A COMPLETE DC ANALYSIS
OF THE SERIES RESONANT CONVERTER

V. VORPÉRIAN AND SLOBODAN ĆUK

California Institute of Technology
Pasadena, California

ABSTRACT

The dc-to-dc conversion ratio of the series resonant converter has been determined in the general discontinuous and continuous conduction modes. This new analysis gives a complete description of the dc operation of the circuit for any load and for any switching frequency.

1. INTRODUCTION

The series resonant converter shown in Fig. 1 is one class of dc-to-dc converters that, while quite widely used is not as well understood as the class of pulse-width-modulated converters (buck, boost, Ćuk, etc.). The state-space averaging technique [1,2,3], that has proved to be so fruitful in the analysis of pulse-width-modulated converters, is not applicable to resonant converters because the principal time constants are not long compared to the switching period. Nevertheless, the same kind of analytical results are as desirable for resonant converters as for pwm converters, in particular the dc-to-dc conversion ratio.

Previous analyses [4,5,and 6] have considered the operation of this circuit as a function of some, for this purpose, inconvenient parameter such as the diode (D1) conduction angle, and have *not* determined the conversion ratio M explicity.

In this paper a thorough analysis of the conversion ratio $M = V/V_g$ as a function of the switching frequency f_s and the load R is given for the series resonant converter in the *general* continuous and discontinuous conduction modes. Experimental results obtained for a resonant converter with resonant frequency $f_0 = 35.7$ kHz and $f_0 = 157$ kHz are in good agreement with predicted values.
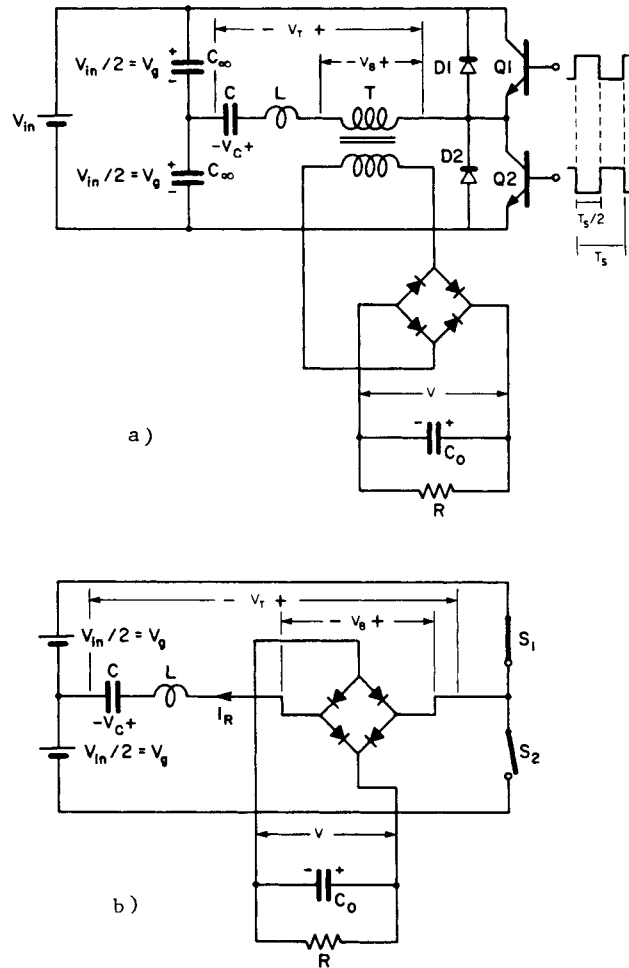
a)

b)

Fig. 1. (a) *The series resonant converter and* (b) *its ideal equivalent circuit.*

2. THEORY OF OPERATION

The circuit in Fig. 1b is equivalent to the circuit in Fig. 1a if we assume that the components are ideal and that $C_\infty \gg C$. The various voltages V_T, V_B, V_C, V and the current I_R are defined according to Fig. 1b. The bidirectional switches

S1 and S2 operate at 50% duty ratio for a duration $T_s/2$, where T_s is defined as the switching interval. The switching frequency f_s is given by

$$f_s = \frac{1}{T_s} \triangleq \text{ switching frequency} \qquad (1)$$

When Q1 is turned ON and simultaneously Q2 turned OFF, the diode D2 will become reverse biased by $V_{in} = 2V_g$, and consequently S2 will be opened. The pattern repeats when Q2 is turned ON and Q1 is turned OFF. At *no* time Q1 and Q2 are switched OFF simultaneously. Later in Section 7 we will remove this restriction and discuss the special case when Q1 and Q2 are switched OFF simultaneously.

The most general current and voltage waveforms during $T_s/2$ (S1 ON and S2 OFF) are shown in Figs. 2-3. The waveforms during the following cycle (S2 ON and S1 OFF), parts of which are shown, are the same as the waveforms in the preceding cycle but of opposite sign because of the symmetry of the circuit.

The voltage V_B at the input side of the bridge is controlled by the sign of the resonant current I_R and its peak value is equal to the output voltage V. The resonant current I_R consists of *piecewise* currents each of frequency equal to the resonant frequency given by

$$f_0 = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{T_0} \triangleq \text{ resonant frequency} \qquad (2)$$

From the current waveforms in Figs. 2 and 3 we see that there are two different types of discontinuous and continuous conduction modes. In the discontinuous conduction mode the resonant current I_R terminates after completing n full half cycles denoted by $i_1(t)$ through $i_n(t)$ as shown in Figs. 2a,b. This is followed by a period of no conduction even though, we point out that, *the switch S1 is closed and capable of conduction*. The reason for the discontinuity in the current I_R, as will be shown later, is that the bridge becomes reverse biased. The two different discontinuous conduction modes correspond to whether n is odd or even and are appropriately called *odd* or *even type n dcm*. We can see that in general for type n dcm to occur we must have

$$\frac{T_s}{2} > \frac{nT_0}{2} \quad \text{or} \quad 0 < \frac{f_s}{f_0} < \frac{1}{n} \qquad (3)$$

The frequency interval given by (3) is the largest possible interval for type n dcm to occur. For a given load R, however, the interval in which type n dcm occurs is more restricted than (3) and can be written as

$$0 < C_2(R,n) < \frac{f_s}{f_0} < C_1(R,n) \leq \frac{1}{n} , \qquad (4)$$

where $C_1(R,n)$ and $C_2(R,n)$ are constants depending on the load R which will be determined in the next section.

We now define the load parameter Q as follows

$$Q = \frac{\omega_0 L}{R} \qquad (5)$$

This Q of course is not to be confused with the Q_{coil} of the inductor L which we have assumed to be ideal ($Q_{coil} \to \infty$).

In the continuous conduction mode the current I_R never ceases to flow as shown in Figs. 3a,b. For any switching frequency less than the resonant frequency, $f_s < f_0$, the Q determines whether the circuit is operating in discontinuous or continuous conduction mode: when Q is increased above a critical value dependent on f_s, the circuit will enter continuous conduction mode (ccm). In this section we will only describe the operation of the continuous conduction mode and in a later section determine the critical values of Q. There are two different types of continuous conduction mode depending on whether D1 (D2) or Q1 (Q2) conducts first when S1 (S2) is closed. Fig. 3a shows Q1 conducting first at the instant S1 is closed. This is followed by D1 and Q1 conducting alternately for n *full* half cycles after which, while D1 is conducting Q2 is switched ON and S1 is turned OFF. Steady state periodicity of I_R requires that if Q1 conducts first, D1 should conduct last. This implies that n can only be even or zero. This type of continuous conduction mode is called *+type n ccm*, where n refers to the number of full half cycles of I_R while S1 or S2 is ON, and the positive sign refers to the fact that I_R and V_T have the same sign when S1 or S2 is turned ON (or immediately afterwards). Referring to Fig. 3a or Fig. 6a, we note that for this case the time interval $T_s/2 - nT_0/2$ must satisfy the following inequality;

$$\frac{T_0}{2} < \frac{T_s}{2} - \frac{nT_0}{2} < 2 \cdot \frac{T_0}{2} \quad \text{or} \quad \frac{1}{n+2} < \frac{f_s}{f_0} < \frac{1}{n+1} \qquad (6)$$

Eq. (6) gives the frequency interval in which +type n ccm can occur.

Fig. 3b shows D1 conducting first at the instant S1 is closed. This is followed by Q1 and D1 conducting alternately for n *full* half cycles after which Q1 while conducting is switched OFF and Q2 is switched ON (but D2 will conduct first). As before, periodicity requires n to be even or zero. This type of continuous conduction mode is called *-type n ccm* where the negative sign refers to the fact that I_R and V_T have opposite signs when S1 or S2 is turned ON (or immediately afterwards). Referring to Fig. 3b or Fig. 6b we see that for this case the time interval $T_s/2 - nT_0/2$ must satisfy the following inequality,

