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A CONCEPTUAL BASIS FOR MODELLING AND SIMULATION

by

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Introduction

Since its practice lies at the center of all scientific activity, interest in the nature of modelling comes from a multiplicity of directions. Physicists and philosophers of science have long been concerned with the role that models play in fundamental theories of the physical universe [1, 5, 7, 8, 15, 16, 24, 25, 35, 36, 42, 49, 51, 52, 53]. As they become more mathematically oriented, the social sciences have spawned efforts to better understand the formal structure of the description of behavioral data and related explanatory models [16, 17, 30, 34, 50]. In economics, the relatively advanced state of quantification and modelling has encouraged the investigation of the aggregation process and the relation between micro and macro economic systems [28, 4]. More recently, the increased ability to seriously consider complex multicomponent systems, made possible through computer simulation [9, 20, 40, 54, 55, 56], has given rise to new modelling approaches in such areas as management science, biology, psychology and ecology (to name only a few) with attendant efforts to provide solutions to the serious problems of validation and credibility such simulations raise [6, 10, 14, 17, 18, 22, 39, 43, 45, 47, 48]. Finally, in control engineering where mathematical modelling has a long history, efforts to automate model construction as part of adaptive design have necesitated the development of practical and effective parameter identification procedures [2, 4].

Given such diversity and elaboration of its aspects, achieving a unified understanding of modelling may well be a Herculean task. But this broad scope of interest also increases the potential impact of any fundamental contribution. Indeed, given the relative isolation in which each of the above efforts proceed, even the bringing to attention of developments in one area to workers in another area may be justification enough for unification attempts.

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There is precedent for the development of a theory of modelling and simulation in the attempts of general systems theorists to provide the conceptual basis for all special systems theories [3, 31, 33]. Indeed, the long range goals of general systems theory and of the theory of modelling and simulation cannot really be separated, since the issues raised in the latter are almost by definition general systems issues. However, as a matter of fact, the thrust of formal developments in general systems theory has been to provide mathematical formalisms for dealing with the abstract concepts of structure and behavior which have come to characterize the area [29, 38, 57, 59]. In contrast, the concerns of modelling and simulation tend to focus theoretical attention on such issues as the relations between systems, validation and inference [12, 13, 46] which have received relatively little emphasis thus far by general systems theorists.[1]

In this paper, I attempt to ferret out some of the foundational ideas that have been supporting previous work in modelling theory (my own [60-67][2] and others). A system or black box is conceived of as being knowable at several different levels which form a sort of inclusion hierarchy.

The first level provides the *a priori* framework for modelling, while the second observational level is the only level at which direct knowledge about the black box may be acquired. The question then is: how are we able to achieve knowledge at the higher, more structural levels?

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[1] Note Klir's comments in his introduction to [32]

[2] These papers should be consulted for background development of any concepts close to the heart of the author but unfamiliar to the reader.
For a more introductory presentation of system levels see Klir [31].

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The solution is given in terms of "justifying conditions". The search for these kinds of conditions (under which implications become equivalences) is a standard mathematical ploy but recognition of the importance of these conditions for modelling theory seems to be novel.

Using this knowledge hierarchy, two common models of knowledge acquisition are examined. The first, epitomized by the discovery procedures of linguistics, attempts to climb up the hierarchy driven by the observation data alone. The second methodology constructs models at the higher levels for validation at the observational level. The prior assumptions necessary for the success of these philosophies of modelling begin to be understandable with the help of the knowledge hierarchy construct.

The development of these ideas will take place here within the automata theory formalism. This is so, largely as a matter of convenience and ease of exposition since the basic concepts can be replicated within more general systems formalisms. The problems raised concerning availability of decision procedures and practical algorithms would have to be considered anew in each formal context.

The knowledge hierarchy is described within the, by now common, black box conceptual framework. However, this framework must be (and is) extended radically in order to adequately study the modelling and simulation methodologies.

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The Knowledge Hierarchy

Let us start with the proverbial black box. After reviewing some of the standard concepts about such black boxes, we shall be in a position to come to a more adequate account of the nature of modelling. The development will proceed in terms of "levels of knowledge" about the nature of the black box.

Level of Knowledge 1 - We know the box to be a deterministic *automaton* of the Moore type [41], i.e., the black box is specified by a structure $M = \langle X_M, Q_M, Y_M, \delta_M, \lambda_M \rangle$ where X_M, Q_M, Y_M are nonempty, not necessarily finite sets of inputs, internal states and outputs, $\delta_M: Q_M \times X_M \to Y_M$ is the transition function and $\lambda_M: Q_M \to Y_M$ is the output function.

This structure determines a set of input-output (I-O) functions indexed by the internal states. Thus for each q in Q if we start M off in state q and apply an input sequence $x_1x_2\ldots x_t$ we would observe a unique output sequence $y_1y_2\ldots y_t$. $\tilde{\beta}_q$, the *I-O function associated with state* q, is the set of all such pairs $(x_1x_2\ldots x_t, y_1y_2\ldots y_t)$ associated with state q.[3]

The set of all input-output pairs associated with M is called the *I-O relation* R_M. $R_M \subseteq X^+ \times Y^+$ equals $\bigcup_{q \in Q} \tilde{\beta}_q$. In other words,

$(x_1x_2\ldots x_t, y_1y_2\ldots y_t) \in R_M \iff$ there is a state $q \in Q$ such that $\tilde{\beta}_q(x_1x_2\ldots x_t) = y_1y_2\ldots y_t$.

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[3] $\tilde{\beta}_q: X^+ \to Y^+$ is computable as follows: Let $\tilde{\delta}_M: Q \times X^+ \to Q$ be the extended transition function determined from δ_M by iteration: $\tilde{\delta}_M(q,s) = \delta_M(q,s)$ for $s \in X$ and $\tilde{\delta}(q,xs) = \delta_M(\tilde{\delta}_M(q,x),s)$ for $x \in X^+$. Then $\tilde{\beta}_q(x_1x_2\ldots x_t) = \lambda_M(q)\lambda_M(\tilde{\delta}_M(q,x_1))\lambda_M(\tilde{\delta}_M(q,x_1x_2))\ldots\lambda_M(\tilde{\delta}_M(q,x_1x_2\ldots x_{t-1}))$.

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<u>Level knowledge 2</u> - So far we know only that our black box is an automaton - we have no knowledge of its components X_M, Q_M, Y_M, δ_M, λ_M.

Level 2 is to be characterized by knowledge of inputs and outputs as some known particular sets X and Y respectively i.e. $X_M = X$ and $Y_M = Y$.

We are now ready to do experiments on our box. An experiment consists of subjecting the system to an input sequence $x_1x_2\ldots x_t$ and observing an output sequence $y_1y_2\ldots y_t$. Since our machine was in some (unknown) state at the beginning of the experiment we know that $\tilde{\beta}_q(x_1x_2\ldots x_t) = y_1y_2\ldots y_t$ for some unknown function $\tilde{\beta}_q$.

Since we have no further knowledge about the box after the experiment, the best we can do is try another experiment. After a finite amount of experimentation we will have a finite subset R' of the I-O relation R_M.

<u>Level knowledge 3</u> - The I-O relation R_M is completely known. R_M represents the most that can be known about the black box by experimentation at level 2. Since R_M is an infinite set, it is the unattainable limit of such experimentation.

<u>Level knowledge 4</u> - The set $B_M = \{\tilde{\beta}_q | q \in Q\}$ of I-O functions is completely known. This set B_M, called the *behavior* of M, is to be distinguished from the I-O relation R_M. Since R_M is the *union* of functions in B_M, it is always possible to recover R_M from B_M but the reverse process of expressing R_M as a union of sequential functions (i.e., I-O functions of an automaton) is not at all obvious. Indeed, it constitutes one of the fundamental problems of mathematical systems theory [59].

Though it should be taken with a grain of salt, the following piece of mental imagery (Moore [41], Gill [19]) can help fix the difference between level's 3 and 4. Instead of being given a single black box as we have previously assumed, we are to be given an infinite supply of identical black boxes. Since the boxes are identical we can be assured that prior

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to any input-output experiment all boxes will be in the same initial state. Each box is subjected to a different input sequence.
The unattainable limit of such experiments is a sequential function β' ($= \tilde{\beta}_q$ for the initial state q common to all boxes). We call this the *identical copies assumption.*

Now to obtain the behavior B_M we need to repeat the above procedure for each possible state of the box.[4] Thus we can imagine an infinite supply of identical boxes except that each comes stamped with a label indicating its initial state, and that every state of the state space has its identifying label.[5]

The difference between Levels 3 and 4 then, is represented by the stamped supply of boxes - one may jump from Level 3 to Level 4 with the help of a benign demon who copies and appropriately labels boxes (though we shall shortly be looking for other ways to make this jump).

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[4]Actually, the experiments need only be done for states not accessible from any other state.

[5]A more sophisticated version of this picture has the boxes being painted, but not labelled, according to the criteria of Zadeh [59].

