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A Computational System-Level Model of Oculomotor Pathways Accounting for the Representation of Eye Biomechanics in the Cerebellar Vermis

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ABSTRACT In this paper a comprehensive system-level computational model of oculomotor pathways is presented. This model shows the necessity of embedding internal models of muscles biomechanics in the cerebellar Vermis to realize fast saccadic eye movements based on predicting the changes in muscles lengths. First, the eye biomechanics are described by nonlinear equations during ''slow'' and ''fast'' movements. Afterward, by analyzing these equations, a computational model, is deduced. Furthermore, each part of this model is interpreted as a possible function of an element in the oculomotor pathways based on physiological and anatomical pieces of evidence. In this model, two internal feedback loops compensate two types of error: 1- error between desired and estimated values of eye position, calculated by Superior Colliculus, and 2- error between desired and estimated torque, calculated by Cerebellar pathways. Simulations of this circuit produce signals similar to the actual neuronal activities in the corresponding sites of the oculomotor pathways during saccades. Effects of bilateral lesions of Fastigial nuclei, Vermis, Prepositus Hypoglossi, the stimulation of Omni-Pause Neuron and Superior Colliculus are studied. Furthermore, the model ability in performing smooth pursuit eye movements is investigated. Finally, the ''main sequence'' is reproduced. This model is the first one to derive both the cerebellar function and the bilateral connectivity of the oculomotor pathways from calculations based on physical hypotheses. The proposed model is useful to better understand computational functions of different parts of the oculomotor pathways, and also using in robotics application for controlling fast movement inspired by the brain.

INDEX TERMS Internal model, inverse function approximation, oculomotor pathways, saccade, cerebellum, fast movements.

I. INTRODUCTION

During recent decades, many experimental and theoretical studies have been performed to comprehend the principles of motor control and better understand motor diseases [1]–[6]. Moreover, Biological motor control systems are robust, adaptive and versatile, and they are also accurate, although sensory information is always delayed due to the transmission and processing delay of sensory pathways [3], [7]–[11]. Remarkably, fast and accurate movements can even be performed without any sensory feedback. Thus, principles of biological motor control might be used to improve robots movements [11]–[14].

Fast movements such as saccadic eye movements, are defined as those lasting less than the total duration of the transmission and processing delays in the sensory-motor

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pathways [5], [15]–[20]. Therefore, they cannot be controlled by a closed-loop controller based on sensory feedback signals.

To use an open-loop controller instead of a closed-loop one, an estimation of the inverse function of the plant is necessary [5], [7], [16], [18]–[28]. But many functions have no inverse one, because although a direct function is deterministic, an inverse function is generally not. For instance, biomechanical functions describing muscle contraction or relaxation and limb dynamics are deterministic according to the laws of classical mechanics, but various configurations of the arm can perform similar hand displacements. Thus, similar effects can result from different causes, and a cause-effect relationship is generally not bijective, but is rather a surjection from the domain of the causes to that of the effects [5], [7], [16], [18]–[23], [25]–[28]. Therefore, no general method permits to infer a single cause from a definite effect.

To overcome the ill-posed problem of inverse function calculation, forward internal models are learned and built in the brain to represent the causal relationships between the input and the output variables of the biomechanical functions. A forward internal model predicts the next state of a commanded system, given the current state and the motor command [5], [7], [16], [18]–[20], [23], [25]–[28].

Fast and slow eye movements are generated by neural activity propagating through distinct pathways named oculomotor pathways. Oculomotor pathways has some interesting characteristics due to eye's specifications:

1) The eye globe is an almost homogeneous sphere rotating in the orbit around its center so that its inertia is constant and insensitive to its orientation in the gravity field;
2) It carries its own weight;
3) In the case of fixed head, eye movements are not perturbed, and thus no stabilizing reflexes control muscle forces or lengths (an involuntary head movement can be regarded as a perturbation, and in the case of nystagmus, movements can be out of control leading to diseases).

Due to these characteristics along with interesting ability of fast movements generation, many mathematical models of the saccadic motor pathways have been designed. The common empirical practice is to start from the anatomical pathways between brain regions, and define a circuit whose elements are linked by connections matching these pathways; then, local signal processing in each element is expressed by a function defined on purpose to produce a simulated signal resembling the observed time-course of the corresponding neuronal activity (review in [29], [30]).

In most previous studies, linear functions are generally used. In seminal works published since 1975, David Robinson and his followers chose to simplify drastically the biomechanics and neglect the nonlinear behavior of the muscles [31]. Thus, the whole biomechanics of the eye globe, connecting tissues and muscles, can be described together by a single linear function, liable to a *Laplace* transform, which reduces to a first-order low-pass filter. In accordance, inverse function can be approximated by two parallel pathways in the Brainstem: a direct path and an integrator whose time constant matches that of the eye biomechanics, so that poles and zeroes cancel out at low frequencies. This engineer description of the command of eye saccadic eye movements was an achievement at the time. But it is possible only due to the simplicity of the eye biomechanics: constant inertia, no sensitivity to gravity, no charge, no perturbations. Since the biomechanical functions of the eye globe, connecting tissues and muscles are not distinguished, the knowledge about oculomotor pathways cannot be generalized to limb motor pathways. In particular, the function of the Cerebellum is not accounted for.

In the ''bang-bang model'' of the saccadic pathways, proposed by Robinson [31], a feedback loop in the saccadic pathways acts as a rough inner model of the eye biomechanics. The difference between desired and current positions is calculated and serves as a motor signal. A solution for optimal control is proposed: to go as fast as possible up to about half the amplitude, and then switch the command to brake the movement. After this pioneering work, several sites where signals are compared have been identified. During saccadic movements, motor pathways take into account the biomechanics of the eye, including the ocular globe, muscles and connective tissues [32], [33]. Since eye movements in the orbit are never disturbed, and since obstacle avoidance is not necessary, the limitation on eye velocity is that acceleration is limited. Thus, command orders are likely limited by a motor learning, similar to learning of the biomechanics of the muscles in the Cerebellum.

Internal estimations of movement variables can be used instead of sensory signals when those are unavailable [5], [16], [18]–[20], [23]–[28], [34]–[36]. Signals encoding estimates of angular velocity and position, recorded in the Prepositus Hypoglossi (PH) nucleus, reach the Superior Colliculus (SC) and the Cerebellum [37], and signals encoding predicted muscle lengths, recorded in the Vermis, reach the Fastigial nucleus. By means of these inner signals, motor commands can be calculated in a closed loop manner based on internal estimated feedback without delay, instead of using real sensory feedback received by delay [38].

Most models describe the ''saccadic generator'' within the Pontine Paramedian Reticular Formation (PPRF), some focus on the cerebellar Vermis and Fastigial nucleus, or on the Superior Colliculus, but few consider all these parts together. A mathematical model able to reproduce experimental results is the outcome toward which any quantitative research on a phenomenon tends. It is indispensable to express any quantitative theory and check the consistency of its hypotheses. A model formalizes principles, allows to interpret observations and helps to transmit knowledge. Experimenters might use it to design experiments, although this is not an indispensable criterion of validity. Any acceptable model of neural pathways should be represented by a circuit endowed with a structure consistent with anatomy, each element of the circuit

corresponding to a group of neurons. It should be able to reproduce physiological and experimental observations.

Since movements are determined by the laws of classical mechanics, models of sensorimotor neural pathways should be based on these laws and on explicit command principles. Indeed, the value of the accurate force must have been calculated and translated into the action potentials train. The first step of modeling is to choose the level of precision intended in the study and write a set of differential equations expressing the physical relationships between the variables and the command signals. Kinematics and Biomechanics of muscles and limbs are described by non-linear equations so that Laplace transforms can no longer be used. Thus, to prepare a future study of limb movements command, and progress toward a theory of motor control, a command circuit of the eye was derived from a mathematical analysis of a well-known set of equations describing the eye biomechanics. Constraints on the motor pathways, which iteratively compute ocular motor signals, were then deduced. This method is reverse and complementary to the usual empirical design of models.

This article presents a comprehensive model of saccadic pathways, and it is organized as follows: first in section II a brief survey of the main anatomical and physiological features of the saccadic pathways is presented, which must be accounted for by any realistic model of the eye motor command circuit. At the end of section II, a brief summary of the hypotheses used in the previous studies are presented. Afterwards, mathematical analysis of the biomechanics of the eye, ocular muscles, and connective tissues, is performed in section III to precise the physical constraints on the processing of the motor signals. Next, the modeling of the saccadic command pathways is presented. Section IV includes the computer simulations and comparison of the time-course of the signal issued from each element to the neuronal activity in the corresponding site of the oculomotor pathways. Model suggestions, its main differences with previously proposed models, and interpretation of computational functions of its different parts are presented in section V. Finally, the paper is concluded by conclusions in section VI.

II. PHYSIOLOGICAL AND ANATOMICAL REVIEW

The entire oculomotor system is composed of two parts:

1) The biomechanical system, consisting of the eye, the muscles, and the connective tissues;
2) The neural motor circuit which prepares the motor commands sent to the muscles, and is composed of functionally articulated sub-circuits exchanging messages encoded in neural signals. This part includes areas of cerebral cortex, Superior Colliculus, Cerebellum, Nucleus Reticularis Tegmenti Pontis (NRTP), Prepositus Hypoglossi (PH) nucleus, and saccade generators.

Any realistic model should be compatible with the anatomical and physiological features which are hereafter reviewed. Only the necessary pieces of information are presented.

A. AREAS OF CEREBRAL CORTEX

The Lateral Intraparietal area of Cerebral Cortex receives sensory signals encoding the location of possible targets. Neuronal activities in the Parietal lobe represent the body scheme and potential actions on objects in the surrounding. The Frontal Eye Field, Supplementary Eye Field and dorsomedial portion of the Frontal Cortex, are involved in selecting one relevant target toward which the gaze is to be oriented, and the Frontal Eye Field plans either large saccades of the head and eyes towards memorized targets located out of the visual field, or voluntary saccades toward a direction, including saccades of the eyes only while the head remains immobile. These signals are sent to the neural motor circuit in the Brain Stem termed ''saccadic generator'', to the Superior Colliculus and to the pre-cerebellar nuclei, such as the Nucleus Reticularis Tegmenti Pontis (NRTP) [39].

B. SUPERIOR COLLICULUS

The firing rate of some neurons in the Superior Colliculus is modulated by the deviation of the eyes in the orbit [40]. The locus of activity is proposed to encode a two-dimensional ''motor error'', the angular distance in space between the target and the current gaze orientation [41], [42].

C. NUCLEUS RETICULARIS TEGMENTI PONTIS (NRTP)

The caudal part of Nucleus Reticularis Tegmenti Pontis (NRTP), which is the main pre-cerebellar nucleus, receives signals from the motor Cerebral Cortex, the Superior Colliculus, the contralateral Fastigial and Vestibular Nuclei and the region surrounding the oculomotor nuclei [43]. Before and during saccades, patterns of activity of neurons in the NRTP resemble those of Superior Colliculus (SC) neurons encoding eye position in the orbit [44] or eye velocity [45]. The NRTPs project to the Flocculus and Medial Vestibular Nucleus, to the oculomotor Vermis and to the Fastigial nucleus. Thus, they take part in a short control loop Cerebellum/NRTP.

D. CEREBELLUM

The Cerebellum is involved in control and learning of fast, smooth, and fine-tuned movements, both in sensory closed-loop and motor open-loop control. Since the Cerebellum adapts motor commands to various conditions, by means of short-term signals and long-term changes in synaptic weights, it has been compared to the controllers used in robotics ([5], [11], [16], [19]–[23], [28]).

Cerebellar Nuclei and Cortex receive similar excitatory signals via the mossy fibers, and the Cerebellar Cortex inhibits the Cerebellar Nuclei via the axons of the Purkinje cells [46], [47]. Most Purkinje cells simple spike activities are correlated with muscular tensions but show variability, commonly interpreted as fitting the predictive signal to the variable fatigue of muscular fascia [48], [49].

