



A COMPLEX PROPORTIONAL ASSESSMENT METHOD FOR GROUP DECISION MAKING IN AN INTERVAL-VALUED INTUITIONISTIC FUZZY ENVIRONMENT

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Abstract. Multi-criteria decision making is an implicational field that concerns with selecting or designing the best scenarios among a finite set of scenarios based on a finite set of criteria. Different methods and techniques for handling this issue have been proposed. Complex proportional assessment is an analytical tool for solving multi-criteria decision making problems. Originally, the COPRAS method has been developed for decision making under a deterministic environment. Since uncertainty is an unavoidable property of decision making due to a lack of knowledge, this paper suggests an extended form of the COPRAS method used for group decision making problems in an uncertain environment where such uncertainty is captured through a generalized form of fuzzy sets - the so called interval valued intuitionistic fuzzy sets. An algorithmic scheme for the COPRAS-IVIF method has been introduced thus examining its application with reference to two numerical examples. It seems that the recommended framework of COPRAS-IVIF can be satisfactorily implemented in decision making problems under ambiguous and ill-defined conditions.

Keywords: multiple criteria decision making, uncertainty, COPRAS, interval valued intuitionistic fuzzy sets.

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Introduction

Multiple criteria decision making (MCDM) is an applicable branch of operation research which means ‘*decision making considering multiple affecting criteria*’. This class is further divided into multi-objective decision making and multi-attribute decision making (MADM) (Climaco 1997). The problem of MADM often arises when the issue of a choice or comparison emerges. Due to frequent numerous and antithetic criteria established dealing with actual decision making problems, MCDM methods have become one of the most commonly used branches of operation research science during last decades (Triantaphyllou 2000; Figueira *et al.* 2005; Zavadskas, Turskis 2011; Antucheviciene *et al.* 2011; Kou *et al.* 2012; Bragge *et al.* 2012). A formal definition of the MADM problem can be stated as follows: Suppose that we have a nonempty and finite set of decision alternatives, that their desirability will be judged according to a finite set of goals, attributes or criteria. The aim of MADM is to determine an optimal alternative having the highest degree of desirability with respect to all relevant goals (Zimmerman 1987). Hwang and Yoon (1995) categorized MADM techniques as two compensatory and non-compensatory models. A compensatory class allows interchange among criteria. Compensatory models themselves fall into three main subgroups: scoring sub-models, a compromising subgroup and a concordance subgroup.

Each method of MADM can be classified into deterministic and indeterministic methods. The challenge of uncertainty in decision making is the provenance of such classification. Uncertainty is frequently an output of the unfamiliarity of decision makers with alternatives or their situation regarding different criteria. In fact, most of decisions are not made on the basis of the well-known calculations and there is much ambiguity and uncertainty in decision making problems (Riabacke 2006). Under these conditions, exact data are inadequate to model real-life problems. In response to such complexity and ambiguity, some researchers suggest the application of structures such as fuzzy sets and grey numbers in decision making. The fuzzy set theory, introduced by Zadeh (1965), is the generalization of the classic set theory that assigns a single membership degree (function) to each element. Grattan-Guinness (1976) discovered and later Gau and Buehrer (1993) pointed out that the presentation of a linguistic expression in the form of a fuzzy set was not enough. Atanassov (1986) introduced the notion of an intuitionistic fuzzy set (IFS) as the generalization of Zadeh’s fuzzy sets. In addition to the membership degree of each element in ordinal fuzzy sets, the IFS assigns a degree of non-membership to each element. Later, Atanassov and Gargov (1989) extended it to an interval valued intuitionistic fuzzy set (IVIFS). The IFS and IVIFS have wide applications to analyze problems with uncertainty, including questions related to MADM.

Li (2005) investigated MADM using the IFS and proposed a method for decision making in such environment. Li (2008) further developed linear programming techniques for multidimensional analysis of preference (LINMAP) under the IFS presented by Atanassov. Li *et al.* (2009) and Boran *et al.* (2009) adopted the TOPSIS method under the IFS. Li (2010a) generalized OWA aggregation operators for the IFS and developed a method for MADM. Verma *et al.* (2010), Ye (2010) and Park *et al.* (2011) promoted the TOPSIS method under the IVIFS. Li (2010b) worked out a methodology for solving MADM problems. The ratings of alternatives for both attributes and weights are expressed using the IVIFS. Xu (2012) investigated MADM problems when attribute values were the IFS. Vahdani *et al.* (2012)

improved the ELECTRE method for group MADM based on the IFS. Chen *et al.* (2012) proposed a MADM method based on the IVIFS weighted averaging operator and a fuzzy ranking method for the IFS.