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Level of Knowledge 5 - Level 4 represents the highest level at which knowledge of our black box can be gained by external observation alone. Level 5 knowledge is the equivalent of being allowed to peek inside the box to the extent that its internal specifications Q_M, δ_M and λ_M (in addition to X_M and Y_M) are known as abstract sets and mappings, i.e. $Q_M = Q$, $\delta_M = \delta$, $\lambda_M = \lambda$ for particular Q, δ and λ. At this point we know which particular Moore automaton our box is specified by. But we have yet to reach the complete knowledge of our box's insides.

Level of Knowledge 6 - At this level we know the *(coordinatized) structure* of the sets X,Q,Y. One way of saying this, is that we know the component input, state, output variables which are used to describe elements of $\{X, Q, Y\}$. Another equivalent idea is that we have partial knowledge of the automaton as a network of interconnected (Moore type) automata. The external excitable (input) wires of the network are named by the *input* variables, the external (output) wires whose signals are determined by the network are named by the *output* variables and the internal state sets of the component machines become the *state* variables. Our knowledge is incomplete, since while we know the characterizing variables of network we do not know its interconnection structure.

Level of Knowledge 7 - At this, the highest level of knowledge to be considered, we know the *(local) structure* of the functions δ and λ (in addition to the structures of the sets X, Q and Y). From the network point of view we now know the nature of the component automata (i.e., we have a level 5 description of each as an abstract automaton) and the connection diagram of the network, Alternatively, we know the local transition and output functions and their arguments. (Each state variable α has as associated *trans-*

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ition function δ_α (called local to distinguish it from the global transition function δ) which determines its next state as a function of the present states of its *external* and *internal neighbors* (the external neighbors are those *input* variables which appear as arguments in δ_α and the internal neighbors are those *state* variables which so appear). Similarly the neighbors of an output wire are the state variables which determine its output signals.

The 7 knowkedge levels are summarized in Table 1.

Table 1.

Level of Knowledge	Additional Attribute Known
1	Black Box is a Moore automaton $M = \langle X_M, Q_M, Y_M, \delta_M, \lambda_M \rangle$
2	X_M (input), Y_M (output) are known : $X_M = X$, $Y_M = Y$. Finite subset R' of R_M is known.
3	R_M, the I-O relation is known: $R_M = R$
4	B_M, the I-O behavior is known: $B_M = B$.
5	State space, transition and output functions are known: $Q_M = Q$, $\delta_M = \delta$, $\lambda_M = \lambda$.
6	Structures of Q, X and Y (state, input and output variables) are known: $Q \subseteq \times_{\alpha\varepsilon D} Q_\alpha$, $X \subseteq \times_{\beta\varepsilon E} X_\beta$, $Y \subseteq \times_{\gamma\varepsilon F} Y_\gamma$.
7	Structures of δ and λ (the complete net structure) are known: δ determined by a set $\{\delta_\alpha \mid \alpha\varepsilon D\}$, λ determined by a set $\{\lambda_\gamma \mid \gamma\varepsilon F\}$ (local transition and output functions).

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The Problem of Stuctural Inference

These levels of knowledge form a hierarchy in the following sense :

a) Implication downward. Level i+1 knowledge implies level i knowledge. This is true since information about the black box is retained (by definition) as one advances from i to i+1.

b) Non-implication upward. Level i+1 knowledge cannot be uniquely inferred from level i knowledge. Thus, at each level, additional independent information is acquired.

The rest of this section is devoted to formalizing this idea. The following section will then provide evidence that the levels of knowledge form a hierarchy as defined.

Let M_i be the class of possible structured automata consistent with knowledge at level i. Then by a) and b) the sequence

$$M_1 \subset M_2 \quad \ldots \subset M_7$$

forms a strict inclusion chain.

Explicity given,

M_1 is the set of structured Moore automata;

$M_2 = \{M \in M_1 | X_M = X,\ Y_M = Y,\ R_M \supseteq R'\}$;

$M_3 = \{M \in M_2 | R_M = R\}$;

$M_4 = \{M \in M_3 | B_M = B\}$;

$M_5 = \{M \in M_4 | Q_M = Q,\ \delta_M = \delta,\ \lambda_M = \lambda\}$;

$M_6 = \{M \in M_5 | Q \subseteq \times_{\alpha \in D} Q_\alpha,\ X \subseteq \times_{\beta \in E} X_\beta,\ Y \subseteq \times_{\gamma \in F} Y_\gamma\}$

$M_7 = \{M \in M_6 | \delta$ is determined $\{\delta_\alpha\}$ and λ by $\{\lambda_\gamma\}\}$

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I take the following philosophical position : Level 2 knowledge is the only direct empirical, secure knowledge we can obtain about the black box. (Level 1 knowledge provides the apriori logical framework within which Level 2 knowledge is to be interpreted.) Thus the question arises : If Level 2 knowledge is the only secure knowledge we can obtain, in what sense are we able to acquire knowledge of the black box at the higher levels 3-7.

The answer I propose is in terms of "justifying conditions" which enable us to "climb up" from one level of knowledge to the next.

As an illustration consider the situation at level 3. Referring to Fig.1., we have the set M_3 of automata each having the same I-O relation R. Some of these, namely M_4 (which contains our black box), have an I-O behavior B. Others have different I-O behaviors B', B'',.... All these behaviors B, B', B'',... are consistent with R in the sense that $R = \bigcup_{\beta \in B} \beta = \bigcup_{\beta \in B'} \beta = \bigcup_{\beta \in B''} \beta = \ldots$ so that the automata in M_4 cannot be distinguished from the rest knowing only R.

Suppose, however, that we assume that our black box is a strongly connected finite automaton.[6] Then it is known that in this case, only one behavior can be consistent with R. Since this must be B we have been able to "climb up" from Level 3 knowledge (which states that the black box lies in M_3) to Level 4 knowledge (which asserts the stronger claim that the black box lies in M_4). The justifying condition employed here was that the black box was finite and strongly connected.[7] A better way of stating this is that the framework of admissible models for the black box, namely M_1, has been restricted to finite strongly connected automata.

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[6] M is *strongly connected* if for evey pair q, q' ε Q there is an input sequence taking q into q'.

[7] More than one justifying condition may be available, e.g., the above theorem is also true for *linear* systems (Harrison [23]).

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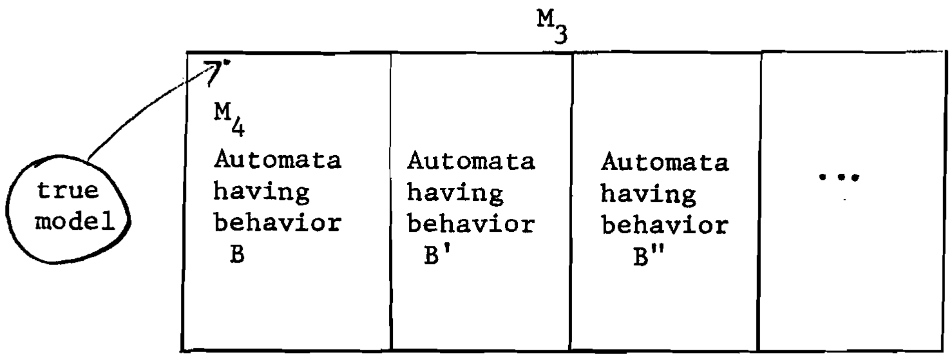

Figure 1.

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The theorem used to make the level leap is the following [49]:

For any two strongly connected automata M_1 and M_2,

$R_{M_1} = R_{M_2} \iff B_{M_1} = B_{M_2}$.

Examining the theorem we see that it takes the following form. We have an equivalence relation on $\mathcal{M}_1$, call it $\underset{3}{\equiv}$, such that two automata M_1 and M_2 are 3-equivalent, if, and only if, they have the same input, output and I-0 relation, i.e.,

$$M_1 \underset{3}{\equiv} M_2 \iff X_{M_1} = X_{M_2},\ Y_{M_1} = Y_{M_2} \text{ and } R_{M_1} = R_{M_2}$$

(3-equivalence is often called "indistinguishability" or "weak equivalence"). At the same time we have an equivalence relation on $\mathcal{M}_1$, call it $\underset{4}{\equiv}$, such that M_1 and M_2 are 4-equivalent, if, and only if, they have the same input, output and I-0 behavior, i.e.,

$$M_1 \underset{4}{\equiv} M_2 \iff X_{M_1} = X_{M_2},\ Y_{M_1} = Y_{M_2} \text{ and } B_{M_1} = B_{M_2}$$

(4-equivalence is often called "equivalence" or "strong equivalence").

Normally 4-equivalence properly refines 3-equivalence i.e., $\underset{4}{\equiv} < \underset{3}{\equiv}$.[8] This means that having the same I-0 behavior implies having the same I-0 relation but not conversely (see [59] for an example of a pair of finite automata which have the same I-0 relation but have different sets of I-0 functions).

