# A Consensus Model for Group Decision Making with Incomplete Fuzzy Preference Relations

E. Herrera-Viedma, S. Alonso, F. Chiclana, and F. Herrera

***Abstract*— Two processes are necessary to solve group decision making problems: a consensus process and a selection process. The consensus reaching process is necessary to obtain a final solution with a certain level of agreement between the experts; and the selection process is necessary to obtain such a final solution. In [19], we present a selection process to deal with group decision making problems with incomplete fuzzy preference relations, which uses consistency measures to estimate the incomplete fuzzy preference relations. In this paper we present a consensus model. The main novelty of this consensus model is that of being guided by both consensus and consistency measures. Also, the consensus reaching process is guided automatically, without moderator, through both consensus and consistency criteria. To do that, a feedback mechanism is developed to generate advice on how experts should change or complete their preferences in order to reach a solution with high consensus and consistency degrees. In each consensus round, experts are given information on how to change their preferences, and to estimate missing values if their corresponding preference relation is incomplete. Additionally, a consensus and consistency based induced ordered weighted averaging operator to aggregate the experts' preferences is introduced, which can be used in consensus models as well as in selection processes. The main improvements of this consensus model is that it supports the management of incomplete information and it allows to achieve consistent solutions with a great level of agreement.**

***Index Terms*— Group Decision Making, Fuzzy Preference Relations, Consensus, Aggregation.**

## I. INTRODUCTION

Group decision making (GDM) problems consist in finding the best alternative(s) from a set of feasible alternatives $X = \{x_1, ..., x_n\}$ according to the preferences provided by a group of experts $E = \{e_1, ..., e_m\}$. Due to their apparent merits when aggregating experts' preferences into group preferences [20], [22], [38], we assume that experts provide fuzzy preference relations [6], [14], [22], [26], [32], [36].

A difficulty that has to be addressed when dealing with real GDM problems is the lack of information. Indeed, there may be cases where an expert would not be able to efficiently express any kind of preference degree between two or more of the available options. This may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, or because that expert is unable to discriminate the degree to which some options are better than others. Experts in these situations would rather not guess those preference degrees and as a consequence they might provide incomplete information [1], [9], [19], [27], [28], [40].

Usually, GDM problems are faced by applying two different processes before a final solution can be given [5], [16], [18], [24]: 1) *the consensus process* and 2) *the selection process*. The consensus process refers to how to obtain the maximum degree of consensus or agreement between the set of experts. Usually, the consensus process is guided by a human figure called moderator [16], [23], [24]. The selection process obtains the final solution according to the preferences given by the experts. It involves two different steps [17], [33]: aggregation of individual preferences and exploitation of the collective preference. Clearly, it is preferable that the experts had achieved a high level of consensus concerning their preferences before applying the selection process.

In [1], [19] we introduce a selection process to deal with the GDM problems with incomplete fuzzy preference relations. In this selection process we present a consistency based procedure which is able to estimate all missing values from the known preferences. In this paper, we focus on the consensus process. In the literature, we can find many approaches to model the consensus processes in GDM [3]–[5], [7], [10], [11], [16], [18], [23]–[25], [29], [35], [37], [46]. Most of these approaches use only consensus measures to control and guide the consensus process. If a consensus process is seen as a type of persuasion model [8], then other criteria could be used to guide the consensus reaching processes as it could be, for example, the cooperation or consistency criterion. A first approach to consensus using a consistency criterion can be found in [12], although preference relations were assumed to be complete. Also, in the context of the analytical hierarchy process (AHP) [34], consistency has been used in GDM [2], [39].

The aim of this paper is to present a consensus model for GDM problems with incomplete fuzzy preference relations. This consensus model will not only be based on consensus measures but also on consistency measures. As in [16], we use two kinds of consensus measures to guide the consensus reaching processes, consensus degrees (to evaluate the agreement of all the experts) and proximity degrees (to evaluate the agreement between the experts' individual preferences and the group preference). To compute them, firstly, all missing values of the incomplete fuzzy preference relations are estimated using the consistency based estimation procedure presented in [19]. Afterwards, some consistency measures for each expert are computed. Both consensus measures and consistency mea-

Enrique Herrera-Viedma, Sergio Alonso and Francisco Herrera are with the Department of Computer Science and Artificial Intelligence, University of Granada, 18071 Granada, Spain (email: viedma@decsai.ugr.es; herrera@decsai.ugr.es; salonso@decsai.ugr.es).

Francisco Chiclana is with the Centre for Computational Intelligence, School of Computing, De Montfort University, Leicester LE1 9BH, UK (email: chiclana@dmu.ac.uk).

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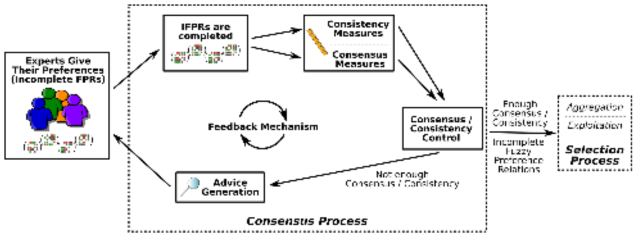


Fig. 1. Consensus Model for GDM with Incomplete Information

sures are used to design a feedback mechanism that generates advice to the experts on how they should change and complete their fuzzy preference relations to obtain a solution with a high consensus degree (making experts' opinions closer), but also maintaining a certain consistency level on their fuzzy preference relations (avoiding self contradiction). This feedback mechanism is able to substitute the actions of the moderator. Figure 1 depicts this consensus model. The experts provide their preferences by means of incomplete fuzzy preference relations. After the fuzzy preference relations are completed, the system computes consistency and consensus measures. If they satisfy a condition based on a consensus and consistency threshold value, then the selection process to obtain the final solution of the problem is applied; otherwise, the system will generate advice to the experts to help them make their opinions closer, more consistent and complete. Additionally, we also introduce an Induced Ordered Weighted Averaging (IOWA) operator [31], [42]–[44] to aggregate the experts' preferences in the whole decision process which uses both consensus and consistency criteria as inducing variable. The main novelty of this consensus model is that it supports the management of incomplete information allowing to achieve consistent solutions with a high consensus degree.

This paper is set out as follows. Section II deals with the preliminaries necessary to develop our consensus model. Section III introduces the consensus model for GDM problems with incomplete fuzzy preference relations. Finally, in Section IV we draw our conclusions.

## II. Preliminaries

In this section, we briefly present the tools necessary to design the consensus model, that is, the concept of incomplete fuzzy preference relation, consistency measures, and the consistency based procedure to estimate missing values.

### A. Incomplete Fuzzy Preference Relations

Among the different representation formats that experts may use to express their opinions, fuzzy preference relations [6], [14], [22], [26], [32], [36] are one of the most used because of their effectiveness as a tool for modelling decision processes and their utility and easiness of use when we want to aggregate experts' preferences into group ones [20], [22], [38].

*Definition 1:* A fuzzy preference relation $P$ on a set of alternatives $X$ is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P \colon X \times X \longrightarrow [0,1]$.

When cardinality of $X$ is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ik})$, being $p_{ik} = \mu_P(x_i, x_k)$ $(\forall i, k \in \{1, \dots, n\})$ interpreted as the preference degree or intensity of the alternative $x_i$ over $x_k$: $p_{ik} = 1/2$ indicates indifference between $x_i$ and $x_k$ $(x_i \sim x_k)$, $p_{ik} = 1$ indicates that $x_i$ is absolutely preferred to $x_k$, and $p_{ik} > 1/2$ indicates that $x_i$ is preferred to $x_k$ $(x_i \succ x_k)$. Based on this interpretation we have that $p_{ii} = 1/2 \;\; \forall i \in \{1, \dots, n\}$ $(x_i \sim x_i)$.

It has been common practice in research to model GDM problems in which all the experts are able to provide all the required preference values, that is, to provide all $p_{ik}$ values. This situation is not always possible to achieve. Experts could have some difficulties in giving all their preferences due to lack of knowledge about part of the problem, or simply because they may not be able to quantify some of their degrees of preference. In order to model such situations, we define the concept of an *incomplete fuzzy preference relation* [19].

*Definition 2:* A function $f \colon X \longrightarrow Y$ is partial when not every element in the set $X$ necessarily maps onto an element in the set $Y$. When every element from the set $X$ maps onto one element of the set $Y$ then we have a total function.

*Definition 3:* An incomplete fuzzy preference relation $P$ on a set of alternatives $X$ is a fuzzy set on the product set $X \times X$ that is characterized by a partial membership function.