The Complex Proportional Assessment (COPRAS) method as a new method of MADM was suggested by Zavadskas *et al.* (1994). The COPRAS method determines a solution and the ratio to the ideal solution and the ratio to the worst-ideal solution, and therefore can be regarded as a compromising method. The COPRAS method is applied for solving numerous problems by its exhibitors and their colleagues. Andruskevicius (2005) applied the COPRAS method to evaluate contractors based on 26 criteria. Malinauskas and Kalibatas (2005) analyzed the manner of selecting an optimal building technological project in a few examples referring to the COPRAS method. Zavadskas *et al.* (2007) developed and implemented a methodology for multi-attribute assessing of multi-alternative decisions in road construction. Following a rough overview of multi-attribute decision support for assessing road design alternatives, the COPRAS approach was chosen. Banaitiene *et al.* (2008) considered the application of a methodology for the multivariate design and multiple criteria analysis of the life cycle of a building based on COPRAS. Kanapeckienė *et al.* (2011) used the COPRAS method for assessing the market value in the analysis of construction and retrofit projects. Medineckiene and Björk (2011) used COPRAS for preferences regarding renovation measures. Kildiene *et al.* (2011) used the entropy and COPRAS method for a comparative analysis of capabilities owned by European country management within the construction sector at the time of crisis. Chatterjee *et al.* (2011) employed the COPRAS method for selecting materials.

The inherent nature of uncertainty regarding the attribute value and their weights persuaded researchers to extend the COPRAS method under this condition. Zavadskas *et al.* (2008) developed and applied the COPRAS-G method dealing with the problem of matching managers to construction projects. Zavadskas *et al.* (2009), in terms of an assertion about the uncertainty of quantitative and qualitative assessment, used the methodology of grey relations for defining the utility of an alternative and proposed the application of the COPRAS-G method. Bindu Madhuri *et al.* (2010) put forward a multi attribute model for comparing the use of different websites and offered the application of the COPRAS-G method. Rezaeiniya *et al.* (2012) used ANP and COPRAS-G methods for greenhouse locating. Hashemkhani Zolfani *et al.* (2012) suggested a hybrid AHP and COPRAS-G method for selecting a quality control manager. Hashemkhani Zolfani *et al.* (2011) used AHP and COPRAS-G methods for finding the locations of forest roads. Yazdani *et al.* (2011) applied fuzzy COPRAS for the risk analysis of critical infrastructure. Maity *et al.* (2012) employed COPRAS-G for selecting materials. Fouladgar *et al.* (2012a) put forward a fuzzy MADM method based on COPRAS and AHP. Fouladgar *et al.* (2012b) used a new hybrid model for the evaluation of working strategies applying ANP and fuzzy COPRAS.

The aim of this paper is to extend the COPRAS method when the ratings of alternatives on attributes, attribute weights or both are expressed by the IVIFS. The paper is organized as follows: Section 1 reviews the original COPRAS method. Section 2 gives a brief review on the concept of IVIFSs and their operational rules. Section 3 introduces the extended COPRAS-IVIF method. Section 4 provides two applications of the introduced method. The final section consists of conclusions and proposals for future work.

1. The COPRAS method

Multiple attribute decision aid provides several powerful and effective tools. Hwang and Yoon (1995), Vincke (1997), Figueira *et al.* (2005) and Zavadskas and Turskis (2011) presented a comprehensive overview of different MADM techniques. The COPRAS method introduced by Zavadskas *et al.* (1994) is an MADM technique that will be overviewed in this section. Suppose that we have a decision making problem consisting of m alternatives that must be evaluated based on n criteria and x_{ij} is the value of the i^{th} alternative in the j^{th} criterion. The algorithm of the COPRAS method consists of the steps introduced below:

Step 1. Select the available set of the most important attributes describing the alternatives.

Step 2. Prepare decision-making matrix X :

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}. \quad (1)$$

Step 3. Determine the weights of attributes w_j .

Step 4. Normalize decision-making matrix \bar{X} . The normalized values of the matrix are calculated as

$$\bar{x}_{ij} = x_{ij} / \sum_{i=1}^m x_{ij}; \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n. \quad (2)$$

Step 5. Calculate weighted normalized decision-making matrix \hat{X} . Weighted normalized values \hat{x}_{ij} are calculated as

$$\hat{x}_{ij} = \bar{x}_{ij} \cdot w_j; \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n. \quad (3)$$

In formula (3), w_j is the weight of the j^{th} attribute. Next, the weighted normalized decision-making matrix is obtained.

Step 6. Sum up attribute values where higher values are more preferable (maximization is a direction for optimization) and name P_i for each alternative i ($i = 1, 2, \dots, m$):

$$P_i = \sum_{j=1}^k \hat{x}_{ij}. \quad (4)$$

In Eq. (4), k is the number of attributes that must be maximized (it is assumed that in decision-making matrix columns, attributes having a maximum optimal direction are placed before a minimum optimal direction).

Step 7. Sum up attributes values where lower values are more preferable (minimization is a direction for optimization) and name R_i for each alternative i ($i = 1, 2, \dots, m$):

$$R_i = \sum_{j=k+1}^n \hat{x}_{ij}. \quad (5)$$

Step 8. Determine the minimal value of R_i :

$$R_{\min} = \min_i R_i; \quad i = 1, 2, \dots, m. \quad (6)$$

Step 9. Calculate the relative weight of each alternative Q_i :

$$Q_i = P_i + \left(R_{\min} \sum_{i=1}^m R_i \right) / \left(R_i \sum_{i=1}^m R_{\min} / R_i \right). \tag{7}$$

Eq. (7) can be written as follows:

$$Q_i = P_i + \left(\sum_{i=1}^m R_i \right) / \left(R_i \sum_{i=1}^m 1 / R_i \right). \tag{8}$$

Step 10. Determine optimality criterion K :

$$K = \max_i Q_i; i = 1, 2, \dots, m. \tag{9}$$

Step 11. Determine the priority of the projects. Greater weight Q_i for alternative i shows a higher priority (rank) of the alternative. In case of Q_{\max} , the satisfaction degree is the highest.

