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A Complete Model for Operations on Integers

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A Complete Model for Operations on Integers

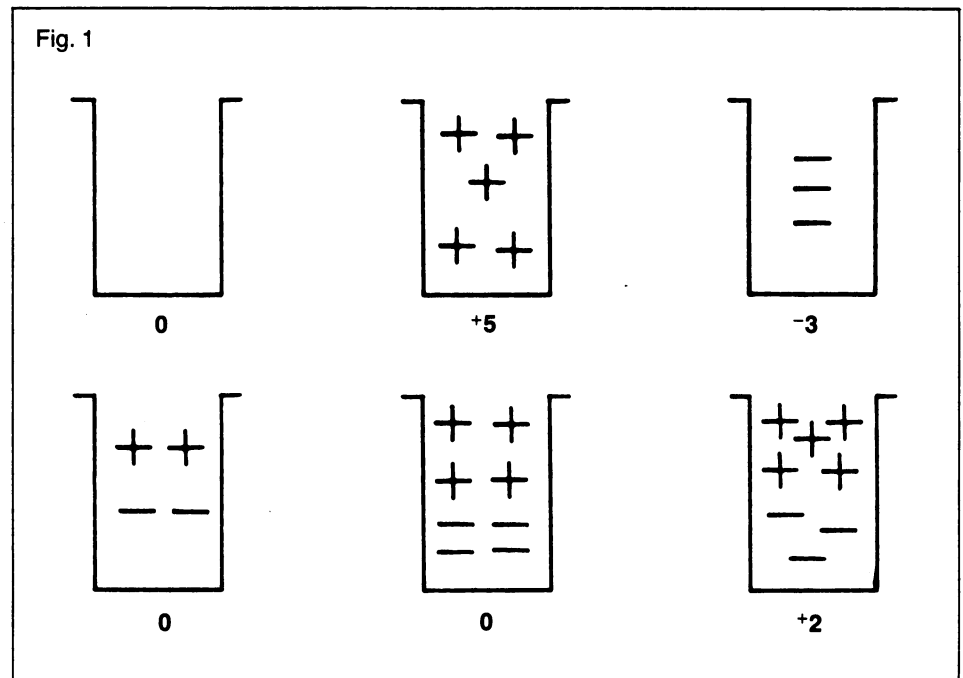
By Michael T. Battista

Teaching students the four basic operations on the set of integers in a meaningful way is a difficult task. The task could be made easier, however, if there was a single physical model for the integers in which all four basic operations could be represented. The number-line model, though useful, has serious shortcomings and is incomplete. As an alternative, some authors have suggested the "positive-negative charge" model (Frاند and Granville 1978, Grady 1978), but they used this model only for representing addition and subtraction. In this article I will describe the "charge" model and demonstrate how it can be extended to all four operations on the set of integers.

Integers as Collections of Charges

To use the model you will need a small set of transparent jars of the same size and a good supply of two colors of poker chips or counters. Let us assume that the chips are red and white. On each white chip paste or use a magic marker to write a "+" sign. These are the positive "charges." On each red chip put a "-" sign. These are the negative "charges."

We will be concerned, not with



individual charges, but with collections of charges in jars. When we speak of the *charge on a jar*, we will be referring to the integer represented by the collection of charges inside it. So, if a jar is empty or contains an equal number of positive and negative charges, it has charge zero. (In the latter case, every positive charge "cancels" exactly one negative charge.) If a jar contains five positive charges, it has charge +5. If it contains three negative charges, it has charge -3. If a jar contains five positive charges and three negative charges, the charge on the jar is +2. (Three positive charges cancel three negative charges and we find the charge on the remaining collection.) See figure 1.

Thus, in this model, *integers are*

represented by collections of charges. Every jar of charges represents an integer, and every integer can be represented by a jar of charges. Note though, that although a jar of charges represents a unique integer, an integer can be represented in an infinite number of ways. See figure 2.

It should be stressed at this point that before proceeding to operations on integers, students must have a firm understanding of how integers are represented in the model. Teachers introducing the model should emphasize from the outset that integers have multiple representations. Students should be able not only to recognize what integer is represented by a given jar of charges, but also to construct multiple jar representations for any given integer.