### B. Consistency Measures

In real GDM problems with fuzzy preference relations some properties about the preferences expressed by the experts are usually assumed desirable to avoid contradictions in their opinions, that is, to avoid inconsistent opinions. One of these properties is the *transitivity property*, which represents the idea that the preference value obtained by directly comparing two alternatives should be equal to or greater than the preference value between those two alternatives obtained using an indirect chain of alternatives. There are several possible characterizations for the transitivity property (see [20]). In this paper, we make use of the *additive transitivity property*. The mathematical formulation of the *additive transitivity* was given by Tanino in [38]:

$$(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5) \quad \forall i, j, k \in \{1, \dots, n\} \tag{1}$$

The underlying concept on which the additive transitivity property is based has been applied in both Saaty's AHP [34] and Fishburn SSB Utility Theory [13]. In the first case, as shown in [20], additive transitivity for fuzzy preference relations can be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations. In the second case, as shown in [12] if we represent the degree of preference of $x_i$ over $x_j$ by means of a Skew-Symmetric Bilinear function $\phi(x_i, x_j) \in R$ the consistency condition can be stated as

$$\phi(x_i, x_j) + \phi(x_j, x_k) = \phi(x_i, x_k)$$

which corresponds to expression (1), taking into account that Fishburn represents indifference with the value of 0. We acknowledge that additive transitivity is a condition difficult to be satisfied by experts' preferences. However, as shown in [20], [30] additive transitivity can be used to obtain more consistent fuzzy preference relation from a given one, and as shown in [1], [19] it is also a valuable concept for incomplete fuzzy preference relations as it reduces experts' uncertainty when choosing values to estimate their unknown ones, which is not the case if other types of weaker transitivity conditions were to be used.

Additive transitivity implies additive reciprocity. Indeed, because $p_{ii} = 0.5 \ \forall i$, if we make $k = i$ in equation (1) then we have: $p_{ij} + p_{ji} = 1 \ \ \forall i, j \in \{1, \ldots, n\}$. Then, equation (1) can be rewritten as:

$$p_{ik} = p_{ij} + p_{jk} - 0.5 \ \ \forall i, j, k \in \{1, \ldots, n\} \tag{2}$$

We will consider a fuzzy preference relation to be "*additive consistent*" when for every three options in the problem $x_i, x_j, x_k \in X$ their associated preference degrees $p_{ij}, p_{jk}, p_{ik}$ fulfil (2).

Expression (2) can be used to calculate an estimated value of a preference degree using other preference degrees in a fuzzy preference relation. Indeed, the preference value $p_{ik}$ $(i \neq k)$ can be estimated using an intermediate alternative $x_j$ in three different ways:

1) From $p_{ik} = p_{ij} + p_{jk} - 0.5$ we obtain the estimate
$$(cp_{ik})^{j1} = p_{ij} + p_{jk} - 0.5 \tag{3}$$
2) From $p_{jk} = p_{ji} + p_{ik} - 0.5$ we obtain the estimate
$$(cp_{ik})^{j2} = p_{jk} - p_{ji} + 0.5 \tag{4}$$
3) From $p_{ij} = p_{ik} + p_{kj} - 0.5$ we obtain the estimate
$$(cp_{ik})^{j3} = p_{ij} - p_{kj} + 0.5 \tag{5}$$

The overall estimated value $cp_{ik}$ of $p_{ik}$ is obtained as the average of all possible $(cp_{ik})^{j1}$, $(cp_{ik})^{j2}$ and $(cp_{ik})^{j3}$ values:

$$cp_{ik} = \frac{\sum_{j=1; i\neq k\neq j}^{n} (cp_{ik})^{j1} + (cp_{ik})^{j2} + (cp_{ik})^{j3}}{3(n-2)} \tag{6}$$

When the information provided is completely consistent then $(cp_{ik})^{jl} = p_{ik} \ \forall j, l$. However, because experts are not always fully consistent, the information given by an expert may not verify (2) and some of the estimated preference degree values $(cp_{ik})^{jl}$ may not belong to the unit interval $[0, 1]$. We note, from expressions (3–5), that the maximum value of any of the preference degrees $(cp_{ik})^{jl}$ $(l \in \{1, 2, 3\})$ is 1.5 while the minimum one is -0.5. Taking this into account, we define the error between a preference value and its estimated one as follows:

*Definition 4:* The error between a preference value and its estimated one in $[0, 1]$ is computed as:

$$\varepsilon p_{ik} = \frac{2}{3} \cdot |cp_{ik} - p_{ik}| \tag{7}$$

Thus, it can be used to define the consistency level between the preference degree $p_{ik}$ and the rest of the preference values of the fuzzy preference relation.

*Definition 5:* The consistency level associated to a preference value $p_{ik}$ is defined as

$$cl_{ik} = 1 - \varepsilon p_{ik} \tag{8}$$

When $cl_{ik} = 1$ then $\varepsilon p_{ik} = 0$ and there is no inconsistency at all. The lower the value of $cl_{ik}$, the higher the value of $\varepsilon p_{ik}$ and the more inconsistent is $p_{ik}$ with respect to the rest of information.

Easily, we can define the consistency measures for particular alternatives and for the whole fuzzy preference relation:

*Definition 6:* The consistency measure associated to a particular alternative $x_i$ of a fuzzy preference relation $P$ is defined as

$$cl_i = \frac{\sum_{\substack{k=1 \\ i\neq k}}^{n} (cl_{ik} + cl_{ki})}{2(n-1)} \tag{9}$$

with $cl_i \in [0, 1]$.

When $cl_i = 1$ all the preference values involving the alternative $x_i$ are fully consistent, otherwise, the lower $cl_i$ the more inconsistent these preference values are.

*Definition 7:* The consistency level of a fuzzy preference relation $P$ is defined as follows:

$$cl = \frac{\sum_{i=1}^{n} cl_i}{n} \tag{10}$$

with $cl \in [0, 1]$.

When $cl = 1$ the preference relation $P$ is fully consistent, otherwise, the lower $cl$ the more inconsistent $P$.

*Example 1:* Suppose the following complete fuzzy preference relation

$$P = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ 0.8 & - & 0.9 & 0.7 \\ 0.4 & 0.1 & - & 0.3 \\ 0.6 & 0.3 & 0.7 & - \end{pmatrix}$$

The computation of the consistency level of the preference value $p_{43}$ is as follows:

$$\begin{aligned}
(cp_{43})^{11} &= p_{41} + p_{13} - 0.5 = 0.6 + 0.6 - 0.5 = 0.7 \\
(cp_{43})^{21} &= p_{42} + p_{23} - 0.5 = 0.3 + 0.9 - 0.5 = 0.7 \\
(cp_{43})^{12} &= p_{13} - p_{14} + 0.5 = 0.6 - 0.4 + 0.5 = 0.7 \\
(cp_{43})^{22} &= p_{23} - p_{24} + 0.5 = 0.9 - 0.7 + 0.5 = 0.7 \\
(cp_{43})^{13} &= p_{41} - p_{31} + 0.5 = 0.6 - 0.4 + 0.5 = 0.7 \\
(cp_{43})^{23} &= p_{42} - p_{32} + 0.5 = 0.3 - 0.1 + 0.5 = 0.7
\end{aligned}$$

$$cp_{43} = 0.7 \Rightarrow \varepsilon p_{43} = \tfrac{2}{3} \cdot |cp_{43} - p_{43}| = 0 \Rightarrow \\ cl_{43} = 1 - \varepsilon p_{43} = 1$$

The same consistency value 1 is obtained for all the preference values of this fuzzy preference relation, which means that it is a completely additive consistent fuzzy preference relation.

When working with an incomplete fuzzy preference relation, expression (6) cannot be used to obtain the estimate of a known preference value.

If expert $e_h$ provides an incomplete fuzzy preference relation $P^h$, the following sets can be defined [19]:

$$\begin{array}{l} A = \{(i,j) \mid i,j \in \{1,\ldots,n\} \wedge i \neq j\} \\ MV^h = \left\{(i,j) \in A \mid p^h_{ij} \text{ } is\ unknown\right\} \\ EV^h = A \setminus MV_h \\ H^{h1}_{ik} = \left\{j \neq i,k \mid (i,j),(j,k) \in EV^h\right\} \\ H^{h2}_{ik} = \left\{j \neq i,k \mid (j,i),(j,k) \in EV^h\right\} \\ H^{h3}_{ik} = \left\{j \neq i,k \mid (i,j),(k,j) \in EV^h\right\} \\ EV^h_i = \{(a,b) \mid (a,b) \in EV^h \ \wedge \ (a = i \vee b = i)\} \end{array}$$

