Engineering Journal of Qatar University, Vol. 1, 1988.

# "A COMPRESSIVE STRENGTH CRITERION FOR ANISOTROPIC ROCK MATERIALS"

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**ABSTRACT**

This paper puts forward a general compressive strength criterion for anisotropic rock materials under multiaxial state of stress. The proposed criterion, is a generalization of the Von Mises yield criterion for ductile metals, which criterion has previously been used for brittle fracture as well, both being looked upon as limits of linear elastic behaviour. The proposed criterion takes into consideration the effects, on rock material failure under multiaxial stress state, of confining pressure, presence of various stress components, and material anisotropy. Relevant failure envelopes, for several types of rock materials, are compared with corresponding experimental results; close agreement being obtained in all cases. This result confirms the versatility and applicability of the proposed strength criterion in representing the compressive strength behaviour of anisotropic rock materials under complex multiaxial state of stress.

## 1. INTRODUCTION

In many engineering applications, rational design with engineering materials requires the use of a suitable strength criterion for estimating the load carrying capacity of the material under complex multiaxial loading. Numerous strength criteria, are at present, available for predicting stress states that cause yielding or fracture of engineering materials (1-9). The strength of an engineering material is, in general, a function of many factors such as stress state, temperature and strain rate. Current failure criteria are commonly based on the assumption that, at constant temperature and strain rate, the strength depends solely on the state of stress in the material.

In geotechnical engineering practice there are currently several strength theories for rock materials which are in common use (7). Most of these hypotheses, for example Coulomb-Navier, Mohr, Griffith, and modified Griffith criteria, take into consideration the long established confining pressure dependency of rock strength. Nevertheless, in many cases none of these criteria is satisfactory. They fail to correlate experimental results under complex stress states (10, 11). There have been more recently several attempts to develop more adequate criteria such as those depicted in (12, 13).

It is noted that all of the aforementioned hypotheses do not consider the effect of intermediate principal stress, $\sigma_2$, on failure in a multiaxial stress state. This effect, however, is important as has been demonstrated experimentally. It should then be considered in any adequate failure criterion (7, 8). Moreover all these theories inherently assume the material to be isotropic, a condition not satisfied in many types of rocks. As is well known, rock usually occurs in its natural state as a flawed, inhomogeneous, anosotropic and discontinuous material.

One of the criteria which considers the $\sigma_2$ effect is the Von Mises yield criterion for ductile metals. Extensive investigations have shown the high applicability of Von Mises criterion for yielding of ductile metals (1). This criterion states that for failure:

$$\tau_{oct} = (1/3)\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} = \text{constant} \quad (1)$$

in which $_{oct}$ denotes the octahedral shear stress and $\sigma_1$, $\sigma_2$, $\sigma_3$ represent principal stresses. The Von Mises criterion can be interpreted as stating that yielding will occur when the distortional strain energy reaches a certain value. This yield criterion has also been used as a strength criterion for brittle fracture as both may be looked upon as limits of linear elastic behaviour.

The Von Mises theory is applicable to the case in which the yield stress or the brittle fracture stress is independent of the confining pressure. There are, however, several cases involving metals in which the yield strength apparently increases with confining pressure (14). Extensive experimentation on rock materials has clearly shown that both yield and failure strength of many rocks are appreciably affected by the confining pressure.

In order to achieve good correlation between theory and experiment, Mogi (11) proposed a modified von Mises criterion for brittle fracture of rocks as follows:

For failure $\tau_{oct} = f(\sigma_1 + \sigma_3)$ (2)

in which f is a monotonically increasing fu⅛ction of maximum and minimum principal stresses, $\sigma_1$ and $\sigma_3$. Based on this criterion, good correlation was possible with experimental results for a number of rocks. Mogi's experimental results indicated that f approximately takes the form:

$$f(\sigma_1, \sigma_3) = C_0 + C_1 (\sigma_1 + \sigma_3) \tag{3}$$

$C_0$ and $C_1$ being constants. Mogi's criterion, however, inherently assumed an isotropic material. Such assumption would severely limit its application in many cases of rock materials which exhibit clear anisotropic behaviour with regard to fracture under both uniaxial and multiaxial states of stress.

