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A Conglomerating approach for Model Order Reduction of Continuous Large Scale Systems

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ABSTRACT: In this note, a method is presented for the order reduction of continuous approach for order reduction of complex discrete uncertain systems is proposed. Using Interval arithmetic Routh Stability arrays are formed to obtained numerator and denominator of reduced order model. The developed approach preserves the stability aspect of reduced system if higher order uncertain system is stable. A numerical example is included to illustrate the proposed algorithm along with the comparison with existing techniques

KEYWORDS: Interval system, Inverse distance measure, Kharitonov's polynomials, Model reduction, Factor division, Pole-clustering method.

I. INTRODUCTION

The modelling of the physical system was done to define the characteristics of the model which helps to find out the size, physical behaviour, utility, etc. The required accuracy of the model largely depends on the purpose for which the model is intended. As VLSI technology advances, integrated circuits are designed for reduced sizes and with high performing ability which having consequences to interconnect with the effects that it have an increasing impact on many critical design criteria. Therefore, accurate modelling of the interconnected circuits has become very important. A typical interconnect model usually involves thousands or even millions of components whose direct simulation can stretch the limit of computing resources. We have to reduces the complexity of the interconnected model for better understanding of the system which issues the topic i.e. reduction of the original model to an appropriate model with far lesser variables. It is highly desirable that this approximate model inherit many of the properties of the original system.
Due to several advantages e.g. reduced computational effort in simulation, simplified understanding of system, simpler control laws etc., and model reduction has been ample area of research. Several methods have been proposed for reduction of continuous-time systems (Shamash, 1974; Aoki, 1968; Glover, 1984; Sinha & Kuszta, 1983; Huttan & Friedland, 1975; Shamash, 1975; Rao el al., 1978; Singh el al., 2004; Singh et al., 2004). Among them, Padé approximation method (Shamash, 1974) has found to be very useful in theoretical physics research (Baker & Graves-Morris, 1981; Baker, 1975) due to being computationally simple but the reduced-order model obtained using Padé approximation method often leads to be unstable even though the high-order system is stable. To overcome limitation, many improvements have been proposed (Huttan & Friedland, 1975; Shamash, 1975; Rao el al., 1978; Singh el al., 2004; Singh et al., 2004) in literature. The order reduction of HOISs has also attracted researchers since the pioneering work by Kharitonov (Kharitonov, 1978). Some methods have been presented for order reduction of HOISs (Bandyopadhyay et al., 1994; Dolgin and Zeheb, 2003; Bandyopadhyay et al., 1997; Sastry et al., 2000; Ismail & Bandyopadhyay, 1995)

Among these, Routh-Padé approximation (Bandyopadhyay et al., 1994) has been presented for continuous HOISs in which the numerator is obtained by matching the time moments and the denominator is obtained by direct truncation of the Routh table. However, Dolgin and Zeheb (Dolgin and Zeheb, 2003) showed that a stable interval polynomial may provide an unstable interval polynomial if it is obtained by direct truncation of the Routh table as in (Bandyopadhyay et al., 1994). In (Bandyopadhyay et al., 1997), a ROIM is proposed for model reduction of continuous HOISs using parameters. But this method becomes complex as it requires both and tables for obtaining ROIM. An improvement (Sastry et al., 2000) in which the numerator and denominator of ROIM both are obtained using parameters is suggested over (Bandyopadhyay et al., 1997).
In this paper, a method conglomerate by pole clustering and factor division method for order reduction of continuous HOIS is proposed where the clusters are obtained with grouping of poles from denominator of transfer function of

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HOIS in which the poles of denominator is first calculated then grouped in order of ROIS with the IDM measure. The numerator is obtained by factor division of first r terms of HOIS to those of its ROIM. The briefing of this paper is as follows: section-II holds the procedure for reduction method for pole clustering techniques, section-III contains algorithm for reduction of original uncertain system, section-IV contains numerical example with error indices showing superiority of this method and conclusion is given in section-V

II. REDUCED METHOD

Rules for obtaining the modified pole clustering for HOIS

(i) Separate clusters should be made for real and complex poles.