However, the theorem says, that under certain conditions (for example, when the domain is restricted to finite, strongly connected automata) the two normally distinct relations become identical, i.e. $\underset{4}{\equiv} = \underset{3}{\equiv}$.

Going back to our levels of knowledge the statement that the black box has an I-0 relation R amounts to the statement that the black box lies within

[8] For two relations, R and R' over the same domain, R refines R', $R \leqslant R' \iff (\forall x,y)(xRy \Rightarrow xR'y)$ and R properly refines R', $R < R' \iff R \leqslant R'$ but not conversely (i.e. there are x^o, y^o which are R' -related but not R-related). $R = R' \iff R \leqslant R'$ and $R' \leqslant R$.

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an equivalence class M_3 of the relation $\underset{3}{\equiv}$[9]. The justifying conditions, which make 3 and 4 equivalence identical, enable us to asset the level 4 knowledge that the black box lies within equivalence class M_4 (now equal to M_3). In other words, given the I-0 relation R, there is a unique set of I-0 functions which is consistent with it (if the justifying conditions hold).

In general, the problem of structural inference is illustrated in Fig.2 and is formulated as follows. To each level of knowledge i, we associate an equivalence relation $\underset{i}{\equiv}$. These equivalence relations are to form a hierarchy in the sense that

$$\underset{7}{\equiv} < \underset{6}{\equiv} < \underset{5}{\equiv} < \ldots \underset{2}{\equiv} < \underset{1}{\equiv} .$$

(This is a formal restatement of the inclusion chain property of what are seen to be equivalence classes; recall that the proper refinement of $\underset{i}{\equiv}$ by $\underset{i+1}{\equiv}$ means that independent additional knowledge resides at level i+1.)

To "climb up" this hierarchy is to locate the unknown black box in successively higher indexed equivalence classes. A justifying condition at level i, $J_{i \to i+1}$ is a condition which forces the equality of $\underset{i}{\equiv}$ and $\underset{i+1}{\equiv}$. The problem of structural inference is thus formulated as the problem of obtaining justifying conditions which facilitate the "climbing up" of the knowledge hierarchy.

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[9] Considering R to be a variable we see that a 3-equivalence class can be identified as M_3^R. Similarly the 4-equivalence class corresponding to a given B can be identified as M_4. In general, we can superscript any M_i to identify it as an equivalence class of i-equivalence.

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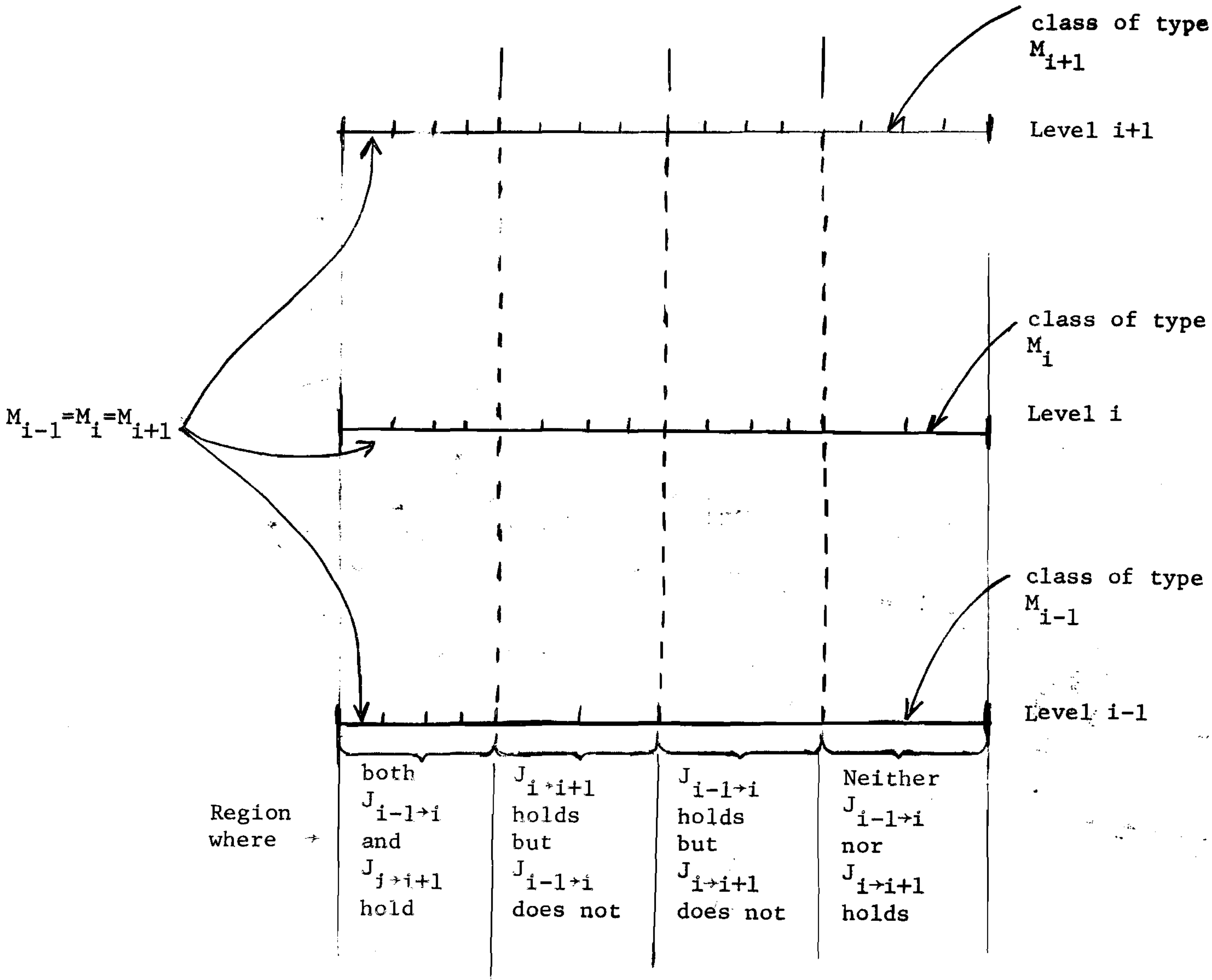

Figure 2.

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Proof of Hierarchy and Justifying Conditions

I shall briefly discuss the appropriate equivalences and justifying conditions at each level.

Level 1 The 1-equivalence can be regarded as trivial equivalence in which all elements are in the same block, i.e.,

$$M_1 \underset{1}{\equiv} M_2 \Leftrightarrow M_1 \text{ and } M_2 \text{ are structured Moore automata.}$$

Note however that this block delineates the class of models from which the eventual model of the black box must come and canalizes the inference processes by which it will be determined.

Clearly there can be no meaningful justifying conditions at this level. Such conditions would be specified in advance of empirical data and thus could be directly incorporated in the definition of 1-equivalence.

Level 2 At this level we really have not one, but a family, of equivalences, each corresponding to a particular known data set R'. Let us say that M is compatible with R' if $R' \leq R'_M$. Then for each R' we have two disjoint classes — those compatible, and those not compatible, with R'.[10] With continued experimentation the set R' may be expected to grow (thus we really will have an inclusion sequence of R' s) and the class of compatible automata (candidates for the "real" model) concomittantly to diminish. Though the actual sequence will be heavily dependent on the experimental circumstances, we can consider, for illustration purposes, the length determined sequence as follows:

For each integer n = 1, 2, 3,... define (2,n)-equivalence by

$$M_1 \underset{(2,n)}{\equiv} M_2 \Leftrightarrow Y_{M_1} = X_{M_2},\ Y_{M_1} = Y_{M_2} \text{ and } R^n_{M_1} = R^n_{M_2}$$

[10] More formally $M_1 \underset{R'}{\equiv} M_2 \Leftrightarrow (R' \subseteq R_{M_1} \Leftrightarrow R' \subseteq R_{M_2}$

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where

$$R_M^n = \{(x_1x_2\ldots x_t, y_1y_2\ldots y_t) \mid (x_1x_2\ldots x_t, y_1y_2\ldots y_t)$$

$$\varepsilon\ R_M \text{ and } t \leq n\}.$$

(the subset of R_M containing all I-0 pairs at most of length n).

Clearly, for any n = 0, 1, 2,...

$$\underset{3}{\equiv} \leq \underset{(2,n+1)}{\equiv} \leq \underset{(2,n)}{\equiv} .$$

Proper refinement of (2,n) - equivalence by (2,n+1) - equivalence for each n (hence also by 3-equivalence) can be demonstrated by an appropriate sequence of automata.[11]

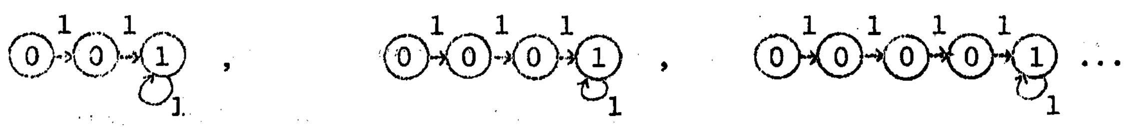
The justifying conditions at level 2 enable one to uniquely infer a complete I-0 relation R_M on the basis of some appropriate finite subset of it. This is possible for finite memory automata [19].