$MV^h$ is the set of pairs of alternatives whose preference degrees are not given by expert $e_h$, $EV^h$ is the set of pairs of alternatives whose preference degrees are given by the expert $e_h$; $H^{h1}_{ik}$, $H^{h2}_{ik}$, $H^{h3}_{ik}$ are the sets of intermediate alternative $x_j$ $(j \neq i,k)$ that can be used to estimate the preference value $p^h_{ik}$ $(i \neq k)$ using equations (3), (4), (5) respectively; and $EV^h_i$ is the set of pairs of alternatives whose preference degrees involving the alternative $x_i$ are given by the expert $e_h$. Then, the estimated value of a particular preference degree $p^h_{ik}$ $\left((i,k) \in EV^h\right)$ can be calculated as follows [19]:

$$if\ (\#H^{h1}_{ik} + \#H^{h2}_{ik} + \#H^{h3}_{ik}) \neq 0 \quad \Rightarrow \quad cp^h_{ik} = \frac{\sum_{j \in H^{h1}_{ik}} (cp^h_{ik})^{j1} + \sum_{j \in H^{h2}_{ik}} (cp^h_{ik})^{j2} + \sum_{j \in H^{h3}_{ik}} (cp^h_{ik})^{j3}}{(\#H^{h1}_{ik} + \#H^{h2}_{ik} + \#H^{h3}_{ik})} \tag{11}$$

In decision-making situations with incomplete information, the notion of completeness is also an important factor to take into account when analyzing the consistency. Clearly, the higher the number of preference values provided by an expert the higher the chance of inconsistency [19]. So, we can define the consistency level associated to a preference value in a incomplete fuzzy preference relation as follows:

*Definition 8 ([19]):* The consistency level $cl^h_{ik}$ associated to a preference value $p^h_{ik},\ (i,k) \in EV^h$, is defined as

$$cl^h_{ik} = (1 - \alpha^h_{ik}) \cdot (1 - \varepsilon p^h_{ik}) + \alpha^h_{ik} \cdot \frac{C^h_i + C^h_k}{2} \quad ; \ \alpha^h_{ik} \in [0,1] \tag{12}$$

where $C^h_i$ is the completeness level of the alternative $x_i$ according to the preferences provided by the expert $e_h$ which is defined as the ratio between the actual number of preference values known for $x_i$, $\#EV^h_i$, and the total number of possible preference values in which $x_i$ is involved with a different alternative, $2(n-1)$, i.e., $C^h_i = \dfrac{\#EV^h_i}{2(n-1)}$; and $\alpha^h_{ik}$ a parameter to control the influence of completeness in the evaluation of the consistency levels for $e_h$ defined as

$$\alpha^h_{ik} = 1 - \frac{\#EV^h_i + \#EV^h_k - \#(EV^h_i \cap EV^h_k)}{4(n-1) - 2} \tag{13}$$

*Remark 1:* Note that $\alpha^h_{ik}$ decreases with respect to the number of known preference values. In such a way, $\alpha^h_{ik} = 0$ if all possible preference values between $x_i$ and $x_k$ are known, in which case the completeness concept lacks any meaning, and $\alpha^h_{ik} = 1$ if no values are known.

Clearly, expression (12) is an extension of expression (8), because when $P$ is complete both $EV$ and $A$ coincide and $\alpha_{ik} = 0\ \forall i,k$.

### *C. Estimation Procedure of Missing Values for Incomplete Fuzzy Preference Relations*

As we have already mentioned, missing information is a problem that has to be addressed because experts are not always able to provide preference degrees between every pair of possible alternatives. Therefore, it is necessary to estimate the missing values before the application of a consensus model or a selection model. To do that, we use the estimation procedure of missing values for incomplete fuzzy preference relations developed in [19]. This procedure estimates missing information in an expert's incomplete fuzzy preference relation using only the preference values provided by that particular expert. It is an iterative procedure that is designed using the expression (11). The procedure estimates missing values by means of two different tasks:

1) *Establish the elements that can be estimated in each step of the procedure*
Given an incomplete fuzzy preference relation $P^h$, the subset of missing values $MV^h$ that can be estimated in step $t$ is denoted by $EMV^h_t$ and defined as follows:

$$EMV^h_t = \{ \ (i,k) \in MV^h \setminus \bigcup_{l=0}^{t-1} EMV^h_l \mid i \neq k \ \wedge \ \exists j \in \{H^{h1}_{ik} \cup H^{h2}_{ik} \cup H^{h3}_{ik}\} \ \} \tag{14}$$

and $EMV^h_0 = \emptyset$ (by definition). When $EMV^h_{maxIter} = \emptyset$ with $maxIter > 0$ the procedure will stop as there will not be any more missing values to be estimated. Moreover, if $\bigcup_{l=0}^{maxIter} EMV^h_l = MV^h$ then all missing values are estimated, and consequently, the procedure is said to be successful in the completion of the incomplete fuzzy preference relation.

2) *Estimate a particular missing value*
In order to estimate a particular value $p^h_{ik}$ with $(i,k) \in EMV^h_t$, the following function $estimate\_p(h,i,k)$ is used

```
function estimate_p(h,i,k)
a) (cp^h_ik)^1 = 0, (cp^h_ik)^2 = 0, (cp^h_ik)^3 = 0
b) if #H^h1_ik ≠ 0 then (cp^h_ik)^1 = Σ_{j∈H^h1_ik} (cp^h_ik)^j1
c) if #H^h2_ik ≠ 0 then (cp^h_ik)^2 = Σ_{j∈H^h2_ik} (cp^h_ik)^j2
d) if #H^h3_ik ≠ 0 then (cp^h_ik)^3 = Σ_{j∈H^h3_ik} (cp^h_ik)^j3
e) Calculate cp^h_ik = ((cp^h_ik)^1 + (cp^h_ik)^2 + (cp^h_ik)^3) / (#H^h1_ik + #H^h2_ik + #H^h3_ik)
end function
```

Then, the complete iterative estimation procedure is the following

ITERATIVE ESTIMATION PROCEDURE

```
0. EMV_0^h = ∅
1. t = 1
2. while EMV_t^h ≠ ∅ {
3.     for every (i,k) ∈ EMV_t^h {
4.         estimate_p(h,i,k)
5.     }
6.     t++
7. }
```

*Example 2:* Suppose the following incomplete fuzzy preference relation

$$P^1 = \begin{pmatrix} - & \mathbf{0.2} & \mathbf{0.6} & \mathbf{0.4} \\ x & - & x & x \\ x & x & - & x \\ x & x & x & - \end{pmatrix}$$

The application of the estimation procedure is provided:

**Step 1:** The set of elements that can be estimated are:

$$EMV_1^1 = \{(2,3),(2,4),(3,2),(3,4),(4,2),(4,3)\}$$

After these elements have been estimated, we have:

$$P^1 = \begin{pmatrix} - & \mathbf{0.2} & \mathbf{0.6} & \mathbf{0.4} \\ x & - & 0.9 & 0.7 \\ x & 0.1 & - & 0.3 \\ x & 0.3 & 0.7 & - \end{pmatrix}$$

As an example, to estimate $p_{43}^1$ the procedure is as follows:

$$\begin{array}{lll} H_{43}^{11} = \emptyset & \Rightarrow & (cp_{43}^1)^1 = 0 \\ H_{43}^{12} = \{1\} & \Rightarrow & (cp_{43}^1)^{12} = p_{13}^1 - p_{14}^1 + 0.5 = \\ & & 0.6 - 0.4 + 0.5 = 0.7 \Rightarrow (cp_{43}^1)^2 = 0.7 \\ H_{43}^{23} = \emptyset & \Rightarrow & (cp_{43}^1)^3 = 0 \end{array}$$

$$cp_{43}^1 = \frac{0 + 0.7 + 0}{1} = 0.7$$

**Step 2:** The set of elements that can be estimated are: $EMV_2^1 = \{(2,1),(3,1),(4,1)\}$

After these elements have been estimated, we have the following completed fuzzy preference relation:

$$P^1 = \begin{pmatrix} - & \mathbf{0.2} & \mathbf{0.6} & \mathbf{0.4} \\ 0.8 & - & 0.9 & 0.7 \\ 0.4 & 0.1 & - & 0.3 \\ 0.6 & 0.3 & 0.7 & - \end{pmatrix}$$

As an example, to estimate $p_{41}$ the procedure is as follows:

$$\begin{array}{lll} H_{41}^{21} = \emptyset & \Rightarrow & (cp_{41}^1)^1 = 0 \\ H_{41}^{22} = \emptyset & \Rightarrow & (cp_{41}^1)^2 = 0 \\ H_{41}^{23} = \{2,3\} & \Rightarrow & \left\{ \begin{array}{l} (cp_{41}^1)^{23} = p_{42}^1 - p_{12}^1 + 0.5 = 0.6 \\ (cp_{41}^1)^{33} = p_{43}^1 - p_{13}^1 + 0.5 = 0.6 \end{array} \right\} \\ & \Rightarrow & (cp_{41}^1)^3 = 1.2 \end{array}$$

$$cp_{41}^1 = \frac{0 + 0 + 1.2}{2} = 0.6$$

*Remark 2:* We should point out that although the estimation procedure of missing values is based on the additive consistency property, this does not mean that a fuzzy preference relation emerging from its application is necessarily additive consistent.