(ii) Clusters of the poles in the left half s -plane should not contain any pole of the right half s-plane and vice-versa.

(iii) Poles on the jw-axis have to be retained in the reduced order model.

(iv) Poles at the origin have to be retained in the reduced order model.

The brief algorithm for realizing the denominator polynomial an interactive computer oriented algorithm has been developed, which automatically finds the modified cluster center and is give as follows:

Step 1: Let r real poles in a cluster be $|p_1| < |p_2| < \cdots < |p_r|$

Step 2: Set j = 1

Step 3: Find pole cluster center $c_j = \left[\sum_{i=1}^{r} \left(\frac{-1}{|p_i|} \right) \div r \right]^{-1}$

Step 4: Set j =j+1

Step 5: Now find a modified cluster center from $c_j = \left[\left(\frac{-1}{|p_i|} + \frac{-1}{|c_{j-1}|} \right) \div 2 \right]^{-1}$.

Step 6: Is $r = j$? , if No, and then go to Step-4otherwise go to Step-7.

Step 7: Take a modified cluster center of the k^{th}-cluster as. $p_{ek} = c_j$

III. PROBLEM FORMULATION

Consider a stable continuous HOIS given by the transfer function

$$G(s) = \frac{[a_{k-1}^-, a_{k-1}^+]s^{k-1} + [a_{k-2}^-, a_{k-2}^+]s^{k-2} + ... + [a_0^-, a_0^+]}{[b_k^-, b_k^+]s^k + [b_{k-1}^-, b_{k-1}^+]s^{k-1} + ... + [b_0^-, b_0^+]} = \frac{N(s)}{D(s)} \quad (1)$$

Where $[a_i^-, a_i^+]$ for (i=0,1,2,…,n-1) and $[b_i^-, b_i^+]$ for (i=0,1,2,…,n) are the interval parameters for numerator and denominator respectively.

Step 1: The HOIS has four fixed Kharitonov's polynomial for numerators and denominators are

Numerators:

$$N^1(s) = a_0^- + a_1^- + a_2^+ + a_3^+ + \cdots\cdots \quad (2)$$

$$N^2(s) = a_0^+ + a_1^- + a_2^- + a_3^+ + \cdots\cdots \quad (3)$$

$$N^3(s) = a_0^+ + a_1^- + a_2^- + a_3^+ + \cdots\cdots \quad (4)$$

$$N^4(s) = a_0^+ + a_1^+ + a_2^- + a_3^- + \cdots\cdots \quad (5)$$

Denominators:

$$D^1(s) = b_0^- + b_1^- + b_2^+ + b_3^+ + \cdots\cdots \quad (6)$$

$$D^2(s) = b_0^+ + b_1^- + b_2^- + b_3^+ + \cdots\cdots \quad (7)$$

$$D^3(s) = b_0^+ + b_1^- + b_2^- + b_3^+ + \cdots\cdots \quad (8)$$

$$D^4(s) = b_0^+ + b_1^+ + b_2^- + b_3^- + \cdots\cdots \quad (9)$$

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Step 2: Taking all the combination of numerator and denominators forms fixed coefficient HOS as

$$G^{ij}(s)=\frac{a_0^i+a_1^i s+a_2^i s^2+\cdots\cdots+a_{k-1}^i s^{k-1}}{b_0^j+b_1^j s+b_2^j s^2+\cdots+b_{k-1}^j s^{k-1}+b_k^j s^k}=\frac{N^i(s)}{D^j(s)} \tag{10}$$

where (i,j=1,2,3,4)

Step 3: From fixed coefficient of denominator polynomial the poles can be arranged as follows:

$-\lambda_1<-\lambda_2<-\lambda_3<\cdots\cdots<-\lambda_k$ are the poles of higher order system

where ($\lambda_i = p_i + jq_i$)