More precisely,

If M_1 and M_2 have finite memory of order n[12] then

$$M_1 \underset{(2,n)}{\equiv} M_2 \Leftrightarrow M_1 \underset{3}{\equiv} M_2$$

[11] The sequence of automata



M_1 M_2 M_3 ...

is such that $M_n \underset{(2,n)}{\equiv} M_{n+1}$ is true and $M_n \underset{(2,n+1)}{\equiv} M_{n+1}$ is false for each n = 0, 1, 2, 3... .

[12] M has finite memory of order n if its next output depends only on its n previous inputs and outputs, i.e., its (reduced) state space takes essentially the special form $Q = (X \times Y)^n$. The finite memory restriction is not a

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It is also known [19] that the I-O behavior of any member of the class of all finite strongly connected compatible machines with state set cardinalities bounded by some integer k can be identified with a sequence of length ℓ exponentially related to k. Thus for this class,

$$M_1 \underset{(2,\ell)}{\equiv} M_2 \iff M_1 \underset{3}{\equiv} M_2$$

Finally, in the identical copies assumption, the characterizing relation is a sequential function β_M. Gray and Harrison [21] show that in this case, if M has more than n states, then β_M is completely specified by its restriction to the first 2n strings of X^+. Thus we have

$$M_1 \underset{(2,2n)}{\equiv} M_2 \iff M_1 \underset{3}{\equiv} M_2 .$$

Thus some justifying conditions at level 2 are

$J_{2\to3}$: a) M_1 restricted to finite memory automata or

b) M_1 restricted to automata having no more than some specified number of states.

c) The identical copies assumption with state bounds as in b).

Level 3 We have already discussed this level. For completeness

$$M_1 \underset{3}{\equiv} M_2 \iff X_{M_1} = X_{M_2} ,\ Y_{M_1} = Y_{M_2} \text{ and } R_{M_1} = R_{M_2}$$

$J_{3\to4}$: a) M_1 restricted to finite state strongly connected automata or

b) M_1 restricted to linear automata.

Level 4 Recall that the 4-equivalence is given by

$$M_1 \underset{4}{\equiv} M_2 \iff X_{M_1} = X_{M_2} ,\ Y_{M_1} = Y_{M_2} \text{ and } B_{M_1} = B_{M_2} .$$

[12] limitation on state behavior since any machine can be made to have finite memory (of order 1) by allowing the state to appear as output.

Autonomous machines having n states are finite memory of order 2n.

It is not hard to see how knowing all I-O pairs of length n, one can uniquely determine the output strings possible for any given input string. See [59] for an example.

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$J_{4\to 5}$: M_1 restricted to reduced automata (to be amplified below).

Level 5 Automata can be identified at most up to isomorphism from their external behavior. Thus 5-equivalence is isomorphism, i.e.,

$$M_1 \underset{5}{\equiv} M_2 \iff X_{M_1} = X_{M_2},\ Y_{M_1} = Y_{M_2} \text{ and}$$

there is a 1-1 correspondence h between Q_{M_1} and Q_{M_2} which preserves both the transition and output functions.[13]

It is well known that the isomorphic machines have the same I-O behavior. Two machines having the same behavior need not be isomorphic (hence $\underset{5}{\equiv} < \underset{4}{\equiv}$) but this is isomorphism is forced if the machines are reduced[14] [44].

There are no natural automata theory justifying conditions for the jump to level 6. This has important consequences for modelling methodologies as we shall see later.

Level 6 The appropriate 6-equivalence is given in terms of the weak structure preserving morphism in [60]. Essentially, M_1 and M_2, assumed to be 5-equivalent, are to be 6-equivalent, if and only if, their component input, state and output variables can be placed in a one-one correspondence which agrees with the isomorphism underlying the 5-equivalence.[15] We may refer to 6-equivalence as *weak structure equivalence*.

6-equivalence refines 5-equivalence by definition. The proper refinement is a consequence of the fact that an abstract set can be given many distinct co-ordinatizations. A well known class of examples is that of linear systems where

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[13] $\delta_{M_2}(h(q),s) = h(\delta_{M_1}(q,s))$ and $\lambda_{M_2}(h(q)) = \lambda_{M_1}(q)$ for all $q \in Q_1$, $s \in X_{M_1}$.

[14] M is reduced if Q_M is in 1-1 correspondence with B_M, i.e., every distict state q has a distinct I-O behavior β_q.

[15] Let the structures for Q_{M_1} and Q_{M_2} be given by $Q_{M_1} \subseteq \underset{\alpha^1 \in D_1}{X} Q^1_{\alpha^1}$ and $Q_{M_2} \subseteq \underset{\alpha^2 \in D_2}{X} Q^2_{\alpha^2}$.

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in order that similar (hence isomorphic) automata also be weak structure equivalent, the similarity transformation must be limited to a permutation of the basis vectors and a change in scale (cf., the notion of similarity in Klir and Valach, [33]). Thus change of basis to the eigencoordinates does not generally preserve 6-equivalence.[16]

The justifying conditions at level 6 enable one to infer strong structure equivalence from weak structure equivalence. In other words, if we know the structures of Q, X and Y and the (global) transition and output functions δ and λ (level 6 knowledge) under what conditions can we uniquely determine the complete net structure, i.e., the local transition and output function structures (level 7 knowledge). These justifying conditions were developed in [60] and are stated as:

$J_{6\to 7}$: M_1 restricted to structured automata in reduced form (1) whose input and state coordinatizations are irredundant(2).

Condition (1) essentially requires that there be no local functions with dummy arguments (or equivalently, every wire in the net must serve some essential informational purpose). Condition (2) requires essentially that there cannot be multiple sources for input and state information. This forces the external and internal neighbors of a module to be uniquely identifiable.

Then weak structure equivalence $\underset{6}{\equiv}$ requires that the isomorphism h be dedefinable as follows:
There is a 1-1 correspondence $d: D_1 \to D_2$ between coordinate sets. For each $\alpha^1 \varepsilon D_1$, there is a 1-1 correspondence $h_{\alpha 1}: Q^1{}_{\alpha^1} \to Q^2{}_{d(\alpha^1)}$ such that for all $(q_{\alpha_1}, q_{\alpha_2}, q_{\alpha_3}, \ldots) \varepsilon Q_{M_1}$; $h(q_{\alpha_1}, q_{\alpha_2}, q_{\alpha_3}, \ldots) = (h_{d(\alpha_1)}(q_{\alpha_1}), h_{d(\alpha_2)}(q_{\alpha_2}), h_{d(\alpha_3)}(q_{\alpha_2}), \ldots)$. Thus, up to relabelling, Q_{M_1} and Q_{M_2} represent the same abstract set with the same coordinatization. Similar requirements hold for input and output coordinatization.

[16] For example, the autonomous linear state machines $M_i = \langle R \times R, \delta_i \rangle$, $i = 1,2$ (R = real numbers) where $\delta_i(q) = A_i q$ and

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Level 7 As can be gathered, 7-equivalence will be called *strong structure equivalence* and will be a formalized version of the definition:

$$M_1 \underset{7}{\equiv} M_2 \iff \text{the complete structures of } M_1 \text{ and } M_2 \text{ are the same up to relabelling.}$$

In other words, M_1 and M_2 are strong structure equivalent if they are essentially the same networks of essentially the same components connected in essentially the same way.

Clearly, strong structure equivalence ($\underset{7}{\equiv}$) refines weak structure equivalence ($\underset{6}{\equiv}$). The proper refinement is displayed by a pair of automata which are weakly but not strong structure equivalent, as discussed in [60].

A full duscussion of the relation between the weak and strong structure concepts is available in the cited references [60-61].

$A_1 = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$, $A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ are similar under the transformation $P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ (i.e., $A_1 = PA_2P^{-1}$) but while $P:R^2 \to R^2$ is a 1-1 correspondence it does more than permute basis vectors as can be seen from $P\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a+b \\ a-b \end{bmatrix}$.

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Methodologies

So far we have identified a hierarchy of levels at which knowledge of a black box may be acquired. It has been shown how, with the use of justifying conditions, it is possible to attain higher level knowledge, even though direct knowledge is available only at the experimental level 2. However three crucial elements have not yet received attention. One concerns the status of justifying conditions — must they be assumed *a priori* or are they subject to empirical check. The second issue bears upon the use of the justifying conditions. $J_{i \to i+1}$ forces equality of i-equivalence and i+1-equivalence., i.e., it assures us that given an i-equivalence class M_i there is a unique restriction to an i+1-equivalence class M_{i+1}. But the condition itself does not necessarily supply practical algorithms for selecting out the smaller class M_{i+1}. The third problem is that referred to already — how to deal with the jump from level 5 to level 6 knowledge.