Lobules VI and VII of the Vermis project to the caudal part of the Fastigial Nucleus. These parts of the Cerebellum receive, via the NRTP, information on the command signals

issued from the Cerebral Cortex, from the Superior Colliculus, and from the ipsi- and contra-lateral Fastigial nuclei. The Vermis receives also signals from the Prepositus Hypoglossi nuclei, encoding the velocity and position of the eye. When the Superior Colliculus is anesthetized and its output pathway electrically stimulated, the triggered saccades are almost unchanged, regarding the adjustment of the command signals via a cerebellar internal feedback loop [50]. The variability of the neuronal activities in the Vermis and Fastigial nucleus would reflect the compensation by the Cerebellum of the fatigue of the eye muscles, and also of the variability of the activity in other parts of the oculomotor pathways, so that the movement be always performed correctly [51].

In the Fastigial nuclei, close but separate regions process motor signals for saccades, pursuit and stabilizing movements [52]. These nuclei project contralaterally to the NRTPs and EBNs [53]. Bursts of some Fastigial neurons last after the end of contraversive saccades with head fixed [54], [55]. The peak of neuronal activity in the Fastigial Nucleus contralateral to the saccade precedes, by some tenths of milliseconds, that in the Fastigial Nucleus ipsilateral to the saccade [54], [56], [57]. Kleine *et al.* (2003) state: ''faster saccades are typically accompanied by bursts that start and peak earlier, have a larger number of spikes that tend to occur in a shorter time and, consequently, show higher peak discharge rates than slower saccades of the same amplitude.'' Thus signals issued from the Fastigial Nuclei ''reflect the increase in acceleratory drive and in braking force required to steer the saccades'', so that the Cerebellum ''helps to accelerate contralateral and decelerate ipsilateral saccades by means of the early and late bursts, respectively, provided by the Fastigial neurons''. The late activity in the Fastigial Nucleus ipsilateral to the saccade could re-excite the contralateral EBN at the end of the saccade and help to stop it. Thus, activity appears related to the acceleration of the saccade in the contralateral Fastigial and to its deceleration in the ipsilateral Fastigial.

E. SACCADIC GENERATOR

The neurons of the circuit called *''saccadic generator''*, located in the Paramedian Pontine Reticular Formation (PPRF), are active only during saccades [58]. *''Excitatory Burst Neurons''* (EBNs) receive signals from the contralateral Superior Colliculus (SC) (via Long Lead Burster Neurons (LLBN) in Primates [59]) and from contralateral Fastigial Nucleus. In most EBNs, the number of spikes emitted during a saccade is correlated with the amplitude of the angular rotation of one eye [60]. They project to the motoneurons related to the ipsilateral lateral rectus muscle, to the internuclear interneurons which innervate the contralateral medial rectus muscle, and to the neurons of the Prepositus Hypoglossi Nucleus (PH). The saccadic generator dispatches also the motor commands among the motor pathways of the opponent muscles, by means of the phasic *''Inhibitory Burst Neurons''* (IBNs), which inhibit the contralateral EBNs, IBNs, motoneurons, and neurons of the PH.

On the contrary, the activity of *''Omni-Pause Neurons''* OPNs are hypothesized to prevent eye movements during fixations. Just before saccades, neurons in the SC activate Long Lead Burst Neurons (LLBN) close to the NRTP, which activate ''latch neurons'' which in turn inhibit OPNs and thus trigger the saccades [61], [62]. LLBNs dispatch signals of the Colliculus to cerebellar pathways [63], [64], like NRTP, and to the OPNs [61]. In the rostral SC, ''Superior Colliculus fixation neurons'' (SCFNs) pause during saccades and discharge during fixations; they activate OPNs [65]. Electrical stimulations of SCFNs interrupt eye saccades, while renewed activity correlates with saccade end [66].

F. PREVIOUS MODELS OF SACCADIC AND CEREBELLAR PATHWAYS

The most important hypotheses of previously proposed models of eye motor pathways, induced from experimental studies, are summarized as follows [67], [68]:

1) Direct motor pathways overcome the viscosity of the connective tissues and of the muscles;
2) Motor velocity signals are integrated by the Prepositus Hypoglossi nuclei into a position signal, reaching the Superior Colliculus, the Cerebellum, and the motoneurons to overcome the stiffness of connective tissues and muscles, and maintain the eye in the current position;
3) Saccadic movements accuracy is ensured by means of feedback loops, via sites where differences between desired and estimated variables are calculated:
   - The difference (error) between desired and estimated current gaze positions is calculated in the Superior Colliculus (SC) [69]–[71];
   - The difference between desired and estimated muscle lengths is calculated in the Cerebellum [50];
4) Saccadic movement duration is minimized by some optimization;
5) The precision of a gesture depends on the anticipating aptitude of sensorimotor looped pathways. During fast movements, sensory signals are not available but replaced by estimates calculated in predictive circuits. Functionally, motor commands are thus calculated in closed loop even if the information is encoded in predictive rather than sensory signals.

In this study, these largely accepted hypotheses are considered to propose the model.

III. PROPOSED MODEL

A. ANALYSIS OF THE BIOMECHANICS OF THE EYE

In this section, a high-level model of eye biomechanics is presented. Afterwards, based on analyzing the model in both slow and fast movements a mathematical model of oculomotor circuit is presented. In the next section, different parts of these mathematical model are interpreted as the possible models for different parts of the oculomotor circuit, using the physiological and anatomical facts reviewed in section II.

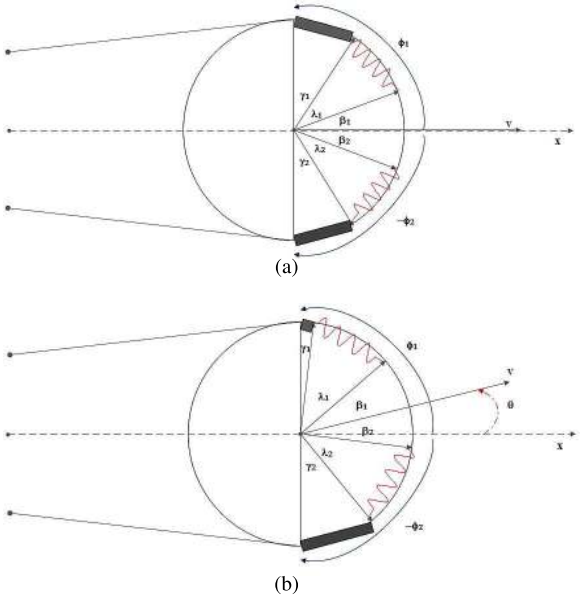

FIGURE 1. Scheme of the eye in the rest position (a) and after an angular movement θ (b). The length of the muscle is represented by black thick line and the spring in red.

1) KINEMATICS OF EYE MOVEMENTS

The length of a spring at equilibrium and in the "rest" position is denoted l_0. If the spring is stretched by a value Δl, the length becomes l and the spring exerts a force $F = K.\Delta l = K.(l - l_0)$. For antagonistic muscles (numbered 1 and 2), λ_1 and λ_2 denote the lengths of the elastic elements of the muscles and connective tissues (Fig. 1). At each angular position of the eye in the orbit, these lengths depend on those of the muscles γ_1 and γ_2. These lengths are also dependent on the angular positions of the insertions of the muscles in the sclera β_1 and β_2. Moreover these lengths depend on the positions of the tangency points of the muscles on the eye ϕ_1 and ϕ_2. Consequently we have:

$$\begin{cases} \lambda_1 = \phi_1 - \beta_1 - \gamma_1 \\ \lambda_2 = \phi_2 - \beta_2 - \gamma_2 \end{cases} \tag{1}$$

β_i as well as ϕ_i are constant, while $\lambda_i(\theta)$ as well as $\gamma_i(\theta)$ are functions of θ ($i = 1, 2$). At the "rest" position (visual field "straight ahead"), the lengths of the muscles are λ_{01} and λ_{02} and the forces exerted by the muscles are: $F_1(0) = F_2(0) = F_0$ (Fig. 1-a). After making a change in muscles lengths in order to move the eye, the line of sight v has rotated by an angle θ (Fig. 1-b):

$$\begin{cases} \lambda_1(\theta) = \phi_1 - \beta_1 - \gamma_1(\theta) - \theta \\ \lambda_2(\theta) = \phi_2 - \beta_2 - \gamma_2(\theta) + \theta \end{cases} \tag{2}$$

Therefore, the forces $F_1(\theta)$ and $F_2(\theta)$ exerted by the springs as follows:

$$\begin{cases} F_1(\theta) - F_0 = K_s.\Delta\lambda_1 = K_s.(\gamma_{01} - \gamma_1(\theta) - \theta)) \\ F_2(\theta) - F_0 = K_s.\Delta\lambda_2 = K_s.(\gamma_{02} - \gamma_2(\theta) + \theta)) \end{cases} \tag{3}$$

Two auxiliary positive variables $y_1(\theta)$ and $y_2(\theta)$ are defined as follows:

$$\begin{cases} y_1(\theta) = (\gamma_{01} - \gamma_1(\theta)) > 0 \\ y_2(\theta) = (\gamma_2(\theta) - \gamma_{02}) > 0 \end{cases} \tag{4}$$

Their initial values are zero. Now, we can rewrite equation (3) by substituting y_1 and y_2 from equation (4):

$$\begin{cases} F_1(\theta) - F_0 = K_s.\Delta\lambda_1 = K_s.(y_1(\theta) - \theta) \\ F_2(\theta) - F_0 = K_s.\Delta\lambda_2 = K_s.(\theta - y_2(\theta)) \end{cases} \tag{5}$$

The radius of the eye being constant, the forces and torques are not distinguished thereafter. Based on equation (5), at the angular position θ, the torque exerted on the eye is calculated as follows:

$$F_1(\theta) - F_2(\theta) = K_s.\,(y_1(\theta) + y_2(\theta) - 2\theta) \tag{6}$$

2) BIOMECHANICAL SYSTEM

Biomechanics of the eyeball, muscles and connective tissues are represented by a second-order equation [72]–[75]. Muscular biomechanics are usually represented by the behavioral Hill's model, which does not describe biochemical mechanisms such as actin/myosin interpenetration, but which is simple and has been used to model extra-ocular muscles [74], [76].

The muscle model used here is a Hill-based model, adapted to represent the biomechanics of the eye muscles [74]. Anatomical pulleys set the pulling direction of the eye muscles, but this has no incidence on dynamics [76]. During horizontal saccades toward side 1, the signals $[\alpha_1, \alpha_2]$ of the motoneurons induce respectively contraction of lateral (external) and relaxation of the medial (internal) rectus muscle. The state of the system is considered as:

$$X = (F_{a1}, F_{a2}, F_p, F_o, y_1, y_2, \dot{y}_1, \dot{y}_2, \theta, \dot{\theta}),$$

which includes:

- The torque of active forces (F_{a1}, F_{a2}) that the muscles exert;
- The resistant torques F_p and F_o exerted respectively by the muscles and connective tissues;
- The angular positions (y_1, y_2) of the ends of the muscles (expressed as angles, since the radius of the eye is constant) and the velocities of muscle shortening $(\dot{y}_1, \dot{y}_2)$;
- The angular position and the velocity of the eye in the orbit $(\theta, \dot{\theta})$.