## III. A Consensus Model for GDM with Incomplete Preference Relations

A consensus process can be viewed as an iterative process with several consensus rounds, in which the experts accept to change their preferences following the advice given by a moderator. The moderator knows the agreement at each moment of the consensus process by means of the computation of some consensus measures. As aforementioned, consensus measures are used to guide and control most of the consensus models developed up to now [3]–[5], [7], [10], [11], [16], [18], [23]–[25], [29], [35], [37], [46].

To solve GDM problems with incomplete fuzzy preference relations, firstly it is necessary to deal with the missing values [27], [28], [40]. The previous consistency based procedure of missing values allows us to measure the consistency levels of each expert. This consistency information is used in this section to propose a consensus model based not only on consensus criteria but also on consistency criteria. We consider that both criteria are important to guide the consensus process in an incomplete decision framework. In such a way, we get that experts change their opinions toward agreement positions in a consistent way, which is desirable to achieve consistent and consensus solutions. In [12] an additive consistency based consensus model was proposed, although in the context of complete fuzzy preference relations.

The proposed consensus model is designed with the aim of obtaining the maximum possible consensus level while trying to achieve a high level of consistency in experts' preferences. Thus, we try to maintain a balance between both. Moreover, we not only achieve a solution with certain consensus and consistency degrees simultaneously, but also we get to deal with incomplete fuzzy preference relations, giving personalised advice to the experts on how to complete them.

In GDM situations, the search for consistency often could lead to a reduction of the level of consensus, and viceversa. Therefore, whether to proceed from consistency to consensus or viceversa is a matter that has to be addressed. We have decided to proceed from consistency to consensus because in GDM situations consensus between experts is usually searched using the basic rationality principles that each expert presents. To simulate this, the consistency criteria is first applied in our model to fix the rationality of each expert and afterwards it searches to meet experts' preferences to reach consensus. If we were to secure consensus and only thereafter consistency, we could destroy the consensus in favour of the individual consistency and the main aim of our process, which is consensus, would be distorted.

Figure 2 depicts this consensus model. We assume that experts provide their opinions on a set of alternatives by means of incomplete fuzzy preference relations. These are completed by using the above estimation procedure. Later, consistency and consensus measures are computed from the completed fuzzy preference relations. These measures are used in a consistency/consensus control step to determine if an appropriate consistency/consensus level has been reached. If so, the consensus reaching process finishes and a selection process is applied to obtain the solution. Otherwise, the

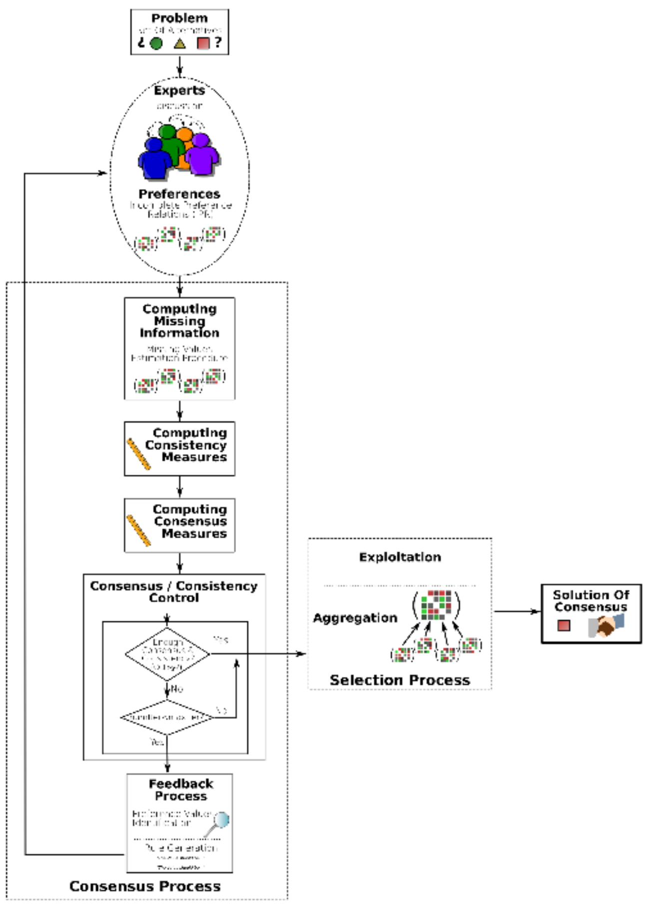


Fig. 2. Consensus Model Based on Consistency and Consensus Criteria

consensus reaching process activates a feedback mechanism, where the preference values which are not contributing to obtain a high consensus/consistency level are detected and some easy rules about how to alter them are generated to help the experts to change and complete their opinions.

The steps of this consensus model are the following:

1) Computing Missing Information
2) Computing Consistency Measures
3) Computing Consensus Measures
4) Controlling the Consistency/Consensus State
5) Feedback Mechanism

They are presented in detail in the following subsections, along with a step-by-step example which illustrates the computations that are being carried out. For the sake of simplicity, we will assume a low number of experts and alternatives.

*Example 3:* Let us suppose that four different experts $\{e_1, e_2, e_3, e_4\}$ provide the following incomplete fuzzy preference relations over a set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$:

$$P^1 = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ x & - & x & x \\ x & x & - & x \\ x & x & x & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & x & 0.7 & x \\ 0.4 & - & x & 0.7 \\ 0.3 & x & - & x \\ x & 0.4 & x & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & 0.3 & x & 0.75 \\ 0.6 & - & x & x \\ x & x & - & x \\ 0.3 & 0.4 & x & - \end{pmatrix}$$

$$P^4 = \begin{pmatrix} - & x & 0.6 & 0.3 \\ 0.4 & - & 0.4 & 0.2 \\ 0.5 & 0.6 & - & 0.3 \\ 0.7 & 0.7 & 0.7 & - \end{pmatrix}$$

### A. Computing Missing Information

In this first step each incomplete fuzzy preference relation is completed by means of the estimation procedure described in subsection II-C. Therefore, for each incomplete fuzzy preference relation $P^h$ we obtain its corresponding complete fuzzy preference relation $\overline{P}^h$.

*Example 4 (Example 3 continuation):* The complete fuzzy preference relations associated to $P^1$, $P^2$, $P^3$ and $P^4$ are:

$$\overline{P}^1 = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ 0.8 & - & 0.9 & 0.7 \\ 0.4 & 0.1 & - & 0.3 \\ 0.6 & 0.3 & 0.7 & - \end{pmatrix}$$

$$\overline{P}^2 = \begin{pmatrix} - & 0.62 & 0.7 & 0.8 \\ 0.4 & - & 0.6 & 0.7 \\ 0.3 & 0.4 & - & 0.57 \\ 0.25 & 0.4 & 0.45 & - \end{pmatrix}$$

$$\overline{P}^3 = \begin{pmatrix} - & 0.3 & 0.54 & 0.75 \\ 0.6 & - & 0.69 & 0.87 \\ 0.46 & 0.31 & - & 0.73 \\ 0.3 & 0.4 & 0.27 & - \end{pmatrix}$$

$$\overline{P}^4 = \begin{pmatrix} - & 0.6 & 0.6 & 0.3 \\ 0.4 & - & 0.4 & 0.2 \\ 0.5 & 0.6 & - & 0.3 \\ 0.7 & 0.7 & 0.7 & - \end{pmatrix}$$

### B. Computing Consistency Measures

To compute consistency measures, firstly, for each $\overline{P}^h$ we compute its corresponding fuzzy preference relation $CP^h = \left(cp^h_{ik}\right)$ according to expression (6). Secondly, we apply expressions (8)–(10) to $(\overline{P}^h, CP^h)(\forall h)$ to compute the consistency measures $CL^h = \left(cl^h_{ik}\right), cl^h_i, cl^h \ \forall i, k \in \{1, ..., n\}$. Finally, we define a global consistency measure among all experts to control the global consistency situation.