I shall consider two distinct methodologies of modelling. The first is the approach naively suggested by the black box framework. In this approach, which can be identified with what is currently called realization theory [29], one passively observes the I-O behavior of the black box and then attempts to adduce a structure (ulimately at level 7) whcih will generate this behavior. We will call this the "discovery" approach.

The second approach is a formulation of the more common process of modelling and simulation which differs from the first in the primacy of the use of a model to guide (and be corrected by) experimentation. This will be called the "postulational" approach since a model is first postulated and then checked again the data, rather than having been discovered from the data.

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[The distinction between the two methodologies has its undertones in the empiricist/rationalist, behaviorist/introspectionist, pre-Chomskian/Chomskian controversies. The first of these pairs leans toward the "discovery" philosophy, claining as its justification a) its minimal prejudice of the data, and b) its safeguards against unwarranted attribution to the data of non-verifiable interpretations. The counterclaims of the "postulational" adherents are that any methodology must prejudice the data since in the process of data collection and reduction, inevitable decisions concerning what to consider and what to neglect are made, and these are based largely on prior expectations. Thus it is better to acknowledge, formalize and allow for these expectations rather than be at their mercy. Moreover, it is argued that non-observable constructs are verifiable, or better disconfirmable, provided that they have observable consequences. Finally, it is claimed that progress in understanding is contingent upon the existence of a rich theoretical framework with which form is given to the data.

I do not wish to deal directly with these controversies here, though I feel that the concepts and discussion of this paper help to set the ground for their formal analysis].

The problems just raised concerning the status, and use of the justifying conditions will be examined first with respect to the "discovery" approach. We shall see that some fundamental limitations arise with this approach. Thse same test will be put to the "postulational" approach, but in order to do so, it will be necessary to make a quite radical departure from the black box conceptual framework to one much more detailed and complete.

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The Discorvey Approach

We now consider the status and use of the justifying conditions at each transition for the discovery approach.

Transition $2 \rightarrow 3$

At the outset we require identification of the relevant input and output sets of the black box. This has been given little formal consideration since the black box is almost always considered to come with input and output terminals built in. In actual fact however, the choice of actions to influence the box (inpit) and the choice of instruments and/or classification schemes by which to observe the box (output) are crucial decisions in the modelling process.[17]

Both approaches must face this problem, but with the usual black box framework there is little that can be said about it. However, in the expanded framework to be given soon, it is more amenable to systematic treatment.

Having decided on $X_M = X$ and $Y_M = Y$, we are in a postion to collect the data $R' \subseteq R_M$. When should such data collection stop? In the discovery mode we are constrained to stop at some point and refrain from further experimentation from then on (hoping that enough data has been gathered). But the stopping criteria (viz, the justifying conditions $J_{2\rightarrow 3}$) are all phrased in terms of a bound on the memory capacity of the box (i.e., order of finite memory or number of states). There is no *a priori* way to establish this bound and it can never be established for certain after finite experimentation. So much

[17] Actually, there are now two problems here 1) the choice of discriptive variables for the box and 2) the designation of these as input or output variables. Assuming the first choice to be arbitrary, Klir [31] suggests a procedure for solving the second (orientation) problem. However, in effect the procedure disallows consideration of internal states, the introduction of which would again make the orientation decision an arbitrary one.

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is always true, but in the discovery mode we have no way of iteratively estimating the bound based on past observation.

With a negative answer to the question of *a priori* checkability we come to the second issue — that of practical algorithms. Assuming that the justifying condition is satisfied, the infinite cardinality of R_M precludes our being able to construct all of it in a finite time. However, it is possible to settle for a less demanding means of "knowing" R_M, for example, that it be recursive, or weaker just recursively enumerable. It is not hard to see that R_M is recursively enumerable (in the finite memory case) and recursive (in the finite automaton case). The catch is that the initial data required (R_M^n) grows exponentially with n, thus rendering the algorithms impractical for all but small n.

So again we come up with a negative conclusion — even if we could assure ourselves that the justifying condition is satisfied, we could not practically employ this knowledge.

Transition $3 \rightarrow 4$

Justifying condition $J_{3\rightarrow 4}$ a) requires that the black box model come from the strongly connected finite automata class. Considering the checkability issue, the finiteness condition is not a restriction in the sense that it can never be refuted with only finte I→O data. (Given any finite set R' or R_M, there is always a finite automaton M' which will reproduce R', i.e., $R' \subseteq R_{M'}$).
However, if we take the view that we have arrived at level 3 with a complete I-O relation R_M (having submerged the finite R' which gave rise to it) then the finite state condition is certainly a restriction since there may well be no models in this class which can reproduce R_M. In this case and depending on presentation form of R_M, finite state-ness may be effectively decidable.

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However, the strong connectivity requirement clearly cannot be checked directly since it is a property of state transition structure and not of the I-O behavior which is available. Thus the more general condition [$J_{3\to4}$ a)] must be assumed without possibility of empirical check.[18]

The most striking problem concerns issue 2, namely, the construction of B_M from R_M. The proof underlying $J_{3\to4}$ are not constructive in the sense of providing an algorithm for paritioning the set of pairs in R_M into a unique covering set of functions B_M. The existence of such an algorithm will be relative to the form in which R_M is given. So far only a form of presentation not likely to be encountered in the "discovery" mode of modelling has been studied.[19]

Transition 4 → 5

The construction of a machine which realizes a given behavior function has received much attention. Thus, given the set of functions B_M, it is possible to abstractly specify a unique reduced automaton M' such that $B_{M'} = B_M$.[20]

An algorithm to concretely specify M' exists when it is possible to

[18] On the other hand, the linearity required by $J_{3\to4}$ b) can be ascertained at the I-O level in the sense that if R_M is linear then there is a linear automaton M' such that $R_M = R_{M'}$. (For the linearity or non-linearity of R_M to make sense, X and Y must be embedded in vector spaces over some field.) However, verifying linearity of the infinite set R_M may involve decision problems depending on its form of presentation.

[19] Gray and Harrison [21] show that if R is given as the set accepted by an automaton over X×Y then R can be effectively checked for finite state-ness and if it is, a finite state machine M such that $R = R_M$ can be effectively constructed. As we have seen, this machine cannot in general be unique, a fact which is demonstrated explicitly in the choice points of Gray's algorithm.

[20] B_M has the property that it is closed under taking of derivitives, i.e.,

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decide the equality of any two functions $\beta, \beta' \varepsilon B_M$.[21] This is possible, for example, when the β's are described by regular expressions though again the practicality of this algorithm is open to question.

In the case corresponding to $J_{2\to3}$ c) B_M is the set of derivitives of a single function β. Gray and Harrison provide an effective procedure, which given an integer n and the first 2n I-0 pairs in β, will construct the unique minimal machine just mentioned, if in fact this machine has no more than n states; if the function β necessitates more than n states, this too will be discovered by the procedure. Again, because of the exponential growth in data requirements, this procedure is impractical, but it suggests that it may be possible to develop discovery procedures with the universality and scope of automata (and systems) theory, which provide at the same time, error estimates of a kind akin to statistical estimation procedures[4].

Transition 5 → 6

As remarked earlier, there are no natural nontrivial automata theoretic conditions constraining the co-ordinatization (structure assignment) of an abstract automaton. Trivially, the embedding cross-product space must be large enough to accommodate the set to be co-ordinatized but this still permits a large range of possibilities. To cut down the range of possibilities, one can impose a measure of complexity upon the space of structured automata M_1 and choose from those having minimum complexity. This is the approach of logical designers, and has led to mathematical theories of decomposition. These attempt to guide the choice of structure assignment so that an appropriate measure of complexity, usually involving component interaction, is minimized.

$\beta \varepsilon B_M \Rightarrow (\forall S \varepsilon X)(\beta \circ L_S \varepsilon B_M)$ where $\beta \circ L_S(x) = \beta(sx)$. The machine $M' \underline{\underline{M}} \langle X, B_M, Y, \delta', \lambda' \rangle$ where $\delta'(\beta, s) \underline{\underline{S}} \beta \circ L_S$ and $\lambda'(\beta) = \beta(\Lambda)$. Then the I-0 function associated with state $\beta \varepsilon B_M$ turns out to be just β.

[21] One must be able to determine, given the list $B_M = \{\beta_i\}$ and any i and $s \varepsilon X$, the j such that $\beta_j = \beta_i \circ L_s$.

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While algorithms do exist in this approach, they become impractical for all but small n (state set size). The combinatorial problem involved in such optimization can be appreciated in the present conceptual framework as follows: Given an abstract automaton at level 5 i.e. an equivalence class M_5, in order to uniquely specify a complete net structure at level 7, one must in effect search through all of the 7-equivalence classes M_7 contained in M_5.[22]

Transition 6 → 7

There are algorithms for checking whether or not a co-ordinatization is irredundant and for obtaining reduced structure assignments $\{\delta_\alpha\},\{\lambda_\alpha\}$ for abstract functions δ, λ. As compared with the transition 5-6 case, the search space is certainly reduced, and there is hope for some practical algorithms in this area.