The system dynamics are described by the system of nonlinear differential equations, presented below in the order of the causes (the first two equations express muscle activation), to the effects (the following equations express the biomechanics of the muscles, connective tissues and eye).

$$\dot{F}_{a1} = \frac{1}{\sigma_1}.(\alpha_1 - F_{a1}) \tag{7}$$

$$\dot{F}_{a2} = \frac{1}{\sigma_2}.(\alpha_2 - F_{a2}) \tag{8}$$

$$F_1 = K_s.(y_1 - \theta) \tag{9}$$

$$F_2 = K_s.(\theta - y_2) \tag{10}$$

$$\dot{y}_1 = \begin{cases} H.\dfrac{F_{a1} - F_1}{0.25F_{a1} + F_1} & F_{a1} \geq F_1 \\ H.\dfrac{F_{a1} - F_1}{3F_{a1}} & F_{a1} < F_1 \end{cases} \tag{11}$$

$$\dot{y}_2 = \begin{cases} -H.\dfrac{F_{a2} - F_2}{3F_{a2}} & F_{a2} \leq F_2 \\ -H.\dfrac{F_{a2} - F_2}{0.25F_{a2} + F_2} & F_{a2} > F_2 \end{cases} \tag{12}$$

$$\dot{F}_p = \frac{1}{\tau}.(K_p\theta + B_p\dot{\theta} - F_p)\ \theta > 0 \tag{13}$$

$$\dot{F}_o = \frac{1}{\tau}.(K_o\theta + B_o\dot{\theta} - F_o)\ \theta > 0 \tag{14}$$

$$J.\ddot{\theta} = F_1 - F_2 - F_p - F_o \tag{15}$$

This system of equations describe the eye muscle dynamics as follows:

- Equations (7) and (8) describe the effects of signals α_1 and α_2 of the motoneurons on the recruitment and activation of muscle fibers. Active muscular forces F_{a1} and F_{a1} develop according to temporal dynamics equivalent to low-pass filters characterized by the time constants σ_1 and σ_2;
- Equations (9) and (10) describe the current forces, F_1 or F_2, exerted by the muscles on the eyeball according to the length $(y_1-\theta)$ or $(y_2-\theta)$ of each ensemble formed by a muscle and its tendons. The stiffness coefficient is K_s. The length of the elastic element is the variable which causes the force;
- Hill's equations (11) and (12) describe the length variation rate $\dot{y}_1$ or $\dot{y}_2$ of each muscle, depending on the active forces, F_{a1} or F_{a2}, and the current force exerted, F_1 or F_2 [74]. Two formulas describe the behavior of the muscles according to whether they contract (first line) or relax (second line). Hill's constant H is the maximum speed of muscle contraction;
- Equations (13) and (14) express, as functions of the position and velocity of the eye, the dynamics of the passive resistive forces of the muscles (F_p) and connective orbital tissues (F_o, characterized by the same time constant τ. The coefficient of passive muscle stiffness is K_p the coefficient of the orbital connective tissues is K_o. The viscosity coefficients are respectively B_p and B_o;
- Finally, equation (15) expresses the principle of *inertia*: the product of the moment of inertia of the eye J and the angular acceleration $\ddot{\theta}$ is equal to the algebraic sum of the active and resistant torques exerted on the joint.

Equations (13) to (15) can be combined into a single second-order equation expressing the principle of inertia applied to the biomechanical function of the eye, denoted $g(\theta, \dot{\theta}, \ddot{\theta}, t)$:

$$\begin{aligned} g(\theta, \dot{\theta}, \ddot{\theta}, t) &= F_1(t) - F_2(t) \\ &= K_s.(y_1(\theta(t)) - \theta(t)) \\ &\quad - K_s.(\theta(t) - y_2(\theta(t))) \end{aligned} \tag{16}$$

By applying Laplace transform on equation (16) we have the following equation in the Laplace domain which is helpful in the remaining analysis:

$$\begin{aligned} F_1(s) - F_2(s) = J.s^2.\theta + (B_p + B_o).\frac{s}{1+s\tau}\theta \\ + (K_p + K_o).\frac{1}{1+s\tau}\theta \end{aligned} \tag{17}$$

B. CONTROL SYSTEM

The control system is proposed to operate differently for slow and fast movements.

1) FUNCTION OF SACCADIC GENERATOR CIRCUIT

Since slow movements last longer than total duration of transport and processing of the sensory signals, they are controlled by using sensory signals including vestibular or visual. In equations (7) and (8), the changing rate of forces ($\dot{F}_1$ and $\dot{F}_2$) are small. The velocities of shortening ($\dot{y}_1$) or lengthening ($\dot{y}_2$) are small. Therefore the current force F_1 or F_2 exerted by each muscle is almost equal to the active force F_{a1} or F_{a2}, which are directly controlled, at any instant, by the signals α_1 and α_2 of the motoneurons: $\alpha_1 - \alpha_2 = F_1 - F_2 = K_s.(y_1 - \theta) - K_s.(\theta - y_2)$.

Since in equation (17), the term of inertia $J.s^2.\theta$ for eye is really negligible compared to those expressing stiffness and viscosity, this term can be ignored. Furthermore, for slow movements, due to the negligible velocities of muscle shortening or lengthening, the effect of the low-pass filter $1/(1 + s.\tau)$ can also be ignored:

$$F_1 - F_2 = \left((B_p + B_o) + (K_p + K_o).\frac{1}{s}\right).s.\theta \tag{18}$$

If we consider that the desired angular velocity of the eye is given to control system as the reference signal ($\dot{\theta}^D$ where 'D' stands for "Desired"), the desired position could be calculated by integrating this input signal in the oculomotor circuit (θ^C where 'C' stands for "Calculated"). Now, since during slow movements the biomechanics of the muscles do not modify the motor commands, it is not necessary to take into account them in the computation of these commands. Therefore, the system can be controlled by signals $\alpha_1(t)$ and $\alpha_2(t)$ such that:

$$\alpha_1(t) - \alpha_2(t) = F_1 - F_2 = (B_p + B_o).\dot{\theta}^D(t) + (K_p + K_o).\theta^C(t).$$

2) FUNCTION OF SUPERIOR COLLICULUS

For fast movements, the term of inertia $J.s^2.\theta$ can also be neglected, but the effect of the low-pass filter $1/(1 + s.\tau)$ cannot. Rotating the eye in the orbit requires that the torque $F_1 - F_2$ be calculated as follows:

$$\begin{aligned} F_1(t) - F_2(t) = (B_p + B_o).\frac{s}{1+s\tau}\theta^D \\ + (K_p + K_o).\frac{1}{1+s\tau}\theta^C \end{aligned} \tag{19}$$

Different parts of equation (19) can be interpreted as functions of different parts of oculomotor circuit utilizing physiological and anatomical facts reviewed in section II.

For saccadic command pathways in Pontine Reticular Formation, these hypotheses can be proposed as follows:

- The EBNs ipsilateral to the saccade receive a signal encoding the desired position change $\theta^D(t)$ from the contralateral Cerebral Motor Cortex and produce the desired velocity signal $\dot{\theta}^D(t)$. In the Laplace domain, their dynamics are expressed by the function: $s/(1+s\tau)$;
- This signal is sent to the ipsilateral motoneurons with a gain $B_p + B_o$ equal to the sum of the passive viscosities of the muscles and the connective tissues viscosities. It is also sent with a negative gain, via the IBNS, to the contralateral neurons of the saccadic system;
- The EBN velocity signal is sent to the neurons of the ipsilateral nucleus Prepositus Hypoglossi and integrated. The calculated position signal $\theta^C(t)$ is in turn sent to the ipsilateral motoneurons with a gain $K_p + K_o$, equal to the sum of the passive stiffness of the muscles and connective tissues stiffness. It is also sent, with a negative gain, to the contralateral motoneurons. Thus the relaxing muscle exerts no active force but only a passive force.

Such a simple organization that includes the function of Pontine Reticular Fromation (which is similar to the model proposed since 1975 by Robinson [31]), approximates the inverse biomechanical functions expressing passive forces (equations (13) and (14)). An important question raised here is *"What are the functions of other parts of oculomotor circuit, including the Vermis, Fastigial, Superior Colliculus, and NRTP?* Here, we will try to find the appropriate answer for this question by revising equation (19), considering the anatomical and physiological facts reviewed in section II.

Command signals issued from the Cerebral Cortex are sent via a direct pathway to the Pontine Reticular Formation, and complemented by signals sent via indirect, closed looped, pathways and computed in two "proportional controllers":

- A proportional controller of gaze orientation lies in the Superior Colliculus, where the difference between desired and estimated (estimation of current position) gaze positions is computed as an error signal [41], [77]. During coordinated eye-head movements, the Superior Colliculus controls gaze trajectory i.e. space displacement of the line of sight (not studied here). If the head is fixed, gaze rotation in space is equal to eye rotation in the orbit.
- A proportional controller of muscles lengths, which coordinates muscles contractions, lies in the cerebellar pathways, composed of the NRTP, the Vermis, and the Fastigial nucleus. The cerebellar Vermis is assumed to embed learned internal models of the biomechanics of the eye muscles. Such an internal model allows prediction of muscles lengths variations and the forces they exert. The difference between the desired and predicted muscle lengths is calculated in the Fastigial nuclei.

During a saccade, the high velocity (up to 1000 degrees per second) prevent from utilizing sensory information, therefore it is necessary to utilize estimated values, calculated by the internal models. Thus, the gaze directing control is performed in a closed loop manner, but with estimated values (sensory information). The signal encoding the desired angular position, issued from the Cerebral Cortex, is completed, at the entry of the EBNs, by corrective signals issued from mentioned "proportional controllers".

To consider the corrective effect of Superior Colliculus, signal θ^D is replaced by $\theta^D + k_c.(\theta^D - \theta^C)$, where k_c is a constant coefficient, θ^D is the desired angular position, θ^C is an estimate of angular eye position (from Prepositus Hypoglossi). Indeed Superior Colliculus calculates the difference between desired and calculated positions as a predicted error, used as a compensator.

C. CEREBELLAR COMPENSATORY SIGNALS

The biomechanics of the muscles, expressed by Hill's equations (11) and (12), can change during fast movements, and also over time (for example as a result of fatigue of motoneurons or muscle). Therefore, the dynamics of muscle contraction should be taken into account by the control unit. The signals issued from the motor Cerebral Cortex and the Superior Colliculus are complemented by signals, computed in the Cerebellum according to the estimated state of the muscles, and sent from the contralateral Fastigial nucleus to the EBNs and NRTP.

In this model it is proposed that the Cerebellar Cortex computes a signal encoding the predicted change of length of each muscle according to the estimated state of the muscle and to the current motor commands. In the Cerebellar Nucleus, this signal is compared to the premotor signal encoding the change of length necessary to complete the movement. In the case of saccadic eye movements, the difference between the necessary and predicted values is calculated in the contralateral Fastigial nucleus and sent to the NRTP and to the EBNs, where it complements the signals issued from the motor Cerebral Cortex and from the Superior Colliculus.

Possible computational rules are presented based on a set of four functional hypotheses:

1) Differential signals are used to adjust the motor commands;
2) The Cerebellum contains predictive models of biomechanical functions;
3) Differential signals are calculated by neurons in the Fastigial Nuclei;
4) Identical calculations are not made twice in the nervous system.

Note: Hereafter, 'D', 'C', and 'E' stand for "Desired", "Estimated", and "Calculated" values.

1) HYPOTHESIS I: DIFFERENTIAL SIGNALS ARE USED TO ADJUST THE MOTOR COMMANDS

Copies of motor commands sent to the motoneurons are sent to the Prepositus Hypoglossi (PH), where the torque to be exerted $(F_1(\theta) - F_2(\theta))^C = g^C(\theta, \dot{\theta}, \ddot{\theta}, t)$ is estimated. This is sent to the Vermis and to the Fastigial Nuclei. Simultaneously, the Cerebellum estimates the torque $(F_1(\theta) - F_2(\theta))^E$

as a prediction of the real generated torque by these motor, given the estimated states of the muscles. The current movement is predicted not to be large enough if:

$$(F_1(\theta) - F_2(\theta))^E < (F_1(\theta) - F_2(\theta))^C$$

The anticipated error is thus: $\Delta C = (F_1(\theta) - F_2(\theta))^C - (F_1(\theta) - F_2(\theta))^E > 0$. Signal which is sent from the Cerebellum to the EBNs would be a signal proportional to ΔC that is able to prevents a movement error.

