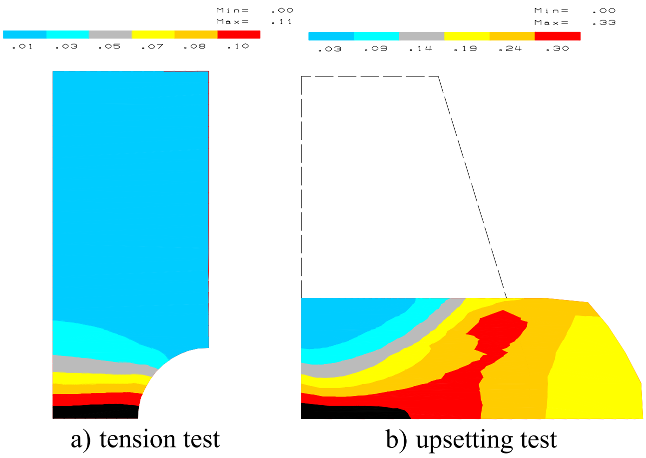
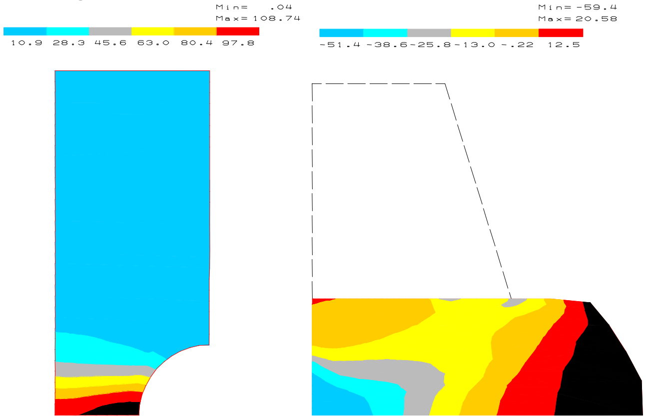


Fig. 6. *Criterion based on the geometry of defects – Rice & Tracey (1969).*

3.2.2. Growth mechanism

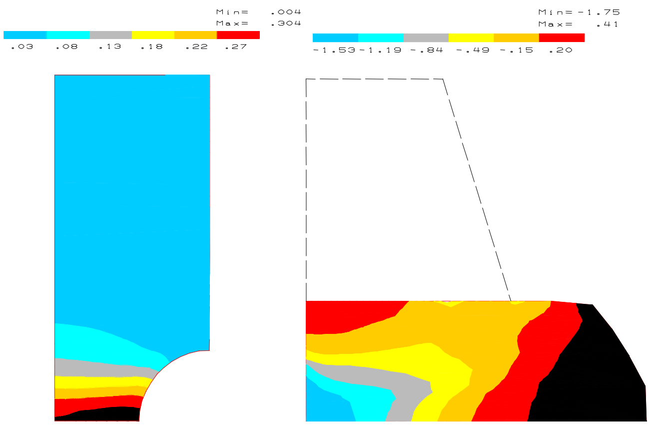
The criteria in which the growth of damage is based on the principal stress (Cockcroft and Latham, 1968, Brozzo *et al.*, 1972), Figures 7 and 8, predict "correctly", although in a very diffusive way, the fracture initiation for the case of the compression test where the principal stress plays an important role, but fail on the tension test where the triaxiality stress state is the main factor at failure.

The criteria based on the hydrostatic stress (Norris *et al.*, 1978, Atkins, 1981) fail in both tests, Figures 9 and 10. This fact indicates that although hydrostatic stress is an important factor for fracture initiation it must not be taken exclusively as a predicting factor.



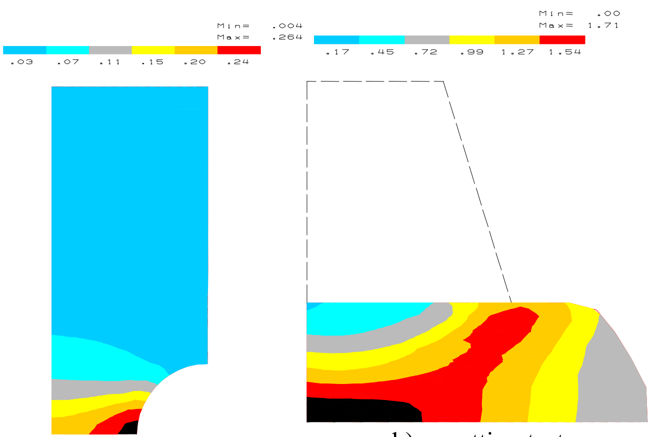
a) tension test **b)** upsetting test

Fig. 7. *Criterion based on the growth of defects due to the maximum principal stress (Cockcroft & Latham, 1968).*