Formal Characterization of Modelling and Simulation

In contrast with the discovery approach is the postulational approach whereby a model is constructed and its behavior is compared with real system behavior. This has been elaborated to a formulation of the basic conceptual framework illustrated in Figure 3 [65].

A simulation instance is taken to involve four elements - a *real system*,

[22] Consider the following estimate: Given a transition function $\delta:Q \to Q$ where Q has 2^n elements. There are $2^n!$ coding of Q onto $\{0,1\}^n$; these can be partitioned in 6-equivalence classes each of which has 2^n members. Thus the number of 6-equivalence classes is $(2^n-1)!$. This is a (not very good) lower bound on the total number of classes M_6 contained in M_5, since we will get more classes with each manner of coding Q. Of course, there are still more 7-equivalence classes contained in M_5.

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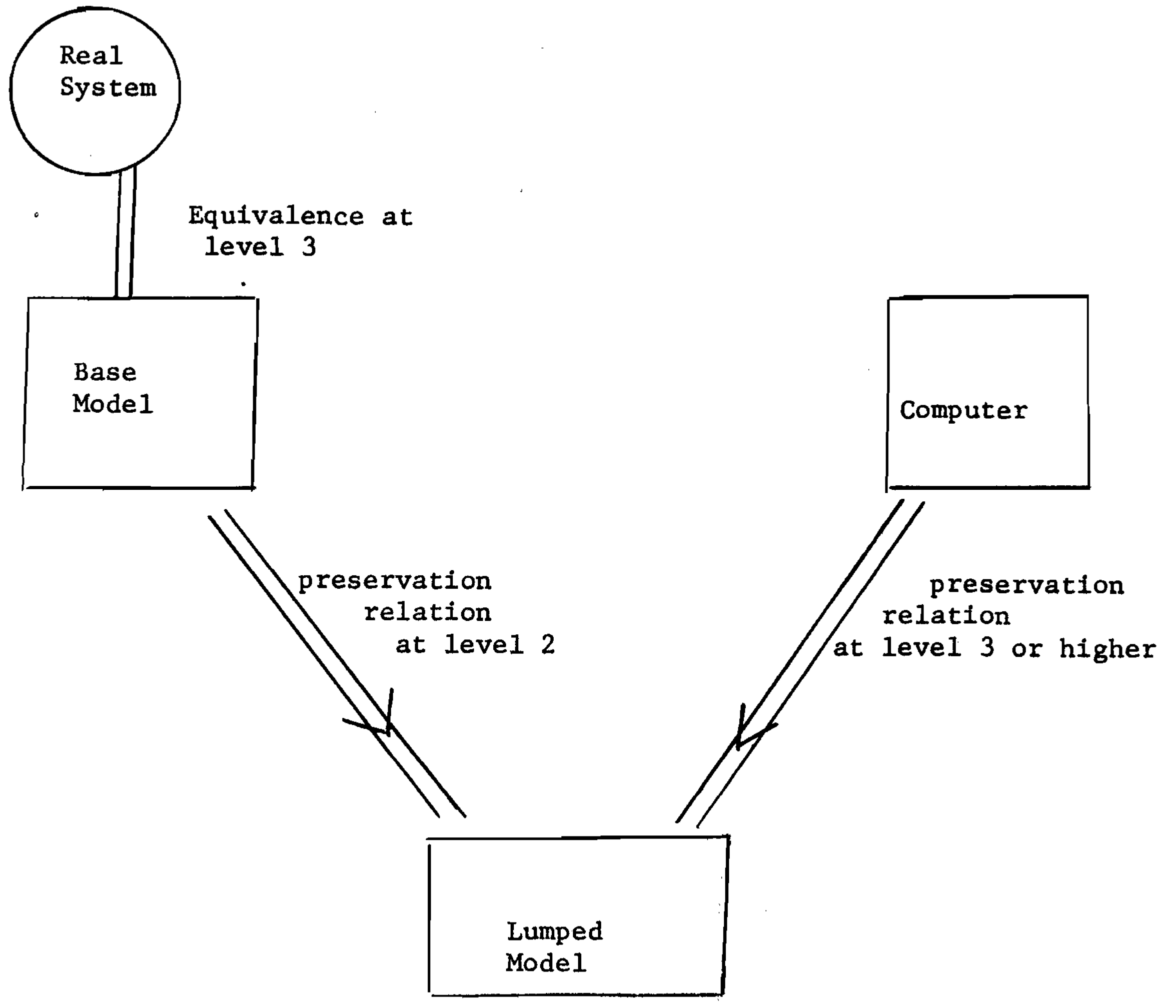

Figure 3.

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a *base model* of the real system, an abstract or *lumped model* and a *computer* (for the present discussion, all assumed to be finite automata, though extension to general systems is not precluded [63]). The real system is presumed to be knowable only at the observational level, i.e. as a set R of input-output pairs. The base model represents an internal structure of the real system postulated at a level of detail intrinsic to the subject matter and unconstrained by considerations of the complexity that processing such a model might entail. Examples of levels are given in Table 2. Since computational complexity considerations can be shown often to preclude actual computer processing of such a model, it becomes necessary to construct lumped models which *can* be feasibly processed. Thus the table also gives representative levels at which lumped models have been constructed. We see then that the computer must be taken as a basic element in a formulation of modelling and simulation since only in this way can the computational complexity constraints that are placed on possible models be explicitly considered. (Here I shall not dwell on the role of the computer, for further development, see [61].)

The distinction we have made between base model and lumped model opens the way toward fuller characterization of their interrelation. Clearly the set R must represent the outcomes of all possible experiments which can be performed on the base model; conversely, the base model must fully account for all the input-output pairs contained in R. However, first we shall want to adequately represent the set of possible experiments. We attempt this as follows (please refer to Fig.4):

The base model is to be a structured state machine $M_B = \langle I,Q,\delta \rangle$ where $\delta:Q \times I \to Q$, i.e an underlying transition structure is attributed to the real system complete with an identified set of input variables $\{I_\alpha\}$, state

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TABLE 2.

SYSTEM	Level of	
	Base Model	Lumped Model
Bacterial Cell	Molecular Species	Pools of Molecules
Cortex	Neurons	Cubes of Nerve Tissue
Grosslands	Species	Compartments (Classes of Species)

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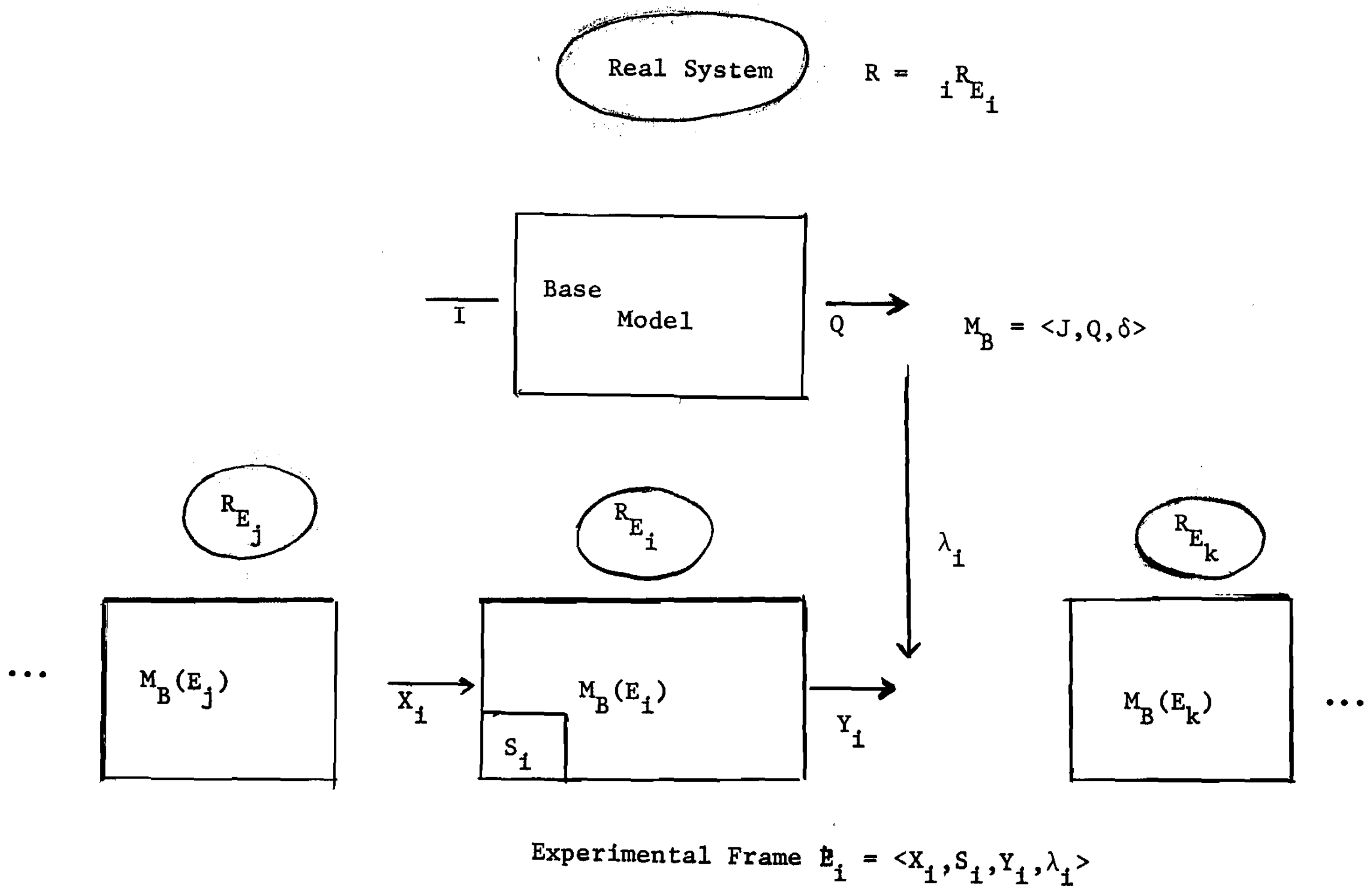
Real System
R = $_iR_{E_i}$
I
Base Model
Q
M_B = <J,Q,δ>
λ_i
R_{E_j}
R_{E_i}
R_{E_k}
...
$M_B(E_j)$
X_i
$M_B(E_i)$
S_i
Y_i
$M_B(E_k)$
...
Experimental Frame E_i = $<X_i, S_i, Y_i, \lambda_i>$