*Definition 9:* The global consistency measure is computed as follows:

$$CL = \frac{\sum_{h=1}^{m} cl^h}{m} \tag{15}$$

*Example 5 (Example 3 continuation):* Global consistency measure

I) The corresponding fuzzy preference relations $\{CP^h\}$ for

$P^1$, $P^2$, $P^3$ and $P^4$ are:

$$CP^1 = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ 0.8 & - & 0.9 & 0.7 \\ 0.4 & 0.1 & - & 0.3 \\ 0.6 & 0.3 & 0.7 & - \end{pmatrix}$$

$$CP^2 = \begin{pmatrix} - & 0.63 & 0.72 & 0.77 \\ 0.4 & - & 0.6 & 0.67 \\ 0.3 & 0.42 & - & 0.58 \\ 0.25 & 0.35 & 0.45 & - \end{pmatrix}$$

$$CP^3 = \begin{pmatrix} - & 0.45 & 0.51 & 0.7 \\ 0.6 & - & 0.62 & 0.89 \\ 0.48 & 0.41 & - & 0.64 \\ 0.33 & 0.1 & 0.42 & - \end{pmatrix}$$

$$CP^4 = \begin{pmatrix} - & 0.6 & 0.5 & 0.35 \\ 0.4 & - & 0.45 & 0.2 \\ 0.5 & 0.55 & - & 0.3 \\ 0.65 & 0.8 & 0.7 & - \end{pmatrix}$$

II) The consistency measures for every pair of alternatives in the experts' preferences are:

$$CL^1 = \begin{pmatrix} - & 1.0 & 1.0 & 1.0 \\ 1.0 & - & 1.0 & 1.0 \\ 1.0 & 1.0 & - & 1.0 \\ 1.0 & 1.0 & 1.0 & - \end{pmatrix}$$

$$CL^2 = \begin{pmatrix} - & 0.99 & 0.98 & 0.97 \\ 1.0 & - & 1.0 & 0.97 \\ 1.0 & 0.98 & - & 0.99 \\ 1.0 & 0.95 & 1.0 & - \end{pmatrix}$$

$$CL^3 = \begin{pmatrix} - & 0.85 & 0.97 & 0.95 \\ 1.0 & - & 0.93 & 0.98 \\ 0.96 & 0.9 & - & 0.91 \\ 0.97 & 0.7 & 0.86 & - \end{pmatrix}$$

$$CL^4 = \begin{pmatrix} - & 1.0 & 0.9 & 0.95 \\ 1.0 & - & 0.95 & 1.0 \\ 1.0 & 0.95 & - & 1.0 \\ 0.95 & 0.9 & 1.0 & - \end{pmatrix}$$

III) The consistency measure that each expert presents in his/her preferences are:

$$cl^1 = 1.0 \ ; \ cl^2 = 0.99 \ ; \ cl^3 = 0.91 \ ; \ cl^4 = 0.97$$

IV) The global consistency level is:

$$CL = \frac{1.0 + 0.99 + 0.91 + 0.97}{4} = 0.97$$

### C. Computing Consensus Measures

We compute several consensus measures for different fuzzy preference relations. In fact, as in [16], [21] we compute two different kinds of measures: consensus degrees and proximity measures. Consensus degrees are used to measure the actual level of consensus in the process, whilst the proximity measures give information about how close to the collective solution every expert is. These measures are given on three different levels for a fuzzy preference relation: pairs of alternatives, alternatives and relations. This measure structure will allows us to find out the consensus state of the process at different levels. For example, we will be able to identify which experts are close to the consensus solution, or in which alternatives the experts are having more trouble to reach consensus.

*1) Consensus Degrees:* Firstly, for each pair of experts $(e_h, e_l)$ $(h < l)$ we define a similarity matrix $SM^{hl} = \left(sm_{ik}^{hl}\right)$ where

$$sm_{ik}^{hl} = 1 - |\overline{p}_{ik}^h - \overline{p}_{ik}^l| \qquad (16)$$

Then, a collective similarity matrix, $SM = (sm_{ik})$ is obtained by aggregating all the $(m-1) \times (m-2)$ similarity matrices using the arithmetic mean as the aggregation function $\phi$:

$$sm_{ik} = \phi(sm_{ik}^{hl}) \ ; \ \forall h, l = 1, ..., m \mid h < l. \qquad (17)$$

Once the similarity matrices are computed we proceed to calculate the consensus degrees in the three different levels:

**Level 1.** *Consensus degree on pairs of alternatives*. The consensus degree on a pair of alternatives $(x_i, x_k)$, denoted $cop_{ik}$, is defined to measure the consensus degree amongst all the experts on that pair of alternatives:

$$cop_{ik} = sm_{ik} \qquad (18)$$

**Level 2.** *Consensus degree on alternatives*. The consensus degree on alternative $x_i$, denoted $ca_i$, is defined to measure the consensus degree amongst all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1;k\neq i}^{n}(cop_{ik} + cop_{ki})}{2(n-1)} \qquad (19)$$

**Level 3.** *Consensus degree on the relation*. The consensus degree on the relation, denoted $CR$, is defined to measure the global consensus degree amongst all the experts' opinions:

$$CR = \frac{\sum_{i=1}^{n} ca_i}{n} \qquad (20)$$

*Example 6 (Example 3 continuation):* Computation of consensus degrees

Following with our example, we need to compute the 6 possible similarity matrices between every pair of different experts (not included for simplicity), and the collective one, which is:

$$SM = \begin{pmatrix} - & 0.74 & 0.92 & 0.69 \\ 0.77 & - & 0.74 & 0.67 \\ 0.89 & 0.74 & - & 0.74 \\ 0.73 & 0.8 & 0.74 & - \end{pmatrix}$$

From $SM$ we obtain the following consensus degree on the relation

$$CR = 0.76.$$

*2) Proximity Measures:* To compute proximity measures for each expert we need to obtain the collective fuzzy preference relation, $P^c$, which summarizes preferences given by all the experts. To obtain $P^c$ we use an IOWA operator [42]–[44], which uses both consensus and consistency criteria as inducing variable. In such a way, we obtain each collective fuzzy preference degree according to the most consistent and consensual individual fuzzy preference degrees.

*Definition 10 ([43]):* An IOWA operator of dimension $n$ is a function $\Phi_W : (\mathbb{R} \times \mathbb{R})^n \to \mathbb{R}$, to which a weighting vector is associated, $W = (w_1, \ldots, w_n)$, with $w_i \in [0,1]$, $\Sigma_i w_i = 1$, and it is defined to aggregate the set of second arguments of a list of $n$ 2-tuples $\{\langle u_1, p_1 \rangle, \ldots, \langle u_n, p_n \rangle\}$ according to the following expression,

$$\Phi_W \left( \langle u_1, p_1 \rangle, \ldots, \langle u_n, p_n \rangle \right) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)}$$

$\sigma$ being a permutation of $\{1, \ldots, n\}$ such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}, \forall i = 1, \ldots, n-1$, i.e., $\langle u_{\sigma(i)}, p_{\sigma(i)} \rangle$ is the 2-tuple with $u_{\sigma(i)}$ the i-th largest value in the set $\{u_1, \ldots, u_n\}$.
In the above definition, the reordering of the set of values to be aggregated, $\{p_1, \ldots, p_n\}$, is induced by the reordering of the set of values $\{u_1, \ldots, u_n\}$ associated to them, which is based upon their magnitude. Due to this use of the set of values $\{u_1, \ldots, u_n\}$, Yager and Filev called them the values of an order inducing variable and $\{p_1, \ldots, p_n\}$ the values of the argument variable [42]–[44].

Following Yager's ideas on quantifier guided aggregation [41], we could compute the weighting vector of an IOWA operator using a linguistic quantifier $Q$ [45] as

$$w_h = Q\left(\frac{\sum_{j=1}^{h} u_{\sigma(j)}}{T}\right) - Q\left(\frac{\sum_{j=1}^{h-1} u_{\sigma(j)}}{T}\right) \tag{21}$$

being $T = \sum_{j=1}^{n} u_j$ and $\sigma$ the permutation used to produce the ordering of the values to be aggregated.

Thus, to obtain each collective fuzzy preference degree $p_{ik}^c$ according to the most consistent and consensual individual fuzzy preference degrees we propose to use an IOWA operator with the consistency/consensus values, $\{z_{ik}^1, z_{ik}^2, \ldots, z_{ik}^m\}$, as the values of the order inducing variable, i.e.,

$$p_{ik}^c = \Phi_W(\langle z_{ik}^1, p_{ik}^1 \rangle, \cdots, \langle z_{ik}^m, p_{ik}^m \rangle) = \sum_{h=1}^{m} w_h \cdot p_{ik}^{\sigma(h)} \tag{22}$$

where

- $\sigma$ is a permutation of $\{1, \ldots, m\}$ such that $z_{ik}^{\sigma(h)} \geq z_{ik}^{\sigma(h+1)}, \forall h = 1, \ldots, m-1$, i.e., $\left\langle z_{ik}^{\sigma(h)}, p_{\sigma(i)} \right\rangle$ is the 2-tuple with $z_{ik}^{\sigma(h)}$ the h-th largest value in the set $\{z_{ik}^1, \ldots, z_{ik}^m\}$;
- the weighting vector is computed according to the following expression

$$w_h = Q\left(\frac{\sum_{j=1}^{h} z_{ik}^{\sigma(j)}}{T}\right) - Q\left(\frac{\sum_{j=1}^{h-1} z_{ik}^{\sigma(j)}}{T}\right) \tag{23}$$

  with $T = \sum_{j=1}^{m} z_{ik}^j$;
- and the set of values of the inducing variable $\{z_{ik}^1, \ldots, z_{ik}^m\}$ are computed as

$$z_{ik}^h = (1-\delta) \cdot cl_{ik}^h + \delta \cdot co_{ik}^h, \tag{24}$$

  being $co_{ik}^h$ a consensus measure for the preference value $p_{ik}$ expressed by expert $e_h$ and $\delta \in [0,1]$ a parameter to control the weight of both consistency and consensus criteria in the inducing variable. Usually $\delta > 0.5$ will be used to give more importance to the consensus criterion. We should note that in our framework, each value $co_{ik}^h$ used to calculate $\{z_{ik}^1, \ldots, z_{ik}^m\}$ is defined as

$$co_{ik}^h = \frac{\sum_{l=h+1}^{n} sm_{ik}^{hl} + \sum_{l=1}^{h-1} sm_{ik}^{lh}}{n-1} \tag{25}$$