2) HYPOTHESIS II: THE CEREBELLUM CONTAINS PREDICTIVE MODELS OF BIOMECHANICAL FUNCTIONS

The torque estimated in the Cerebellum is denoted, with the negative sign:

$$-(F_1(t) - F_2(t))^E = K_s.\Big((\theta^D(t) - y_1^E(t)) + (\theta^D(t) - y_2^E(t))\Big) \quad (20)$$

In equation (20), the signal $\theta^D(t)$ is the sum of all copies of motor signals that activate EBNs and which are sent to the Fastigial nucleus. Predictive signals are assumed to encode the shortening $y_1^E(t)$ of tendons and muscle 1, and the lengthening $y_2^E(t)$ of tendons and muscle 2. They are likely calculated in the neural networks of lobules VI and VII of the Vermis, which receive sensory and motor signals before and during movements.

3) HYPOTHESIS III: THE DIFFERENTIAL SIGNALS ARE CALCULATED BY NEURONS IN THE FASTIGIAL NUCLEI

Assuming that the input signals to the EBNs have the physical dimension of an angle, the signal sent from the Cerebellum to the EBNs, as well as to the contralateral Fastigial and NRTP in Laplace domain is calculated as follows (z is decomposed to z_1 and z_2 to show the effect of two sides, contralateral and ipsilateral):

$$\begin{aligned} z = z_1 + z_2 &= \frac{1}{K_s}.\Delta C \\ &= \frac{1}{K_s}\Big(g^C(\theta, s\theta, s^2\theta) + (\theta^D - y_1^E) + (\theta^D - y_2^E)\Big) \end{aligned} \quad (21)$$

Now, if motor commands reached symmetrically both sides during a saccade, activities of EBNs could be, in Laplace:

$$\begin{cases} EBN_1 = 0.5.\dfrac{s}{1+s.\tau}\Big(\theta^D + k_c.(\theta^D - \theta^C) \\ \quad + k_f.\dfrac{1}{1+s.\delta}.\big(g^C(\theta, s\theta, s^2\theta) + 2.(\theta^D - y_1^E)\big)\Big) \\ EBN_2 = 0.5.\dfrac{s}{1+s.\tau}\Big(\theta^D + k_c.(\theta^D - \theta^C) \\ \quad + k_f.\dfrac{1}{1+s.\delta}.\big(g^C(\theta, s\theta, s^2\theta) + 2.(\theta^D - y_2^E)\big)\Big) \end{cases} \quad (22)$$

where k_c and k_f are two positive gains, δ is the transfer delay. But, due to the inhibition of neurons on the side contralateral to the saccade, the activities of EBNs are, for the almost entire duration of a saccade to the left:

$$\begin{cases} EBN_1 = \dfrac{s}{1+s.\tau}\Big(\theta^D + k_c.(\theta^D - \theta^C) \\ \quad + k_f.\dfrac{1}{1+s.\delta}.\big(g^C(\theta, s\theta, s^2\theta) + 2.(\theta^D - y_1^E)\big)\Big) \\ EBN_2 = 0 \end{cases} \quad (23)$$

By expressing explicitly $g^C(\theta, s\theta, s^2\theta)$ and noting Δ as the transmission delay from the PH to the Cerebellum, the motor signal issued from EBN_1 can be expressed in turn as:

$$\begin{aligned} EBN_1 = H_{EBN}.\Big((\theta^D + k_c.(\theta^D - \theta^C)).(1 + s.\delta) \\ + k_f.(\theta^D - y_1^E + \theta^D - y_2^E)\Big) \end{aligned} \quad (24)$$

where, the transfer function H_{EBN} is a second order high-pass filter expressed as:

$$\begin{aligned} H_{EBN} &= \frac{s.K_s}{H(s)} \\ H(s) &= (1 + s.\tau).(1 + s.\delta).K_s \\ &\quad - k_f.(1 - s.\Delta).\big((B_p + B_o).s + K_p + K_o\big) \end{aligned} \quad (25)$$

Thus, the signal EBN_1 has the physical dimension of an angular velocity, which is expected [46].

4) HYPOTHESIS IV: IDENTICAL CALCULATIONS ARE NOT MADE TWICE IN THE NERVOUS SYSTEM

Each Fastigial nucleus, on the side i,(i=1 or 2), sends to contralateral EBNs a signal $z_i(t)$ which could be written generally as follows:

$$\begin{cases} z_1(t) = \dfrac{1}{k}.\Big(\dfrac{a}{K_s}.g^C(\theta, \dot\theta, \ddot\theta, t) + (\theta^D - y_1^E) + c.z_2(t - \delta)\Big) \\ z_2(t) = \dfrac{1}{k}.\Big(\dfrac{1-a}{K_s}.g^C(\theta, \dot\theta, \ddot\theta, t) + (\theta^D - y_2^E) + c.z_1(t - \delta)\Big) \end{cases} \quad (26)$$

where k,c and a are constant coefficients. The coefficient a is in (0,1). In the Laplace domain, a short delay is represented by a decreasing exponential $e^(-\delta.s)$ and these equations could be rewritten in Laplace domain as follows:

$$\begin{cases} z_1 = \dfrac{1}{k}.\Big(\dfrac{a}{K_s}.g^C(\theta, s.\theta, s^2.\theta) + (\theta^D - y_1^E) + c.z_2.e^{-\delta.s}\Big) \\ z_2 = \dfrac{1}{k}.\Big(\dfrac{1-a}{K_s}.g^C(\theta, s.\theta, s^2.\theta) + (\theta^D - y_2^E) + c.z_1.e^{-\delta.s}\Big) \end{cases} \quad (27)$$

Sum of z_1 and z_2 which is the final effect of the cerebellum, based on the approximation according to the Taylor expansion $e^{-\delta.s} \approx 1 - s.\delta$:

$$\begin{aligned} z = z_1 + z_2 = \frac{1}{k - c + c.s.\delta}\Big(\frac{1}{K_s}.g^C(\theta, s.\theta, s^2.\theta) \\ + (\theta^D - y_1^E) + (\theta^D - y_2^E)\Big) \end{aligned} \quad (28)$$

To have a stable format (all polls are one the left hand side of s-plan), assume a general form for k: $k = d + s.\tau$ where τ is the time constant value, we have:

$$\frac{1}{k - c + c.s.\delta} = \frac{1}{(d - c) + (\tau + c.\delta).s}.$$

If these coefficients are chosen to have $d - c = 1$ and $\tau + c.\delta = T$ (T is an arbitrary time constant), equation (27) would be rewritten as follows:

$$z = z_1 + z_2 = \frac{1}{1 + s.T}\left(\frac{1}{K_s}.g^C(\theta, s.\theta, s^2.\theta) + (\theta^D - y_1^E) + (\theta^D - y_2^E)\right) \quad (29)$$

which is a stable low-pass filter if $T \geq 0$, and as a result: $\frac{1}{k} = \frac{1}{d} \cdot \frac{1}{1+s.\tau/d}$. Therefore, by replacing this new k is equation (27 we have:

$$\begin{cases} z_1 = \frac{\frac{1}{d}}{1 + s.\frac{\tau}{d}} \cdot \left(\frac{a}{K_s}.g^C(\theta, s.\theta, s^2.\theta) + (\theta^D - y_1^E) + c.z_2.e^{-\delta.s}\right) \\ z_2 = \frac{\frac{1}{d}}{1 + s.\frac{\tau}{d}} \cdot \left(\frac{1 - a}{K_s}.g^C(\theta, s.\theta, s^2.\theta) + (\theta^D - y_2^E) + c.z_1.e^{-\delta.s}\right) \end{cases} \quad (30)$$

To have $T = 0$ (an ideal form which is not a low-pass filter), we should set $d = 1 + c$ and $\tau = -c.\delta$. Therefore, by considering time constant τ as a positive value, c must be a negative value. In this case, an interneuron is assumed to inhibit the contralateral Fastigial Nucleus. A possible solution is $c = -0.5$ and $a = 0.5$:

$$\begin{cases} z_1 = \frac{2}{1 + s.\delta} \cdot \left(\frac{1}{2K_s}.g^C(\theta, s.\theta, s^2.\theta) + (\theta^D - y_1^E) - 0.5.z_2.e^{-\delta.s}\right) \\ z_2 = \frac{2}{1 + s.\delta} \cdot \left(\frac{1}{2K_s}.g^C(\theta, s.\theta, s^2.\theta) + (\theta^D - y_2^E) - 0.5.z_1.e^{-\delta.s}\right) \end{cases} \quad (31)$$

where in this equation:

- $g^C(\theta, s.\theta, s^2.\theta)$: Efference Copy of the current motor command calculated in PH;
- y_1^E &y_2^E: Predicted consequences calculated in Vermis and received by Purkinje cells;
- θ^D: Desired Consequence received by Mossy Fibers;
- z_1&z_2: Compensating signals calculated in Fastigial nuclei of different sides;
- δ: The transmission time delay between Fastigial nuclei in different sides

Therefore, the Vermis, based on learned internal model of eye biomechanics and an efference copy of motor command, predicts the consequences of the current motor command. Next, Fastigial nuclei calculate the compensation signal by receiving the prediction of consequences by the Purkinje cell and the desired consequence by Mossy fibers. Each Fastigial nucleus, on the side i,(i=1 or 2), sends to contralateral EBNs a signal $z_i(t)$ which are defined in the Laplace domain as equation 31.

The whole processing computes an approximate inverse function of muscles biomechanics. Fig. 2 shows the unilateral and abstract block diagram of the model. The proposed model is a bilateral model, composed of two sides, either involved in controlling movements in one direction. Fig. 3 shows the complete bilateral model, proposed and implemented in this article, realizes the anatomical structure and connections of the oculomotor circuit.

5) PATHWAYS VIA EBN, IBN, AND OPN

The Excitatory Burst Neurons (EBNs) receive inputs issued from contralateral regions, via three pathways:

- A direct or indirect by Long-Lead Burst Neurons (LLBN) pathway from the Oculomotor Cortex [46];
- An indirect pathway from the Superior Colliculus (SC), which conveys an estimated position error;
- An indirect pathway from the Fastigial Nucleus, which conveys an anticipated error of lengthening or shortening of the muscles, calculated from a torque error.

The sum θ^D of the input signals received via these three pathways is processed by the function of the EBNs, modeled as: $s/(1 + s.\tau)$ as expressed by equations (19) and (23). The gain $(B_p + B_o)$ of the pathway from the EBNs to the motoneurons is equal to the sum of the coefficients of viscosity of the muscles and connective tissues. Moreover, EBNs activate IBNs neurons, which send inhibitory signals to the neurons of the contralateral eye motor pathways.

Furthermore, when a saccade is decided, Omni Pause Neurons (OPNs) are inhibited directly by neurons in the SC, and indirectly by the EBNSs, via the latch neurons. The saccade starts and lasts till the motor error becomes smaller than a threshold value so that the inhibition is released. OPNs are modeled by empirical equations, in which γ is an activation threshold and $f(x)$ is a sigmoid function (here OPN_{max} is a constant value that shows the OPN output when it is active and k_c is a constant coefficient shows the effect of Superior Colliculus (SC)):

$$\begin{cases} f(x) = \frac{OPN_{max}}{1 + e^{-x}} \\ EBN_i(x) = \begin{cases} \frac{s}{1 + s.\tau}(x) & \text{Ipsilateral side} \\ 0 & \text{Contralateral side} \end{cases} \\ IBN_i = EBN_j \\ SC_i = \begin{cases} k_c.(\theta^D - \theta^C) & \text{Contralateral side} \\ 0 & \text{Ipsilateral side} \end{cases} \\ Tr_i = f(SC_i - \gamma) \\ OPN = OPN_{max} - EBN_1 - EBN_2 - Tr_1 - Tr_2 \end{cases} \quad (32)$$

where $i, j \in 1, 2$.