Step 12. Calculate the utility degree of each alternative:

$$N_i = \left(Q_i / Q_{\max} \right) 100\%, \tag{10}$$

where Q_i and Q_{\max} – the weights of the projects obtained from Eq. (8).

2. Interval valued intuitionistic fuzzy sets

Atanassov (1986) introduced the notion of intuitionistic fuzzy sets in the following way.

Definition 1. Let set E be fixed. Intuitionistic fuzzy set A in E is defined as an object of the following form:

$$A = \left\{ \left\langle x, \mu_A(x), \nu_A(x) \right\rangle \mid x \in E \right\}. \tag{11}$$

Where functions

$$\mu_A : E \rightarrow [0, 1], \tag{12}$$

and

$$\nu_A : E \rightarrow [0, 1], \tag{13}$$

define the degrees of the membership and non-membership of element $x \in E$, and for every $x \in E$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{14}$$

Atanassov and Gargov (1989) generalized the IFS to interval valued intuitionistic fuzzy sets as follows.

Definition 2. Let $D[0, 1]$ be a set of all closed subintervals of interval $[0, 1]$. Let X be a given non-empty set. The IVIFS in X is an expression given by $\tilde{A} = \left\{ \left\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \right\rangle \mid x \in X \right\}$ where $\mu_{\tilde{A}} : X \rightarrow D[0, 1]$, $\nu_{\tilde{A}} : X \rightarrow D[0, 1]$ with condition $0 < \sup_x \mu_{\tilde{A}}(x) + \sup_x \nu_{\tilde{A}}(x) \leq 1$.

Intervals $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ denote the degrees of the membership and non-membership of element x in set A . Thus, for each $x \in X$, $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are closed intervals the lower and upper end points of which are denoted $\mu_{AL}(x)$, $\mu_{AU}(x)$, $\nu_{AL}(x)$ and $\nu_{AU}(x)$.

IVIFS A is denoted by

$$A = \left\{ \left\langle x, \left[\mu_{AL}(x), \mu_{AU}(x) \right], \left[\nu_{AL}(x), \nu_{AU}(x) \right] \right\rangle \mid x \in X \right\}, \tag{15}$$

where $0 < \mu_{AU}(x) + \nu_{AU}(x) \leq 1$, $\mu_{AL}(x), \nu_{AL}(x) \geq 0$. For convenience, the value of the IVIFS is denoted by $\tilde{A} = ([a, b], [c, d])$ and called an interval valued intuitionistic fuzzy number (IVIFN).

Definition 3. Let $\tilde{A}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{A}_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFNs. Then, their operational laws are defined as follows (Xu 2007):

$$\tilde{A}_1 + \tilde{A}_2 = ([a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2], [c_1c_2, d_1d_2]); \tag{16}$$

$$\tilde{A}_1 \cdot \tilde{A}_2 = ([a_1a_2, b_1b_2], [c_1 + c_2 - c_1c_2, d_1 + d_2 - d_1d_2]); \tag{17}$$

$$\lambda \tilde{A}_1 = \left(\left[1 - (1 - a_1)^\lambda, 1 - (1 - b_1)^\lambda \right], [c_1^\lambda, d_1^\lambda] \right), \lambda \geq 0. \tag{18}$$

Definition 4. Let $\tilde{A} = ([a, b], [c, d])$ be an IVIFN. Then,

$$s(\tilde{A}) = \frac{1}{2}(a - c + b - d), \tag{19}$$

is called the score function of \tilde{A} , where $s(\tilde{A}) \in [-1, 1]$ and

$$h(\tilde{A}) = \frac{1}{2}(a + c + b + d), \tag{20}$$

is called the accuracy function of A , where $h(\tilde{A}) \in [0, 1]$ (Xu 2007).

Definition 5. Let \tilde{A}_1 and \tilde{A}_2 be any two IVIFNs. Therefore,

1. If $s(\tilde{A}_1) < s(\tilde{A}_2)$, then \tilde{A}_1 is smaller than \tilde{A}_2 , $\tilde{A}_1 < \tilde{A}_2$.

2. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, then

2.1. If $h(\tilde{A}_1) = h(\tilde{A}_2)$, then $\tilde{A}_1 = \tilde{A}_2$.

2.2. If $h(\tilde{A}_1) < h(\tilde{A}_2)$, then \tilde{A}_1 is smaller than \tilde{A}_2 , $\tilde{A}_1 < \tilde{A}_2$ (Xu 2007).