*Example 7 (Example 3 continuation):* Computation of the collective fuzzy preference relation

I) To compute the proximity measures it is necessary to obtain the consistency/consensus values of the inducing variable of the IOWA operator. To do so, firstly we compute the consensus values matrices $co^h = (co_{ik}^h)$:

$$co^1 = \begin{pmatrix} - & 0.69 & 0.95 & 0.72 \\ 0.67 & - & 0.66 & 0.78 \\ 0.92 & 0.66 & - & 0.77 \\ 0.75 & 0.8 & 0.77 & - \end{pmatrix}$$

$$co^2 = \begin{pmatrix} - & 0.75 & 0.88 & 0.68 \\ 0.8 & - & 0.8 & 0.78 \\ 0.85 & 0.8 & - & 0.77 \\ 0.72 & 0.87 & 0.77 & - \end{pmatrix}$$

$$co^3 = \begin{pmatrix} - & 0.76 & 0.91 & 0.72 \\ 0.8 & - & 0.8 & 0.66 \\ 0.92 & 0.8 & - & 0.66 \\ 0.75 & 0.87 & 0.65 & - \end{pmatrix}$$

$$co^4 = \begin{pmatrix} - & 0.76 & 0.95 & 0.65 \\ 0.8 & - & 0.67 & 0.44 \\ 0.88 & 0.67 & - & 0.77 \\ 0.68 & 0.67 & 0.77 & - \end{pmatrix}$$

II) With values $co_{ik}^h$ and $cl_{ik}^h$ (Example 5), the inducing variable values for each expert, $z^h = (z_{ik}^h)$ (we assume that $\delta = 0.75$), are obtained:

$$z^1 = \begin{pmatrix} - & 0.77 & 0.96 & 0.79 \\ 0.75 & - & 0.75 & 0.83 \\ 0.94 & 0.75 & - & 0.83 \\ 0.81 & 0.85 & 0.83 & - \end{pmatrix}$$

$$z^2 = \begin{pmatrix} - & 0.81 & 0.91 & 0.76 \\ 0.85 & - & 0.85 & 0.83 \\ 0.89 & 0.85 & - & 0.82 \\ 0.79 & 0.89 & 0.83 & - \end{pmatrix}$$

$$z^3 = \begin{pmatrix} - & 0.78 & 0.92 & 0.77 \\ 0.85 & - & 0.83 & 0.74 \\ 0.93 & 0.83 & - & 0.72 \\ 0.81 & 0.82 & 0.7 & - \end{pmatrix}$$

$$z^4 = \begin{pmatrix} - & 0.82 & 0.94 & 0.73 \\ 0.85 & - & 0.74 & 0.58 \\ 0.91 & 0.74 & - & 0.83 \\ 0.75 & 0.73 & 0.83 & - \end{pmatrix}$$

III) Using the following fuzzy linguistic quantifier *"most* *of" $Q$*:

$$Q(r) = \begin{cases} 0 & \text{if } r < 0.3 \\ \frac{r-0.3}{0.8-0.3} & \text{if } 0.3 \leq r < 0.8 \\ 1 & \text{if } r \geq 0.8 \end{cases}$$

to compute the weighting vector of the IOWA operator, the collective fuzzy preference relation $P^c$ is:

$$P^c = \begin{pmatrix} - & 0.43 & 0.58 & 0.74 \\ 0.42 & - & 0.77 & 0.78 \\ 0.46 & 0.23 & - & 0.46 \\ 0.31 & 0.37 & 0.57 & - \end{pmatrix}$$

Once we have computed $P^c$, we can compute the proximity measures in each level of a fuzzy preference relation:

**Level 1.** *Proximity measure on pairs of alternatives*. The proximity measure of an expert $e_h$ on the pair of alternatives $(x_i, x_k)$ to the group one, denoted $pp^h_{ik}$, is calculated as

$$pp^h_{ik} = 1 - |\overline{p}^h_{ik} - p^c_{ik}| \tag{26}$$

**Level 2.** *Proximity measure on alternatives*. The proximity measure of an expert $e_h$ on alternative $x_i$ to the group one, denoted $pa^h_i$, is calculated as:

$$pa^h_i = \frac{\sum_{k=1;k\neq i}^{n}(pp^h_{ik} + pp^h_{ki})}{2(n-1)} \tag{27}$$

**Level 3.** *Proximity measure on the relation*. The proximity measure of an expert $e_h$ on his/her preference relation to the group one, denoted $pr^h$, is calculated as:

$$pr^h = \frac{\sum_{i=1}^{n} pa^h_i}{n} \tag{28}$$

*Example 8 (Example 3 continuation):* Computation of proximity measures

I) The proximity measures on pairs of alternatives for each expert are:

$$pp^1 = \begin{pmatrix} - & 0.77 & 0.98 & 0.66 \\ 0.62 & - & 0.87 & 0.92 \\ 0.94 & 0.87 & - & 0.84 \\ 0.71 & 0.93 & 0.87 & - \end{pmatrix}$$

$$pp^2 = \begin{pmatrix} - & 0.81 & 0.88 & 0.94 \\ 0.98 & - & 0.83 & 0.92 \\ 0.84 & 0.83 & - & 0.89 \\ 0.94 & 0.97 & 0.88 & - \end{pmatrix}$$

$$pp^3 = \begin{pmatrix} - & 0.87 & 0.96 & 0.99 \\ 0.82 & - & 0.92 & 0.91 \\ 0.98 & 0.92 & - & 0.73 \\ 0.99 & 0.97 & 0.7 & - \end{pmatrix}$$

$$pp^4 = \begin{pmatrix} - & 0.83 & 0.98 & 0.56 \\ 0.98 & - & 0.63 & 0.42 \\ 0.96 & 0.63 & - & 0.84 \\ 0.61 & 0.67 & 0.87 & - \end{pmatrix}$$

II) The proximity measures on alternatives for each expert are:

$$pa^1 = \begin{pmatrix} 0.78 & 0.83 & 0.9 & 0.82 \end{pmatrix}$$

$$pa^2 = \begin{pmatrix} 0.9 & 0.89 & 0.86 & 0.92 \end{pmatrix}$$

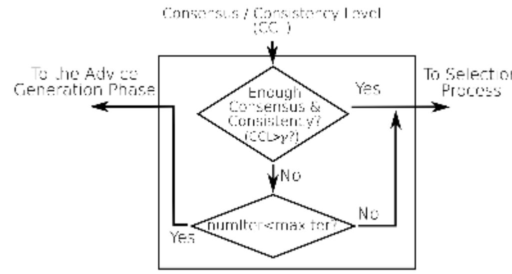


Fig. 3. Consensus/Consistency State Control Routine

$$pa^3 = \begin{pmatrix} 0.94 & 0.9 & 0.87 & 0.88 \end{pmatrix}$$

$$pa^4 = \begin{pmatrix} 0.82 & 0.69 & 0.82 & 0.66 \end{pmatrix}$$

III) The proximity measures on the relation for each expert are:

$$pr^1 = 0.83\ ;\ pr^2 = 0.89\ ;\ pr^3 = 0.90\ ;\ pr^4 = 0.75.$$

### *D. Controlling Consistency/Consensus State*

The consistency/consensus state control process will be used to decide when the feedback mechanism should be applied to give advice to the experts or when the consensus reaching process has to come to an end. It should take into account both the consensus and consistency measures. To do that, we define a new measure or level of satisfaction, called *consistency/consensus level (CCL)*, which is used as a control parameter:

$$CCL = (1-\delta)\cdot CL + \delta \cdot CR \tag{29}$$

with $\delta$ the same value used in (24). When $CCL$ satisfies a minimum satisfaction threshold value $\gamma \in [0,1]$, then the consensus reaching process finishes and the selection process can be applied.

Additionally, the system should avoid stagnation, that is, situations in which consensus and consistency measures never reach an appropriate satisfaction value. To do so, a maximum number of iterations $maxIter$ should be fixed and compared to the actual number of iterations of the consensus process $numIter$.

The consensus/consistency control routine follows the schema shown in Figure 3: first the consistency/consensus level is checked against the minimum satisfaction threshold value. If $CCL > \gamma$ the consensus reaching process ends. Otherwise, it will check if the maximum number of iterations has been reached. If so, the consensus reaching process ends, if not it activates the feedback mechanism.

