A Concept of Combined Micro and Macro Fracture Mechanics to Brittle Fracture

By
T. Yokobori,
Professor of Mechanical Engng, Tohoku Univ., Sendai, Japan,
A. Kamei,
Dr. of Mechanical Engng, Ibid.,
and T. Kogawa,
Kawasaki Heavy Industry Res. Lab., Kobe.

§ 1. Introduction

Real materials contain inevitably both stress concentrators as cracks or notches and as dislocations, that is, both macroscopic and microscopic one. From this viewpoint, a concept of combined micro and macro fracture mechanics is proposed to brittle fracture.

§ 2. Local critical tensile stress criterion

Let us denote local tensile stress σ_ℓ in the body under simple tension. Then whatever the detailed model may be, local tensile stress criterion is written as:

$$|\sigma_{\ell n} + \alpha \sigma_{\ell d}|_{max} = R_{SI}, \qquad (1)$$

where $\sigma_{\ell n}$ and $\sigma_{\ell d}$ are local tensile stresses at the site concerned caused by microscopic and macroscopic defects, respectively, and α is geometrical factor.

This is general argument. Then, in the present section, for example, let us assume a model of plastically induced propagation of the crack as shown in Fig.1. That is, at the crack tip dislocations are initiated and spread into the body and the tip stress is relaxed. On the other hand, dislocations are formed by multiplication near by and move toward the crack tip, and may pile up at the obstacle, such as a grain boundary. Then high local tensile stress appears near the dislocation pile-up, and the crack extends to this point. The process will be repeated, and thus the crack propagation will occur. In this case of materials showing strain hardening, when the plastic zone is very small compared with the crack length, the plastic tensile stress distribution in the direction at a point $(x(\ll c), 0)$ near the crack tip is as shown in Fig.1*. On the other hand, stress relaxation will occur at the crack tip. Let us denote by **a** the zone of stress relaxation, then the real plastic tensile

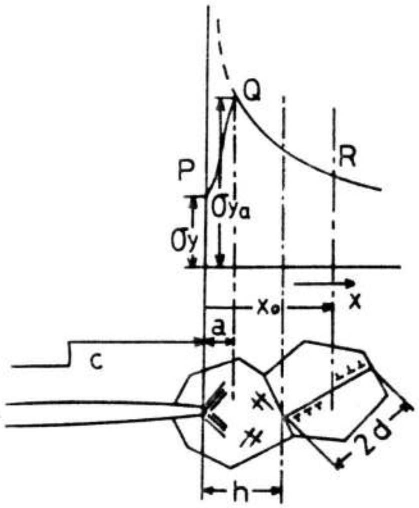

Fig.1 Schematic illustration of an example of a model of combined micro-and-macro fracture mechanics approach

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* For the case the plastic zone size is far much smaller than the crack length, elastic approximation was used in the previous articles.[1) 2)]

stress distribution is as shown in Fig.1. It is to be noted that σ_l is not obtained by simple sum of σ_{ln} and σ_{ld} when the crack tip and the piled up point are close[1)] as in this case, although it may be understood that $|\sigma_l|_{max}$ will be near the piled-up. Taking into interaction of the two, and using approximation, we get finally the brittle fracture stress σ_c of this type as follows:

$$\sigma_c = \frac{\alpha^*}{\sqrt{c}} \left\{ \frac{R^*}{\sqrt{d}} + \tau_i - \frac{Z}{2} \right\} , \qquad (2)$$

where $\alpha^* = 2\sqrt{2x_o}/A_o$, $R^* = \sqrt{2(h-x)}R_{s1}$. Z and A_o = constants dependent on material and loading mode. It can be seen that the calculated value of σ_c (by Eq.(2)) exceeds atomic cohesive force when using reasonable values of parameters.