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Fig. 2

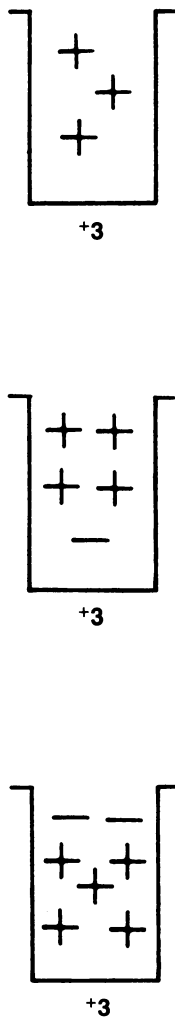
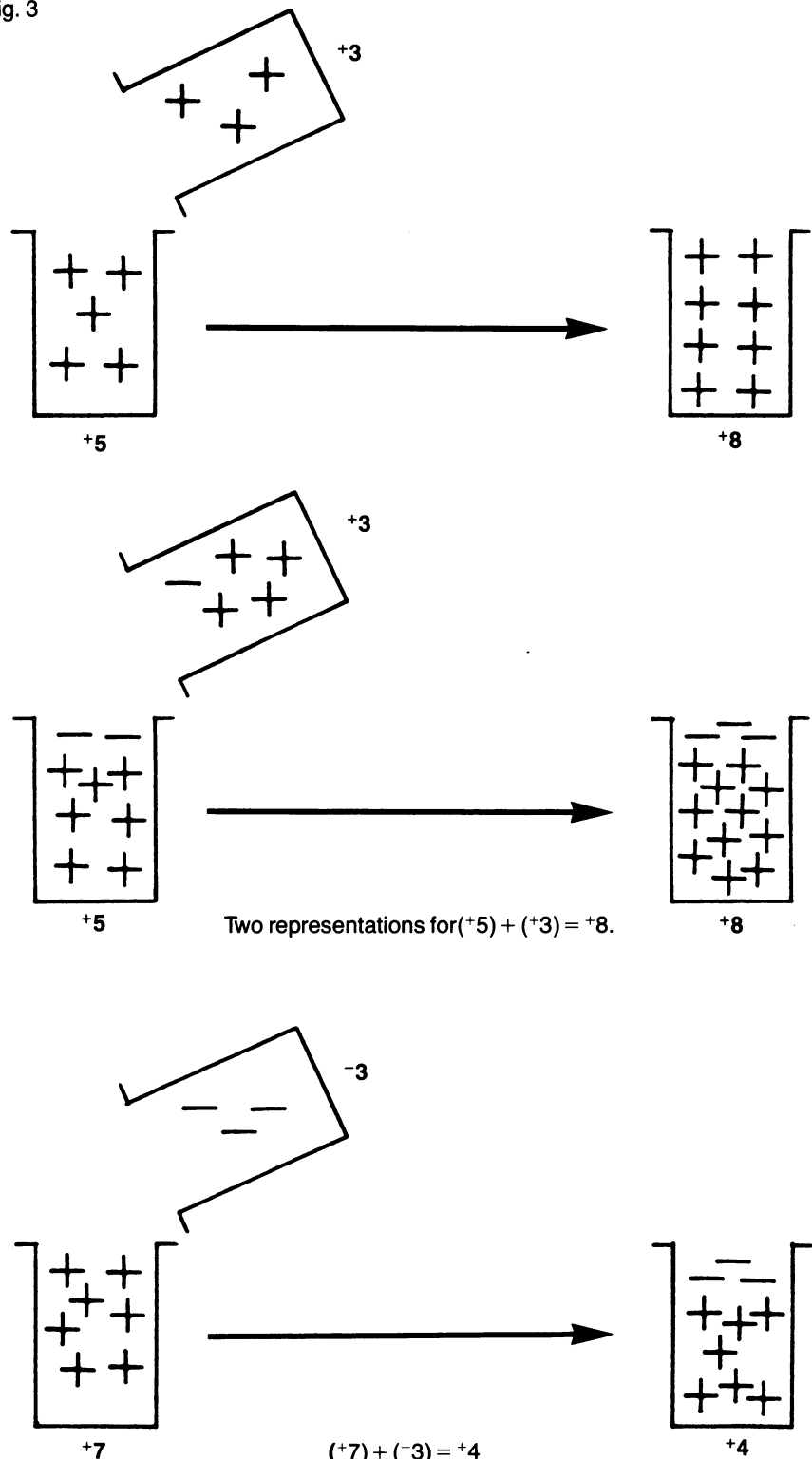


Fig. 3



Two representations for $(+5) + (+3) = +8$.