Fig. 4 shows the block diagram of functions of Superior Colliculus (SC) and EBN.

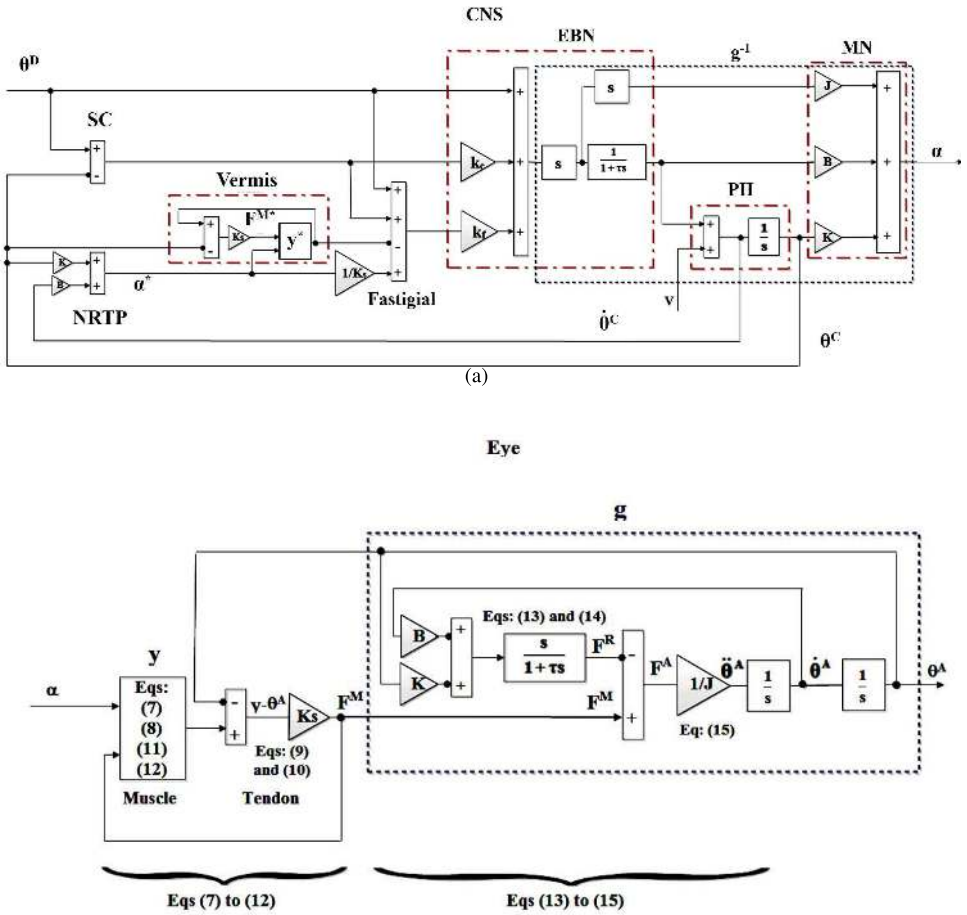

FIGURE 2. A lumped representation of the command circuit of the eye, with a simplified anatomical interpretation. (a) The whole functional command circuit of the model assumed to take place in the CNS. (b) The biomechanics of the eye. K: the sum of K_p and K_o, B: the sum of B_p and B_o. Frames delimited by red interrupted lines indicate anatomical interpretations. The biomechanics of the eye is denoted g, and the inverse function g^{-1}, delimited by black dotted lines, is assumed to be computed in oculomotor pathways. The input to the PH denoted V is issued from the Vestibular Nuclei and encode the desired velocity stabilizing the gaze in space. Here, 'D', 'A', 'C' stand for "Desired", "Actual", and "Calculated" values.

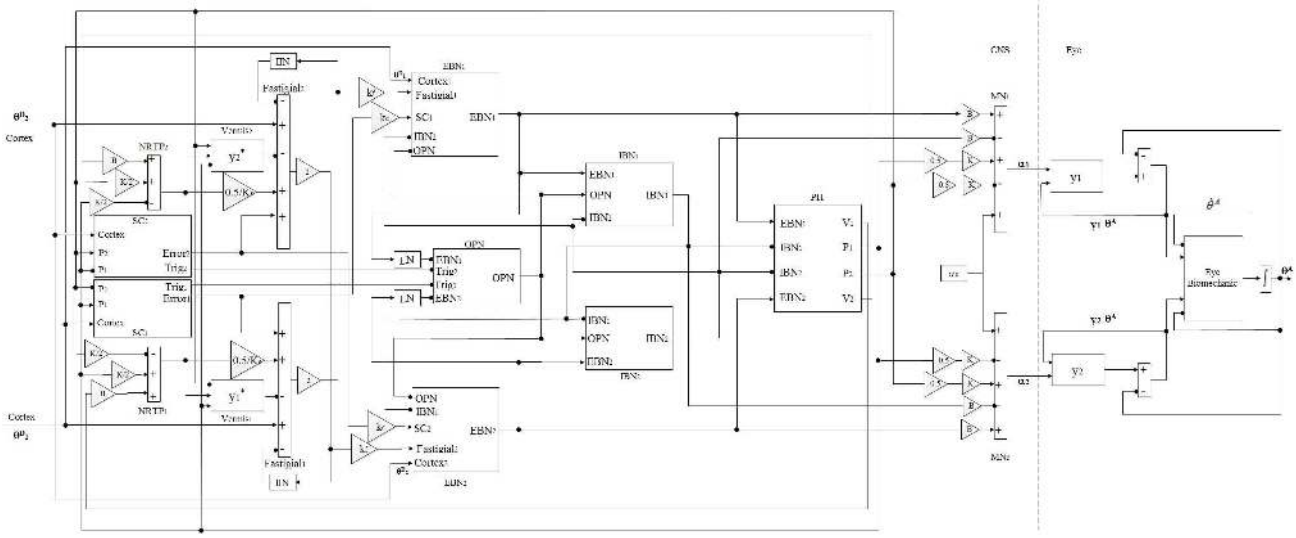

FIGURE 3. The bilateral connectivity of the circuit. K: the sum of K_p and K_o, B: the sum of B_p and B_o, LN: Latch Neuron, IIN: Internal Inhibitory Neuron; V_i: signals encoding velocity; P_i: signals encoding position..

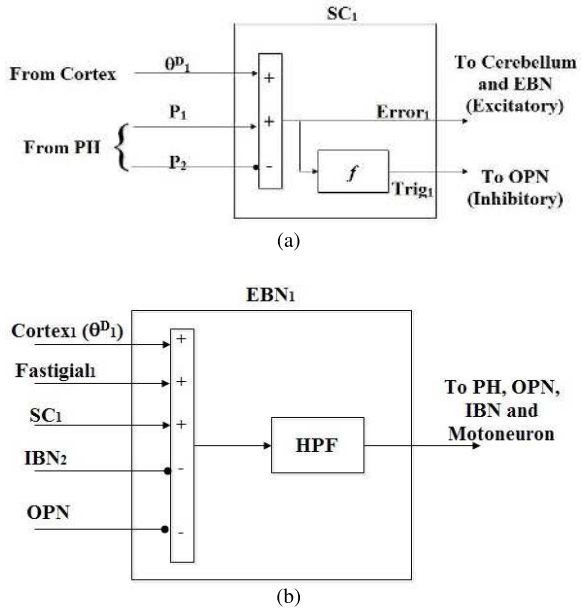

FIGURE 4. Function and connectivity simulated for two regions: (a) SC, (b) EBN; HPF: High-pass filter.

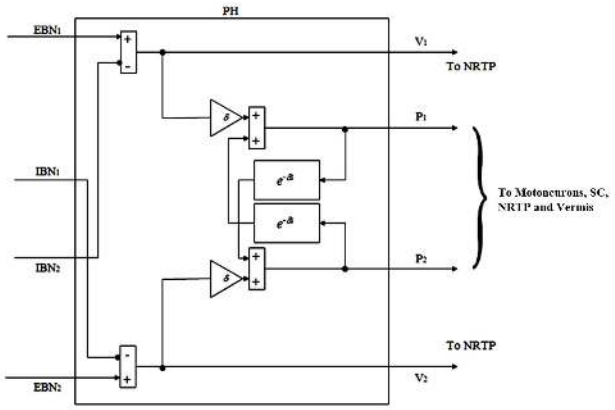

FIGURE 5. Function and connectivity simulated for PH.

6) PATHWAYS VIA THE PREPOSITUS HYPOGLOSSI NUCLEUS

Input signals to the Prepositus Hypoglossi (PH) are copies of velocity signals sent to the motoneurons from the EBNs and the IBNs, and from the Vestibular Nuclei (which is not considered in this study). The sum of these signals together encodes the desired eye velocity. PH neurons are assumed to compute signals encoding eye velocity and position, which are sent not only to the motoneurons but also to the Superior Colliculus, the NRTP, and the Cerebellum. Therefore, it is necessary to have an integration mechanism in PH.

Anatomical studies show the existence of reciprocal interaction between PH nuclei in two sides. Here, we consider this reciprocal interaction as a mechanism to integrate received desired angular velocity for calculating the angular position.

The velocity command signals issued from the EBNs are integrated into looped circuit, endowed with short delays δ, between the PH nuclei of the two sides, as shown in Fig. 5. In Laplace notation, equations are very simple:

$$\begin{cases} P_1 = \delta.(EBN_1 - IBN_2) + P_2.e^{-\delta.s} \\ P_2 = \delta.(EBN_2 - IBN_1) + P_1.e^{-\delta.s} \end{cases} \tag{33}$$

In VI it is shown that these equations approximate an integrator.

IV. SIMULATION

A. SIMULATION SETUP

The proposed model is used to simulate the command of a simulated eye with two antagonistic muscles. For these simulations, MATLAB 2018 and Simulink are used. Most figures below show results for saccades of amplitudes of 10 and 20 degrees.

Detailed neuronal dynamics are not modeled: for instance, in equation (23) activation of each neuronal type are represented by a summing element followed by a low-pass filter endowed with a time constant set between 5 and 15 ms. Similar results were obtained whatever the precise value of this time constant. Adding stochastic noise at several sites was checked, but did not change the results, since looped circuits ensure stability and reduce noise. Values of different parameters of the model are determined in Table 1 [74].

B. SIMULATION RESULTS

Figures 6 to 14 show simulations of experimental results, which match with the observations summarized in the bibliographic survey. Indeed, the time-courses of the simulated signals in elements of the circuit are similar to the mean neuronal activities measured experimentally in the corresponding sites of the brain.

1) NEURONAL ACTIVITIES OF THE DIFFERENT PARTS OF THE MODEL

Simulated eye movement variables and neuronal activities during saccades from the rest position (while looking forward) to 10 (black lines) and 20 degrees (red lines) to the left are shown in Fig. 6.

Fig. 6-a, the simulated curves reproduce the time-courses of changes of eye position in the orbit, for two saccades of 10 and 20 degrees. Fig. 6-b, the velocity time-courses of the eye for these two movements are shown. Note that, for the fastest movement, the acceleration is greater than the deceleration, as in actual saccades. Two dashed vertical line represent the end-time of these movements. It is important to note that the profile of actual velocity is not symmetric and the duration of movements with different amplitudes are not equal, movements with higher amplitudes have longer duration, here about 15 ms (see Fig. 6-a and 6-b). It is shown that the deceleration time is longer for larger movements (about 15 ms). These results are in accordance to the experimental results on monkeys [78].