Figure 4.

variables $\{Q_\beta\}$ and local transition functions $\{\delta_\beta\}$. In many situations there are natural choices for such variables as illustrated in Table 2. It is assumed that the sets $\{I_\alpha\}$ and $\{Q_\beta\}$ are known but that δ and hence $\{\delta_\beta\}$ are not known. Indeed, the primary aim of investigation is to discover what they are.

An *experimental frame* consists of a triple $E=(X,Y,S,\lambda)$ where $X\subseteq I^+$, Y is a set , $S\subseteq Q$ is a set of starting states and $\lambda:S\to Y$. The observations possible within an experimental frame E are given by $R_E = \bigcup_{q\in S} \tilde{\beta}_q^E$ where $\tilde{\beta}_q^E:X^+\to Y$ is defined by $\tilde{\beta}_q^E(x_1x_2\ldots x_n) = \lambda(q)\lambda(\tilde{\delta}(q,x_1))\cdots\lambda(\tilde{\delta}(q,x_1x_2\ldots x_n)$ for $x_i\in X$. The map $\lambda:Q\to Y$ is interpreted as characterizing direct observation by an instantaneous measuring instrument or other classification procedure m_λ. Using m_λ one can know the true state of the base model only up to membership in one of the output value-defined equivalence classes.[23] The subset $X\subseteq I^+$ characterizes two aspects of experimentation: a) the set of possible experimenal input sequences within a particular (real) experimental framework is strictly limited (thus X is a *subset* of I^+), and b) there is a time scale associated with an experimental framework since observations are made only at certain intervals. (Here if $X\subseteq I^n$, this corresponds to observational access to the base model every n base time units.)[24] Finally, the set S represents a set of possible starting states for the base model. We require the closure of S under X (i.e, $\tilde{\delta}(S,X)\subseteq S$), so as to allow for the fact that if q is a possible starting state and one can force q to a state q' by some sequence of input sequences from X then q' is also a possible starting state.

[23] i.e. an element of the level partition $\pi_\lambda = \{\lambda^{-1}(y)\,|\,y\in Y\}$

[24] c.f. Klir's notion of space time resolution levels [31].

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Let E be a set of experimental frames. Then E may be chosen to represent the set of possible experimental frames in which experimentation with the real system (as represented by the base model) can be undertaken. The experimental limitations embodied in E may be attributed to one or a combination of:

a) Technological factors — the state of the art of instrumentation will determine what measurements, observations, etc. are possible, and to what level of resolution these can be made;

b) Factors intrinsic to the approach — the point of view taken and the questions asked about a system determine the modes of observation (λ) and stimulation (X,S) it will be subjected to. For example, the psychological and neurophysiological approaches to human brain function can be distinguished as having different experimental frames (E) relative to the same model M_B;

c) Factors due to state of understanding — availability of appropriate observational modes may hinge upon the degree to which relevant aspects of the system behavior have been isolated. For example, technology may ultimately make it possible to obtain simultaneous recordings of all the 10^{10} neurons in the human cortex but it is doubtful that such a mode λ would be useful by itself. More likely, progress will hinge upon discovering relevant aggregates (i.e., coarser modes λ') of this information.[25]

Given a base model M_B and a set of experimental frames E, it is possible to characterize the real system R by taking $R=\bigcup_{E\epsilon E} R_E$. R is thus the set of all input-output segments that could be observed of the real system in any of the admissible frames in E. Note that R represents, as it should, level 3 knowledge. Of course at any point in time, only a finite subset R or R' will

[25] λ is the finer than λ' (λ' is coarser than λ) if $\pi_\lambda \leq \pi_{\lambda'}$, i.e. for all $q_1, q_2 \epsilon Q$, $q_1 \pi_\lambda q_2 \Rightarrow q_1 \pi_{\lambda'} q_2$.

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be known (constituting level 2 knowledge)

To sum up, R_E represents what can be observed about the real system having prior knowledge only about which inputs X and which measuring instrument m_λ (and its range Y) are to be used. (The other parameters $\{S,\lambda\}$ need not be known at the time of observation. Indeed, these parameters cannot be "known" since they are structural (level 5) in nature and not accessible at level 2.) R represents the sum total of all knowledge attainable within a set of environmental frames E.

Lumped Models

The lumped model will be taken to be an automaton $M_L = \langle X_L, Q_L, Y_L, \delta_L, \lambda_L \rangle$. M_L will be valid with respect to a given experimetal frame if it can account for the real system behavior observed in this frame. More specifically if $E=\langle X,S,Y,\lambda\rangle$ then M_L is *valid with respect to* E if there are 1-1 correspondences $e:X_L \to X$ and $d:Y \to Y_L$ such that $R_{M_L} = \tilde{e} \circ R_E \circ \tilde{d}$, i.e., (x,y) is an I-O pair of the lumped model M_L $\iff$ $(\tilde{e}(x), \tilde{d}^{-1}(y))$ is an I-O pair observable in the experimental frame E.[26] In other words, M_L is valid for E if it accounts for all and only the I-O pairs in R_E as they are viewed under the mappings d and e through which the pairs $\{X,X_L\}$ and $\{Y,Y_L\}$ are put in correspondence.

Within this framewrok we can see how there may exist valid models whose state space is smaller than that of the base model We have taken the validity of a model to be judged relative to an experimental frame E. Within such a frame the base model M_B need not be reduced and there may exist nontrivial classes of equivalent states (states with the same I-O function) even if M_B is reduced with respect to all frames in E.[27]

---

[26] $\tilde{e}$ is e extended to X^+, i.e. $e(s_1 s_2 \ldots s_n) = e(s_1)e(s_2)\ldots e(s_n)$, similarly $\tilde{d}$ is d extended to Y^+.

[27] M_B is reduced in E if for every pair $q,q' \in Q$ there is an $E \in E$ with

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Relation betweein Base and Lumped Models

Let us hold fixed a particular experimental frame $E=<X,S,Y,\lambda>$. We can consider the base model relative to this frame to be an automaton $M_B(E) = $ $= <X,S,Y,\delta^E,\lambda>$ where $\delta^E: S \times X \to S$ is given by $\delta^E(q,x) = \tilde{\delta}(q,x)$ for all $q \in S$ and $x \in X$.[28] $M_B(E)$ is called a *macro machine* since it represents the behavior of M_B as it is projected through an observation map λ and through a compounding of transition steps arising from the restriction to sequences $X \subseteq I^+$.

We are now in a position to deal with $M_B(E)$ as a black box at level 2. There are however two important connections between $M_B(E)$ and M_B. The first is that since $M_B(E)$ is a time compounded version of M_B, their relative complexities may be different. This will not be further explored here. The second is that S inherits the structure of its parent set Q, i.e. if Q is coordinatized by $\{Q_\alpha | \alpha \in D\}$ then so is S but now the coordinatization need not be Cartesian with the consequence that it may not be irredundant.[29]

We can now see how the justifying conditions at level 5 are handled in the postulational approach. A base model $M_B = <I,Q,\delta>$ is postulated in which I and Q are taken to be known structured sets. For example, in the model of a living cell, molecules may be taken as the coordinates of the state space; for brain models, neurons may be taken as the components. See Table 1 for a more complete description. The transition function δ however is not known,

---

$E = <X,Y,S,\lambda>$ and an $x \in X^+$ such that $q,q' \in S$ and $\tilde{\beta}^E_q(x) \neq \tilde{\beta}^E_q(x)$.