$(+7) + (-3) = +4$

The Four Operations

Addition

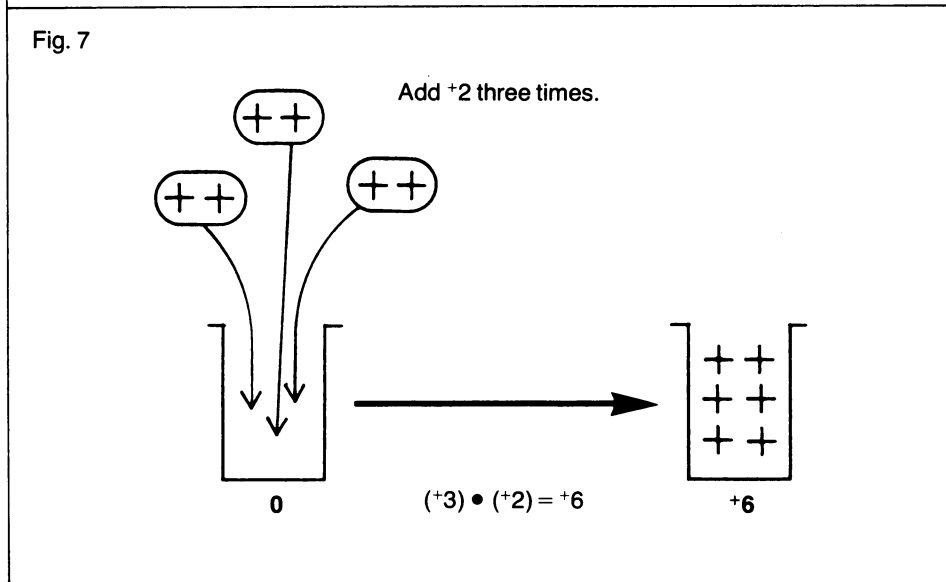
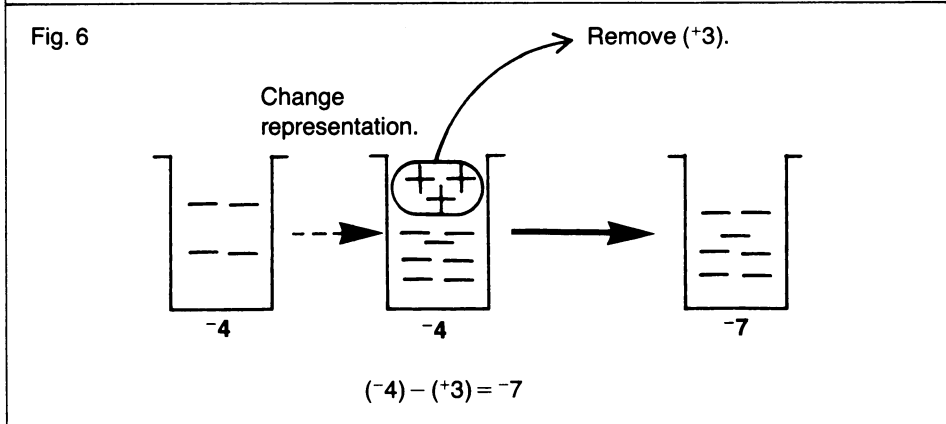
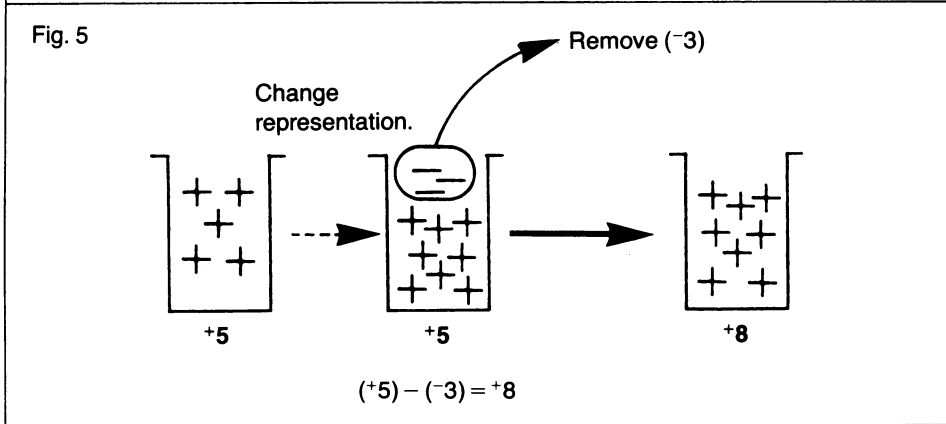
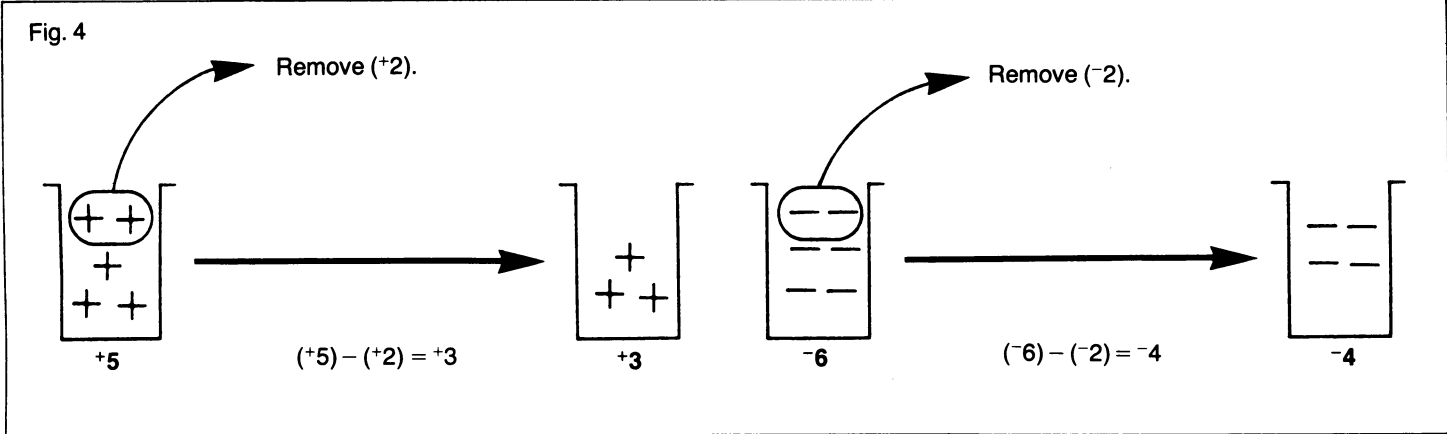
Representing addition with the charge model is easy. It is simply an extension of the familiar, cardinal number model of whole-number addition. In order to add two integers, represent each integer with its own jar of charges. The operation of addition is represented by the *action* of joining the contents of the two jars. So pour the contents of one jar into the other and find the charge on the latter jar. This is the sum. See figure 3. Note that an addition problem can be represented in more than one way.