$$0 < \frac{T_s}{2} - \frac{nT_0}{2} < \frac{T_0}{2} \quad \text{or} \quad \frac{1}{n+1} < \frac{f_s}{f_0} < \frac{1}{n} \qquad (7)$$
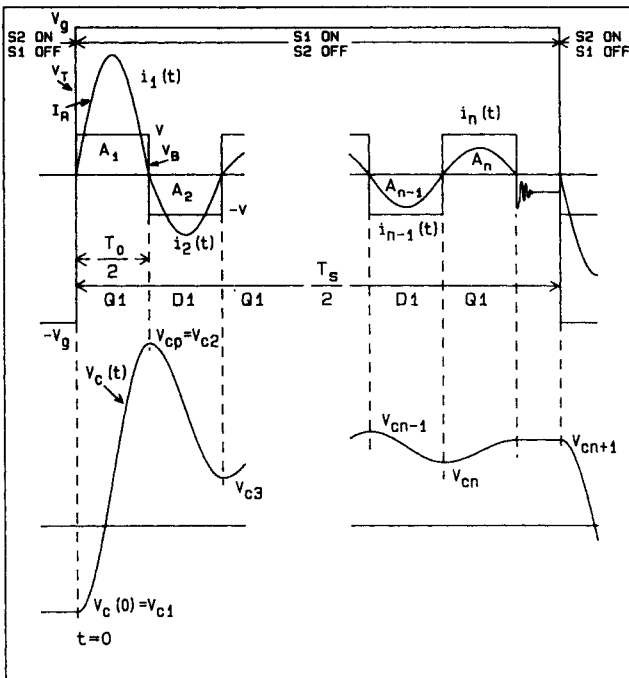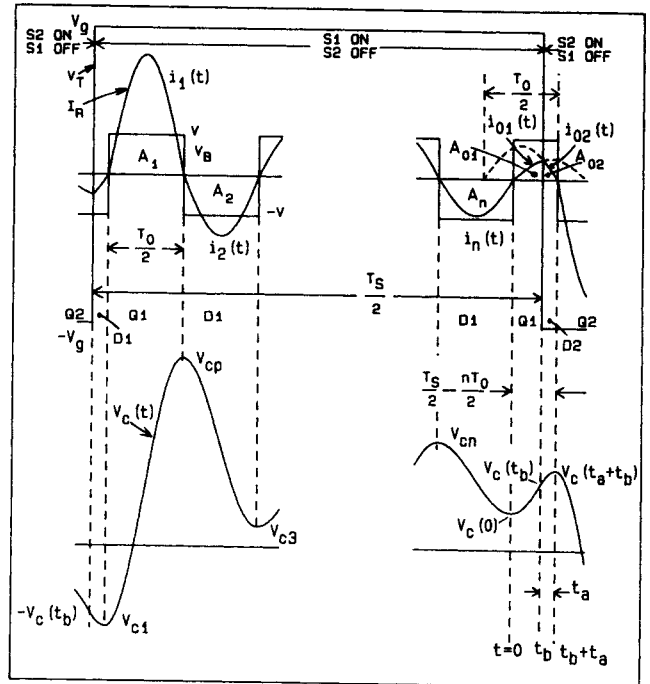
Eq. (7) gives the frequency interval in which
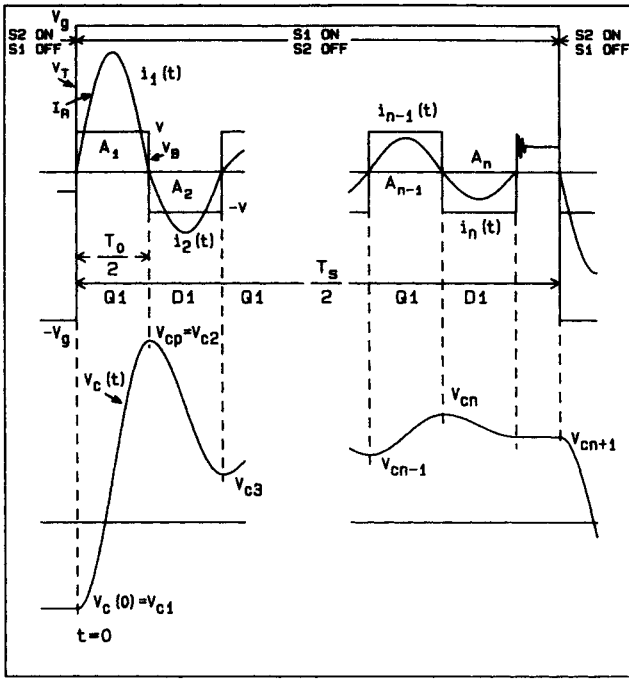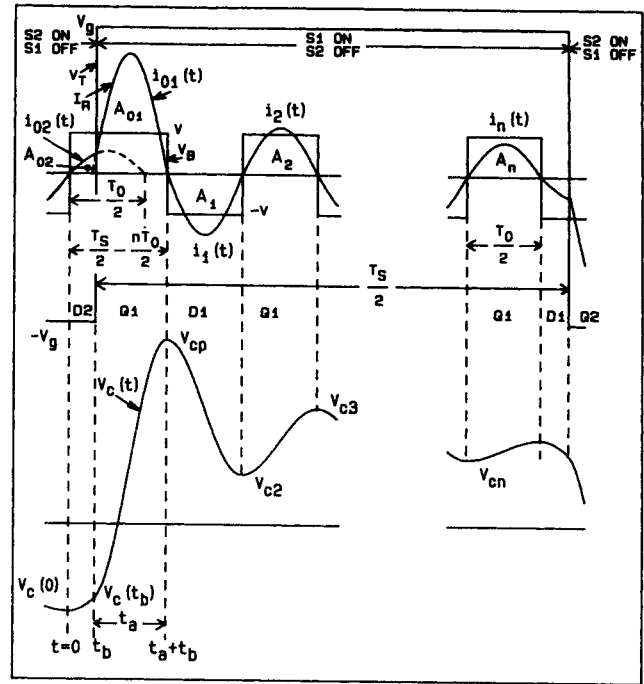
Fig. 2. *Voltage and current waveforms in the discontinuous conduction mode (a) even type n dcm and (b) odd type n dcm.*

Fig. 3. *Voltage and current waveforms in the continuous conduction mode (a) +type n ccm (b) -type n ccm.*

-type n ccm can occur. The frequency intervals given by Eqs. (6) and (7) can be more restricted depending on the load R. This is shown in Table 1 where $B_1(Q,n)$ and $B_2(Q,n)$ are constants depending on n and Q and will be determined in Section 6.

Before going on to the analysis, we would like to show the consistency of the nomenclature of the two types of continuous and discontinuous conduction modes. We have

Even +type n dcm	Even type n dcm
Odd +type n dcm	Odd type n dcm
Even +type n ccm (a)	+type n ccm (b)
Even -type n ccm	-type n ccm

Since the positive sign with dcm and the word even with ccm are redundant, we omit them and obtain the nomenclature given above in (b).

The results are summarized in Table 1.

3. ANALYSIS OF THE CONVERSION RATIO IN THE DISCONTINUOUS CONDUCTION MODE

This mode of operation, as mentioned earlier, occurs when the circuit is excited below resonance, and the reason for its occurence is that the output rectifier bridge becomes reverse biased. In this section we will show that the conversion ratio M, for the even discontinuous conduction mode, is a *linear* function of the switching frequency and the load. For the odd discontinuous conduction mode, it will be shown that M is insensitive to variations in load and switching frequency.

We define now two additional parameters:

$$\gamma = \frac{f_0}{f_s} \pi \quad \text{or} \quad \frac{f_s}{f_0} = \frac{\pi}{\gamma} \tag{8}$$

$$K = \frac{Q \cdot \gamma}{2} = \frac{\pi}{2RC\omega_s} \tag{9}$$

The conversion ratio is defined as

$$M = \frac{V}{V_g} , \tag{10}$$

where $V_g = V_{in}/2$ as shown in Fig. 1b. We proceed now with the analysis of M in both types of discontinuous conduction mode.

Even type n dcm (n = even)

The output current I_0 is the average rectified resonant current $|I_R(t)|$:

$$I_0 = \frac{V}{R} = \frac{2}{T_s} \int_0^{T_s/2} |I_R(t)| dt \tag{11}$$

Referring to Fig. 2a this integral can be expressed in terms of the areas, A_n, under each full half cycle of I_R. We can then write (11) as

$$\frac{V}{R} = \frac{2}{T_s} \sum_{m=1}^{n} A_m . \tag{12}$$

Using definition (9) we can write (12) as

$$\sum_{m=1}^{n} A_m = 2CVK . \tag{13}$$

Next we consider the power delivered to the circuit, P_{in}, and the power delivered to the load, P_0, which in the ideal case are equal:

$$P_{in} = \frac{2}{T_s} \int_0^{T_s/2} V_g I_R(t) dt = \frac{2}{T_s} V_g \sum_{m=1}^{n} A_m (-1)^{m+1} \tag{14}$$

Mode	Conduction Sequence	Frequency Interval
even type n dcm	Q1 D1 ...... Q1 D1 $\underbrace{\qquad\qquad}$ n full half cycles	$0 < C_2(Q,n) < \frac{f_s}{f_0} \leq C_1(Q,n) \leq \frac{1}{n}$ n even
odd type n dcm	$\overbrace{\text{Q1 D1 ...... Q1 D1 Q1}}$	$0 < C_2(Q,n) < \frac{f_s}{f_0} \leq C_1(Q,n) \leq \frac{1}{n}$ n odd
+ type n ccm	Q1 $\underbrace{\text{D1 Q1 ...... D1 Q1}}$ D1 n full half cycles	$\frac{1}{n+2} \leq \frac{f_s}{f_0} \leq B_1(Q,n) \leq \frac{1}{n+1}$ n = 0,2,4...even
-type n ccm	D1 $\overbrace{\text{Q1 D1 ...... Q1 D1}}$ Q1	$\frac{1}{n+1} \leq \frac{f_s}{f_0} \leq B_2(Q,n) \leq \frac{1}{n}$

Table 1

$$P_0 = \frac{2}{T_s} \int_0^{T_s/2} V|I_R(t)|dt = \frac{2}{T_s} V \sum_{m=1}^{n} A_m \qquad (15)$$

Equating (14) and (15) we obtain for $M = V/V_g$

$$M = \frac{\sum_{m=1}^{n} A_m(-1)^{m+1}}{2CVK} \qquad n \text{ even} \quad (16)$$

The equations for the currents $i_1(t)$ through $i_n(t)$ can be determined by straightforward circuit analysis:

$$i_m(t) = \omega_0 C[V_g + (-1)^m V - V_{cm}]\sin\omega_0(t-(m-1)\frac{T_0}{2}) \qquad (17)$$

$$A_m = \int_{(m-1)T_0/2}^{mT_0/2} |i_m(t)|dt = 2C[V_g-V_{cm}+(-1)^m V](-1)^{m+1} \qquad (18)$$

where,

$V_{cm} \triangleq$ the initial voltage across the capacitor C at the beginning of the mth half cycle (Fig. 2a).