Fig. 6-c, shows the time-course of activity of one neuronal type in the Superior Colliculus encoding the motor error i.e.

TABLE 1. Parameters values for simulations.

Parameter	Value	Parameter	Value	Parameter	Value
σ_1	$4.10^{-3}s$	σ_2	$8.10^{-3}s$	K_s	$2gf.deg^{-1}$
H	$900deg.s^{-1}$	J	$4.10^{-5}gf.s^2.deg^{-1}$	τ	$10^{-1}s$
OPN_{max}	100	K_p	$3.10^{-1}gf.deg^{-1}$	K_o	$3.10^{-1}gf.deg^{-1}$
B_p	$2.10^{-2}gf.s.deg^{-1}$	B_o	$6.10^{-2}gf.s.deg^{-1}$	δ	$10^{-3}s$

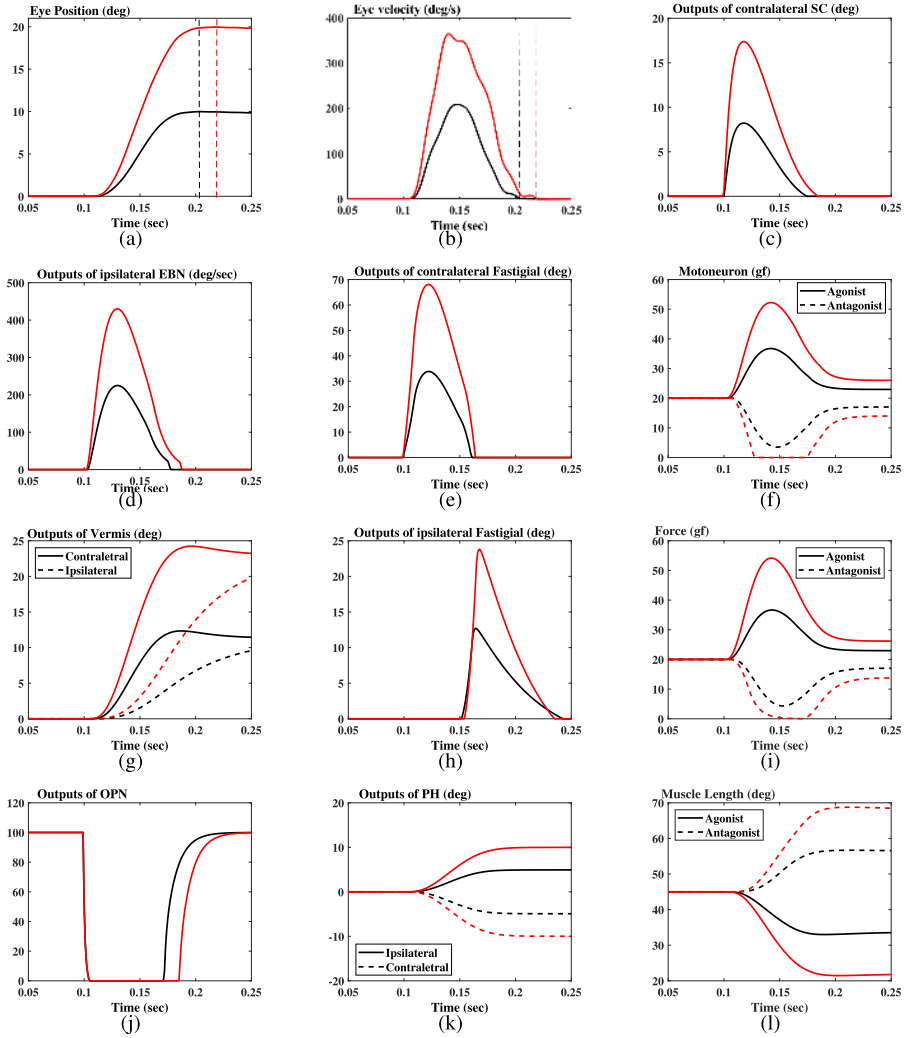

FIGURE 6. Time-courses of activities in various sites of the circuit from the rest position (while looking forward) to 10 (black lines) and 20 degrees (red lines) to the left. Tow vertical dashed lines in (a) and (b) represent the end-time of saccades, the black line for 10 deg (finished at 0.204 s) and the red line for 20 deg movements (finished at 0.218 s). The difference between these two lines is about 15 ms. In this figure, "gf" is "gram-force", a unit of measuring force.

the difference between the desired and estimated eye angular positions in the orbit. Note that this activity characterizes only one type of collicular neuron in the deep layer since superficial layers contain various neuronal types (visual, quasi-visual, etc.).

Fig. 6-d, the time-courses of activities of the EBNs are shown. Bursts of activity are followed by progressive declines.

Figs. 6-e and 6-h, show the time-courses of the mean activity in the contralateral and Ipsilateral Fastigial nuclei, respectively. The time-shift between the activities in the contralateral Fastigial nuclei is due to the transmission delay from one side to the other.

Fig. 6-f, shows the pulse-step activity profiles of the ipsilateral motoneurons and the reduction of activity of contralateral motoneurons: note that it is clipped to zero during large saccades due to the inhibition exerted by the IBNs. Similarly, Fig. 6-i shows the profile of muscular force, generated based on the applied motor commands. Consequently, Fig. 6-l shows the profile of changing muscles length due to applying the generated force. Note that the muscles slacken at the beginning of fixations.

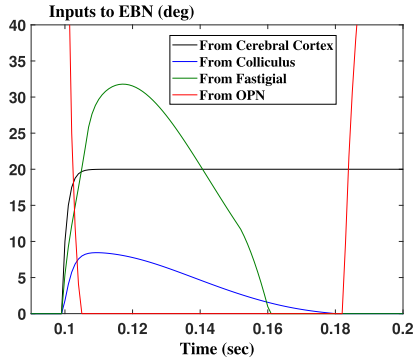

FIGURE 7. Different inputs from different sources to the EBN. Since the inputs are assumed to have the physical dimension of an angle, they are expressed in degrees.

Fig. 6-g, the simulated curves show the activities in the oculomotor Vermis. Note that the timescale is different is this case.

Fig. 6-j, OPN neurons stop firing during saccades, due to their inhibition by neurons of the Superior Colliculus.

Fig. 6-k, PH neurons integrate the velocity signal into a position signal, sent to the motoneurones, the Superior Colliculus and the Cerebellum.

Fig. 7 shows the time-courses and relative amplitudes of the various input signals to the EBNs expressed in degrees since these input signals have the physical dimension of an angle. Note that the magnitudes of the signals issued from different origins are comparable.

On fig. 8, the left diagram shows the eye movements during saccades for amplitudes10, 15, 20, and 25 degrees. The right diagram shows, the amplitude/velocity relationship. These results match the experimental results published by Optican and Miles (Fig. 11 in [79]).

2) LESION STUDY

In this part, we simulate the lesions in different parts of oculomotor circuit by removing the corresponding connections in the model, to show and evaluate the effect of different lesions by reproducing the experimental studies. Here lesions in Fastigial, Superior Colliculus, and Prepositus Hypoglossi are studied.

Eye movements in healthy conditions and after a lesion, either in the contralateral or ipsilateral Fastigial, are shown by the three curves on Fig. 9-a. The eye movements in healthy conditions and after a lesion either in the contralateral or in the ipsilateral Vermis are compared on Fig. 9-b. The results of these simulations reproduce qualitatively the experimental results recalled above in the bibliographic survey

Fig. 10 compares the eye movements in healthy conditions and after lesions in the output pathways of the Superior Colliculus. As expected, the lack of the signal issued from the Superior Colliculus reduces slightly the amplitudes of the saccades. Finally, Fig. 11 shows the lack of velocity integration which results from a lesion in the PHs nuclei.

3) STIMULATION STUDY

In this section the effects of stimulation of different parts, including Superior Colliculus and OPN are studied to evaluate the model.

Fig. 12, reproduces the effects of stimulations of the superficial layers of the Superior Colliculus by electrical pulses lasting for 5, 10 or 15 ms. The amplitude of the movement increases with the duration of the stimulation. These layers are also activated when sensory cues indicate the sudden entry of a target of interest in the visual or auditory surrounding.

Fig. 13 reproduces, on the upper row, the effects of stimulating the OPNs during the first half of the saccades, by means of electrical pulses lasting respectively for 5, 10 or 15 ms. Stimulations in the course of the first half of saccades, shown on the upper row, perturb more the movements than stimulations in the course of the second half. This matches the experimental results recalled above in the bibliographic survey.

4) MAIN SEQUENCE STUDY

The relationship between saccade velocity and amplitude is termed ''main sequence'' [80], [81]. Figure 14 plots the relationships (main-sequence) between peak velocities of the saccades (a), duration of the saccades (b) and their product (c) with their amplitudes. On figure 14-c, the slope of the regression line (red line), which could be interpreted as the ratio of peak velocity to mean velocity, is equal to 1.4 which is close to the estimated range of 1.54-1.80 from experimental data (Harwood *et al.* 1999 [82]). It is important to not that the sketch is in *logarithmic scale*. The results are also similar to the experimental results published in [81].

5) TRACKING MOVEMENT STUDY

Another interesting issue is to study model's behavior in performing slow movements including tracking a moving stimulus. Previous experimental studies showed that the typical oculomotor behavior of a naive monkey tracking a moving visual target with a constant speed, would be a sequence of saccades [83], [84]. This behavior would become smoother incrementally by training monkeys [83], [84].

Fig. 15 shows the study of oculomotor behavior as a response to tracking a visual target moving by 20 deg/s constant velocity ([83], [84]). Since learning is not considered in this study directly and the synaptic gains are determined as fixed values obtained from mathematical analysis, to exhibit the effect of learning, synaptic weight of trainable synapses (including Parallel Fibers to Purkinje Cell and Fastigial nuclei outputs) are set initially to some other values.

As it is shown in Fig. 15, the tracking movement is initially a sequence of saccades. By learning the mentioned synaptic weights (which are adaptive weights), the movement become smoother and tends to saccade free movement.

V. DISCUSSION

The purpose of this section is to focus on model's predictions, their interpretations in terms of physiology and engineering

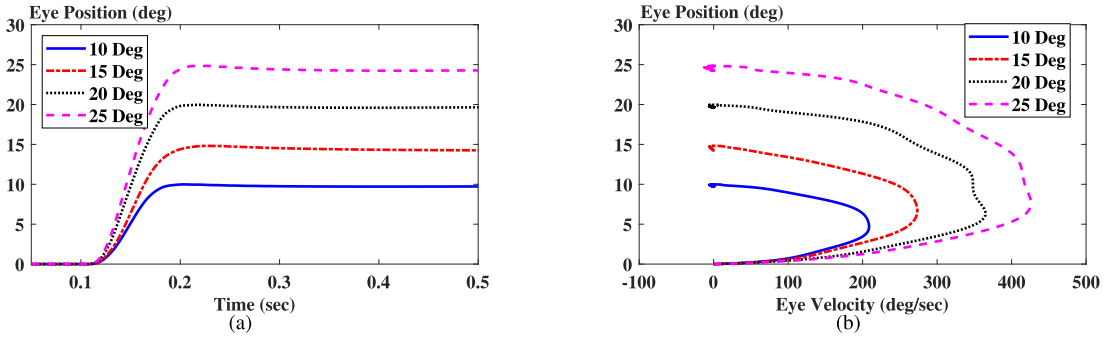

FIGURE 8. Times courses of eye position, and position/velocity diagram, during saccades of different amplitudes. These simulations reproduce the measures made experimentally.