Finally, a generalized Von Mises criterion for brittle fracture of coal was proposed by Ashour and Hsu, (15). Based on this criterion, it was possible to establish failure envelopes for several types of coal under biaxial and triaxial compression, and to obtain good correlation with experimental results. For generally anisotropic rock materials, however, a more general strength criterion is needed to adequately represent strength behaviour of such materials under general multiaxial states of stress.

In the light of these concepts, it seems worthwhile to develop a general compressive strength criterion for anisotropic rock materials under multiaxial states of stress with adequate experimental verification. Several test results from various sources covering a wide variety of rock materials and various types of failure, are herein correlated with the proposed criterion.

## 2. AN ANISOTROPIC STRENGTH CRITERION

The isotropic Von Mises criterion, as represented by Equation (1), can be reformulated as follows:

$$(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6\,\tau_{xy}^2 + 6\,\tau_{xz}^2 + 6\,\tau_{yz}^2 = \text{constant} \tag{4}$$

x, y, and z being three arbitrary mutuallyperpendicular axes. $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{xz}$ and $\tau_{yz}$ are stress components in the x, y, z coordinate system. For anisotropic materials, Equation (4) can be generalized to assume the form:

$$A\ \sigma_x^2 + B\ \sigma_y^2 + C\ \sigma_z^2 + D\,\sigma_x\,\sigma_y +$$

$$E\,\sigma_x\,\sigma_z + F\,\sigma_y\,\sigma_z + G\ \tau_{xy}^2 + H\ \tau_{xz}^2 + I\ \tau_{yz}^2 \qquad (5)$$

in which A, B, C, D, E, F, G, H, and I are experimentally determined constants. Other generalizations have been proposed by several investigators (6).

As mentioned earlier, there has been ample experimental evidence that the compressive strength of rock materials increases with confining pressure. Consequently, for generally anisotropic rock materials, the strength criterion given in Equation (5) can be further generalized to read as follows:

$$A\ \sigma_x^2 + B\ \sigma_y^2 + C\ \sigma_z^2 + D\,\sigma_x\,\sigma_y + E\,\sigma_x\,\sigma_z +$$

$$F\,\sigma_y\,\sigma_z + G\ \tau_{xy}^2 + H\ \tau_{xz}^2 + I\ \tau_{yz}^2 = f(\sigma_x, \sigma_y, \sigma_z) \qquad (6)$$

f denoting a monotonically increasing function of the normal compressive stresses $\sigma_x$, $\sigma_y$, $\sigma_z$. In the present investigation, the following form for the function f is proposed:

$$f(\sigma_x, \sigma_y, \sigma_z) = C_o + C_1\sigma + C_2 e^{(\alpha\,\sigma)} \qquad (7)$$

$\sigma = (\sigma_x + \sigma_y + \sigma_z)$. The coefficients A, B, C, D, E, F, G, H, I, $C_0$, $C_1$, $C_2$ and $\alpha$ are experimentally determined constants. Equations (6) and (7) are the proposed compressive strength criterion for anisotropic rock materials which can be stated as follows:

For failure, $\tilde{E} \geqslant f(\sigma)$ (8)

where $\tilde{E} = A\ \sigma_x^2 + B\ \sigma_y^2 + C\ \sigma_z^2 + D\ \sigma_x\,\sigma_y +$

$$E\ \sigma_x\,\sigma_z + F\ \sigma_y\,\sigma_z + G\,\tau_{xy}^2 + H\ \tau_{xz}^2 + I\,\tau_{yz}^2$$