Definition 6. Let $\tilde{A}_j = ([a_j, b_j], [c_j, d_j])$, $j = 1, 2, \dots, n$ be a collection of IVIFNs. Then, the generalized interval intuitionistic fuzzy weighted average $GIIFWA_w(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)$ is defined as follows:

$$GIIFWA_w(A_1, A_2, \dots, A_n) = \left(w_1 \tilde{A}_1^\lambda + w_2 \tilde{A}_2^\lambda + \dots + w_n \tilde{A}_n^\lambda \right)^{1/\lambda}, \tag{21}$$

where $\lambda > 0$, and $w = (w_1, w_2, \dots, w_n)^T$ is weight vector with $w_j \geq 0$, $j = 1, 2, \dots, n$, and $\sum_{j=1}^n w_j = 1$.

It can be shown that GIIFWA is also an IVIFN and can be calculated as follows (Zhao et al. 2010):

$$GIIFWA_w(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \left[\left[\left(1 - \prod_{j=1}^n (1 - a_j^\lambda)^{w_j} \right)^{1/\lambda}, \left(1 - \prod_{j=1}^n (1 - b_j^\lambda)^{w_j} \right)^{1/\lambda} \right], \right. \\ \left. \left[1 - \left(1 - \prod_{j=1}^n (1 - (1 - c_j)^\lambda)^{w_j} \right)^{1/\lambda}, 1 - \left(1 - \prod_{j=1}^n (1 - (1 - d_j)^\lambda)^{w_j} \right)^{1/\lambda} \right] \right], \quad (22)$$

if $\lambda = 1$, then GIIFWA is turned into interval intuitionistic fuzzy weighted average (IIFWA).

3. COPRAS with IVIF information

In this section, the COPRAS method is extended under the condition that information on the decision making problem has appeared in the form of the IVIFS.

Suppose a decision making problem, as defined in Section 3, containing m alternatives $A_i, i = 1, 2, \dots, m$ and n criteria $C_j, j = 1, 2, \dots, n$ for evaluating those alternatives. Assume that a group of K experts participate in the decision making process. Also, the ratings of alternatives on attributes and/ or attribute weights are not determined exactly and are expressed by IVIFNs. The process of group decision making applying the COPRAS-IVIF method is developed following the below steps.

Step 1. Determine the importance of decision makers. While the decision is made by a group of decision makers, first, the importance or share of each decision maker considering the final decision is determined. Suppose that $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_K)$ is a vector indicating the importance of decision makers, where $\lambda_k \geq 0, k = 1, 2, \dots, K$ is the importance of the k^{th} decision maker and $\sum_{k=1}^K \lambda_k = 1$. Note that if decision makers are of similar importance, $\lambda_1 = \lambda_2 = \dots = \lambda_K = 1/K$.

Step 2. Individual evaluations. On this step, each expert expresses his or her evaluations regarding the ratings of alternatives on attributes and attribute weights. Suppose that $\tilde{x}_{ij}^k, i = 1, 2, \dots, m; j = 1, 2, \dots, n$ is the k^{th} expert's evaluation of alternative A_i rating on criterion j stated by IVIFN $\tilde{x}_{ij}^k = \left(\left[\mu_{Lij}^k, \mu_{Uij}^k \right], \left[\nu_{Lij}^k, \nu_{Uij}^k \right] \right)$. Then, the decision matrix of expert k is constructed as follows:

$$\tilde{X}^k = \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \dots & \tilde{x}_{1n}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \dots & \tilde{x}_{2n}^k \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1}^k & \tilde{x}_{m2}^k & \dots & \tilde{x}_{mn}^k \end{bmatrix}. \quad (23)$$

Synchronously, expert k expresses his or her judgments regarding the weights of criteria. Suppose that $\tilde{w}_j^k = \left(\left[\mu_{Lj}^k, \mu_{Uj}^k \right], \left[\nu_{Lj}^k, \nu_{Uj}^k \right] \right)$ is the k^{th} expert's judgment on the importance of the j^{th} criterion.

Step 3. Determine the weights of criteria. On this step, the aggregated weights of criteria are determined calculating the IIFWA operator, Eq. (22), of criterion weights defined by decision makers. If $\tilde{w}_j^k, k=1,2,\dots,K$ are the weights of criterion j expressed by decision makers, then, the aggregated weight of criterion $j, \tilde{w}_j = \left(\left[\mu_{Lj}, \mu_{Uj} \right], \left[\nu_{Lj}, \nu_{Uj} \right] \right)$, will be computed as follows:

$$w_j = IIFWA_\lambda \left(\tilde{w}_j^1, \tilde{w}_j^2, \dots, \tilde{w}_j^K \right) = \left(\left[\left(1 - \prod_{k=1}^K (1 - \mu_{Lj}^k) \right)^{\lambda_k}, \left(1 - \prod_{k=1}^K (1 - \mu_{Uj}^k) \right)^{\lambda_k} \right], \left[\prod_{k=1}^K (v_{Lj}^k)^{\lambda_k}, \prod_{k=1}^K (v_{Uj}^k)^{\lambda_k} \right] \right). \tag{24}$$