*Example 9 (Example 3 continuation):* We fix a minimum threshold value $\gamma = 0.85$. Because the consistency/consensus level at this moment is $CCL = (1-0.75)\cdot 0.97 + 0.75 \cdot 0.76 = 0.81$, then the consensus process applies the feedback mechanism.

### *E. Feedback Mechanism*

The feedback mechanism generates personalised advice to the experts according to the consistency and consensus criteria. It helps experts to change their preferences and to complete their missing values. This activity is carried out in two steps: *Identification of the preference values* that should be changed and *Generation of advice*.

*1) Identification of the Preference Values:* We must identify preference values provided by the experts that are contributing less to reach a high consensus/consistency state. To do that, we define the set $APS$ that contains 3-tuples $(h, i, k)$ symbolising preference degrees $p_{ik}^h$ that should be changed because they affect badly to that consistency/consensus state. To compute $APS$, we apply a three step identification process that uses the proximity and consistency measures previously defined.

**Step 1.** We identify the set of experts $EXPCH$ that should receive advice on how to change some of their preference values. The experts that should change their opinions are those whose preference relation level of satisfaction is lower than the satisfaction threshold $\gamma$, i.e.,

$$EXPCH = \{h \mid (1-\delta) \cdot cl^h + \delta \cdot pr^h < \gamma\} \tag{30}$$

**Step 2.** We identify the alternatives that the above experts should consider to change. This set of alternatives is denoted as $ALT$. To do this, we select the alternatives with a level of satisfaction lower than the satisfaction threshold $\gamma$, i.e.,

$$\begin{aligned} ALT = \{(h,i) \mid e_h \in EXPCH \wedge \\ (1-\delta) \cdot cl_i^h + \delta \cdot pa_i^h < \gamma\} \end{aligned} \tag{31}$$

**Step 3.** Finally, we identify preference values for every alternative and expert ($x_i$ ; $e_h \mid (h,i) \in ALT$) that should be changed according to their proximity and consistency measures on the pairs of alternatives, i.e.,

$$\begin{aligned} APS = \{(h,i,k) \mid (h,i) \in ALT \wedge \\ (1-\delta) \cdot cl_{ik}^h + \delta \cdot pp_{ik}^h < \gamma\} \end{aligned} \tag{32}$$

Additionally the feedback process must provide rules for missing preference values. To do so, it has to take into account in $APS$ all missing values that were not provided by the experts, i.e.,

$$APS' = APS \cup \{(h,i,k) \mid p_{ik}^h \in MV_h\} \tag{33}$$

*Example 10 (Example 3 continuation):* Following with our example, the set of 3-tuples $APS$ that experts should change is:

$$APS = \{(4,2,3),(4,2,4),(4,4,1),(4,4,2)\}$$

Taking into account all missing values not provided by the experts, the $APS'$ set is:

$$\begin{aligned} APS' = \ & \{(1,2,1),(1,2,3),(1,2,4),(1,3,1),(1,3,2), \\ & (1,3,4),(1,4,1),(1,4,2),(1,4,3),(2,1,2), \\ & (2,1,4),(2,2,3),(2,3,2),(2,3,4),(2,4,1), \\ & (2,4,3),(3,1,3),(3,2,3),(3,2,4),(3,3,1), \\ & (3,3,2),(3,3,4),(3,4,3),(4,1,2),(4,2,3), \\ & (4,2,4),(4,4,1),(4,4,2)\} \end{aligned}$$

Note that there are so many 3-tuples in $APS'$ because there were many missing values in the incomplete fuzzy preference relations provided by the experts.

*2) Generation of Advice:* In this step, the feedback mechanism generates personalised recommendations to help the experts to change their fuzzy preference relations. These recommendations are based on easy recommendation rules that will not only tell the experts which preference values they should change, but will also provide them with particular values for each preference to reach a higher consistency/consensus state.

The new preference degree of alternatives $x_i$ over alternative $x_k$ to recommend to the expert $e_h$, $rp_{ik}^h$, is calculated as the following weighted average of the preference value $cp_{ik}^h$ and the collective preference value $p_{ik}^c$:

$$rp_{ik}^h = (1-\delta) \cdot cp_{ik}^h + \delta \cdot p_{ik}^c, \tag{34}$$

As previously mentioned, with $\delta > 0.5$ the consensus model leads the experts towards a consensus solution rather than towards an increase on their own consistency levels.

Finally, we should distinguish two cases: the recommendation is given because a preference value is far from the consensus/consistency state; the recommendation is given because the expert did not provide the preference value. Therefore, $\forall (h,i,k) \in APS$:

1) If $p_{ik}^h \in EV_h$ the recommendation generated for the expert $e_h$ is: *"You should change your preference value $(i,k)$ to a value close to $rp_{ik}^h$."*
2) If $p_{ik}^h \in MV_h$ the recommendation generated for the expert $e_h$ is: *"You should provide a value for $(i,k)$ close to $rp_{ik}^h$."*

For each 3-tuple using the recommendation rules we generate a recommendation:

*Example 11 (Example 3 continuation):* The recommendations for our example are:

**To expert** $e_1$ $\Rightarrow$ You should provide a value for (2, 1) close to 0.52
**To expert** $e_1$ $\Rightarrow$ You should provide a value for (2, 3) close to 0.8
**To expert** $e_1$ $\Rightarrow$ You should provide a value for (2, 4) close to 0.76
**To expert** $e_1$ $\Rightarrow$ You should provide a value for (3, 1) close to 0.44
**To expert** $e_1$ $\Rightarrow$ You should provide a value for (3, 2) close to 0.2
**To expert** $e_1$ $\Rightarrow$ You should provide a value for (3, 4) close to 0.42
**To expert** $e_1$ $\Rightarrow$ You should provide a value for (4, 1) close to 0.38
**To expert** $e_1$ $\Rightarrow$ You should provide a value for (4, 2) close to 0.35
**To expert** $e_1$ $\Rightarrow$ You should provide a value for (4, 3) close to 0.6
**To expert** $e_2$ $\Rightarrow$ You should provide a value for (1, 2) close to 0.48
**To expert** $e_2$ $\Rightarrow$ You should provide a value for (1, 4) close to 0.75
**To expert** $e_2$ $\Rightarrow$ You should provide a value for (2, 3) close to 0.73

**To expert** $e_2 \Rightarrow$ You should provide a value for (3, 2) close to 0.28
**To expert** $e_2 \Rightarrow$ You should provide a value for (3, 4) close to 0.49
**To expert** $e_2 \Rightarrow$ You should provide a value for (4, 1) close to 0.29
**To expert** $e_2 \Rightarrow$ You should provide a value for (4, 3) close to 0.54
**To expert** $e_3 \Rightarrow$ You should provide a value for (1, 3) close to 0.56
**To expert** $e_3 \Rightarrow$ You should provide a value for (2, 3) close to 0.73
**To expert** $e_3 \Rightarrow$ You should provide a value for (2, 4) close to 0.81
**To expert** $e_3 \Rightarrow$ You should provide a value for (3, 1) close to 0.46
**To expert** $e_3 \Rightarrow$ You should provide a value for (3, 2) close to 0.28
**To expert** $e_3 \Rightarrow$ You should provide a value for (3, 4) close to 0.5
**To expert** $e_3 \Rightarrow$ You should provide a value for (4, 3) close to 0.53
**To expert** $e_4 \Rightarrow$ You should provide a value for (1, 2) close to 0.47
**To expert** $e_4 \Rightarrow$ You should change your preference value for (2, 3) to a value close to 0.69
**To expert** $e_4 \Rightarrow$ You should change your preference value for (2, 4) to a value close to 0.64
**To expert** $e_4 \Rightarrow$ You should change your preference value for (4, 1) to a value close to 0.39
**To expert** $e_4 \Rightarrow$ You should change your preference value for (4, 2) to a value close to 0.48

Once experts receive the recommendations, another round of the consensus process takes place, with the experts giving new fuzzy preference relations closer to a consensus solution and with higher levels of consistency.

*Example 12 (**Finishing Ex. 3: Second Consensus Round**):* We assume that all the experts follow the recommendations they were given, which implies that the new fuzzy preference relations for the second round of the consensus process are:

$$P^1 = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ 0.52 & - & 0.8 & 0.76 \\ 0.44 & 0.2 & - & 0.4 \\ 0.38 & 0.35 & 0.6 & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & 0.48 & 0.7 & 0.75 \\ 0.4 & - & 0.73 & 0.7 \\ 0.3 & 0.28 & - & 0.49 \\ 0.29 & 0.4 & 0.54 & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & 0.3 & 0.56 & 0.75 \\ 0.6 & - & 0.73 & 0.81 \\ 0.46 & 0.28 & - & 0.5 \\ 0.3 & 0.4 & 0.53 & - \end{pmatrix}$$

$$P^4 = \begin{pmatrix} - & 0.47 & 0.6 & 0.3 \\ 0.4 & - & 0.69 & 0.64 \\ 0.5 & 0.6 & - & 0.3 \\ 0.39 & 0.48 & 0.7 & - \end{pmatrix}$$

Applying the same process (which will not be detailed here) we obtain the following global consistency and consensus levels:

$$CL = 0.91 \text{ and } CR = 0.88.$$

Obviously, the consistency level has decreased a little bit because the process gave more importance to the consensus criteria than the consistency one. However, the consensus level has increased. Finally, as the consistency/consensus level satisfies the minimum consensus threshold value, i.e.,

$$CCL = 0.89 > \gamma = 0.85,$$

then the consensus reaching process ends and a solution of consensus is obtained at this point by applying a selection process.