$$f(\sigma) = C_o + C_1\sigma + C_2 e^{(\alpha\sigma)}$$

$$\sigma = \sigma_x + \sigma_y + \sigma_z$$

## 3. CORRELATION BETWEEN THEORETICAL ENVELOPES AND EXPERIMENTAL RESULTS

The proposed criterion is herein used to construct the failure envelopes for several types of rock materials. In each case, the available test results are used in conjunction with a weighted residuals procedure to determine optimum values of the constants in a non-dimensional form of Equation (8); this would minimize summation of weighted squares of residuals. The optimum value of the coefficient $\alpha$ is further determined by trial and error. In this way, the failure envelope is obtained as a relation between $\bar{E}$ and $\sigma$, Equation (8). Consequently in order to compare the theoretical envelope and the experimental results, a diagram is constructed in each case for the failure envelope together with the experimental results in $(\bar{E} - \sigma)$ coordinates. Quantities in the strength criterion are herein normalised according to the following notation:

a) $\bar{\sigma}_x, \quad \bar{\tau}_{xy}, \ldots = \sigma_x / \sigma_o, \quad \tau_{xy}/\sigma_o$

to be the non-dimensionalized stress components with respect to the lowest uniaxial compressive strength of the material, $\sigma_0$.

b) $\bar{\sigma} = \bar{\sigma}_x + \bar{\sigma}_y + \bar{\sigma}_z$

Thus, the failure criterion in dimensionless form becomes:

For failure, $\bar{E} \geqq f(\bar{\sigma})$

$$\text{in which } \bar{E} = A\bar{\sigma}_x^2 + B\bar{\sigma}_y^2 + C\bar{\sigma}_z^2 + D\bar{\sigma}_x\bar{\sigma}_y + E\bar{\sigma}_x\bar{\sigma}_z + F\bar{\sigma}_y\bar{\sigma}_z + G\bar{\tau}^2_{xy} + H\bar{\tau}^2_{xz} + I\bar{\tau}^2_{yz} \tag{9}$$

$$f(\bar{\sigma}) = C_o + C_1\bar{\sigma} + C_2 e^{(\alpha\bar{\sigma})}$$

$$\bar{\sigma} = \bar{\sigma}_x + \bar{\sigma}_y + \bar{\sigma}_z$$

Four types of rock materials are considered. The first is Green River Shale, a clay shale which has been tested by Chenevert and Gatlin (16) under triaxial compression using cylinderical specimens for a confining pressure range between 0 and 83 MPa. In

Chenevert and Gatlin tests, the critical stress states were determined for various inclination angles between bedding planes and specimen's axis ( $\sigma_1$ direction). Figure (1) presents the theoretical strength envelope for the shale, based on the proposed criterion, together with Chenevert and Gatlin experimental results.

The second type of rock is Pentremawr Coal, a lightly cleated coal which has been tested by Hobbs (17) under triaxial compression using 90 cylindrical specimens for a confining pressure range between 0 and 34 MPa. Specimens tested at each confining pressure were selected at random. Seven to eight specimens were tested at each confining pressure. The theoretical strength envelope for Pentremawr Coal together with Hobbs' experimental results are displayed in Figure (2).