Step 4. Construct the aggregated decision matrix applying IIFWA operators to the elements of individual decision matrices. The aggregated decision matrix will be as follows:

$$\tilde{X}^k = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix}, \tag{25}$$

where element $\tilde{x}_{ij} = \left(\left[\mu_{Lij}, \mu_{Uij} \right], \left[\nu_{Lij}, \nu_{Uij} \right] \right)$ means the rating of alternative A_i on criterion j and is calculated as follows:

$$\tilde{x}_{ij} = IIFWA_\lambda \left(x_{ij}^1, x_{ij}^2, \dots, x_{ij}^K \right) = \left(\left[\left(1 - \prod_{k=1}^K (1 - \mu_{Lij}^k) \right)^{\lambda_k}, \left(1 - \prod_{k=1}^K (1 - \mu_{Uij}^k) \right)^{\lambda_k} \right], \left[\prod_{k=1}^K (v_{Lij}^k)^{\lambda_k}, \prod_{k=1}^K (v_{Uij}^k)^{\lambda_k} \right] \right). \tag{26}$$

Step 5. Calculate the weighted matrix. According to Eq. (3), the weighted matrix is calculated as $\hat{X} = \left[\hat{x}_{ij} \right], i=1,2,\dots,n; j=1,2,\dots,m$, where $\hat{x}_{ij} = \tilde{x}_{ij} \cdot \tilde{w}_j$. Following Eq. (17),

$$\hat{x}_{ij} = \left(\left[\mu_{Lij} \mu_{Lj}, \mu_{Uij} \mu_{Uj} \right], \left[\nu_{Lij} + \nu_{Lj} - \nu_{Lij} \nu_{Lj}, \nu_{Uij} + \nu_{Uj} - \nu_{Uij} \nu_{Uj} \right] \right). \tag{27}$$

Step 6. Sum the values of criteria for benefit. Let $J = \{1,2,\dots,l\}$ be a set of criteria the higher values of which are better. Then, calculate the following index for each alternative:

$$\tilde{P}_i = \sum_{j \in J} \hat{x}_{ij}, \tag{28}$$

where the sum is done based on Eq. (16).

Step 7. Sum the values of cost criteria. Let $J' = \{l+1, l+2, \dots, n\}$ be a set of criteria the lower values of which are better. Then, calculate the following index for each alternative.

$$\tilde{R}_i = \sum_{j \in J'} \hat{x}_{ij}. \tag{29}$$

Step 8. Determine the minimal value of \tilde{R}_i referring to Definition 5.

Step 9. Calculate the relative weight of each alternative (Q_i). Let $s(\tilde{R}_i)$ and $s(\tilde{P}_i)$ be the scores of \tilde{R}_i and \tilde{P}_i respectively.

$$Q_i = s(\tilde{P}_i) + \frac{s(\tilde{R}_{\min}) \sum_{i=1}^m s(\tilde{R}_i)}{s(\tilde{R}_i) \sum_{i=1}^m \frac{s(\tilde{R}_{\min})}{s(\tilde{R}_i)}} \tag{30}$$

Eq. (30) can be written as follows:

$$Q_i = s(\tilde{P}_i) + \frac{\sum_{i=1}^m s(\tilde{R}_i)}{s(\tilde{R}_i) \sum_{i=1}^m \frac{1}{s(\tilde{R}_i)}} \tag{31}$$

Step 10. Determine optimality criterion K .

$$K = \max_i Q_i; i = 1, 2, \dots, m. \tag{32}$$

Step 11. Determine the priority of alternatives. The greater weight Q_i for alternative i shows, the higher is the priority (rank) of the alternative. The utility degree of each alternative is defined as follows:

$$N_i = (Q_i / Q_{\max}) 100\%. \tag{33}$$

4. Numerical examples

This section analyzes some application examples of MCDM problems previously solved with the help of different methods.

Example 1. Wang *et al.* (2009) examined a decision problem consisting of the evaluation of four potential investment alternatives $A = \{A_1, A_2, A_3, A_4\}$ based on four attributes: risk (C_1), growth (C_2), socio-political issues (C_3) and environment impacts (C_4). The fund manager provided each alternative on each attribute as an IVIFN. The provided decision matrix is as follows:

$$\tilde{X} = \begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \left(\begin{matrix} ([0.42, 0.48], [0.4, 0.5]) \\ ([0.4, 0.5], [0.4, 0.5]) \\ ([0.3, 0.5], [0.4, 0.5]) \\ ([0.2, 0.4], [0.4, 0.5]) \end{matrix} \right) & \left(\begin{matrix} ([0.6, 0.7], [0.05, 0.25]) \\ ([0.5, 0.8], [0.1, 0.2]) \\ ([0.1, 0.3], [0.2, 0.4]) \\ ([0.6, 0.7], [0.2, 0.3]) \end{matrix} \right) & \left(\begin{matrix} ([0.4, 0.5], [0.2, 0.5]) \\ ([0.3, 0.6], [0.3, 0.4]) \\ ([0.7, 0.8], [0.1, 0.2]) \\ ([0.5, 0.6], [0.2, 0.3]) \end{matrix} \right) & \left(\begin{matrix} ([0.55, 0.75], [0.15, 0.25]) \\ ([0.6, 0.7], [0.1, 0.3]) \\ ([0.5, 0.7], [0.1, 0.2]) \\ ([0.7, 0.8], [0.1, 0.2]) \end{matrix} \right) \end{matrix} \tag{34}$$

Also, the fund manager determines attribute weights as $w_1 = 0.13$, $w_2 = 0.17$, $w_3 = 0.39$ and $w_4 = 0.31$. While there is only one decision maker, COPRAS-IVIF is started from Step 5.