### F. Analysis of the Consensus Model

In this subsection we provide a discussion on some relevant aspects of our proposed consensus model with regards to other different consensus models.

1) Firstly, we should point out that our model presents two main advantages with respect to others consensus models proposed in the literature [3]–[5], [7], [10], [11], [16], [18], [23]–[25], [29], [35], [37], [46]: (i) Our consensus models deals with decision situations with incomplete information, and (ii) it helps experts to reach consensus with consistency and consensus criteria simultaneously, and therefore, it guides experts in their preference changes allowing them to maintain their basic rationality principles. Also, due to the role of the parameter $\delta$ used in expression (29) for the consistency and consensus levels to guide the consensus reaching process, our consensus model can be seen as a more general model than previous proposed models. Take for example the extreme cases of $\delta = 1$ and $\delta = 0$. In the first one our model is guided using just consensus criteria, while in the latter it would be just the consistency one.
2) The consistency based consensus model proposed in [12], although it presents similarities with our consensus model in that a consistency index of preferences is proposed in order to 'endogenously assign different weights to decision makers,' it differs with respect to our consensus model in that: (i) it is defined in decision situations with complete fuzzy preference relations, (ii) it applies a consensus measure defined over pairwise preference degrees, i.e., it does not incorporate the different consensus levels of a relation and it does not use proximity measures, and more significantly (iii) it provides recommendations indiscriminately to all experts given that it acts dynamically over all experts' preferences.
3) The steps of our consensus model with incomplete information are designed emulating the human behaviour in real group decision making processes. In such processes,

although initially an expert may not be able to provide some preference degrees, however as discussion process progresses this expert may be in a situation of, based on on his own rationality principles and the fact of having known other experts' preferences, providing values for those preferences he was not able before. In our model, to simulate this behaviour we introduce the consistency criterion. By doing this, experts with incomplete information can complete their preferences by using estimate values consistent with his opinions and, therefore, they participate in a better and fully way in the decision process. Also, as a result of this, situations in which one particular expert or group of experts may control and dominate the decision process are avoided. Indeed, using the example provided in this paper to illustrate our consensus model, if those values not given were simply ignored in constructing the collective preference relation, expert $e_1$ would receive too many recommendations based only on the preferences of the rest of the experts, which would decrease her/his real participation in the decision process. Furthermore, in this scenario of ignoring values not given, the more complete a fuzzy preference relation is provided by an expert, the more the decision process would be dominated by that expert. Following with our example, expert $e_4$ would be the most dominant in the decision process and,for example, his preference values $p^4_{23}, p^4_{32}, p^4_{34}$ and $p^4_{43}$ would determine the corresponding preference values for the rest of experts. However, our model overcomes these problems: (i) expert $e_1$ receives a recommendation value of 0.8 for $p^1_{23}$ and not 0.4 (the corresponding value provided by $e_4$); (ii) this recommended value is obtained by taking into account his rationality principles and therefore closer to his consistent estimated preference value of 0.9; (iii) finally, expert $e_4$ receives a recommendation value of 0.69 to change his preference value $p^4_{23}$ to make it closer to the values $p^1_{23}, p^2_{23}, p^3_{23}$, which shows that he does not possess a dominant position in this preference value when the values not given are consistently estimated.

4) Obviously, the feedback mechanism would make the group to move towards the consensus only if their recommendations are taken into account and implemented in each round of the consensus process. An important characteristic of our consensus model is that it does not provide indiscriminate recommendations to experts, which in the end guarantees its convergence. The two processes within the feedback mechanism that guarantee this convergence towards consensus are:
(i) *Preference Identification Process* by which only those experts and their preference values to be considered in the advice process are identified. This is represented by the set APS. In such a way, we get that all experts do not have to change all their preference values in each round of the consensus process and furthermore a minimum consensus level among experts' opinion is established.
(ii) *Advice Process* by which the recommended values are computed from both the corresponding consistent and collective preference values (see expression (34)) in the same proportion $(\delta, 1-\delta)$than the one already fixed and used in expression (29) for the consistency and consensus levels. As a consequence, the acceptance of the recommendation by the experts would lead the decision process towards the consensus because in each round the cardinality of APS would diminish and the achieved consensus level would be greater than in the previous consensus round.
Obviously, the consensus reaching process will depend on the size of the group of experts as well as on the size of the set of alternatives, so that when these sizes are small and when opinions are homogeneous, the consensus level required is easier to obtain.

## IV. Conclusions

In this paper we have presented a new consensus model for GDM problems with incomplete fuzzy preference relations. Contrary to many other previous consensus models, it uses two different kinds of measures to guide the consensus reaching process, consistency and consensus measures, and generates advice to experts in a discriminate way. As a consequence, the consensus model will contribute to achieve consistent and consensus solutions. Furthermore, the consensus model can be developed automatically without the participation of a human moderator.

This consensus model applies a feedback mechanism to give personalised advice to the experts on how to change and complete their fuzzy preference relations. This feedback mechanism could be used like an estimation procedure of missing values because it generates possible values to complete the missing values in the incomplete fuzzy preference relations. Therefore, it could act as an estimation procedure based on consistency/consensus criteria.

In the future, we will refine and extend this consensus model to linguistic decision frameworks.

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**Enrique Herrera-Viedma** was born in 1969. He received the M.Sc. and Ph.D. degrees in Computer Sciences, both from the University of Granada, Spain, in 1993 and 1996, respectively.

Currently, Dr. Herrera-Viedma is a Senior Lecturer of Computer Science in the Dept. of Computer Science and Artificial Intelligence at the University of Granada. He coedited various journal special issues on Computing with Words and Preference Modelling and Soft Computing in Information Retrieval. His research interests include group decision making, decision support systems, consensus models, linguistic modelling, aggregation of information, information retrieval, genetic algorithms, digital libraries, Web quality evaluation and recommender systems.

**Sergio Alonso** was born in 1979 in Granada, Spain. He received the M.Sc. and the Ph.D. degrees in Computer Sciences, both from the University of Granada, Spain, in 2003 and 2006, respectively.

Currently, he is a researcher in the Department of Computer Science and Artificial Intelligence at the University of Granada. His research interests include fuzzy preference modelling, modelling missing information situations, consistency properties of preferences, group decision making, decision support systems, consensus reaching processes, aggregation of information, Web quality evaluation and recommender systems.

In 2005, he was awarded a grant by the Spanish Department for Education and Science to conduct research for a three months period at the Centre for Computational Intelligence (CCI) Research Group at De Montfort University, Leicester, U.K.

**Francisco Chiclana** received the B.Sc. and Ph.D degrees in mathematics, both from the University of Granada, Spain, in 1989 and 2000, respectively.

In August 2003, he left his permanent position as a teacher of mathematics in secondary education in Andalucía (Spain) and joined the Centre for Computational Intelligence (CCI) Research Group at De Montfort University, Leicester, U.K. His research interests include fuzzy preference modelling, decision making problems with heterogeneous fuzzy information, decision support systems, the consensus reaching process, aggregation of information, modelling situations with missing information and modelling variation in automated decision support.

Dr. Chiclana's thesis "The Integration of Different Fuzzy Preference Structures in Decision Making Problems with Multiple Experts" received the *'Outstanding Award for a PhD in Mathematics for the academic year 1999/2000'* from the University of Granada.

**Francisco Herrera** received the M.Sc. degree in Mathematics in 1988 and the Ph.D. degree in Mathematics in 1991, both from the University of Granada, Spain.

He is currently a Professor in the Department of Computer Science and Artificial Intelligence at the University of Granada. He has published over 100 papers in international journals and he is coauthor of the book "*Genetic Fuzzy Systems: Evolutionary Tuning and Learning of Fuzzy Knowledge Bases*" (World Scientific, 2001). As edited activities, he has co-edited three international books and co-edited 15 special issues in international journals on different Soft Computing topics, such as, "*Preference Modelling*", "*Computing with Words*", "*Genetic Algorithms*" and "*Genetic Fuzzy Systems*". He currently serves on the editorial boards of the Journals: Soft Computing, Mathware and Soft Computing, International Journal of Hybrid Intelligent Systems, International Journal of Computational Intelligence Research and Evolutionary Computation. His current research interests include computing with words, preference modelling and decision making, data mining and knowledge discovery, genetic algorithms, and genetic fuzzy systems.