Step 4: The denominator of reduced order is formed by clustering the poles of the original system given in frequency domain

form m real poles in one cluster group $P=\left\{\sum_{i=1}^{m}\left(\frac{1}{p_i}\right)\div m\right\}^{-1}$ (11)

form m complex poles in one cluster group $P+jQ=\left\{\sum_{i=1}^{m}\left(\frac{1}{p_i}\right)\div m\right\}^{-1}+j\left\{\sum_{i=1}^{m}\left(\frac{1}{q_i}\right)\div m\right\}^{-1}$ (12)

then by obtaining cluster centre, the denominator becomes

for real poles: $D_r(s)=\prod_{i=1}^{r}(s-P_i)$ (13)

for complex conjugate poles: $D_r(s)=\prod_{j=1}^{r/2}[s-(P_j+Q_j)][s-(P_j-Q_j)]$ (14)

And

For real and complex conjugate poles: $D_r(s)=(s-P_1)\prod_{j=2}^{(r+1)/2}[s-(P_j+Q_j)][s-(P_j-Q_j)]$ (5.10) (15)

Step 5: Determination of the numerator polynomial of r^{th} order reduced model by factor division method as:

$$N_r(s)=\frac{N(s)\times D_r(s)}{D(s)} \tag{16}$$

Step 6: Coefficient of reduced order numerator from a modified form of moment generating algorithm as:

$$\begin{aligned}
n_0 &= \left\langle \frac{u_0}{a_0} \middle| \begin{matrix} u_0 & u_1 & \cdots & u_{r-1} \\ a_0 & a_1 & \cdots & a_{r-1}\end{matrix}\right. \\
n_1 &= \left\langle \frac{v_0}{a_0} \middle| \begin{matrix} v_0 & v_1 & \cdots & v_{r-2} \\ a_0 & a_1 & \cdots & a_{r-2}\end{matrix}\right. \\
n_2 &= \left\langle \frac{w_0}{a_0} \middle| \begin{matrix} w_0 & w_1 & \cdots & w_{r-3} \\ a_0 & a_1 & \cdots & a_{r-3}\end{matrix}\right. \\
\vdots & \quad \vdots \quad\quad \vdots \quad\quad \vdots
\end{aligned} \tag{17}$$

where

$$v_i = u_{i+1}-\mu_0 a_{i+1}$$
$$w_i = v_{i+1}-\mu_1 a_{i+1}$$
$$\vdots \quad \vdots \quad \vdots$$

Step 7: Reduced transfer function for fixed coefficient is obtained as:

$$G^{ij}(s)=\frac{n_0^i+n_1^i s+n_2^i s^2+\cdots\cdots+n_{r-1}^i s^{r-1}}{d_0^j+d_1^j s+d_2^j s^2+\cdots+d_{r-1}^j s^{n-1}+d_r^j s^r} \tag{18}$$

where (i,j=1,2,3,4)

Step 3: Combining the Reduced transfer function of fixed coefficient for stable ROIS is obtained as:

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$$G_r(s)=\frac{[a_{r-1}^-,a_{r-1}^+]s^{r-1}+[a_{r-2}^-,a_{r-2}^+]s^{r-2}+....+[a_0^-,a_0^+]}{[b_r^-,b_r^+]s^r+[b_{r-1}^-,b_{r-1}^+]s^{r-1}+...+[b_0^-,b_0^+]}=\frac{N_r(s)}{D_r(s)} \qquad (19)$$

Where $[a_i^-,a_i^+]$ for (i=0,1,2,…,r-1) and $[b_i^-,b_i^+]$ for (i=0,1,2,…,r) are the interval parameters for numerator and denominator respectively.

IV. NUMERICAL ILLUSTRATION

Consider the linear structured HOIS [5].

$$G_3(s)=\frac{[2,3]s^2+[17.5,18.5]s+[15,16]}{[2,3]s^3+[17,18]s^2+[35,36]s+[20.5,21.5]} \qquad (20)$$