Since attribute weights are expressed as crisp numbers, the values of \hat{x}_{ij} are computed based on Eq. (18) as follows:

$$\hat{x}_{ij} = \left(\left[1 - (1 - \mu_{Lij})^{w_j}, 1 - (1 - \mu_{Uij})^{w_j} \right], \left[v_{Lij}^{w_j}, v_{Uij}^{w_j} \right] \right). \tag{34}$$

Therefore, the weighted matrix is provided in the following way:

$$\hat{X} = \begin{pmatrix} \left(\left[\begin{matrix} 0.0684, 0.0815 \\ 0.8877, 0.9138 \end{matrix} \right], \left[\begin{matrix} 0.1442, 0.1851 \\ 0.6009, 0.79 \end{matrix} \right] \right), & \left(\left[\begin{matrix} 0.1806, 0.2369 \\ 0.5338, 0.7631 \end{matrix} \right], \left[\begin{matrix} 0.2193, 0.3493 \\ 0.5554, 0.6507 \end{matrix} \right] \right), \\ \left(\left[\begin{matrix} 0.0643, 0.0862 \\ 0.8877, 0.9138 \end{matrix} \right], \left[\begin{matrix} 0.1112, 0.2394 \\ 0.6761, 0.7606 \end{matrix} \right] \right), & \left(\left[\begin{matrix} 0.1299, 0.3005 \\ 0.6253, 0.6995 \end{matrix} \right], \left[\begin{matrix} 0.2473, 0.3115 \\ 0.4898, 0.6885 \end{matrix} \right] \right), \\ \left(\left[\begin{matrix} 0.0453, 0.0862 \\ 0.8877, 0.9138 \end{matrix} \right], \left[\begin{matrix} 0.0178, 0.0588 \\ 0.7606, 0.8558 \end{matrix} \right] \right), & \left(\left[\begin{matrix} 0.3747, 0.4662 \\ 0.4074, 0.5338 \end{matrix} \right], \left[\begin{matrix} 0.1934, 0.3115 \\ 0.4898, 0.6072 \end{matrix} \right] \right), \\ \left(\left[\begin{matrix} 0.0286, 0.0643 \\ 0.8877, 0.9138 \end{matrix} \right], \left[\begin{matrix} 0.1442, 0.1851 \\ 0.7606, 0.8149 \end{matrix} \right] \right), & \left(\left[\begin{matrix} 0.2369, 0.3005 \\ 0.5338, 0.6253 \end{matrix} \right], \left[\begin{matrix} 0.3115, 0.3928 \\ 0.4898, 0.6072 \end{matrix} \right] \right) \end{pmatrix}.$$

Table 1 shows the values of \tilde{P}_i , \tilde{R}_i and \tilde{Q}_i for alternatives while C_1 is considered as the cost criterion and other criteria as benefit.

Table 1. The values of \tilde{P}_i , \tilde{R}_i and \tilde{Q}_i taking into account alternatives

Alternative	\tilde{P}_i	\tilde{R}_i	\tilde{Q}_i	Rank
A_1	([0.4526, 0.5954], [0.1782, 0.3923])	([0.0684, 0.0815], [0.8877, 0.9138])	-0.60574	3
A_2	([0.4178, 0.6337], [0.2071, 0.3662])	([0.0643, 0.0862], [0.8877, 0.9138])	-0.60567	4
A_3	([0.5046, 0.6541], [0.1518, 0.2774])	([0.0453, 0.0862], [0.8877, 0.9138])	-0.47044	2
A_4	([0.5504, 0.6539], [0.1989, 0.3094])	([0.0286, 0.0643], [0.8877, 0.9138])	-0.46831	1

The obtained ranking of the COPRAS-IVIF method, $A_4 \succ A_3 \succ A_1 \succ A_2$, is completely consistent with the results provided by Wang *et al.* (2009).

Note that the membership function can be considered as the satisfaction degree and non-membership degree, as a decision maker’s dissatisfaction, regarding the level of a given alternative, meets the requirements of a certain criterion. In fact, decision makers can make their evaluations in the form of a satisfaction and dissatisfaction degree as IFSS. Next, these IFSSs can be turned into equivalent IVIFSSs employing the method proposed by Bustince and Burillo (1995).

Example 2. Assume that a manufacturing company wants to outsource its annual maintenance operation. The company received four proposals from external maintenance service providers (MSP). A team consisting of three members is formed to choose the best MSP. The team members are of equal importance, i.e. $\lambda = (1/3, 1/3, 1/3)$. Also, four criteria, including the proposed price (C_1), maintenance duration (C_2), reputation (C_3) and workers’ speciality (C_4) are considered to appraise these candidates. The decision making process involves the following steps.