You should also note that we are making an important convention here. For the problem $a + b = \text{---}$, we

pour the contents of the jar representing b into the jar representing a . Thus $(+5) + (+3)$ and $(+3) + (+5)$ have different physical representations. This point should be stressed to students because it is essential for later understanding of the commutative property of addition.

Subtraction

Just as addition is represented by a "joining" action, subtraction is represented by a "take away" action. To subtract one integer from another, represent the first integer (the minuend) with a jar of charges. Then re-



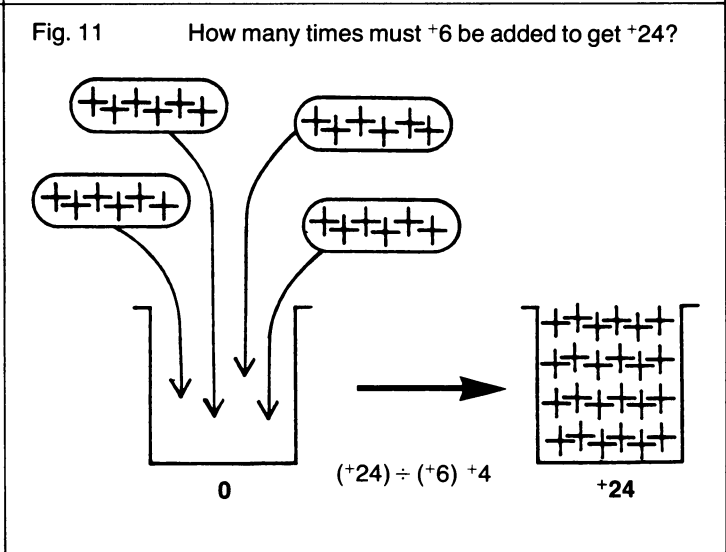
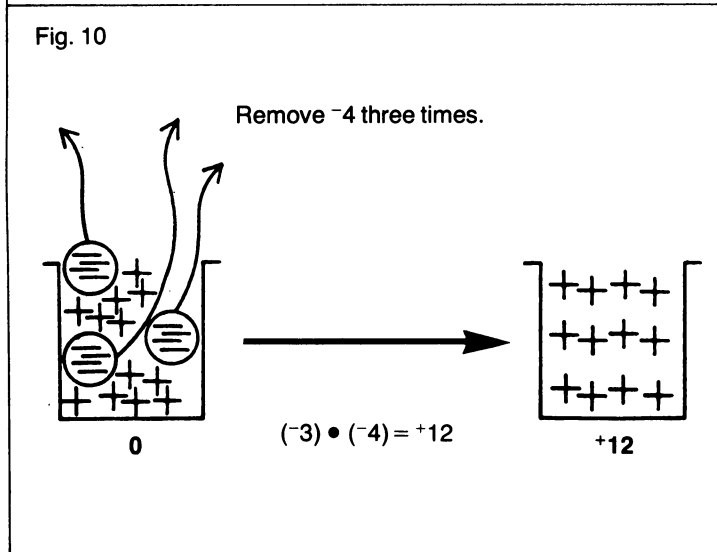
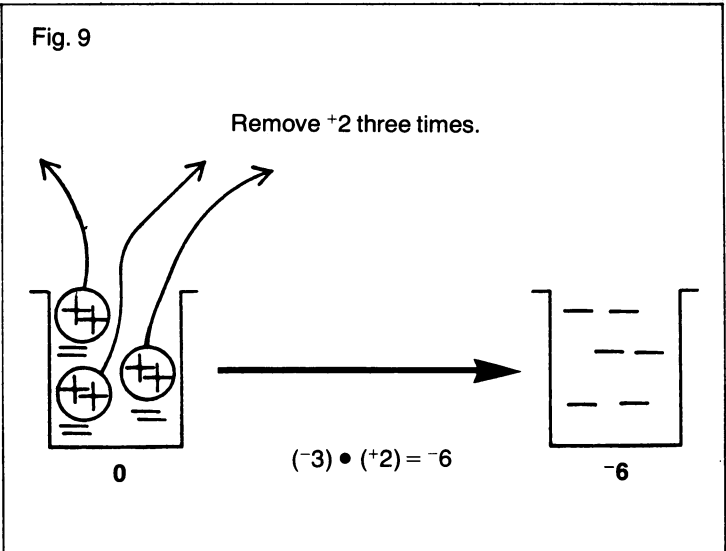
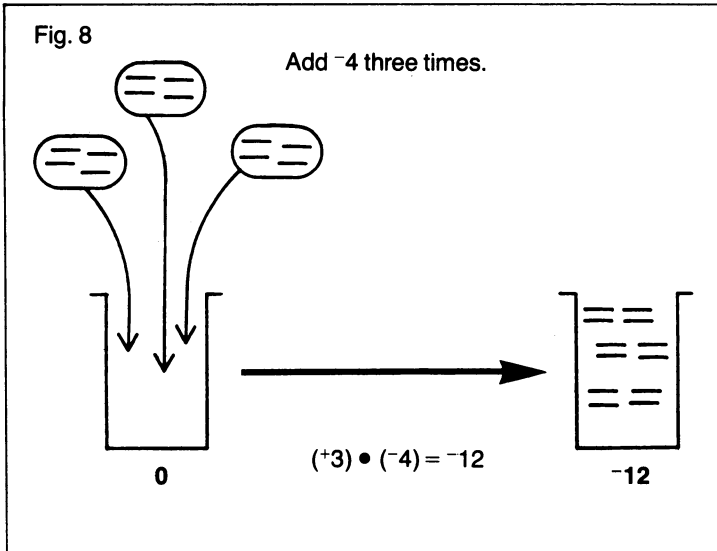
move from this jar a collection of charges representing the second integer (the subtrahend). The new charge on the first jar is the difference. See figure 4.

But what about the problem $(+5) - (-3)$? If $+5$ is represented by a jar that does not contain three negative charges, we cannot physically remove three negative charges from the jar. So, before the subtraction can be performed, the representation for $+5$ must be changed to one that includes at least three negative charges. See figure 5. A similar procedure can be carried out for other subtraction problems requiring a change in representation. See figure 6.