[28] Note that "x" on the left is interpreted as a symbol while "x" on the right is a sequence. this amounts to a compounding of micro operations in M_B into macro operations which corrlarespomd to macro operations of $M_B(E)$.

[29] If $Q = \times_{\alpha \in D} Q_\alpha$ it has received a Cartesian coordinatization by $\{Q_\alpha | x \in D\}$. Since $S \subseteq Q$ we have $S \subseteq \times_{\alpha \in D} Q_\alpha$ so S is structured by $\{Q_\alpha | \alpha \in D\}$.

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indeed, its discovery will be the central task of the modelling and experimentation to be undertaken. Knowledge of δ is of course attained at levels 5 and higher and to the extent that such knowledge is taken to be fully confirmed, δ may be taken to parially known. For example, certain biochemical interactions (e.g. the Krebs cycle) in the bacterial cell may be assumed as part of the base model. Similarly in brain models certain types of cells and their interconnection can be assumed known from experimental evidence.

Now in an experimental frame $E=\langle X,S,Y,\lambda\rangle$, we can take Y to be a structured set. Thus $M_B(E) = \langle X,S,\delta^E,\lambda\rangle$ is taken to be a structured automaton in which the structures of X, S, and Y are known, but δ^E and λ are at most partially known. Thus justifying conditions $J_{5\to 6}$ are built into the scheme with the result that while $M_B(E)$ is a black box lying at level 2 it has been constrained to lie within a subset of M_1 consistent with knowledge of the structures of X, S, and Y.

Now in contrast to the discovery approach, in the postulational approach, inference about the structure of $M_B(E)$ is not attempted directly from I-O observations (i.e., subsets of R_E) but by means of a lumped model M_L. The well known postulate-test-modify may be pursued with the goal of obtaining a lumped model M_L valid in E (Klir [31]). Since validity requires equality of infinite sets, it is not of course truly achievable.

One essential distinction between the approaches is that in the postulational approach, the complete structure of M_L is known. In other words M_L is known at level 7. The question then becomes: under what conditions can the level 7 knowledge of M_L be attributed to $M_B(E)$? Differently put, knowing the structure of M_L, what can we infer about the structure of $M_B(E)$?

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The answer involves justifying conditions as before. The difference however, is that the relations between $M_B(E)$ and M_L are not symmetric and thus the equivalence relations $\underset{i}{\equiv}$, i = 1,2,...7 applicable before are not appropriate now. The appropriate relations in this case are what we shall call *preservation relations* $\underset{i}{\leq}$, i = 2,5...7. For each i, $\leq_i$ is reflexive and transitive so that its intersection with its converse is an equivalence relation which is just $\underset{i}{\equiv}$

i.e. $M_1 \underset{i}{\equiv} M_2 \iff M_1 \leq_i M_2$ and $M_2 \underset{i}{\leq} M_1$.

The following table outlines the appropriate definitions.

$M_1 \underset{(2,n)}{\leq} M_2 \iff X_{M_1} = X_{M_2}, Y_{M_1} = Y_{M_2}$ and $R^n_{M_1} \subseteq R^n_{M_2}$;

$M_1 \underset{3}{\leq} M_2 \iff X_{M_1} = X_{M_2}, Y_{M_1} = Y_{M_2}$ and $R_{M_1} \subseteq R_{M_2}$;

$M_1 \underset{4}{\leq} M_2 \iff X_{M_1} = X_{M_2}, Y_{M_1} = Y_{M_2}$ and $B_{M_1} \subseteq B_{M_2}$;

$M_1 \underset{5}{\leq} M_2 \iff M_1$ is a homomorphic image of a submachine of M_2;

$M_1 \underset{6}{\leq} M_2 \iff M_1$ is a weak structure morphic image of a submachine of M_2.

$M_1 \underset{7}{\leq} M_2 \iff M_1$ is a strong structure morphic image of a submachine of M_2.

It can be shown that these relations form a hierarchy in the same way that the equivalence do. Moreover, there are justifying conditions which are less stringent in the cases of levels 3 and 4. In fact, for these cases one can give conditions which pertain only to M_1 (the left-hand element of a related pair). This is important because in the present application the lumped model M_L plays the role of M_1 and $M_B(E)$ takes the part of M_2. Since the structure of M_L is known, attributes such as "strongly connected" or "reduced" can in principle be checked directly of it.

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The following table outlines some justifying conditions. Conditions involving only M_1 are starred.

$J_{2\to3}$:	a) M_1 and M_2 are finite memory or
	b) M_1 and M_2 are strongly connected with a finite number of states
* $J_{3\to4}$	a) M_1 is finite and strongly connected or
	b) M_1 and M_2 are linear
* $J_{4\to5}$	M_1 is reduced
$J_{5\to6}$	The state space structure of M_1 is a locally specified aggregate mapping of that of M_2 which agrees with the homomorphism of level 5.[30]
$J_{6\to7}$	The coordinatization of Q_1 is irredundant, the homomorphism of level 6 is location preserving[31] and M_1 is in reduced form.

The justifying conditions which pertain to both M_1 and M_2 are interesting because they place limitations on the confidence one can have in making structural inferences form a lumped to a base model. Since the transition structure of the base model is not known, it is impossible to determine whether these conditions do in fact hold. Note however, that the conditions are necessary, in the sense that if they don't hold then the associated inference is unjustified.[32] For example, if any of the subclauses of $J_{6\to7}$ are not satisfied then there are cases in which the lumped model is a homomorphic image of a submachine of the base model, but there are interactions among the components of the lumped model which do not reflect any corresponding

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[30] Same concepts as in footnote 6 except that now d need not be one-one.

[31] This ensures that minimal dependency sets are preserved.

[32] This is a consequence of the proper refinement hierarchy constituted by these relations.

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interactions in the base model. Since the justifying conditions cannot be relaxed without destroying the validity of the inference, they are necessarily a background assumption of every inference from lumped to base model. That is to say, as soon as one attempts to assert that the structure of the model he created (i.e., the lumped model) reflects that of "reality" (i.e., the base model) because the model has been subjected to a variety of behavioral tests in which its behavior compared well with that of the real system, one must necessarily call upon such justifying conditions to justify the assertion. For if such conditions are not satisfied, the assertion may well be false. Now, confidence in the assertion may be bolstered by actually having verified that a justifying condition is satisfied (as is possible when only the lumped model is involved). However, when both lumped and base models are involved there is no other way but to postulate these conditions as assumed axioms. Of course, one may provide grounds for the plausibility of such assumptions but their truth can never be affirmed directly.[33]

Comparison of the Approaches

The essential difference between the discovery and postulation approaches is that in the latter there is a model which has been constructed and is undergoing validation while in the former there is no such *a priori* extant model. In the discovery approach, we have the difficulties previously pointed out in

[33] We should mention one approach to the jump from level 3 to level 4 which has received much attention in the diagnosis of computers but has not yet been expoited in modelling and simulation. This can be explained as follows: The essential difference between level 3 and level 4 knowledge is that at level 3 one has no way of correlating a state of the lumped model with a corresponding state in the base model, while at level 4 this correspondence is known. The approach using homing sequences is to apply a sequence to the real system whose output response can be interpreted as indicating the state that the lumped model should be initialized to, in order that the base and lumped models subsequently begin from corresponding states.

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constructing higher level desriptions from lower level ones. In contrast in the postulational approach, a lumped model at the highest level has already been constructed but the degree to which its structure can be taken to reflect that of the base model stands in need of justification. The advantages of having such a model (actually a base model-lumped model pair) in terms of the ease with which justifying conditions can be checked have been outlined. Moreover, the difficulties encountered in constructing higher level descriptions are entirely obviated.

However, the discussion of the postulational approach has so far not mentioned the problem of selecting a new lumped model once the present one has been disconfirmed. Here, one should more rightly consider the lumped model as a class of models — corresponding to the set of parameters which must be specified to fully define the model. The choice of a new lumped model can then take place by parameter value modification and such an identification process can then proceed until the whole class has been disconfirmed. But what then? Here the discovery approach may be suggestive. Since in it, the I-O data is processed with minimal prior prejudice as to the class of possible models, the realization procedures developed for it may be helpful in suggesting new lumped models to try. Such a two phase strategy has been suggested by Zeitch [68] in another context and merits much further application.

Phrased more broadly the question raised here is that of the convergence properties of inductive inference schemes. Feldman [13] has formalized and studied certain reasonable convergence criteria and has shown that there are inference procedures which meet these criteria. These procedures are essentially model enumeration schemes and thus are not likely to be of a practical nature. The problem may be viewed as that of efficiently searching a space

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(of models) which has been shown in general to require adaptive (non-enumerative) strategies (Holland [22]). The combination of all these lines of research offers exciting possibilities for the develpment of a formal theory of modelling and simulation.

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