Steps 1–2. Each decision maker performs his/her evaluations with regard to the rating of alternatives on attributes. Decision makers have made evaluations in linguistic terms later turned to IVIFNs. Boran *et al.* (2009) developed a scale of performing evaluations based on the IFS. In this case, with reference to the theorem proposed by Bustince and Burillo’s (1995), the scale is transformed into the equivalent IVIFS scale.

Based on the IVIFN scale presented in Table 2, decision makers perform their evaluations of alternatives. Individual decision matrices are presented in Table 3.

Table 2. The IVIFN scale to rate alternatives to criteria

Linguistic term	IVIFNs
Extremely good (EG)/ extremely high (EH)	([1, 1], [0, 0])
Very very good (VVG)/ very very high (VVH)	([0.9, 0.9], [0.1, 0.1])
Very good (VG)/ very high (VH)	([0.7333, 0.825], [0, 0.125])
Good (G)/ high (H)	([0.6333, 0.725], [0.1, 0.225])
Medium good (MG)/ medium high (MH)	([0.5333, 0.625], [0.2, 0.325])
Fair (F)/ medium (M)	([0.4333, 0.525], [0.3, 0.425])
Medium bad (MB)/ medium low (ML)	([0.3333, 0.425], [0.4, 0.525])
Bad (B)/ low (L)	([0.15, 0.2875], [0.45, 0.6375])
Very bad (VB)/ very low (VL)	([0, 0.1375], [0.6, 0.7875])
Very very bad (VVL)/ very very low (VVL)	([0.1, 0.1], [0.9, 0.9])

Table 3. Individual decision matrices

DM1	C ₁	C ₂	C ₃	C ₄
A ₁	MH	M	L	H
A ₂	H	M	H	MH
A ₃	M	H	MH	VH
A ₄	M	MH	M	MH
DM2	C ₁	C ₂	C ₃	C ₄
A ₁	M	ML	MH	H
A ₂	MH	H	H	H
A ₃	MH	VH	H	MH
A ₄	M	M	ML	ML
DM3	C ₁	C ₂	C ₃	C ₄
A ₁	H	MH	ML	VH
A ₂	VH	M	H	H
A ₃	H	MH	MH	MH
A ₄	MH	M	M	M

Step 3. Decision makers also express their judgments with regard to the importance of criteria for evaluating alternatives. Boran *et al.* (2009) also presented an intuitionistic fuzzy scale of judgments that, in our case, are transformed into the IVIF scale based on the theorem developed by Bustince and Burillo (1995). Table 4 shows the IVIF scale to evaluate the weights of criteria.

Table 5 shows the preference of decision makers for the weights of criteria.

Table 4. Linguistic terms for rating the importance of criteria

Linguistic term	IVIFNs
Very important (VI)	$([0.9, 0.9], [0.1, 0.1])$
Important (I)	$([0.4, 0.7625], [0, 0.2115])$
Medium (M)	$([0.15, 0.5125], [0.25, 0.4625])$
Unimportant (U)	$([0, 0.3625], [0.4, 0.6125])$
Very unimportant (VU)	$([0.1, 0.1], [0.9, 0.9])$

Table 5. Individual decision matrices

	C ₁	C ₂	C ₃	C ₄
DM1	M	M	I	I
DM2	I	I	M	I
DM3	I	M	U	I

The aggregated weights of criteria based on Eq. (24) and importance vector λ found by decision makers are determined as follows:

$$\begin{aligned} \tilde{w}_1 &= ([0.3261, 0.6982], [0, 0.2745]) & \tilde{w}_2 &= ([0.2432, 0.6164], [0, 0.3563]) \\ \tilde{w}_3 &= ([0.2010, 0.5805], [0, 0.3913]) & \tilde{w}_4 &= ([0.4, 0.7625], [0, 0.2115]) \end{aligned}$$

Step 4. The aggregated decision matrix is constructed following Eq. (26) and importance vector λ established by decision makers as follows:

$$\tilde{X} = \begin{pmatrix} \begin{pmatrix} [0.5406, 0.6341] \\ [0.1817, 0.3144] \end{pmatrix} & \begin{pmatrix} [0.4392, 0.5321] \\ [0.2884, 0.4170] \end{pmatrix} & \begin{pmatrix} [0.3581, 0.4644] \\ [0.3302, 0.4774] \end{pmatrix} & \begin{pmatrix} [0.6702, 0.7635] \\ [0, 0.1850] \end{pmatrix} \\ \begin{pmatrix} [0.6426, 0.7377] \\ [0, 0.2091] \end{pmatrix} & \begin{pmatrix} [0.5098, 0.6041] \\ [0.2080, 0.3438] \end{pmatrix} & \begin{pmatrix} [0.6333, 0.7250] \\ [0.1, 0.2250] \end{pmatrix} & \begin{pmatrix} [0.6026, 0.6950] \\ [0.1260, 0.2543] \end{pmatrix} \\ \begin{pmatrix} [0.5406, 0.6341] \\ [0.1817, 0.3144] \end{pmatrix} & \begin{pmatrix} [0.6426, 0.7377] \\ [0, 0.2091] \end{pmatrix} & \begin{pmatrix} [0.5693, 0.6618] \\ [0.1587, 0.2875] \end{pmatrix} & \begin{pmatrix} [0.6127, 0.7091] \\ [0, 0.2364] \end{pmatrix} \\ \begin{pmatrix} [0.4688, 0.5610] \\ [0.2621, 0.3886] \end{pmatrix} & \begin{pmatrix} [0.4688, 0.5610] \\ [0.2621, 0.3886] \end{pmatrix} & \begin{pmatrix} [0.4018, 0.4938] \\ [0.3302, 0.4560] \end{pmatrix} & \begin{pmatrix} [0.4392, 0.5321] \\ [0.2884, 0.4170] \end{pmatrix} \end{pmatrix}$$