Multiplication

Now we turn to multiplication. The representation is based on our previously defined representations of addition and subtraction. If the first factor in a multiplication problem is *positive*, we interpret the multiplication as *repeated addition* of the second factor. To find $(+3) \cdot (+2)$, for instance, we start with a jar having charge zero (an empty jar), and add two positive charges to it three times. The product is the charge on the now-filled jar. See figure 7. To find $(+3) \cdot (-4)$, we add four negative charges to an empty jar three times and find the resulting charge. See figure 8.

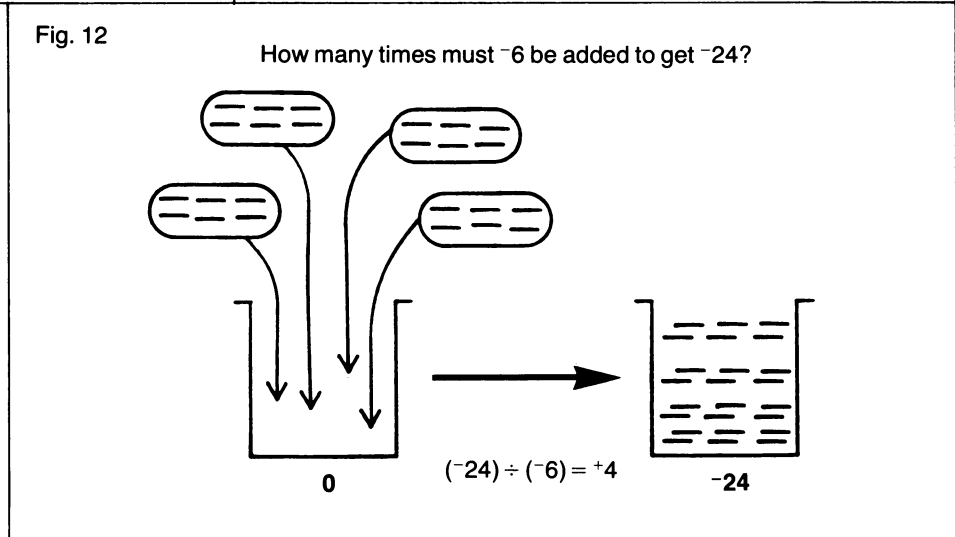
If the first factor in a multiplication problem is *negative*, we interpret the multiplication as *repeated subtraction* of the second factor. For instance, for the problem $(-3) \cdot (+2)$, from a jar having charge zero, we should remove two positive charges three



times, then find the new charge on the jar. We cannot remove charges from an empty jar, however, so a change in representation is necessary. We must start with a zero-charge jar containing at least six (from $3 \cdot 2$) positive and six negative charges. See figure 9. For the problem $(-3) \cdot (-4)$, we must remove four negative charges three times from a jar having charge zero. Here we should start with a zero-charge jar having twelve (from $3 \cdot 4$) positive and twelve negative charges. See figure 10.