In equation (18) the V_{cm} are unknown, however, we need only determine $V_{c1} = V_c(0)$, and a recursive expression for A_m can be determined by successive calculation of $V_{cm} = V_{cm-1} + (-1)^m A_{m-1}/C$ starting with m = 2. The peak-to-peak capacitor voltage V_{cp-p} and V_{c1} are given by

$$CV_{cp-p} = 2CV_{c2} = 2A_1 - \sum_{m=1}^{n} A_m(-1)^{m+1}, \qquad n \text{ even} \quad (19)$$

$$V_{c1} = V_{c2} - \frac{A_1}{C} = -\frac{1}{2C}\sum_{m=1}^{n} A_m(-1)^{m+1} = -MKV \qquad (20)$$

Now the desired recursive relation for A_m can be determined

$$A_m = 2CV_g[1 - (2m-1)M + M^2K] \qquad (21)$$

Now the summation in (16) can be computed

$$\sum_{m=1}^{n} A_m(-1)^{m+1} = 2nV_g CM , \qquad n \text{ even} \quad (22)$$

Substituting (22) into (16) we get for M:

$$M = \frac{n}{K} = \frac{2n}{Q\pi} \cdot \frac{f_s}{f_0} , \qquad n \text{ even} \quad (23)$$

which is a linear function of load and switching frequency. Using (21) in (19) we get for the peak capacitor voltage and V_{c1}

$$V_{cp} = V_g [2 - \frac{2n}{K} + \frac{n^2}{K}], \qquad n \text{ even} \quad (24)$$

$$V_{c1} = -nV \qquad (25)$$

Odd type n dcm (n = odd)

The only difference between this case and the even type n dcm is that in the summation in Eq. (22) n is odd and so we get

$$\sum_{m=1}^{n} A_m(-1)^{m+1} = 2CV_g[1 - nM + KM^2], \qquad n \text{ odd} \quad (26)$$

which when substituted into (16) gives

$$M = \frac{1}{n} \qquad n = \text{odd} \quad (27)$$

This result states that if the circuit is operating in an odd discontinuous conduction mode, then the conversion ratio M is insensitive to load or switching frequency.

The peak capacitor voltage in this case is

$$V_{cp} = V_g[2 - \frac{2}{n} + \frac{K}{n^2}] , \qquad n \text{ odd} \quad (28)$$

4. DISCUSSION ON DISCONTINUOUS CONDUCTION MODE

4.1 Boundary Between Two Discontinuous Conduction Modes

Assume that the converter is operating in even type m dcm. The conversion ratio M according to (23) is given by

$$M_m = \frac{2m}{\pi Q} \frac{f_s}{f_0} \qquad (29)$$

and is plotted in Fig. 4 for three different cases. These are straight lines through the origin with slope $\frac{2m}{\pi Q}$. It is clear that cases II and III correspond to higher values of Q than case I.

For type m dcm to occur we require that
$f_s/f_0 < \frac{1}{m}$, so that (29) is valid in a region to
the left of $f_s/f_0 = 1/m$ on the M vs. f_s/f_0 plot.
In case I, as the switching frequency is increased,
M_m will increase linearly until the converter
enters odd type (m-1) dcm at f_c/f_0. If we continue
to increase f_s/f_0, say up to f_d/f_0, the conversion
ratio M will stay at a constant value of 1/(m-1)
because of operation in odd type (m-1) dcm.
(To complete the discussion for all $f_s > f_c$, we
need to know the results from continuous conduction
mode derived in the next section, but for the
present discussion we need only consider the
results obtained for dcm). If on the other hand
we decrease the switching frequency, M_m will
decrease linearly until the converter enters odd
type (m+1) dcm at f_b/f_0. If we continue to
decrease the frequency, say down to f_a/f_0, the
conversion ratio will stay at a constant value of
1/(m+1). From the foregoing discussion, we see
that case I represents an even type m dcm that is
bounded between odd type (m-1) dcm and odd type
(m+1) dcm. Case II on the other hand represents
an even type m dcm that is bounded between an
odd type (m+1) dcm and +type (m-2) ccm; i.e.,
if we increase f_s beyond 1/m the converter enters
+type (m-2) ccm. In case III we have an even
type m dcm bounded between -type m ccm and +type
(m-2) ccm. From the foregoing discussion we see
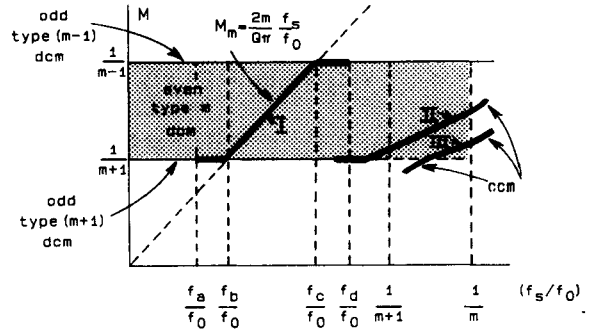that the region of validity of Eq. (29) is the
shaded area shown in Fig. 4.

To determine f_b/f_0 and f_c/f_0 at which
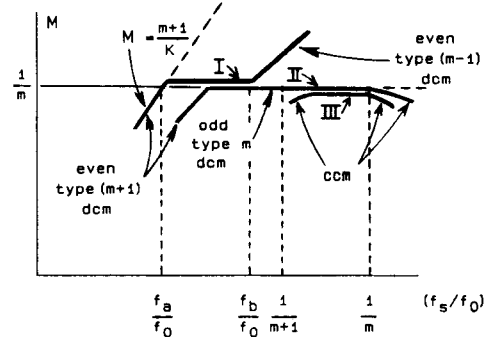operation changes from even type m dcm to odd type
(m+1) dcm and odd type (m-1) dcm respectively, we
equate the conversion ratios of both modes and
we get

$$\frac{f_b}{f_0} = \frac{\pi Q}{2m(m+1)} \qquad (30a)$$

$$\frac{f_c}{f_0} = \frac{\pi Q}{2m(m-1)} \qquad (30b)$$

Referring to Fig. 4, we see that for even
type m dcm we must have $f_b/f_0 \leq 1/m$ (note that f_c/f_0
does not apply to cases II and III), so that from
(30a) we get

$$Q \leq \frac{2(m+1)}{\pi} \qquad (31)$$

This last inequality is very important and
states that for even type m dcm to occur Q must
satisfy (31). For any value of Q larger than
$2(m+1)/\pi$, even type m dcm can not occur. The
condition given in (31) covers all three cases I,
and II and III. If we wish to be particular, we
can determine the condition on Q so that we have
only case I. To do so we require $f_c/f_0 \leq 1/m$ and
we get

$$Q \leq \frac{2(m-1)}{\pi} \qquad (32)$$

With (31) and (32) we now give the intervals
in which even type m dcm occurs along with the
load conditions:

Fig. 4. Boundaries of even type m dcm.

Fig. 5. Boundaries of odd type m dcm.

$$\text{type m dcm} \begin{cases} \dfrac{\pi Q}{2m(m+1)} < \dfrac{f_s}{f_0} < \dfrac{\pi Q}{2m(m-1)} \quad ; \quad Q \leq \dfrac{2(m-1)}{\pi} \\[20pt] \dfrac{\pi Q}{2m(m+1)} < \dfrac{f_s}{f_0} < \dfrac{1}{m} \quad ; \quad \dfrac{2(m-1)}{\pi} < Q < \dfrac{2(m+1)}{\pi} \end{cases}$$

$$(33)$$

The complete description of even type m dcm
is given by (33) which states the condition on
Q for the occurrence of this mode and the intervals
in which it occurs. Comparing (33) to (4) we get:

$$C_2(Q,m) = \frac{\pi Q}{2m(m+1)}$$

$$C_1(Q,m) = \begin{cases} \dfrac{\pi Q}{2m(m-1)} \quad ; \quad Q < \dfrac{2(m-1)}{\pi} \\[20pt] \dfrac{1}{m} \quad ; \quad \dfrac{2(m-1)}{\pi} < Q < \dfrac{2(m+1)}{\pi} \end{cases}$$

Next we consider operation in an odd type m
dcm which according to (27) has a constant gain of
1/m. As in the case of even type m dcm the three
separate cases are shown in Fig. 5. The overlap
region has been shown split in order to identify
the conversion ratio characteristics of each case.

Case I shows an odd type m dcm bounded between even type(m+1) dcm and even type(m-1) dcm. Case II shows an odd type m dcm bounded between even type (m+1) dcm and -type(m-1) ccm. Case III shows an odd type m dcm bounded between -type(m-1) ccm and +type(m-1) ccm. Proceeding exactly as before, we can determine the intervals in which odd type m dcm occurs along with the conditions on Q. The results will be identical to those obtained for the case of even type m dcm, so that (33) is valid for the general odd or even type m dcm.

In this section we have given the boundary between two discontinuous conduction modes and the intervals in which a general type m dcm occurs along with the condition on Q.