It is required to reduce the system such that it follows the similar behaviour as the original system does So as the second order uncertain reduced model closely follows the original HOIS.
From the Kharitonov's theorem numerator and denominator of HOIS, Four transfer functions obtained as:

$$G^{11}(s)=\frac{3s^2+17.5s+15}{3s^3+18s^2+35s+20.5} \qquad (21)$$

$$G^{22}(s)=\frac{3s^2+18.5s+15}{2s^3+18s^2+36s+20.5} \qquad (22)$$

$$G^{33}(s)=\frac{2s^2+17.5s+16}{3s^3+17s^2+35s+21.5]} \qquad (23)$$

$$G^{44}(s)=\frac{2s^2+18.5s+16}{2s^3+17s^2+36s+21.5} \qquad (24)$$

The proposed model order reduction method is applied for the system interval functions $G^{11}(s)$, $G^{22}(s)$, $G^{33}(s)$ & $G^{44}(s)$.From their corresponding reduced order interval systems, the reduced order interval systems of given higher order system can be constituted.
By applying the proposed model order reduction technique, the following reduced order models are obtained.

$$R^{11}(s)=\frac{1.5641s+1.9756}{s^2+3.5975s+2.7} \qquad (25)$$

$$R^{22}(s)=\frac{1.3594s+1.4414}{s^2+2.8878s+1.97} \qquad (26)$$

$$R^{33}(s)=\frac{1.6336s+2.0435}{s^2+3.662s+2.7459} \qquad (27)$$

$$R^{44}(s)=\frac{1.5083s+1.7079}{s^2+3.2162s+2.295} \qquad (28)$$

From the obtained reduced order fixed coefficient transfer function we applying inverse Kharitonov's theorem to get the ROIS as:

$$R_2(s)=\frac{[1.5083,1.5641]s+[1.4414,2.0435]}{s^2+[3.2162,3.5975]s+[1.97,2.7459]} \qquad (29)$$

The comparison of the second order reduced models with original system G(s) are shown in error index i.e. Integral Square Error in which the difference of step response for original system and reduced systems are calculated in the Table I as:

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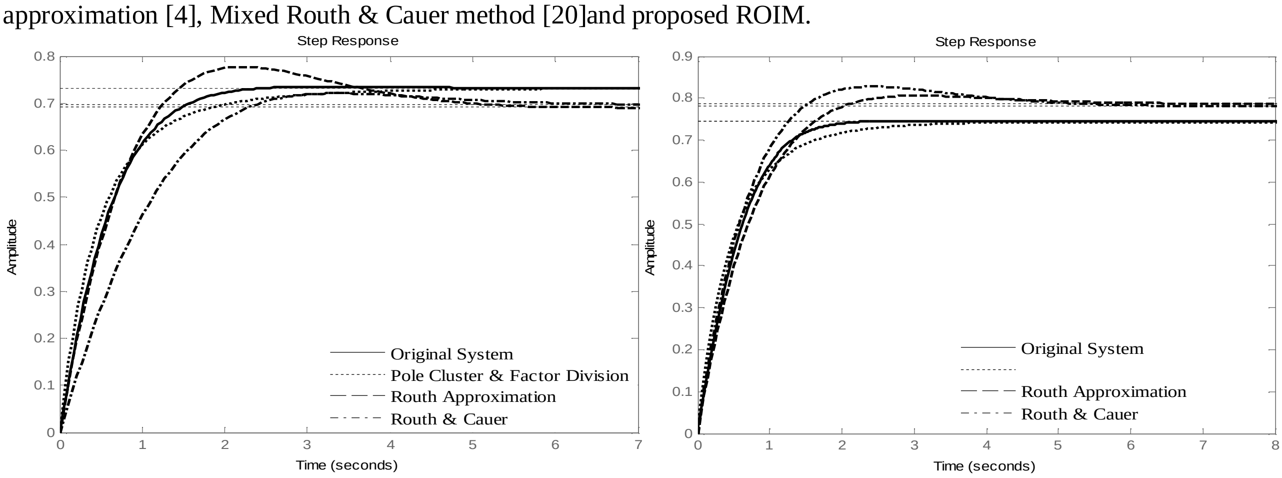
TABLE I. COMPARISON OF ISE FOR REDUCED ORDER MODELS

Model Order Reduction	Reduced Models	ISE (lower)	ISE (upper)
Pole Clustering with Factor Division	$G_2(s)=\frac{[1.5083,1.5641]s+[1.4414,2.0435]}{s^2+[3.2162,3.5975]s+[1.97,2.7459]}$	0.0022	0.0021
Routh Approximation Method [4]	$G_2(s)=\frac{[17.5,18.5]s+[14.1892,16.9143]}{[17,18]s^2+[29.4722,16.9143]s+[20.5,21.5]}$	0.0075	0.0146
Routh&Cauer Method [20]	$G_2(s)=\frac{[9.9631,21.9496]s+[14.3026,16.7811]}{[17,18]s^2+[29.4722,35.7059]s+[20.5,21.5]}$	0.0332	0.0216