Division

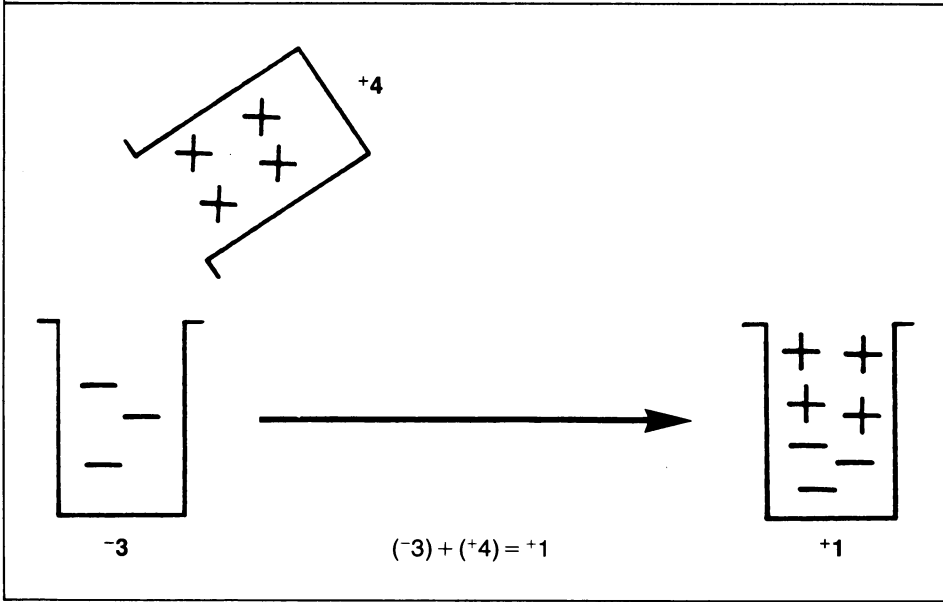
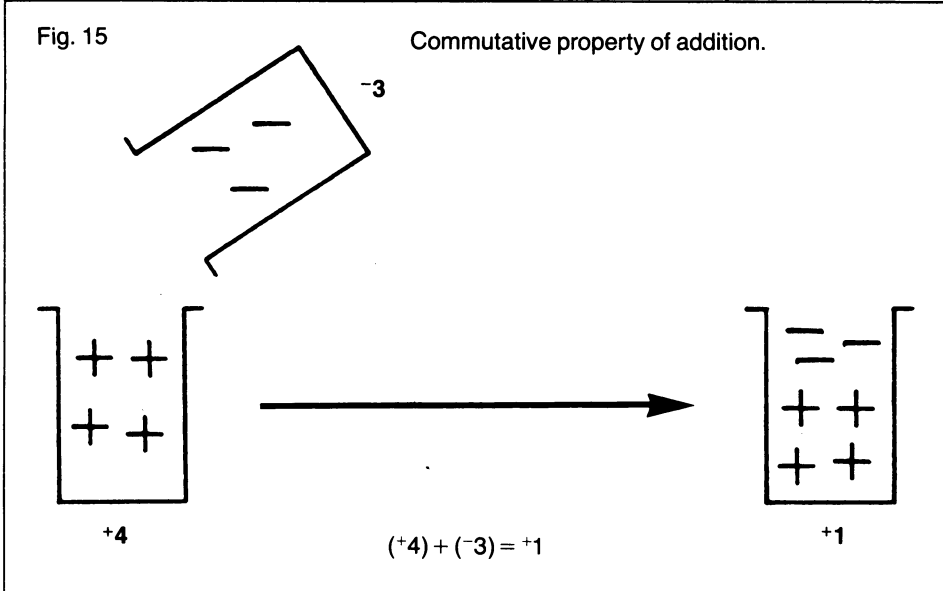
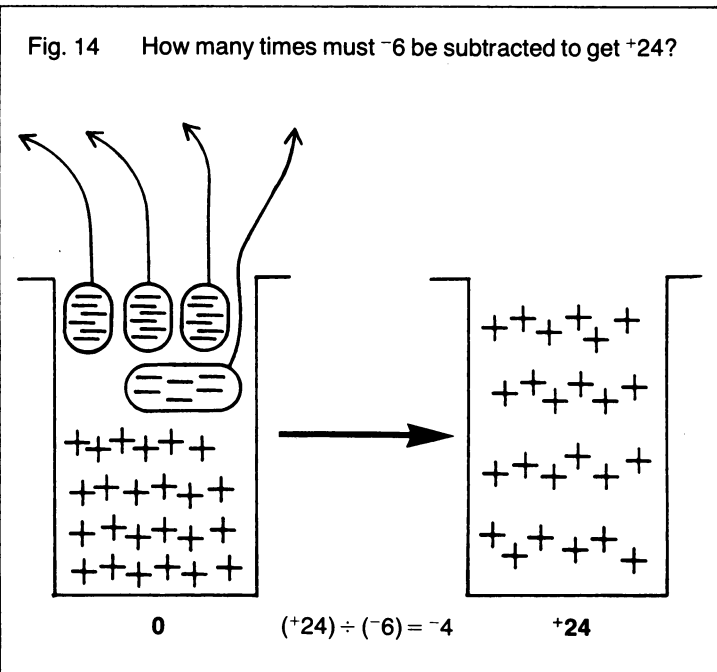
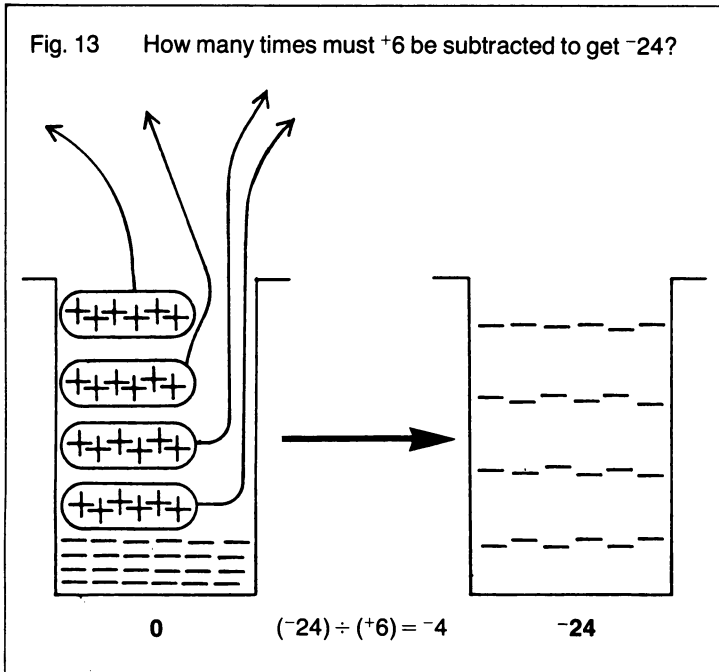
Finally, we examine division. Consider first the case where the dividend and divisor have the same sign. To divide +24 by +6, *count* how many times +6 must be added to a jar having charge zero to get +24. We must *add*