Physical reason for the occurrence of discontinuous conduction mode

Now we will show that, when operating in discontinuous conduction mode, the bridge becomes reversed biased when I_R ceases to flow. Assume that the converter is operating in even type m dcm. Referring to Fig. 4 we can clearly see that

$$M_m > \frac{1}{m+1} \quad \text{for} \quad \frac{f_s}{f_0} > \frac{f_b}{f_0} \tag{34}$$

Referring to Fig. 2a, we see that the capacitor voltage V_c during $I_R = 0$ is $V_{cm+1} = -V_{c1} = mV$ (using Eq. (25)), so that the voltage across the bridge is

$$V_B = V_g - V_{cm+1} = V_g - mV \tag{35}$$

dividing (35) by V we get

$$\frac{V_B}{V} = \frac{V_g}{V} - m = \frac{1}{M_m} - m \tag{36}$$

and using (34) we get

$$\frac{V_B}{V} < 1 \tag{37}$$

which means that the bridge is *reverse biased*, and that I_R ceases to flow even though Q1 is *still ON* or S1 is still closed!

If the converter is operating in odd type m dcm, then by referring to Fig. 5 we have

$$M_m = \frac{1}{m} < \frac{m+1}{K} \quad \text{for} \quad \frac{f_s}{f_0} > \frac{f_a}{f_0} \tag{38}$$

Referring to Fig. 2b we see that the capacitor voltage during $I_R = 0$ is $V_{cm+1} = -V_{c1} = KV/m$ (where this last step is obtained by substituting $M = 1/m$ in (20) which is valid for odd type m dcm as well). The voltage across the bridge, $V_B = V_{cm+1} - V_g$, is

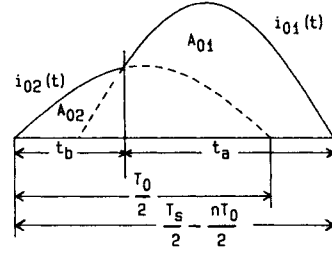$$V_B = \frac{K}{m} V - V_g \quad \text{or} \quad \frac{V_B}{V} = \frac{K}{m} - \frac{1}{M_m} .$$

Fig. 6(a). Expansion of the interval $0 < t < t_a + t_b$ in Fig. 3a.
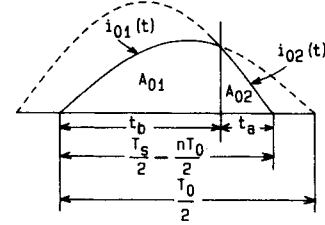

Fig. 6(b). Expansion of the interval $0 < t < t_a + t_b$ in Fig. 3b.

Using (38) we get $\frac{V_B}{V} < 1$ so that the bridge is reversed biased again.

5. ANALYSIS OF THE CONVERSION RATIO IN THE CONTINUOUS CONDUCTION MODE

Operation in continuous conduction mode depends on which of the following two intervals f_s/f_0 falls in:

$$\frac{1}{n+2} < \frac{f_s}{f_0} < \frac{1}{n+1}$$

$$n = 0,2,4,\ldots\text{even} \tag{39}$$

$$\frac{1}{n+1} < \frac{f_s}{f_0} < \frac{1}{n}$$

To determine which interval a given f_s/f_0 falls in, we simply find an *even* n that satisfies only one of the following two inequalities:

$$n + 2 > \frac{f_0}{f_s} > n + 1 \tag{40a}$$

$$n + 1 > \frac{f_0}{f_s} > n \tag{40b}$$

For instance, if $f_s/f_0 = .3781$, then $f_0/f_s = 2.64$ and only n = 2 satisfies (40b), so that $1/3 < f_s/f_0 < 1/2$. If $f_s/f_0 = .323$, then $f_0/f_s = 3.1$ and only n = 2 satisfies (40a), so that $1/4 < f_s/f_0 < 1/3$.

Unlike the case of discontinuous conduction mode, the conversion ratio M in the continuous

conduction mode will be determined numerically.
For each of the two types of continuous conduction
mode we will determine an equation of the form
$F(M,Q,\gamma) = 0$ which will be solved numerically for
a given load (Q) and switching frequency (γ).

+type n ccm: $1/(n+2) < f_s/f_0 < 1/(n+1)$

In this section we will give the most
important steps in determining M for +type n ccm.
Referring to Fig. 3a we proceed by equating input
and output power of the circuit and obtain

$$M = \frac{\sum\limits_{m=1}^{n} (-1)^m A_m + A_{01} - A_{02}}{\sum\limits_{m=1}^{n} A_m + A_{01} + A_{02}} \qquad (41)$$

The output voltage V is

$$V = I_0 R = R \frac{2}{T_s} \left(\sum\limits_{m=1}^{n} A_m + A_{01} + A_{02} \right) \qquad (42)$$

The recursion relation for the areas A_m under
the full half cycles of I_R can be shown to be

$$A_m = 2C(V_{cp} - (2m-1)V - V_g) \qquad (43)$$

From (43) we can determine the summation (recall
n is even or zero)

$$\sum\limits_{m=1}^{n} (-1)^m A_m = - 2nCV \qquad (44)$$

Substituting (44) and (42) into (41) and using
definition (9), we get

$$M = \frac{-2nCV + A_{01} - A_{02}}{2VCK} \qquad (45)$$

To complete the analysis we need to determine the
two voltages $V_c(t_b)$ and $V_c(0)$ and the two currents
$i_{01}(t)$ and $i_{02}(t)$ shown in Fig. 3a and Fig. 6a.
The currents $i_{01}(t)$ and $i_{02}(t)$ in the interval
$0 < t < t_a$ shown in Fig. 3a are redrawn in Fig. 6a
for clarity.

The peak to peak capacitor voltage is

$$CV_{cp-p} = 2CV_{cp} = - \sum\limits_{m=1}^{n} A_m (-1)^m + A_{01} + A_{02} \qquad (46)$$

$$V_c(t_b) = V_{cp} - \frac{A_{01}}{C} = - MKV \qquad (47)$$

where in the last step we made use (44), (45) and
(46). From circuit equations we obtain $i_{02}(t)$ and
A_{02}

$$i_{02}(t) = - \omega_0 C[V_g + V + V_c(0)] \sin\omega_0 t \qquad (48)$$

$$A_{02} = \int_0^{t_b} i_{02}(t)dt = C[V_g + V + V_c(0)][\cos\omega_0 t_b - 1] \qquad (49)$$

but we have

$$V_c(0) = V_c(t_b) - \frac{A_{02}}{C} = - MKV - \frac{A_{02}}{C} \qquad (50)$$

Substituting (50) into (49) and realizing
$t_b = T_s/2 - nT_0/2 - t_a$, we get

$$A_{02} = CV_g[M^2 K - M - 1][\sec(\gamma - \omega_0 t_a) - 1] \qquad (51)$$

$$i_{02}(t) = \frac{\omega_0 CV_g[M^2 K - M - 1]}{\cos(\gamma - \omega_0 t_a)} \sin\omega_0 t \qquad (52)$$

In the same way we obtain $i_{01}(t)$ and A_{01}

$$i_{01}(t) = \frac{\omega_0 CV_g[1 - M + M^2 K]}{\cos\omega_0 t_a} \cdot \sin(\omega_0 t - \gamma) \qquad (53)$$

$$A_{01} = CV_g[1 - M + M^2 K][1 - \sec\omega_0 t_a] \qquad (54)$$

Substituting for A_{01} and A_{02} in (45) we obtain

$$2M(n+1) = [1 + M - M^2 K]\sec(\gamma - \omega_0 t_a)$$

$$- [1 - M + M^2 K]\sec\omega_0 t_a \qquad (55)$$

We must also have $i_{01}(t_b) = i_{02}(t_b)$ which gives

$$\frac{1 - M + M^2 K}{1 + M - M^2 K} \tan\omega_0 t_a = \tan(\gamma - \omega_0 t_a) \qquad (56)$$

Solving (56) and (55) simultaneously to
eliminate "$\omega_0 t_a$" we get

$$\frac{[n+1][1-M+M^2 K]}{1-M(n+1)^2-MK}\cos\gamma \frac{\sqrt{[MK+n][M^2(n+1)^2-1][n+2-MK]}}{1-M(n+1)^2-MK}\sin\gamma$$

$$= \frac{[1+M-M^2 K][n+1]}{1+M(n+1)^2-MK} \qquad (57)$$

The peak capacitor voltage and the peak
resonant current are:

$$V_{cp} = V_g[1 + Mn + \frac{MK - 1}{n + 1}] \tag{58}$$

$$I_{peak} = \frac{\omega_0 CV_g}{n + 1} [M(n+1)^2 + MK - 1] \tag{59}$$

In solving (56) and (55) simultaneously to eliminate "$\omega_0 t_a$" we determine:

$$\cos\omega_0 t_a = \frac{1 - M + M^2 K}{1 - MK - M(n+1)^2}(n+1) \tag{60a}$$

$$\sin\omega_0 t_a = \frac{\sqrt{[MK+n][M^2(n+1)^2-1][n+2 - MK]}}{MK + M(n+1)^2 - 1} \tag{60b}$$

Equation (57) is the desired equation from which we can numerically determine the conversion ratio M for a given load and switching frequency. The results are shown in Fig. 8 for n = 0, 2, and 4 which correspond to the intervals $1/2 < f_s/f_0 < 1$, $1/4 < f_s/f_0 < 1/3$ and $1/6 < f_s/f_0 < 1/5$ respectively. Note that for certain values of Q, M reaches the peak value of $1/(n+1)$ for $f_s/f_0 < 1/(n+1)$ where n = 0,2,4. This happens because operation changes from +type n ccm to odd type (n+1) dcm.

The boundaries between continuous and discontinuous conduction mode are determined in Section 6.