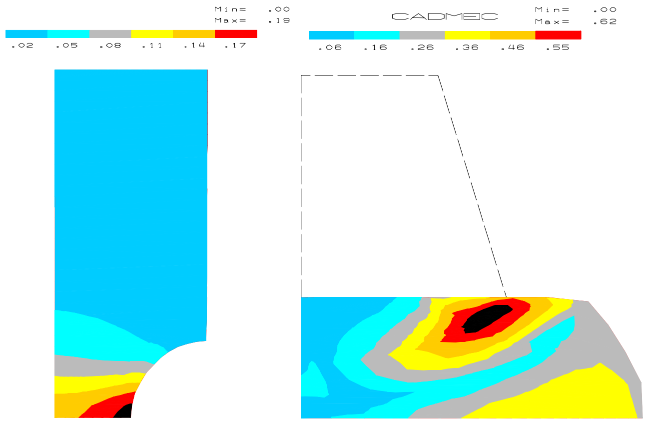
a) tension test **b)** upsetting test

Fig. 8. *Criterion based on the growth of defects due to the maximum principal stress (Brozzo et al., 1972).*



a) tension test b) upsetting test

Fig. 9. *Criterion based on the growth of defects due to hydrostatic stress (Norris et al., 1978).*

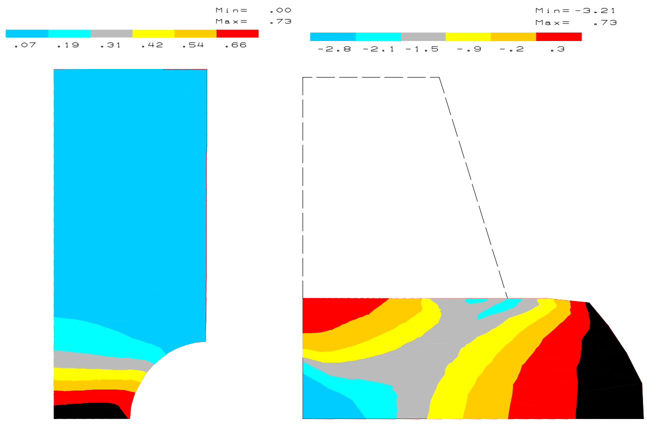


a) tension test b) upsetting test

Fig. 10. *Criterion based on the growth of defects due to hydrostatic stress (Atkins, 1981).*

3.2.3. Material behaviour

The fracture indicator of Oyane *et al.* (1978 and 1980), which is based on the assumption of a constitutive model for porous materials, predicts the region of fracture in both tests but in a very large area as it may seen in Figure 11.

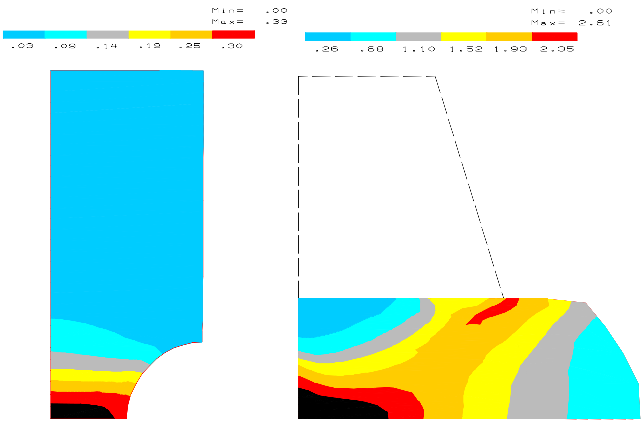


a) tension test b) upsetting test

Fig. 11. *Criterion based on the material model (Oyane et al., 1978 and 1980).*

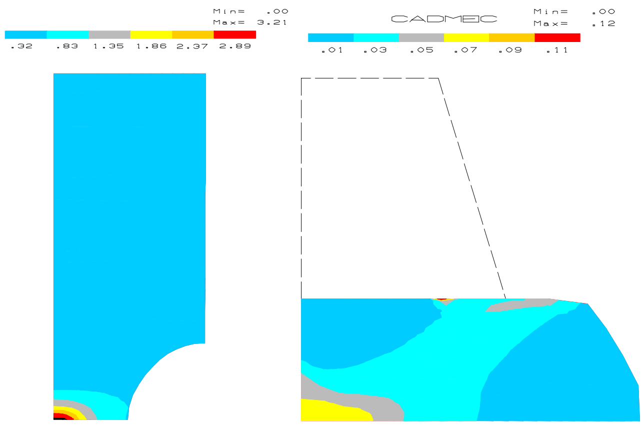
The criterion of Lemaître and Tay and Yang, which is based on the theory of Continuous Damage Mechanics, but with integration on the deformation path, predicts well fracture on the tension test but fails on the compression test as depicted in Figure 12.