+6 to the jar four times, so +24 divided by +6 is *positive* 4. See figure 11. To divide -24 by -6, count how many times -6 must be added to an empty jar to get -24. Since we must add -6 to the jar four times, the quotient is

+4. See figure 12. In both cases, the quotient is positive because we must repeatedly add the divisor to the jar to get the dividend.

Now what about the case where the dividend and divisor have opposite



signs? Let us look at $(-24) \div (+6)$. If we repeatedly add +6 to a jar having charge zero, obviously we will never get -24. But what happens if we repeatedly *subtract* +6 from a zero-charge jar? After subtracting +6 four times we get -24. So -24 divided by +6 is *negative* 4. See figure 13. To divide +24 by -6, count how many times -6 must be subtracted from a jar having charge zero to get +24. Since -6 must be subtracted four times, the quotient is -4. See figure 14. In both cases, the quotient is negative because we must repeatedly subtract the divisor from the jar to get the dividend.

To understand how the sign of a quotient is derived in the model, we must recall that division and multiplication are inverse operations. Each division problem can be rephrased as a multiplication problem by asking the question, What number must the divisor be multiplied by in order to get the dividend? For example, $(+24) \div (+6) = X$ can be rephrased as $X \cdot (+6) = +24$. Thus, by posing each division problem as a multiplication problem, we see that the sign of the quotient is automatically determined by the way our model represents multiplication: If repeated addition is involved, the first factor (the quotient) is positive; if repeated subtraction is involved, the first factor (the quotient) is negative.

Conclusion

We have seen that all four basic operations on the set of integers can be represented with the positive-negative charge model. The model can also be used to illustrate important structural properties of the system of integers such as the commutative and associative properties, the existence of additive and multiplicative identities, and the existence of additive inverses. The commutative property of addition, for instance, can be illustrated as in figure 15. It is clearly seen in the figure that although $(+4) + (-3)$ and $(-3) + (+4)$ are different additions, the resulting sums are the same. A similar demonstration illustrates why there is *no* commutative property of subtraction.

Thus, the positive-negative charge model presents a useful aid for introducing students to operations on integers. The major advantages to using the model are twofold. First, the model is concrete. Many students receiving initial instruction on the integers need concrete representations of the concepts involved. The charge model's closest "competitor," the number line, is usually presented in a pictorial manner. Second, the model is complete; it can effectively represent all four basic operations on the set of integers. As each new operation is introduced, the same familiar model of the integers can be used. Students being introduced to the representations for multiplication and division can build on their knowledge of the representations for addition and subtraction, thus making the learning of the new operations more meaningful. So the completeness of the charge model serves to give students a more meaningful and coherent picture of the workings of the four basic operations on the set of integers.

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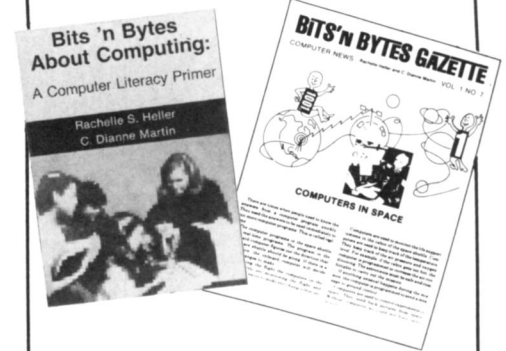
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