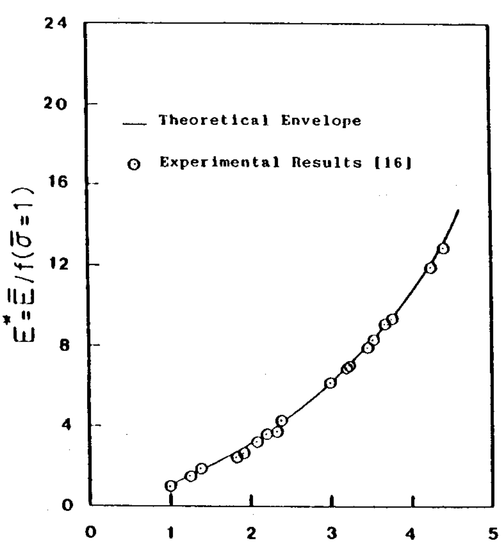


**Figure (1)**

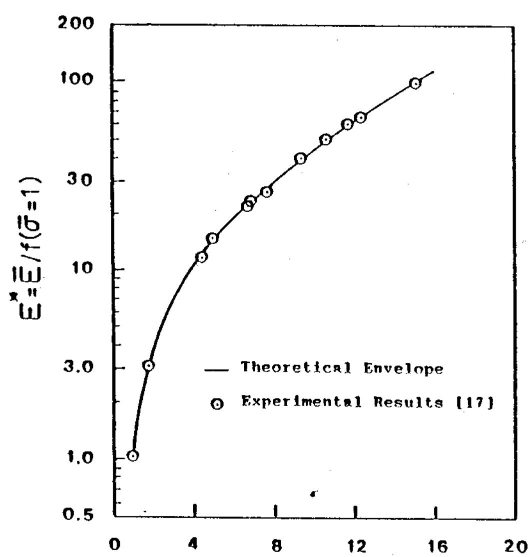


**Figure (2)**

**Figure 1: Theoretical Strength Envelope and Experimental Results for Green River Shale.**

**Figure 2: Thearetical Strength and Experimental Results for Pentremawr Coal.**

The third type of rock is Solenhofen Limestone, a very fine-grained material which has been extensively tested by Handin et al. (10). In these tests, both solid and thin-walled hollow cylindrical specimens were tested at room temperature and at

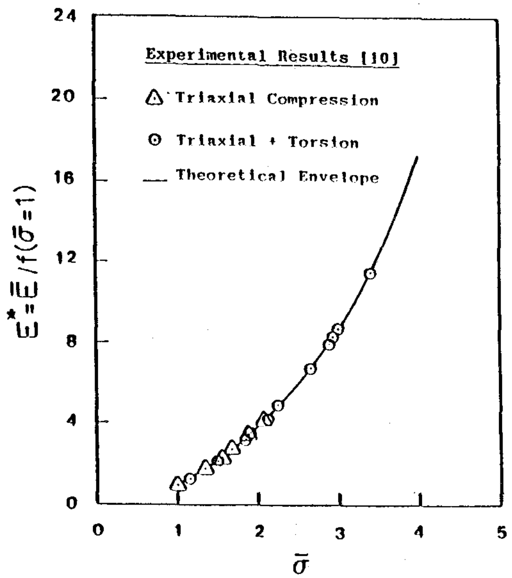


Figure (3)

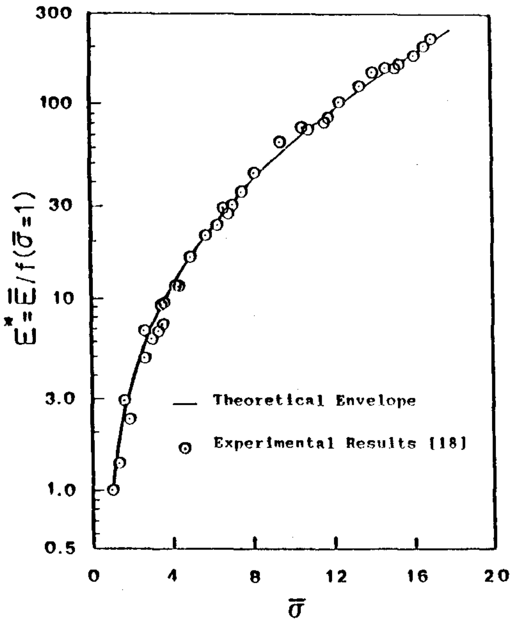


Figure (4)

**Figure 3: Theoretical Strength Envelope and Experimental Results for Solenhofen Limestone.**

**Figure 4: Theoretical Strength Envelope and Experimental Results for a Laminated Silty Mudstone.**

strain rates of the order of $10^{-4}$ per second. The solid cylinders were subjected to triaxial compression at variable levels of confining pressure. The hollow cylinders were subjected, however, to triaxial compression combined with torsion. The thin-walled hollow cylindrical geometry was used in order to maintain a nearly uniform shear stress distribution in the specimen under the applied torque. Figure (3) depicts the theoretical failure strength envelope for limestone, based on the proposed criterion, together with relevant experimental results. It is worth noting that the experimental results for both solid and hollow cylinders, represent either brittle or ductile failure depending on the level of confining pressure.