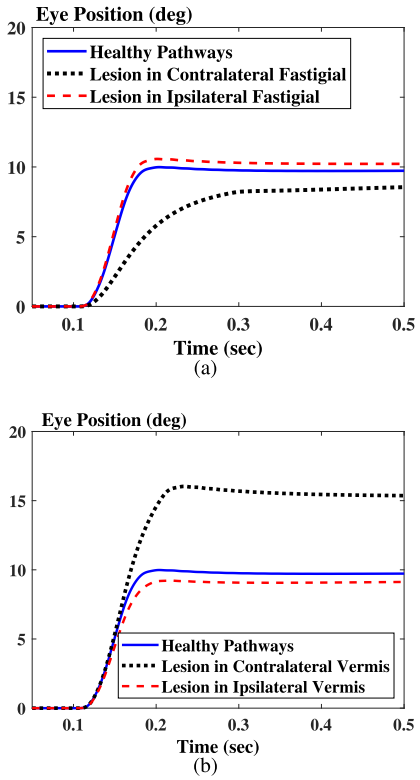

FIGURE 9. Effects of lesions in cerebellar pathways: (a) Lesions in the contralateral or ipsilateral Fastigial nucleus. (b) Lesions in the contralateral or ipsilateral Vermis. These simulations reproduce the experimental results.

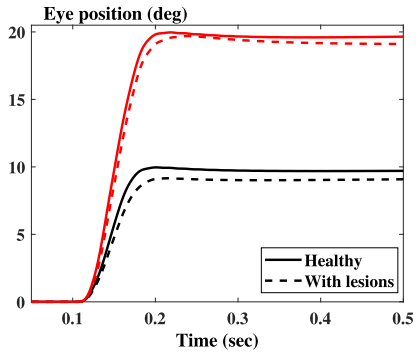

FIGURE 10. Effects of lesions in the connections between the Superior Colliculus and the EBNs. After such a lesion, the error position signal no longer reaches the EBNs, and the amplitude of the saccades are reduced.

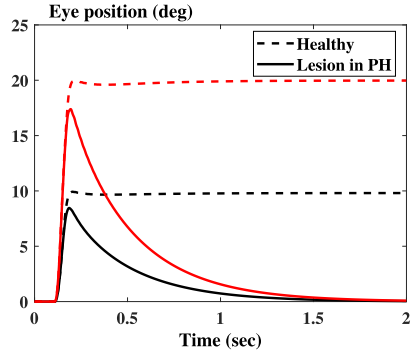

FIGURE 11. Effects of lesions in the PHs: as expected, the integration disappears.

issues, and comparing the results and predictions to predictions of the other hypotheses and models.

A. PROPORTIONAL CONTROL

In this model, the two error signals used for control are the differences between desired and estimated values of eye position (which is calculated by SC) and of torque (which is calculated by Cerebellar pathways).

1) EYE POSITION MOTOR ERROR COMPUTED IN THE SUPERIOR COLLICULUS

The Superior Colliculus (SC) and the Prepositus Hypoglossi nuclei (PH) are active during all movements including slow and fast ones. Signals encoding angular velocity and angular position are issued, via long feed-back loops, from the PHs which receive and integrate copies of the motor signals of the EBNs (and also of head and gaze velocity command signals, from the Vestibular nuclei and from the head motor pathways; not modeled here, since the head remains immobile). An angular motor error based on the difference between the desired final gaze orientation (target direction) and the current estimated orientation, is continuously calculated, likely in a motor map located in the deeper layers of the SC. In addition, the sudden activation of the sensory, superficial, layers is schematically modeled by an activation of the Colliculus.

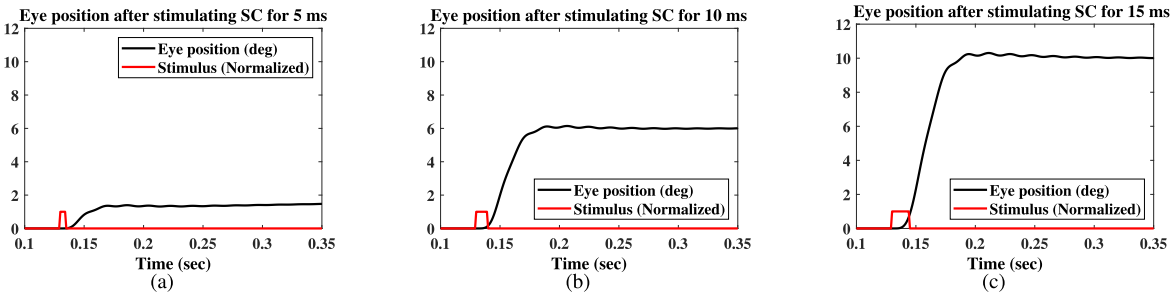

FIGURE 12. Effects of stimulating the SC. This stimulation mimics an express saccade launched by an electrical stimulation or the sudden entry of an object of interest into the visual field. The amplitude of the saccade is proportional to the duration of the simulated stimulation.

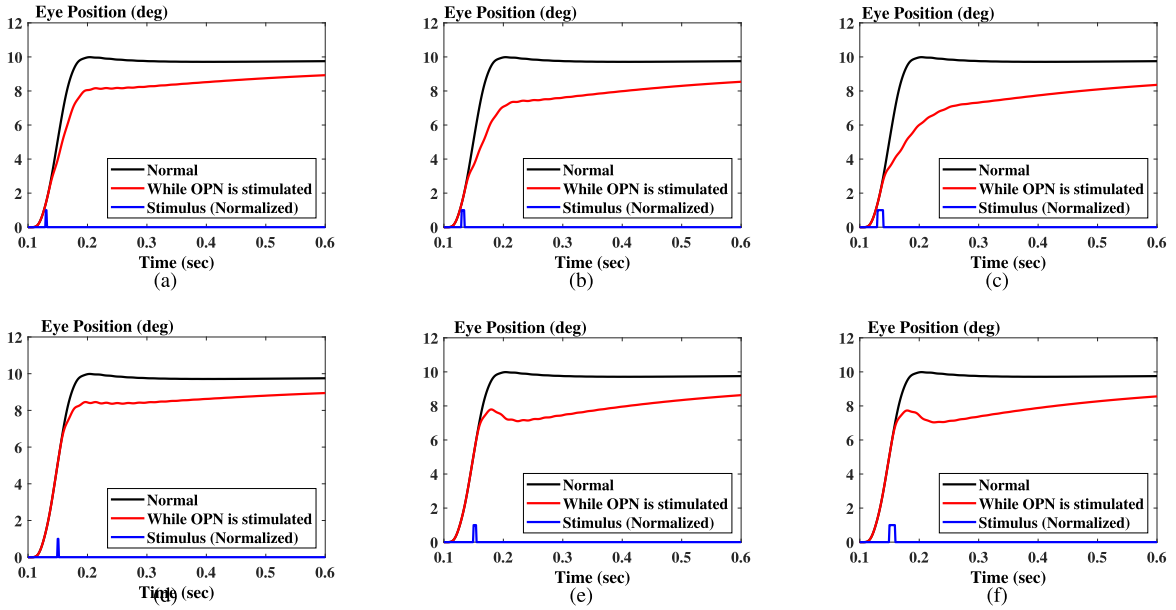

FIGURE 13. Effects of stimulating OPNs: First row: during acceleration, Second row: during deceleration Duration of the stimulation: (a,d): 2 ms, (b,e): 5 ms, (c,f): 10 ms. As experimentally observed, the braking effect is greater the OPNs if stimulated during the acceleration than during the deceleration.

The function of the SC shows two important features:

- Motor errors are encoded in layers mapping the surrounding. Extensive variables are encoded in cortices;
- Since rotations are not commutative, the calculation at each instant of the difference between the scheduled and estimated movements is necessary to synchronize the contractions of the pairs of recti muscles.

2) TORQUE MOTOR ERROR COMPUTED IN THE CEREBELLUM

The Cerebellar Cortex is assumed to embed internal predictive models of the biomechanics of the muscles, and to compute anticipated values of the torques [46]. Representations of the biomechanical functions are likely acquired by motor learning, and stored in the neuronal connectivity, not detailed here. Since several sets of muscle fibers are active during a movement, estimating the torque is an extensive calculation and accordingly takes place in the Cerebellar Cortex of the Vermis.

A torque error, which is the difference between the torque to be exerted and the estimation of the actual torque that the motor commands can drive, is proposed to be calculated in the Cerebellar Fastigial Nucleus. An important result of this study is to establish a correspondence between signal computation (equation (31)) and exchange of signals between Fastigial nuclei on each side, via short neuro-anatomical closed loops passing by the contralateral NRTP. Since a torque error is an intensive variable, its calculation takes place in a nucleus.

After an experimental unilateral lesion of one Fastigial nucleus, saccades toward the side contralateral to the lesion increase, and saccades towards the side ipsilateral to the lesion decrease. Computer simulations allow to reproduce normal saccades accurately, either with positive or negative projections from one Fastigial nucleus to the contralateral one, but the effects of lesions cannot be reproduced with positive reciprocal projections. A prediction of the model is therefore that inhibitory interneurones, denoted *IN* on Fig. 3

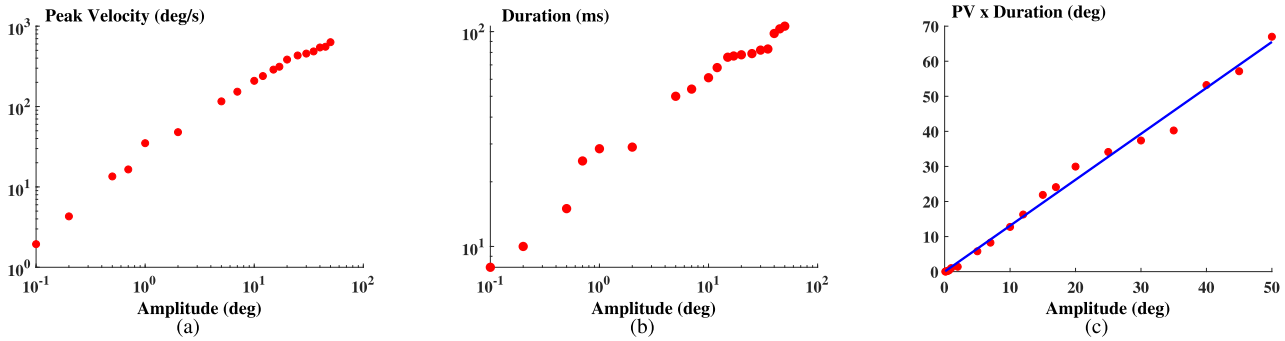

FIGURE 14. The main sequence. Figures (a) and (b) are sketched in logarithmic scale and Figure (c) in linear scale. a: The relation of peak of velocity (PV) with Amplitude of the saccades, b: The relation of duration of saccades with their amplitudes, c: The relation between the product of PV and duration of saccades with their amplitudes.