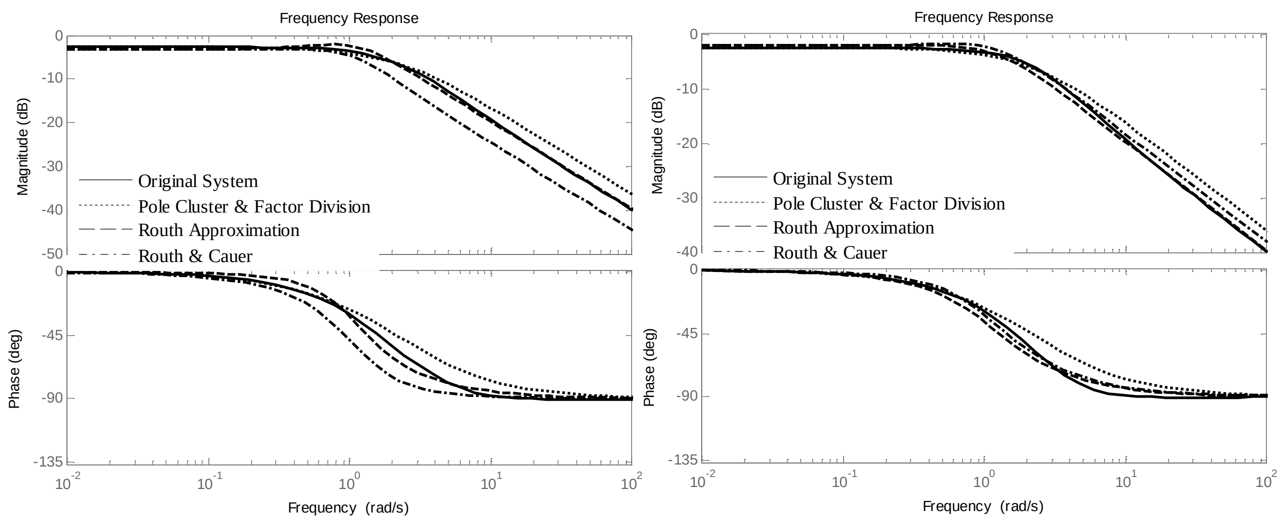
The step responses for the various reduced system are constructed for the lower and upper bound along with original system are shown in Figure 1(a) & 1(b) which having numerator and denominator of HOIS, ROIM in Routh approximation [4], Mixed Routh & Cauer method [20]and proposed ROIM.



(a) (b)

Fig 1. Step response of Original and reduced systems containing proposed and existing techniques (a) Lower bound & (b) Upper bound.

It is clear from Figure 1(a) and 1(b) that the overall time response obtained by proposed method is better than that of existing [4, 20].



(a) (b)

Fig 2. Frequency response of Original and reduced systems containing proposed and existing techniques (a) Lower bound & (b) Upper bound

The frequency responses for different reduced system with the original system are appeared in Figure 2(a) & 2(b). And from Figure 2(a) & 2(b) that the overall frequency responses were occurred to be better. Hence, this confirms the applicability of proposed method to obtain ROIM for continuous HOISs.

V. CONCLUSION

In this method, the denominator polynomial is determined by using pole-clustering algorithm in which clustering of poles is done using IDM criterion while the coefficients of the numerator are obtained by factor division approximations. This method has been tested with numerical example chosen from the literature. Time responses of the original and reduced systems are compared as shown in Figure 1(a) & 1(b). Frequency responses of the original and reduced systems are shown in Figure 2(a) & 2(b).From these comparisons, it has been concluded that the algorithm of the proposed method is simple, efficient and computer oriented. This method is capable of making transient as well as steady state region of the original system. The proposed method is compared with some well-known order reduction methods by using performance indices.

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