Eq.(2) is also rewritten in terms of fracture toughness Kc as:

$$K_c = \alpha^* \left\{ \frac{R^*}{\sqrt{d}} + \tau_i - \frac{Z}{2} \right\} . \qquad (3)$$

It is, however, noted that Kc in Eq.(3) or in this line of consideration is included as the local concentrated stress, but not as the stress field as in usual macroscopic fracture mechanics.[3)] Further in macroscopic fracture mechanics it is fundamentally assumed that the stress at the crack tip could not exceed the yield stress, σ_Y, and thus as for the fracture model, a slipping-off mechanism,[5)] say, rupture named by Orowan[6)] has been used. However, as described above, the stress at the crack tip may exceed atomic cohesion, much larger than the yield stress, and a slipping-off mechanism or critical displacement criterion might not be necessary. Eq.(2) is at least qualitatively in good agreement with the data on the effect of notch length c, and ferrite grain size, d on low-stress brittle fracture of one and the same low carbon steel as shown in Fig.2.

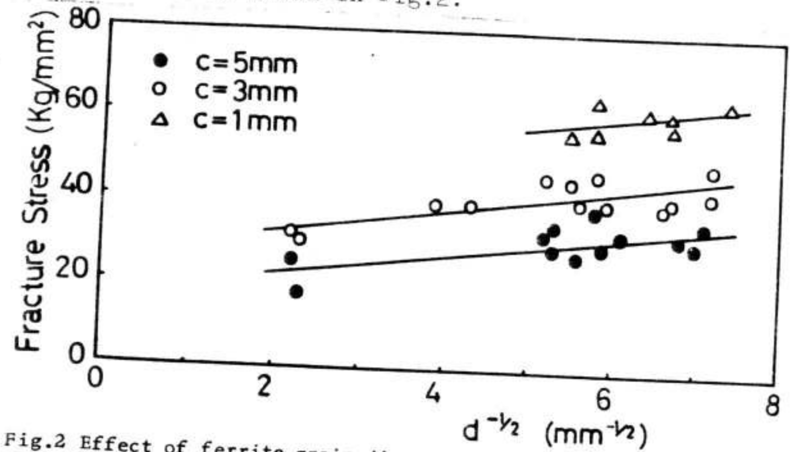

Fig.2 Effect of ferrite grain diameter 2d on brittle fracture stress of the one and the same material as in Fig.3

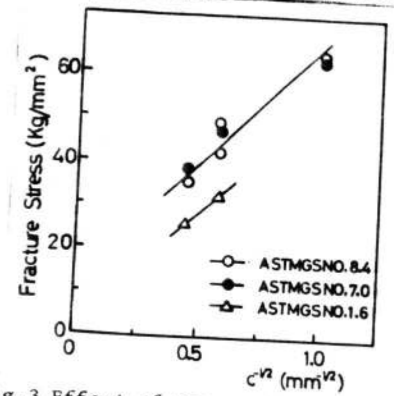

Fig.3 Effect of the crack length c on brittle fracture stress of 0.04% carbon steel.

§ 3. Energy unstable criterion

Knowing the condition fulfiled as described in § 2, it is justified to examine the energy unstable condition.[7] In the case in which the plastic enclave is much smaller than the crack length, if we neglect the interaction of the defects of the two types, then we get[2]

$$K_c \simeq \sqrt{E(\gamma+\gamma_p)}\,, \qquad (4)$$

where γ_p is the energy dissipated by the plastic deformation. Eq.(4) is the same type as Griffith's formula, ans also the formula frequently used in macroscopic fracture mechanics.[4] Thus it should be noted that a fracture criterion assuming a critical value of K, that is, Kc in macroscopic fracture mechanics is one corresponding to a special case of combined micro and macro fracture mechanics.

§ 4. Discussion

It should be noted that the stress intensity factor has a significance as a measure of local stress concentrated for the local tensile stress criterion and, on the other hand, as the crack tip stress field for the energy unstable condition. In general, the two requirements mentioned above should be fulfiled for the crack propagation. The fracture stress will correspond to higher value of the two. Which of the two requirements is critical is future problem needed to be studied.

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