The criterion proposed by Vaz Jr. (1998, 2003), which is also based on the theory of Continuous Damage Mechanics and Lemaitre damage model, is nevertheless substantially different from all referred before because the nominal fracture indicator is evaluated over the damage evolution path. Moreover, it couples damage and plastic deformation at the constitutive level. For the case of the tension test it predicts very precisely the fracture initiation site, Figure 13. Nevertheless this promising new idea failed in the evaluation of the fracture initiation for the case of the compression test, Figure 13, where it predicted the fracture inside the specimen.



a) tension test b) upsetting test

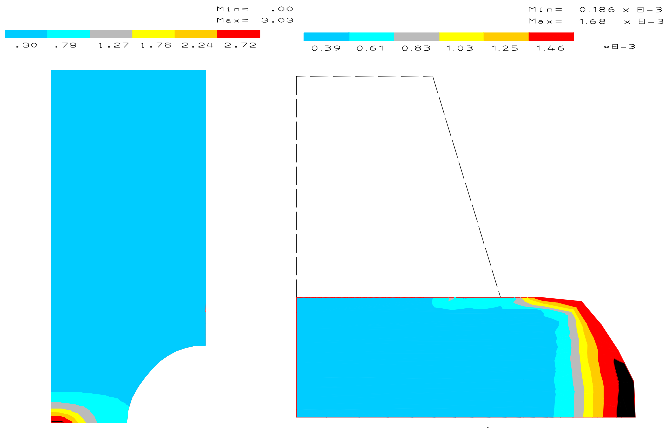
Fig. 12. *Criterion based on Continuous Damage Mechanics (Lemaître, Tay and Yang, 1987).*



a) tension test b) upsetting test

Fig. 13. *Criterion based on Continuous Damage Mechanics (Vaz Jr., 1998, 2001).*

The criterion put forward by Andrade Pires (2001) and César de Sá *et al.* (2002), followed the proposal of Vaz, Jr. (1998) but introduced a slight change in the Lemaitre damage model used, by bringing in a crack closure effect by means of a parameter, *h*, which allows treating differently damage evolution for traction or compression stress states. As it may be seen in Figure 14, in the detection of fracture initiation with this criterion, it is possible to account for the importance of, not only the triaxiality stress state as in the case of the tension test, but also the role played by the principal stress, namely when traction effects are predominant as in the case of the compression test. In both cases this criterion predicts accurately the localization of fracture initiation, as it may be seen in Figure 14.



a) tension test **b)** upsetting test

Fig. 14. *Criterion based on Continuous Damage Mechanics (Andrade Pires, 2001, César de Sá et al, 2002).*

4. CONCLUSIONS

The possibility of predicting ductile fracture plays an important role in the design of components by forging processes. Experimental observations showed that the nucleation, growth and coalescence of voids are the mechanisms that control the initiation and propagation of fracture and that these mechanisms are influenced in different ways by factors like the hydrostatic stress, the equivalent stress or by the maximum principal stress. Many ductile fracture indicators, based on some or all of those factors, are available and used in many practical situations in the design of those components. Most of them are *a-posteriori* criteria, in the sense that they are used after the simulation of a forging process but without taking into account the progressive degradation of the material with the deformation.

In this work a comparative study was made of many of those criteria, using two test examples in which the main factors influencing fracture play different roles. The criteria based on the total plastic work, on the maximum plastic shear work or on the plastic effective strain behaved very poorly in both tests. The *a-posteriori* criteria based on the geometry of voids or on its growth mechanism whether failed in one the tests or showed a very diffused localization of the fracture site. Only the criteria that were based on the theory of Continuous Damage Mechanics, in which damage and deformation are coupled throughout the deformation history, could give a clear localization zone for fracture initiation. In particular only the criterion proposed by Andrade Pires (2001) and César de Sá *et al.* (2002), which is fully based on the criterion proposed by Vaz, Jr (1998) but in which a different damage evolution is assumed for traction or compressive stress states, could give the correct and clear localization for fracture initiation.

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PORÓWNANIE MODELI ZNISZCZENIA DLA PRZEWIDYWANIA PLASTYCZNEGO PĘKANIA W PROCESACH KUCIA

Streszczenie

Możliwość przewidywania plastycznego pękania odgrywa ważną rolę w projektowaniu wyrobów kutych. Badania doświadczalne pokazują, że zarodkowanie, wzrost i łączenie się pustek są mechanizmami kontrolującymi powstawanie i rozprzestrzenianie się pęknięć. Na te mechanizmy oddziałują, w różnym stopniu, takie parametry jak ciśnienie hydrostatyczne, intensywność naprężenia i maksymalne naprężenie główne. Znanych i używanych w praktyce jest wiele kryteriów pękania plastycznego opartych na tych parametrach. W niniejszej pracy te kryteria są porównywane. Spośród najbardziej popularnych wybrano kryteria pękania przynależne do różnych grup, klasyfikowane według podstaw danego kryterium, a więc kryteria oparte na mikrostrukturze materiału, kształcie pustek lub mechanizmie ich wzrostu. Kryteria oparte na mechanizmie kontinuum pękania, w których bierze się pod uwagę sprzężenie między odkształceniem plastycznym i degradacją materiału poprzez analizę różnych możliwości rozwoju zniszczenia dla rozciągającego i ściskającego stanu naprężenia, dają bardziej poprawną lokalizację obszarów, w których następuje inicjacja pęknięcia.

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