–type n ccm: $1/(n+1) < f_s/f_0 < 1/n$

Referring to Fig. 3b we proceed as before by equating input and output power and obtain

$$M = \frac{-\sum\limits_{m=1}^{n} A_m(-1)^m + A_{01} - A_{02}}{\sum\limits_{m=1}^{n} A_m + A_{01} + A_{02}} \tag{61}$$

For the output voltage we have

$$V = <|I_R(t)|> R = \frac{2}{T_s} R [\sum_{m=1}^{n} A_m + A_{01} + A_{02}] \tag{62}$$

The recursion relation for the areas A_m under the full half cycles can be shown to be

$$A_m = 2C[V_g - (2m-1) - V_{c1}] \tag{63}$$

where V_{c1}, as shown in Fig. 3b, is the capacitor voltage at the beginning of the first full half cycle. Next we determine the summation in (61), which for n = even gives:

$$\sum_{m=1}^{n} A_m(-1)^m = -2nCV \tag{64}$$

Substituting (64) and (62) into (61) and making use of definition (9) we get

$$M = \frac{2nCV + A_{01} - A_{02}}{2CVK} \tag{65}$$

To complete the analysis we need to determine $i_{01}(t)$ and $i_{02}(t)$. These are determined in the same way as in the previous case:

$$i_{01}(t) = \frac{V_g \omega_0 C[1 - M - M^2 K] \sin\omega_0 t}{\cos(\gamma - \omega_0 t_a)} \tag{66}$$

$$A_{01} = CV_g[1 - M - M^2 K][\sec(\gamma - \omega_0 t_a) - 1] \tag{67}$$

$$i_{02}(t) = \frac{V_g \omega_0 C[1 + M + M^2 K]}{\cos\omega_0 t_a} \sin(\gamma - \omega_0 t) \tag{68}$$

$$A_{02} = V_g C[1 + M + M^2 K][\sec\omega_0 t_a - 1] \tag{69}$$

Substituting A_{02} and A_{01} into (65) we obtain:

$$(1+M+M^2 K)\sec\omega_0 t_a - (1-M-M^2 K)\sec(\gamma-\omega_0 t_a) = 2M(n+1) \tag{70}$$

Requiring $i_{02}(t_b) = i_{01}(t_b)$ we obtain

$$\frac{1 + M + M^2 K}{1 - M - M^2 K} \tan\omega_0 t_a = \tan(\gamma - \omega_0 t_a) \tag{71}$$

We solve (70) and (71) to eliminate "$\omega_0 t_a$" and we get:

$$\frac{[n+1][1+M+M^2 K]}{1+M(n+1)^2+MK}\cos\gamma + \frac{\sqrt{[MK-n][1-M^2(n+1)^2][n+2+MK]}}{1+M(n+1)^2+MK}\sin\gamma$$
$$= \frac{(1-M-M^2 K)(n+1)}{1-M(n+1)^2 + MK} \tag{72}$$

The peak capacitor voltage is

$$V_{cp} = \begin{cases} V_g MK & n = 0 \\ V_g[1-3M + \frac{1+M(n+1)^2 + MK}{n+1}], & n \text{ even} \end{cases} \tag{73}$$

As in the previous case (72) is the desired equation from which we can determine numerically the conversion ratio M for –type n ccm for a given load
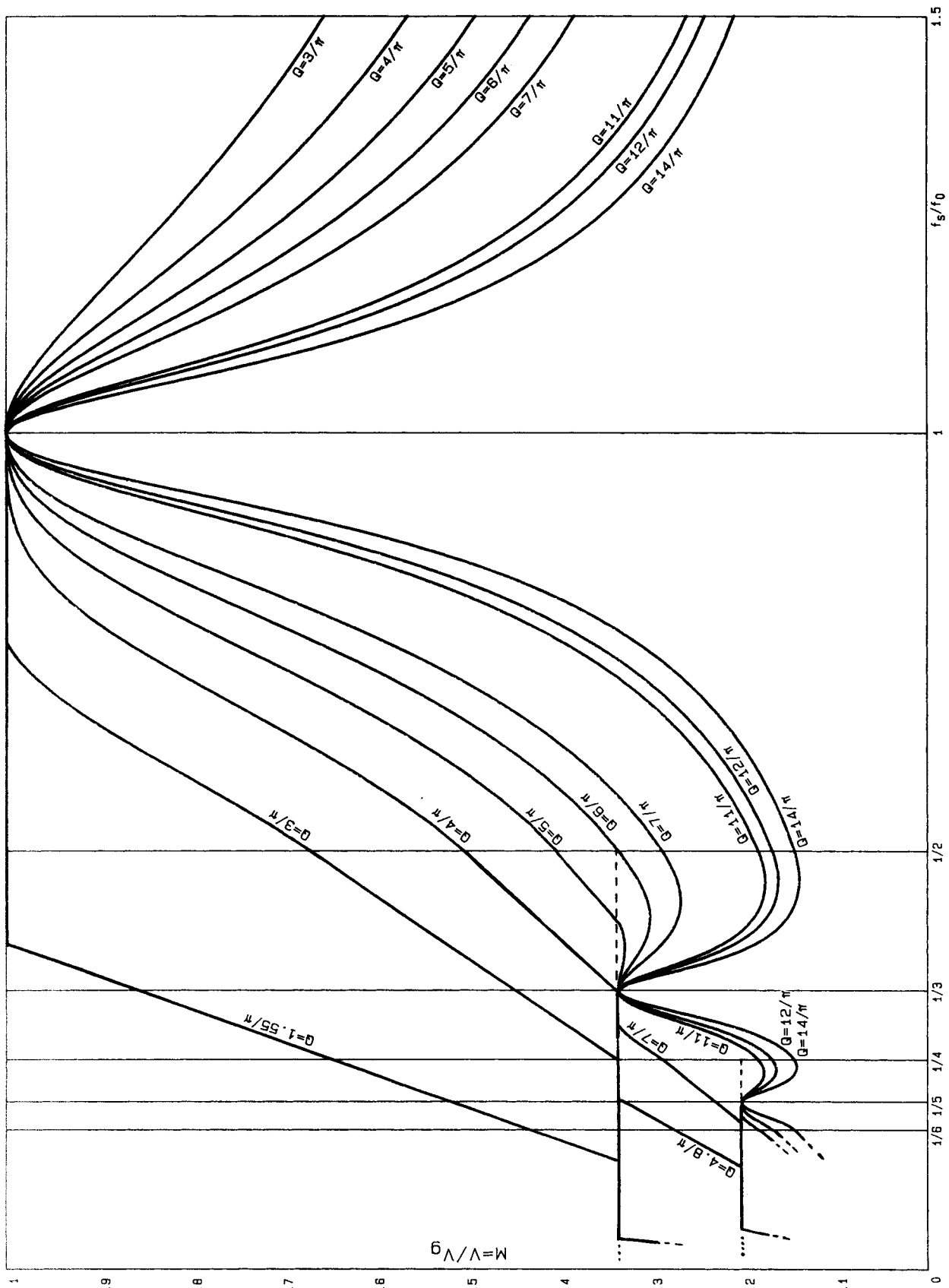
93

Fig. 8.   The general conversion ratio character-
istics of the series resonant converter.

94

and switching frequency. The results are shown in Fig. 8 for n = 0,2, and 4 which correspond to the intervals $f_s/f_0 > 1$, $1/3 < f_s/f_0 < 1/2$ and $1/5 < f_s/f_0 < 1/4$ respectively. The interval $f_s/f_0 > 1$ corresponds to exciting the circuit above resonance and has no discontinuous conduction mode. In the intervals $1/3 < f_s/f_0 < 1/2$ and $1/5 < f_s/f_0 < 1/4$ we see that for certain values of Q operation changes from continuous to discontinuous conduction mode. These boundaries between continuous and discontinuous mode will be given in Section 6. We also note the occurrence of second and third resonant peaks at $f_s/f_0 = 1/3$ and $f_s/f_0 = 1/5$. There are in fact, an infinite number of these resonant peaks located at $f_s/f_0 = 1, 1/3, 1/5, \ldots$, $1/m \ldots$ (m = odd) with the maximum value of M at these peaks being equal to $M_{peak} = 1, 1/3, 1/5, \ldots$ $1/m$ (m = odd) respectively. Of course, in reality these results are valid only for *switching times that are short compared to the output load time constant.*

The peak resonant current is

$$
I_{peak} = \begin{cases} \omega_0 CV_g[1 - M + MK] \quad ; \quad 1 - M - M^2K < 0 \\ \\ \omega_0 CV_g \sqrt{MK(1-M^2)(MK+2)} \, ; 1 - M - M^2K > 0 \end{cases} n=0 \\ \\ \omega_0 CV_g[\frac{1}{n+1} + M(n-1 + \frac{K}{n+1})] \qquad n \neq 0
$$

(74)

The reason V_{cp} for n = 0 is different from $n \neq 0$ can be seen from Fig. 3b. For -type n ccm V_{cp} occurs at the end of the *first full* half cycle of I_R which for n = 0 does not exist and consequently $V_{cp} = V_c(t_a + t_b) = MKV_g$. For +type n ccm V_{cp} occurs at the end of $i_{01}(t)$ which is always present, even when n = 0. This is the same reason for I_{peak} to be different for n = 0 and $n \neq 0$. In the case of I_{peak}, however, we see from Fig. 7 that for -type 0 ccm, I_{peak} depends on the intersection of $i_{01}(t)$ and $i_{02}(t)$. If $i_{01}(t)$ and $i_{02}(t)$ intersect after the occurrence of the peak of $i_{01}(t)$, then I_{peak} is the same as the peak of $i_{01}(t)$. If $i_{01}(t)$ and $i_{02}(t)$ intersect before the occurrence of the peak of $i_{01}(t)$, then $I_{peak} = i_{01}(t_b)$. It can be shown that this break point occurs for $1 - M - M^2K = 0$. It can also be shown that $0 < \omega_0 t_a < \pi/2$ for -type n ccm (n = 0,2,4...), so that the peak of $i_{02}(t)$ never occurs.

a)
I_{peak} $i_{01}(t_b)$
$i_{01}(t) \rightarrow$
$\leftarrow i_{02}(t)$
t_b t_a
$\frac{T_s}{2}$

b)
$i_{01}(t_b) = I_{peak}$
$i_{01}(t) \rightarrow$
$\leftarrow i_{02}(t)$
t_b t_a
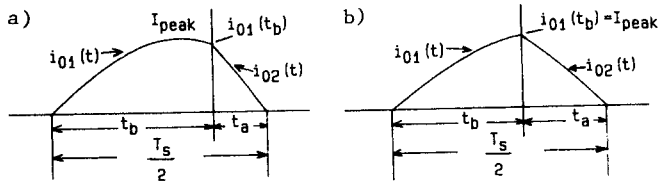$\frac{T_s}{2}$

Fig. 7. (a) I_{peak} *for -type 0 ccm for* $1 - M - M^2K < 0$ *and* (b) *for* $1 - M - M^2K > 0$.