Step 5. Based on Eq. (27), the weighted matrix is formed as follows.

$$\hat{X} = \begin{pmatrix} \begin{pmatrix} [0.1763, 0.4427] \\ [0.1817, 0.5026] \end{pmatrix} & \begin{pmatrix} [0.1068, 0.3280] \\ [0.2884, 0.6247] \end{pmatrix} & \begin{pmatrix} [0.0720, 0.2696] \\ [0.3302, 0.6819] \end{pmatrix} & \begin{pmatrix} [0.2681, 0.5821] \\ [0, 0.3573] \end{pmatrix} \\ \begin{pmatrix} [0.2096, 0.5150] \\ [0, 0.4262] \end{pmatrix} & \begin{pmatrix} [0.1240, 0.3724] \\ [0.2080, 0.5776] \end{pmatrix} & \begin{pmatrix} [0.1273, 0.4209] \\ [0.1, 0.5283] \end{pmatrix} & \begin{pmatrix} [0.2410, 0.53] \\ [0.1260, 0.2543] \end{pmatrix} \\ \begin{pmatrix} [0.1763, 0.4427] \\ [0.1817, 0.5026] \end{pmatrix} & \begin{pmatrix} [0.1563, 0.4547] \\ [0, 0.4909] \end{pmatrix} & \begin{pmatrix} [0.1145, 0.3842] \\ [0.1587, 0.5663] \end{pmatrix} & \begin{pmatrix} [0.2451, 0.5407] \\ [0, 0.3776] \end{pmatrix} \\ \begin{pmatrix} [0.1529, 0.3917] \\ [0.2621, 0.5565] \end{pmatrix} & \begin{pmatrix} [0.1140, 0.3458] \\ [0.2621, 0.6065] \end{pmatrix} & \begin{pmatrix} [0.0808, 0.2866] \\ [0.3302, 0.6689] \end{pmatrix} & \begin{pmatrix} [0.1757, 0.4057] \\ [0.2884, 0.5653] \end{pmatrix} \end{pmatrix}$$

Steps 6–11. Since C_1 and C_2 are cost criteria and C_3 and C_4 are criteria for benefit, the computed values of \tilde{P}_i , \tilde{R}_i and \tilde{Q}_i for each alternative are shown in Table 6.

Table 6. The values of \tilde{P}_i , \tilde{R}_i and \tilde{Q}_i for alternatives

Alternative	\tilde{P}_i	\tilde{R}_i	\tilde{Q}_i	Rank
A_1	([0.3208, 0.6948], [0, 0.2437])	([0.2643, 0.6255], [0.0524, 0.3140])	0.7343	1
A_2	([0.3377, 0.7278], [0.0126, 0.1344])	([0.3076, 0.6956], [0, 0.2462])	0.7001	2
A_3	([0.3315, 0.7172], [0, 0.2138])	([0.3050, 0.6961], [0, 0.2467])	0.6590	3
A_4	([0.2423, 0.5761], [0.0952, 0.3781])	([0.2495, 0.6020], [0.0687, 0.3375])	0.5818	4

Therefore, the final ranking is $A_1 \succ A_2 \succ A_3 \succ A_4$. The example shows the process of decision making applying the proposed COPRAS-IVIF method from the beginning to the end.

Conclusions

Decision making solving practical problems is a often group activity that requires the judgments of decision makers about the importance of a set of criteria and alternatives. These evaluations are always performed with ambiguity based on ill-defined information. Ignoring this ambiguity and uncertainty in decision making may mislead the decision process to an unfair decision. To avoid this problem, scholars developed some procedures to handle uncertainty in decision making. Interval valued intuitionistic fuzzy sets, as a generalized form of fuzzy sets, can be considered as a powerful mean to convey uncertainty in a good manner in decision making. In this paper, the COPRAS method is extended for group decision making in an interval valued intuitionistic fuzzy environment. IVIF data provide a more realistic picture from uncertain aspects of real world problems, and therefore an algorithmic scheme for making decisions employing the above discussed method has been proposed. A combination of IVIFN arithmetic and aggregation operators is used for developing COPRAS-IVIF. Also, some scales are promoted to perform evaluations based on linguistic terms and IVIFNs. The application of the proposed method is examined in two numerical examples. The suggested method can be applied dealing with various selection and ranking problems in different fields as a common uncertain decision analysis method.

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