The fourth type of rock is a laminated silty mudstone which forms the roof rock of the High Main coal seam in County Durham, England. Extensive test results for this mudstone under triaxial compression are reported in (18) for a confining pressure range between 0 and 43 MPa. In these tests, critical stress states were determined for various inclination angles between the laminae planes and the specimen's axis (direction of application of the major principal stress). Theoretical strength envelopes for the mudstone together with the experimental results are exhibited in Figure (4).

Figures (1) to (4) indicate that, in all cases, close agreement exists between the experimental results and the theoretical failure envelopes which are based on the proposed criterion. It is worth noting that in the case of the Solenhofen Limestone, it seems possible, with a single failure envelope, to adequately represent the compressive strength behaviour of the material throughout the entire range of confining pressure. Depending on the level of the confining pressure, the rock experiences either brittle, transitional, or ductile failure (10). A brittle failure is characterized by fracturing and/or faulting. Fracturing is a separation into two or more parts with total loss of cohesion and resistance to differential stress. Faulting is a localized offset along a more or less plane surface of non-vanishing shear stress (19). A transitional failure is characterized by broad fault zones while a ductile failure is characterized by a uniform flow. Uniform flow indicates a macroscopically homogeneous permanent deformation without fracturing or faulting. Moreover, the same single failure envelope adequately represents the compressive strength behaviour of both solid cylindrical specimens under triaxial compression, and hollow cylindrical specimens under triaxial compression combined with torsion.

These results indicate the high applicability and versatility of the proposed criterion in representing the compressive strength behaviour of anisotropic rock materials under complex multiaxial states of stress. Table (1) presents the optimum values of the constant coefficients for the strength criterion for rocks considered. It should be noted that all tests are characterized with vanishing shear stress components $\tau_{xz}$, and $\tau_{yz}$ consequently, no optimum values for the coefficients H and I appear in Table (1).

## 3. CONCLUSIONS

A general compressive strength criterion is herein developed for the failure of anisotropic rock materials under complex multiaxial states of stress. The proposed criterion is capable of representing the compressive strength behaviourn rock materials for a wide range of confining pressure and for various modes of failure.

Based on the proposed anisotropic strength criterion, failure envelopes for a wide variety of rock materials are constructed and close agreement is shown to exist between the failure envelope and the corresponding experimental results in all cases, thus indicating the high applicability, reliability and versatility of the proposed criterion in representing the compressive strength behaviour of anisotropic rock materials.

The proposed criterion may well prove to be a very useful tool in many classical applications dealing with design with rock materials. Morever there are several novel applications for which an adequate anisotropic strength criterion for underground rock formations is deamed necessary. To site but an example, is the novel process of permeability enhancement of underground oil sand formations by a tailored pulse technique, as a step towards the preparation for an efficient enhanced recovery process; this no doubt requires a reliable anisotropic strength criterion as a necessary prerequisite for process optimization.

**Table (1): Optimum Coefficients for the Strength Criterion, Equation (9).**

| Coeff. | Rock Type | | | |
|---|---|---|---|---|
| | **Green RiverShale** | **Pentremawr Coal** | **Solenhofen Limestone** | **Silty Mudstone** |
| A | .537863 | .085239 | .273799 | .014533 |
| B | .540902 | 15.9029 | 5.51250 | .011257 |
| C | 1.95742 | .084413 | 5.51250 | − 13.1359 |
| G | 9.73921 | ——— | − .151026 | 13.4574 |
| D | − 8.60211 | − 15.2370 | − 7.92148 | − 13.4561 |
| E | 7.52071 | 15.0282 | 8.94950 | 13.4082 |
| F | 7.53592 | − 15.2646 | − 9.80103 | 13.4557 |
| $C_0$ | − 1.00000 | − 1.00000 | − 1.00000 | − 1.00000 |
| $C_1$ | .383450 | .074861 | − .168982 | − .077002 |
| $C_2$ | .702209 | .866927 | .877997 | .987627 |
| $\alpha$ | 0.50 | 0.15 | 0.50 | 0.10 |

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