6. BOUNDARY BETWEEN DISCONTINUOUS AND CONTINUOUS CONDUCTION MODE

The conversion ratio characteristics for the general continuous and discontinuous conduction modes are shown in Fig. 9. The line $M = 1/(n+1)$ is the boundary between discontinuous conduction mode and +type n and -type n continuous conduction modes. In this section we will analyze the range of critical values of Q which determine the mode of operation of the converter. Before going into the analysis, let us study the results given in Fig. 9. The two shaded regions in Fig. 9 are determined by the range of critical values of Q.

In region I the conversion ratio character-istics belong to the range of values of $Q\epsilon[2n/\pi, 2(n+1)/\pi]$. The dashed line is the boundary between even type n dcm and -type n ccm. This is precisely case III discussed in Fig. 4. In the interval $1/(n+1) < f_s/f_0 < 1/n$, for this range of values of Q, operation changes from -type n ccm to even type n dcm as we increase f_s/f_0 from $1/(n+1)$ to $1/n$. Upon careful examination of Fig. 9, we realize that because of the periodic nature of the intervals, the converter will be in continuous conduction mode for all $f_s/f_0 > 1/n$ and in discontinuous conduction mode for all $f_s/f_0 < 1/(n+1)$. Moreover, for all these values of Q the conversion ratio will be $1/(n+1)$ in the interval $1/(n+2) < f_s/f_0 < 1/(n+1)$ which is the case of overlapping characteristics as discussed in Fig. 5 (case II). We have the following now:

$$
\frac{2n}{\pi} < Q < \frac{2(n+1)}{\pi} \atop (n = even) \begin{cases} \frac{f_s}{f_0} > \frac{1}{n} & ccm \\ \\ \frac{Q\pi}{2n(n+1)} < \frac{f_s}{f_0} < \frac{1}{n} & \text{Even type n dcm} \\ \\ \frac{1}{n+1} < \frac{f_s}{f_0} < \frac{Q\pi}{2n(n+1)} & \text{-type n ccm} \\ \\ \frac{f_s}{f_0} < \frac{1}{n+1} & dcm \end{cases}
$$

(75)

Compare the second interval in (75) to the second interval given in (33).

Region II corresponds to the range of values of $Q\epsilon[2(n+1)/\pi, 2(n+2)/\pi]$. This is the same as case III shown in Fig. 5. For these values of Q we have:

$$
\frac{2(n+1)}{\pi} < Q < \frac{2(n+2)}{\pi} \atop (n \ even) \begin{cases} \frac{f_s}{f_0} > \frac{1}{n+1} & ccm \\ \\ \frac{Q\pi}{2(n+1)(n+2)} < \frac{f_s}{f_0} < \frac{1}{n+1} & \text{odd type(n+1) dcm} \\ \\ \frac{1}{n+2} < \frac{f_s}{f_0} < \frac{Q\pi}{2(n+1)(n+2)} & \text{+type n ccm} \\ \\ \frac{f_s}{f_0} < \frac{1}{n+2} & dcm \end{cases}
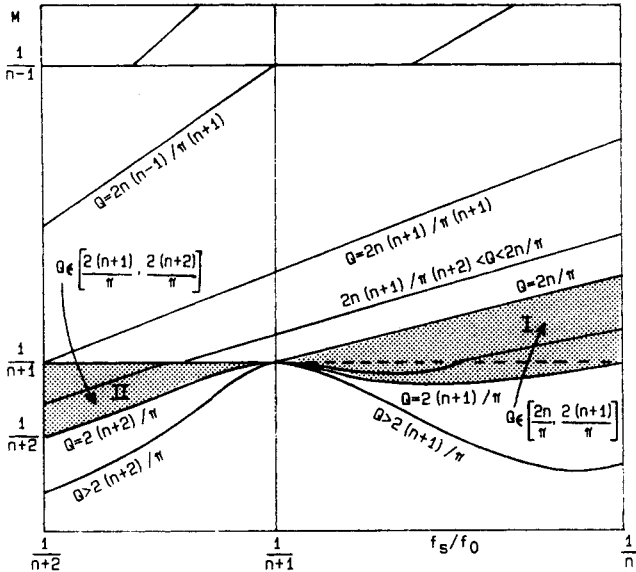$$

(76)

95

Fig. 9. *General boundaries between continuous and discontinuous conduction modes, and the range of critical values of Q.*

To determine which of the two regions a given Q belongs to, we determine an *even* "n" that satisfies only one of the two following inequalities:

$$n + 2 > \frac{Q\pi}{2} > n + 1 \quad \rightarrow \text{Region II} \qquad (77a)$$

$$n + 1 > \frac{Q\pi}{2} > n \qquad \rightarrow \text{Region I} \qquad (77b)$$

If Q = 10, for instance, then only (77a) is satisfied for n = 14, and passage from continuous to discontinuous conduction mode is given by (76). If Q = 9, then only (77b) is satisfied for n = 14 and passage from continuous to discontinuous conduction mode is given by (75).

For a given f_s/f_0 the critical values of Q depend which interval f_s/f_0 falls in:

$$\frac{1}{n+2} < \frac{f_s}{f_0} < \frac{1}{n+1} \begin{cases} Q < \frac{f_s}{f_0} \frac{2(n+1)(n+2)}{\pi} & \text{dcm} \\[2ex] Q > \frac{f_s}{f_0} \frac{2(n+1)(n+2)}{\pi} & \text{+type n ccm} \end{cases}$$

$$\frac{1}{n+1} < \frac{f_s}{f_0} < \frac{1}{n} \begin{cases} Q < \frac{f_s}{f_0} \frac{2n(n+1)}{\pi} & \text{dcm} \\[2ex] Q > \frac{f_s}{f_0} \frac{2n(n+1)}{\pi} & \text{-type n ccm} \end{cases}$$

$$(78)$$

The results just given are derived by studying the radicals in equations (57) and (72). In the interval $1/(n+1) < f_s/f_0 < 1/n$ we see from (72) that we must have

$$MK \geq n \quad \text{and} \quad M \leq \frac{1}{n+1} \qquad (79)$$

Since in this interval the maximum value of M, for the -type n continuous conduction mode, is $1/(n+1)$, we get from (79)

$$K \geq n(n+1) \quad \text{or} \quad \frac{1}{n+1} \leq \frac{f_s}{f_0} \leq \frac{Q\pi}{2n} \cdot \frac{1}{(n+1)} \quad \text{or}$$

$$Q \geq \frac{f_s}{f_0} \cdot \frac{2n(n+1)}{\pi} \qquad (80)$$

Since the maximum value of f_s/f_0 in this interval is $1/n$, then we see from the last inequality in (80) that in order to have -type n ccm in the entire interval $1/(n+1) < f_s/f_0 < 1/n$ we must have $Q > 2(n+1)/\pi$. This is the region below region I in Fig. 9. For values of $Q < 2(n+1)/\pi$ we have continuous conduction mode over an interval given by (80) which is more restricted than $1/(n+1) < f_s/f_0 < 1/n$. However, since the minimum value of f_s/f_0 in this interval is $1/(n+1)$, we must have $Q > 2n/\pi$ for continuous conduction mode as required by (80). The values of $2n/\pi < Q < 2(n+1)/\pi$ define region I, and the results in (75) follow.

From (80) and the preceeding discussion, we can see that $B_2(Q,n)$ in Table 1 of Section 2 is given by

$$B_2(Q,n) = \begin{cases} \dfrac{Q\pi}{2n(n+1)} & ; \quad \dfrac{2n}{\pi} < Q < \dfrac{2(n+1)}{\pi} \\[2ex] \dfrac{1}{n} & ; \quad \quad Q > \dfrac{2(n+1)}{\pi} \end{cases}$$

For the interval $1/(n+2) < f_s/f_0 < 1/(n+1)$, we study the radical in Eq. (57), from which we get

$$MK > n+2 \quad \text{and} \quad M \leq \frac{1}{n+1} \qquad (81)$$

Since in this interval the maximum value of M in the +type n continuous conduction mode is $1/(n+1)$, we get from (81)

$$K \geq (n+1)(n+2) \quad \text{or} \quad Q \geq \frac{f_s}{f_0} \cdot \frac{2}{\pi} (n+1)(n+2) \quad \text{or}$$

$$\frac{1}{n+2} < \frac{f_s}{f_0} < \frac{Q\pi}{2} \cdot \frac{1}{(n+1)(n+2)} \qquad (82)$$

The maximum value of f_s/f_0 in this interval is $1/(n+1)$ so that we see from (82), in order to have continuous conduction mode in the entire interval

$1/(n+2) < f_s/f_0 < 1/(n+1)$, we must have $Q > 2(n+2)/\pi$. The range of values $Q > 2(n+2)/\pi$ defines the region below region II. For values of $Q < 2(n+2)/\pi$ we will have continuous conduction mode in an interval given by (82) which is more restricted than $1/(n+2) < f_s/f_0 < 1/(n+1)$. However, since the minimum value of f_s/f_0 in this interval is $1/(n+2)$ we must have $Q > 2(n+1)/\pi$ for continuous conduction as required by (82). The range of values of $2(n+1)/\pi < Q < 2(n+2)/\pi$ defines region II, and the results in (76) follow.