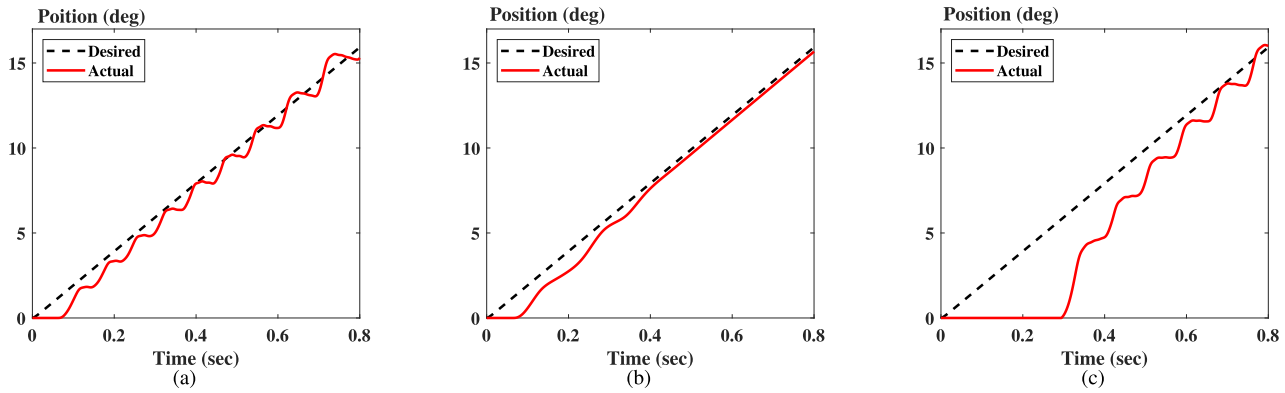

FIGURE 15. The oculomotor behavior of the model during a visual target tracking task. a: The behavior without learning in Cerebellar Cortex (Purkinje cell gain = 0.8, Fastigial nuclei output = 0.5), b: The behavior with obtained parameters of the model based on mathematical analysis (Purkinje cell gain = 1, Fastigial nuclei output = 0.5), c: The behavior without learning in Fastigial nuclei (Purkinje cell gain = 1, Fastigial nuclei output = 1).

and likely located in the NRTPs, would exert a reciprocal inhibition between the two Fastigial nuclei.

In this circuit, two sites of comparison act as proportional controllers, where possible errors are anticipated and actually prevented by means of corrective signals sent to EBNs. At variance, the visual pursuit circuit (not modeled here) would act as a ''proportional integrative controller'' since it receives a velocity signal computed from the foveal retinal slip, and triggers a corrective saccade whenever the cumulative eye drift resulted in a position error harmful for neat vision.

B. COMPARISON WITH OTHER MODELS OF COMMAND OF SACCADES

In early models, different areas of the Cerebellar Cortex were assumed to calculate either direct or inverse function [21]–[24]. But neuronal connectivity is identical over the whole Cerebellar Cortex, whatever the part of the body that each part of the Cerebellum controls, and no area is specialized in computing direct or inverse functions. Mathematically, computing inverse functions can be avoided by assuming that a feedback loop comprises an ''observer'', i.e. an element of a circuit able to predict some effect of the motor command. The neuronal network constituting this predicting element would behave like a direct, and therefore deterministic, internal model of a biomechanical function located in the Cerebellar Cortex [16], [18]–[20], [23]–[27].

The two sites of comparison, which exist in the Superior Colliculus and the Fastigial Nucleus are considered in some previously proposed models of the saccadic pathways as ''dual drive architecture'' [29], [85]–[88]. Nevertheless, the basic assumptions of these previous models are different from the considered principles proposed in our model, as can be seen in the following comparisons:

1) Both the Cerebellar Cortex and Fastigial nuclei were assumed to contain motor maps, coding at different scales the motor commands necessary to drive eye movements. Although they are actually compact, in the previous models Fastigial nuclei were assumed to contain topographically organized maps. On the contrary, in the present model, Fastigial Nuclei are not assumed to contain extensive motor maps but to compare intensive force signals;
2) According to previous models, the earlier start and peak of activity in the Fastigial nucleus contralateral to the saccade than in the Fastigial nucleus ipsilateral with the saccade would reveal the spread of a wave of activity diffusing from the Fastigial nucleus on one side

TABLE 2. Comparison with other models based on modeling the role of different parts of oculomotor pathways and different types of eye movements.

Model	Vermis	Fastigial	NRTP	SC	PH	EBN	IBN	OPN	Bilateral	Pursuit	Saccade
Proposed Model	+	+	+	+	+	+	+	+	+	+	+
Daye et al. (2014) [29]	+	-	+	+	+	+	-	-	-	-	+
Salimi-Badr et al. (2017) [5]	+	+	-	-	-	-	-	-	+	+	+
Arai & Keller (2005) [71]	-	-	-	+	-	-	-	-	-	-	+
Ebadzadeh & Darlot (2003) [19]	+	+	-	-	-	-	-	-	+	-	+
Quaia et al. (1999) [85]	+	+	+	-	+	+	+	+	+	-	+

to the contralateral homologous nucleus, via crossed topographic connections whose existence can be questioned [89]. In the present model, this fact results from an exchange of signals via the NRTPs and Fastigial nuclei, expressed by equation (31).

3) The Cerebellum was assumed to integrate a velocity feed-back signal and stop the movement when the Cerebellar Cortex inhibits the Cerebellar Nucleus. The proposed model contains two proportional controllers, acting respectively on eye position and muscles forces. The Superior Colliculus is assumed to compare desired and current positions, to trigger, reorient and stop the movement. The Cerebellar Vermis is assumed to embed, in its connectivity, inner models of the biomechanics of the muscles, acting as ''observers'' or ''predictors'', which anticipate the values of the torques exerted by the muscles. The Fastigial Nuclei calculate the algebraic sum of their input signals, and act together as a proportional controller issuing corrective signals which set the forces exerted by the antagonist muscles.
4) Elements of the circuit contained equations defined ad hoc to reproduce neuronal activities, and the biomechanics of the muscles were not embedded in a precise site. In the present model, the functioning of the neurons reproduces biomechanical equations (7) to (15). Only equations describing the function of the OPNs in equation (32) are, up to now, empirical, and added on purpose to reproduce the results and match the anatomical connectivity.

A brief comparison of anatomical coverage of different models are presented in Table 2. Here, we restricted this comparison to some previously proposed models and it is concentrated on the different parts of oculomotor pathways included in models and also different types of movements (saccadic and smooth pursuit eye movements).

C. MODEL SUGGESTIONS

The computational study in this article proposes existence of two connections: 1- projection from SC on OPN, and 2- projection between two Fastigial nuclei. These suggestions are made based on the mathematical constraints of the model which are new in comparison to the previously proposed models.

In this model, the error between the desired and current eye position is calculated by the SC which is in accordance to some experimental studies [41], [42]. Therefore, when there is an error (difference between desired and current position), it would be necessary to move to reduce this amount of error by reaching the target point. Generally, OPN prevents eye movement, therefore to have a movement, it is necessary computationally to inform OPN about the presence of error to reduce inhibition. It could be performed by a connection from SC to OPN. Such a connection is proposed by this model based on mathematical constraints. There is no positive nor negative anatomical argument on a connection between the two Fastigial Nuclei. It would not be unexpected that so close nuclei, each located on one side of the symmetry plane, have reciprocal connections. This is surprising that reciprocal connections have been described for long between the two Prepositus Hypoglossi nuclei, which are also close to the symmetry plane. In the proposed model Inhibitory Inter Neuron (IIN) are postulated for stability. Functionally, the significant fact is that a signal encoding the expected force exerted by one muscle (length in the case of eye movements command) is postulated to be sent to the control circuit of the opposite muscle. This piece of information could be issued from another part of the brain, and this postulated connection might be modified in a more developed model including the head movement command circuit. This issue should become more precise in future modelling studies, and perhaps eventually submitted to the sagacity of competent neuro-anatomists.

VI. CONCLUSION

In this article, a new and comprehensive mathematical model of oculomotor circuit is presented. The new insight in the oculomotor system that this study brings is that a cerebellar control is necessary, even at short-term, and this is demonstrated by means of the mathematical analysis.

This model is the first one to derive both the cerebellar function and the bilateral connectivity of the cerebellar pathways from calculations based on physical hypotheses. In most of previous models, the cerebellar pathways were drawn to match the anatomical connectivity and added to an already existent circuit mimicking saccadic pathways [5], [19], [26], [31], [66], [79], [90]. On the contrary, the connectivity of the cerebellar pathways is deduced here from the mathematical study. Of course, this does not mean that only one connectivity pattern would perform the necessary processing, but it shows that the actual neuro-anatomical pathways match the requirements for an iterative mathematical processing. More generally, it suggests that the CNS actually performs step by step calculations by means of looped circuits. This study not only sheds light on the cerebellar pathways controlling eye movements but also to the general function and connectivity of the Cerebellum.

In conclusion, this study shows that:

1) The method of analysis of a set of differential equations describing a dynamic system, commonly used by physicists, can be used to study motor command circuits. It could be used, in the future, to study the command of a limb, for which an engineering approach is not sufficient;
2) Calculations allow deducing a command circuit. Since the nonlinear equations describing the biomechanics of the muscles cannot be inverted (the inverse function is not a unique function [5], [7], [16], [18]–[23], [25]–[28]), a predictive control is necessary. The whole computation is equivalent to an approximate inverse calculation of biomechanical functions;
3) The structure of this command circuit can be identified to the actual neuronal connectivity. This shows the pertinence of the method and suggests that the central nervous system actually performs iterative computation according to the laws of classical mechanics. Biomechanical equations are computed in circuit elements exchanging signals via connections, which determine a structure comparable to the connectivity of saccadic oculomotor pathways. Extensive variables are computed in cortices and intensive variables combined in nuclei;
4) The Cerebellar Cortex of the Vermis embeds internal models of muscle biomechanics and computes predictive signals encoding the expected tensions muscle and lengths;
5) There are two sites of comparison function as proportional controllers: in the Superior Colliculus the desired and calculated gaze positions are compared and in the Fastigial Nuclei the difference between the desired and calculated muscle lengths are calculated;
6) Corrective signals sent to premotor neurons can prevent possible errors. Altogether, motor commands are calculated in a closed loop, although the biomechanical system is commanded in ''open loop'' by the whole circuit, from the Cerebral Cortex to the motoneurons;
7) The command pathways are under continual parametric adjustment, mostly under a cerebellar control. The present study shows which variables the Cerebellum controls. In the rare models which deal with cerebellar control, the cerebellar pathways are added to the direct command pathways, according to neural connectivity. On the contrary, this study demonstrates what the function of the Cerebellum is, and how the cerebellar pathways are articulated with the direct pathways. This can be deduced from physical calculations;
8) Neuronal activities and effects of lesions are reproduced. This validates the method of study of motor control, based on the laws of classical mechanics, which could be applied to the limb motor pathways.

In summary, assuming that motor signal processing follows classic mechanical laws, a physical analysis allows both to deduce the structure of the command pathways and the function of the Cerebellum. The results of the simulations validate this method of study of motor control, which proposes a rationale for the organization of the eye motor pathways, and could be extended to the study of limb motor circuits.

Presenting a more biologically plausible model by reducing the level of model and adding details of neuronal activities based on using Spiky Neural Networks (SNN) is proposed as the main future work of this study. For example, by modeling the Superior Colliculus based on population of spiky neurons, it would be possible to have caudal and rostral parts encoding error related to saccades with different amplitudes. Furthermore, extending model to control vertical and horizontal saccade simultaneously is suggested as another possible future study. Finally, utilizing the idea to propose a model able to control limb movements is proposed as next future work of this study.

APPENDIX
PREPOSITUS HYPOGLOSSI NUCLEI AS AN INTEGRATOR

Here, we show that the mathematical model proposed for Prepositus Hypoglossi (PH) in equation (33) can represent an integrator. The Taylor expansion $e^{-\delta.s} \approx 1 - s.\delta$ allows an approximation, which gives us:

$$\begin{aligned} P_1 + P_2 &= \delta.(EBN_1 - IBN_2) + P_2.(1 - \delta.s) \\ &\quad + \delta.(EBN_2 - IBN_1) + P_1.(1 - \delta.s) \\ &= \delta.(EBN_1 - IBN_2 + EBN_2 - IBN1) \\ &\quad + (1 - \delta.s)(P_1 + P_2) \\ \Rightarrow\ P_1 + P_2 &= \frac{1}{s}.(EBN_1 - IBN_2 + EBN_2 - IBN_1) \end{aligned}$$

Which is an integrator. Now, if $EBN_2 - IBN_1 = 0$ and $IBN_2 = 0$, the sum $P_1 + P_2$ estimates the angular position of the eye to the left:

$$P_1 + P_2 = \theta^C = \frac{1}{s}.EBN_1$$

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