From (82) and the preceeding discussion we can see that $B_1(Q,n)$ in Table 1 of Section 2 is given by

$$B_1(Q,n) = \begin{cases} \dfrac{Q\pi}{2(n+1)(n+2)} & ; & \dfrac{2(n+1)}{\pi} < Q < \dfrac{2(n+2)}{\pi} \\[3mm] \dfrac{1}{n+1} & ; & Q > \dfrac{2(n+2)}{\pi} \end{cases}$$

The results in (78) follow directly from (75) and (76). From (78) we can see that there can be no continuous conduction mode for $Q < 2n/\pi$ in the interval $1/(n+1) < f_s/f_0 < 1/n$, and for $Q < 2(n+1)/\pi$ there can be no continuous conduction mode in $1/(n+2) < f_s/f_0 < 1/(n+1)$.

7. OPERATION IN THE SPECIAL CASE WHEN Q1 AND Q2 ARE OFF SIMULTANEOUSLY

Until now we assumed that Q1 and Q2 were allowed to conduct for the *entire* duration of $T_s/2$. If conduction is inhibited by switching Q1 (or Q2) OFF before the end of $T_s/2$, the converter will be *forced* to operate with a *finite* number of allowed continuous and discontinuous conduction modes over the entire range $f_s/f_0 \in [0,\infty]$. To illustrate this, let us refer to Fig. 2a and assume $n = 6$ for a given Q and f_s/f_0. In Sections 2, 3, and 4 we showed that although the transistors were ON during the entire period $T_s/2$, I_R became discontinuous because the output bridge rectifier became reverse biased. Now we ask what happens if under the same operating conditions the transistors are switched OFF after conducting the current $i_1(t)$. The current $i_2(t)$ which is carried by the diode (D1 or D2) will flow because we can not switch OFF D1 or D2. The current $i_3(t)$ will not be able to flow, however, because Q1 is switched OFF and the circuit will be *forced* to operate in type 2 dcm instead of type 6. Moreover, we see that in the entire range $f_s/f_0 \in [0,\infty]$ the allowed modes of operation are type 1 dcm, type 2 dcm and $\pm$ type 0 ccm. To illustrate further, if we assume for the same Q and f_s/f_0, the transistors are switched OFF after conducting the current $i_3(t)$, the converter will be forced to operate in type 4 dcm instead of type 6 dcm. In this case the allowed modes of operation in the entire range $f_s/f_0 \in [0,\infty]$ are type 1 dcm through type 4 dcm, $\pm$ type 2 ccm and $\pm$ type 0 ccm.

We see from the preceeding discussion that when Q1 and Q2 are OFF simultaneously, the conversion ratio characteristics in the interval $0 < f_s/f_0 < 1/2$ will be different from that in
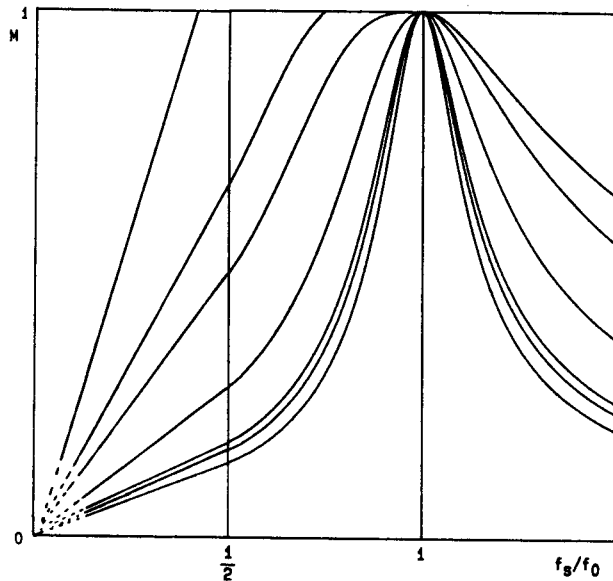

Fig.10. The conversion ratio characteristics in the special case when Q1 and Q2 are allowed to conduct only once.
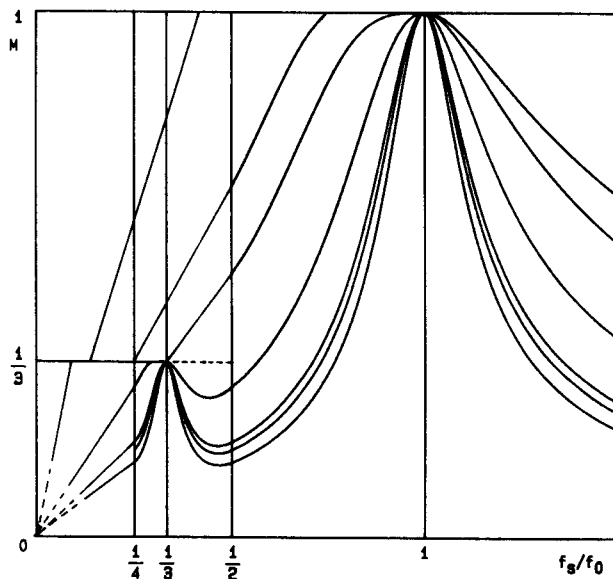

Fig. 11. The conversion ratio characteristics in the special case when Q1 and Q2 are allowed to conduct at most twice.

Fig. 8. In the interval $f_s/f_0 > 1/2$ the conversion ratio characteristics will not be affected because in this interval Q1 and Q2 conduct only once.

Fig. 10 shows the conversion ratio characteristics when Q1 and Q2 are allowed to conduct only once in the period $T_s/2$. Since the only allowed modes of operation in this case are type 1 dcm, type 2 dcm, and $\pm$ type 0 ccm, we can not have the other resonant peaks at $f_s/f_0 = 1/3, 1/5, \ldots$ as

97

shown in Fig. 8. In the entire interval
$0 < f_s/f_0 < 1/2$ M will be given by (23) for type 2
dcm and (27) for type 1 dcm. Moreover, we notice
that the lower bound of the frequency interval in
which type 2 dcm can occur is zero instead of
$C_2(Q,2) = \pi Q/12$ as given by (33). This is obvious
because $f_s/f_0 = C_2(Q,2)$ is the boundary between
type 2 dcm and type 3 dcm which we can *not* have
since we have constrained the converter to operate
with a maximum of type 2 dcm.

In general if the transistor is allowed to
conduct at most only j number of times in the
period $T_s/2$, the maximum allowed discontinuity is
type 2j dcm and consequently $C_2(Q,2j) = 0$.

Fig. 11 shows the conversion ratio
characteristics when Q1 and Q2 are allowed to
conduct only twice in the period $T_s/2$. In this
case the maximum allowed discontinuity is type 4
dcm and $C_2(Q,4) = 0$. The M for type 2 dcm will now
be sandwiched between M = 1/3 and M = 1 because
of the occurrence of type 3 dcm. In addition to
the first peak at $f_s/f_0 = 1$, we also have the
second resonant peak at $f_s/f_0 = 1/3$ but not any
other. It is clear from the above discussion that
if we allow Q1 and Q2 to conduct at most j times
in the period $T_s/2$, we will have only j resonant
peaks at $f_s/f_0 = 1$, 1/3, ... 1/(2j-1).

This completes the D.C. analysis of the
series resonant converter in the general
discontinuous and continuous conduction modes.

8.   EXPERIMENTAL RESULTS

A resonant converter was built with the
resonant frequency selectable either at
$f_0 = 35.7$ kHz or $f_0 = 157$ kHz. The output trans-
former was eliminated in order to avoid extra
parasitic losses. The voltages V_T and V_B shown in
Fig. 1a were measured and the conversion ratio M
was determined by taking the ratio of V_T to V_B, so
that

$$M = \frac{V_T}{V_B} = \frac{V_g - V_s}{V + 2V_D} \simeq \frac{V_g}{V + 2V_D} \qquad (83)$$

V_s is the voltage across S1 or S2 and varies
between V_{sat} and $-V_{D1}$ depending on the direction of
the current I_R. Normally V_g is much greater than
either V_{sat} or V_{D1}, and the approximation in (83)
is good. Accounting for the voltage drops in the
output bridge rectifier is slightly more
complicated and requires modification of the Q.
This is done by considering the power dissipated
at the input terminals of the bridge rectifier:

$$P_{out} = 2V_D \frac{(V_B - 2V_D)}{R} + \frac{(V_B - 2V_D)^2}{R} \qquad (84)$$

The first term in (84) represents the losses
in the bridge and the second term is the power
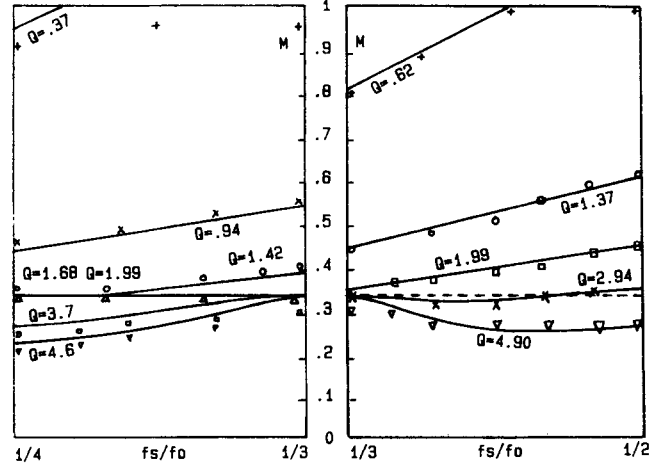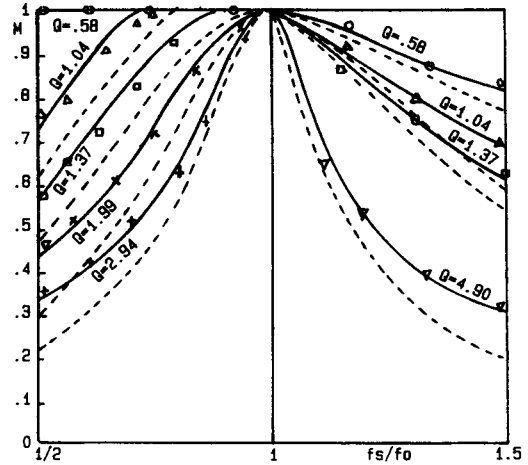delivered to the load R.   Rearranging (84) we get

Fig. 12.   Experimental results in the interval
$1/4 < f_s/f_0 < 1.5$.

$$P_{out} = \frac{V_B^2}{R}\left(1 - \frac{2V_D}{V_B}\right) = \frac{V_B^2}{R}\left(1 - \frac{2V_D}{MV_g}\right) \qquad (85)$$

Eq. (85) implies that when we consider the
conversion ratio $M = V_g/V_B$, then R and Q must be
modified according to

$$R = \frac{R}{1 - 2V_D/MV_g} \quad \text{or} \quad Q = Q[1 - \frac{2V_D}{MV_g}] \qquad (86)$$

To determine M for even type n, with Q
modified, we substitute (86) in (23) and obtain

$$M = \frac{2n}{Q\pi} \frac{f_s}{f_0} + \frac{2V_D}{V_g} \qquad (87)$$

The conversion ratio of the odd type n dcm
remains unaffected by this modification of Q. For
+type n ccm and -type n ccm the correction for Q
to determine M is done numerically. Experimental
results are shown in Fig. 12a,b,c. In Fig. 12a the
dashed lines indicate the conversion ratio
characteristics without modifying Q and the solid

lines after modifying Q as in (86). The boundary
conditions given in (75), (76) and (78) can be
easily modified by changing Q to $Q[1-2V_D(n+1)/V_g]$
as can be seen from (86) in which we let $M = 1/(n+1)$.

Figs. 13a,b,c show how operation changes from
type 1 dcm to type 2 dcm to -type 2 ccm as we change
the load while operating at a fixed switching
frequency. In Fig. 13a we see that although the
diodes D1 and D2 are present, they do *not* conduct
because the load R is not small enough. This can
be seen from (33) in which if we let m = 2, we
require $Q \geq 4f_s/\pi f_0$ for type 2 dcm to occur. When
the Q is modified according to (86), this condition
becomes $Q \geq 4f_s/\pi f_0(1-2V_D/V_g)$. The operating
conditions for Fig. 13a are

$$f_s = 13.9 \text{ kHz} \qquad L = .197 \text{ mH} \qquad V_g = 14V$$

$$f_0 = 35.7 \text{ kHz} \qquad R = 89 \ \Omega \qquad V_D = .7V$$

$$\frac{f_s}{f_0} = .389 \rightarrow n = 2 \text{ and } 1/3 < f_s/f_0 < 1/2$$

$$Q = .496 \cdot \frac{4f_s}{\pi f_0} \frac{1}{1 - 2V_D/V_g} = .55 \qquad (88)$$

We see then from (88) that Q does not satisfy
the condition for type 2 dcm to occur and as a
result the diodes D1 and D2 do not conduct.

If we now change the load to R = 27.0 Ω, the
diodes D1 and D2 begin to conduct and operation
changes to type 2 dcm as shown in Fig. 13b. The
transistors Q1 and Q2, however, do not conduct for
the *second* time in $T_s/2$ even though they are still
ON. This can be seen from (78) in which if we let
n = 2, we require $Q \geq 12 f_s/f_0\pi$ for -type 2 ccm
to occur. When Q is modified according to (86),
this condition becomes for n = 2
$Q \geq 12f_s/f_0\pi(1 - 6V_D/V_g)$. For the operating
conditions in Fig. 13b we have

$$R = 27.\Omega \cdot Q = 1.64 \cdot \frac{12f_s}{\pi f_0(1 - 6V_D/V_g)} = 2.13$$
$$(89)$$

We see then from (89) that Q does not
satisfy the condition for -type 2 ccm to occur and
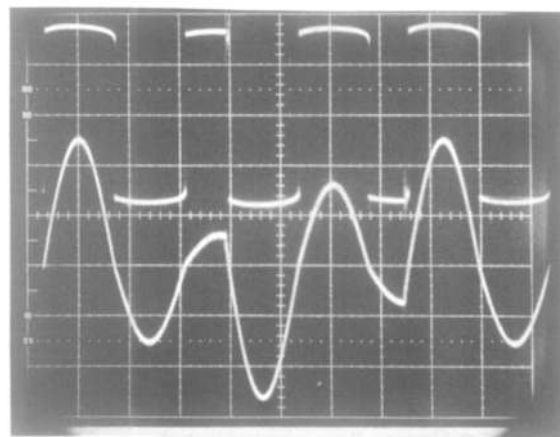as a result Q1 and Q2 do not conduct for the
second time in $T_s/2$.

If we change the load further to R = 11 Ω
then Q = 4.0 > 2.13, and the condition for -type 2
ccm to occur is satisfied. This is seen in Fig. 13c
in which we see the transistors Q1 and Q2
conducting for the second time in the interval $T_s/2$.
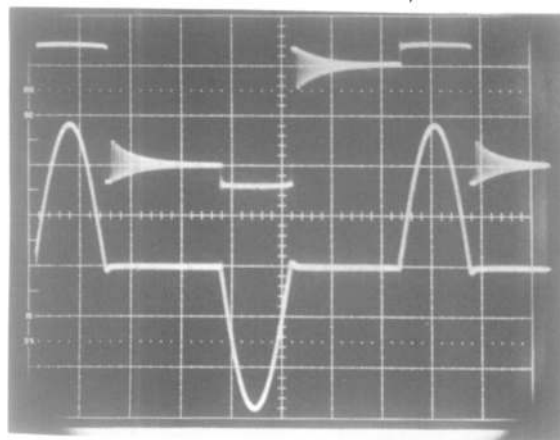
a)
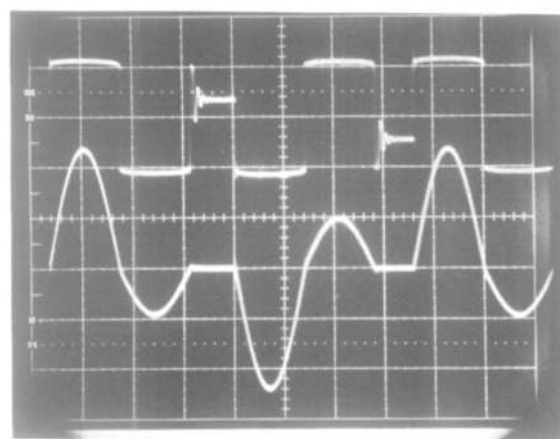
b)

c)

*Fig. 13. Experimental verification of how
operation changes from (a) type 1 dcm
to (b) type 2 dcm to (c) -type 2 ccm as
the load R is reduced from 89Ω to 11Ω.*

Top trace is V_B, bottom trace is I_R

9.   CONCLUSION

In this paper the dc operation of the series resonant converter has been analyzed in terms of two simple parameters: the *load* parameter $Q = \omega_0 L/R$ and the ratio of *switching frequency* to resonant frequency. The *discontinuous* and *continuous* conduction modes have been *identified* in the general case and the conversion ratio M has been determined for each mode of operation.

There are *two* general continuous conduction modes, defined as *+type n ccm* and *-type n ccm*, for which the conversion ratio M is determined numerically.

There are *two* general discontinuous conduction modes defined as *even type n dcm* and *odd type n dcm*. For the even type n dcm the conversion ratio M is a *linear* function of the switching frequency and the load, whereas in the odd type n dcm the conversion ratio is *insensitive* to variation in load and switching frequency.

The *boundaries* between discontinuous and continuous conduction mode are given in terms of the load and switching frequency.

REFERENCES

1.   R. D. Middlebrook and Slobodan Ćuk, "A General Unified Approach to Modelling Switching Converter Power Stages," IEEE Power Electronics Specialists Conference, 1976 Record, pp. 18-34 (IEEE Publication 76CH1084-3 AES); also International J. of Electronics, vol. 42, no. 6, pp. 521-550, June 1977.

2.   Slobodan Ćuk and R. D. Middlebrook, "A General Unified Approach to Modelling Switching Dc-to-Dc Converters in Discontinuous Conduction Mode," IEEE Power Electronics Specialists Conference, 1977 Record, pp. 36-57 (IEEE Publication 77CH1213-8 AES).

3.   R. D. Middlebrook and Slobodan Ćuk, *Advances in Switched-Mode Power Conversion, Vol. I and II*, TESLAco, 1981.

4.   F. C. Schwarz, "An Improved Method of Resonant Current Pulse Modulation for Power Converters," IEEE Power Electroncis Specialists Conference, 1975 Record, pp. 194-204 (IEEE Publication 75CH0965-4 AES).

5.   R. King and T. A. Stuart, "A Normalized Model for the Half-Bridge Series Resonant Inverter," IEEE Transactions on Aerospace and Electronics Systems, vol. AES-17, no. 2, March 1981, pp. 190-198.

6.   E. Kittl, "Transient Waveform Analysis of Switching Converter," DRSEL-TL-PE Tech. Rep. ECOM-4493. U.S. Army Electronics Command, Fort Monmouth, N.J., April 1977.